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Ideal mixing for baryons*

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We suggest that all configuration mixing within the $\underline{70}, 1^-$ multiplet of $SU(6)\times O(3)$ is governed by simple decoupling conditions. We deduce a tentative set of ideal mixing angles and discuss the kind of models that might lead to such mixing.

I. INTRODUCTION

By "ideal mixing" we mean assignment of resonances to $SU(6)\times O(3)$ multiplets via mixing angles determined in first approximation by certain exact decoupling conditions. The concept is familiar in meson spectroscopy where the ω-ϕ and f-f' mixing angles are determined, to a good approximation, by the respective conditions that one physical state should decouple from the $\rho\pi$ or $\pi\pi$ channel.

Our present study concerns baryons and is restricted to the negative-parity resonances that belong to the well-established $\underline{70}, 1^-$ multiplet. However, the phenomenon could well extend to higher-lying baryon multiplets. We take our clue from previous $SU(6)_W$ analyses of decay data[1,2] and in particular from certain approximate decoupling conditions that have already been suggested for the $J^P = \frac{1}{2}^-$ resonances. Notably,

$$\Lambda(1670, \tfrac{1}{2}^-) \nrightarrow N\bar{K}, \Sigma\pi \quad \text{(Ref. 3)},$$

$$N(1700, \tfrac{1}{2}^-) \nrightarrow N\eta \quad \text{(Ref. 3)},$$

$$\Sigma(1750, \tfrac{1}{2}^-) \nrightarrow \Lambda\pi, \Sigma\pi \quad \text{(Ref. 4)}.$$

We refer the reader to Ref. 4 for a uniform discussion of these decoupling conditions and the resultant phenomenology.

The purpose of the present note is to point out that similar decoupling conditions (i.e., involving the same channels) appear to govern $J^P = \frac{3}{2}^-$ mixing too. We suggest a tentative set of ideal mixing angles for all members of the $\underline{70}, 1^-$ multiplet and enumerate the kinds of model that could lead to such a scheme. Our notation and phase conventions follow Ref. 1.

II. IDEAL MIXING ANGLES

A. N^* mixing

In precisely the manner that Petersen's and Rosner's[3] condition $N(1700, \frac{1}{2}^-) \nrightarrow N\eta$ requires an $(8,2) \leftrightarrow (8,4)$ mixing angle of arctan $(1) = 45°$ for the $J^P = \frac{1}{2}^- N^*$ resonances, we notice that the same decoupling condition when applied to $N(1700, \frac{3}{2}^-)$ yields a $J^P = \frac{3}{2}^-$ mixing angle of $\arctan(1/\sqrt{10}) \approx 17\frac{1}{2}°$. These would appear to be good approximations to the fitted mixing angles obtained from decay data, i.e., 43° and 15° (respectively) in Ref. 1 or, more recently, 32° and 10° in Ref. 2.

B. Λ^* mixing

The Peterson-Rosner[3] ideal mixing angles for the S_{01} resonances are

$$\begin{pmatrix} \Lambda(1670, \frac{1}{2}^-) \\ \Lambda(\text{unseen}) \\ \Lambda(1405, \frac{1}{2}^-) \end{pmatrix} = \begin{pmatrix} -(\frac{1}{6})^{1/2} & (\frac{4}{6})^{1/2} & (\frac{1}{6})^{1/2} \\ (\frac{1}{3})^{1/2} & (\frac{1}{3})^{1/2} & -(\frac{1}{3})^{1/2} \\ (\frac{1}{2})^{1/2} & 0 & (\frac{1}{2})^{1/2} \end{pmatrix} \begin{pmatrix} (8,2) \\ (8,4) \\ (1,2) \end{pmatrix} \begin{matrix} \nrightarrow \Sigma\pi, N\bar{K}, (\Sigma^*\pi) \\ \nrightarrow \quad N\bar{K}, (\Lambda\eta) \\ \end{matrix}$$

Here Petersen and Rosner were able to complete the mixing matrix by supplementing $\Lambda(1670, \frac{1}{2}^-) \nrightarrow N\bar{K}, \Sigma\pi$ with the hypothesis that the missing S_{01} resonance has not been seen on account of its decoupling from the $N\bar{K}$ channel. However, we point out that this same mixing matrix automatically implies several other decoupling conditions as indicated in parentheses. Thus any three independent conditions would have yielded the same mixing angles. We have investigated all these possibilities and find that one set provides $J^P = \frac{3}{2}^-$ mixing angles in close agreement with the decay data. That is to say, we postulate that $J^P = \frac{3}{2}^- \Lambda^*$ mixing is determined to a good approximation by the conditions

$$\Lambda(\text{unseen}) \nrightarrow N\bar{K}, \Sigma\pi,$$

$$\Lambda(1690, \tfrac{3}{2}^-) \nrightarrow \Lambda\eta.$$

The resulting classification is

$$\begin{pmatrix} \Lambda(\text{unseen}) \\ \Lambda(1690, \frac{3}{2}^-) \\ \Lambda(1520, \frac{3}{2}^-) \end{pmatrix} = \begin{pmatrix} -(\frac{1}{42})^{1/2} & (\frac{40}{42})^{1/2} & (\frac{1}{42})^{1/2} \\ (\frac{841}{1002})^{1/2} & (\frac{40}{1002})^{1/2} & -(\frac{121}{1002})^{1/2} \\ (\frac{160}{1169})^{1/2} & (\frac{9}{1169})^{1/2} & (\frac{1000}{1169})^{1/2} \end{pmatrix} \begin{pmatrix} (8,2) \\ (8,4) \\ (1,2) \end{pmatrix} \begin{matrix} \dagger\ \Sigma\pi, N\overline{K} \\ \dagger\ \Lambda\eta \\ \\ \end{matrix} ,$$

which should be compared with the results from the Ref. 2 fit to decay data (in our phase convention):

$$\begin{pmatrix} 0.01 & 1.00 & 0.04 \\ 0.92 & 0.00 & -0.39 \\ 0.39 & -0.04 & 0.92 \end{pmatrix}.$$

C. Σ^* mixing

For the Σ^* resonances the experimental situation is much less certain, with only $\Sigma(1670, \frac{3}{2}^-)$ having reasonably cleanly measured properties. However, we notice that by supplementing our previously discussed $J^P = \frac{1}{2}^-$ condition[4] $\Sigma(1750, \frac{1}{2}^-) \dagger \Lambda\pi, \Sigma\pi$ with the hypothesis that one of the missing S_{11} resonances decouples from $\Sigma\eta$, and by applying these same decoupling conditions to $\Sigma(1940, \frac{3}{2}^-)$ and the missing[5] D_{13} resonance we obtain mixing angles in reasonable agreement with those of Ref. 2. Specifically,

$$\begin{pmatrix} \Sigma(1750, \frac{1}{2}^-) \\ \Sigma(\text{missing}) \\ \Sigma'(\text{missing}) \end{pmatrix} = \begin{pmatrix} 0 & (\frac{1}{5})^{1/2} & (\frac{4}{5})^{1/2} \\ (\frac{9}{14})^{1/2} & -(\frac{4}{14})^{1/2} & (\frac{1}{14})^{1/2} \\ (\frac{25}{70})^{1/2} & (\frac{36}{70})^{1/2} & -(\frac{9}{70})^{1/2} \end{pmatrix} \begin{pmatrix} (8,2) \\ (8,4) \\ (10,2) \end{pmatrix} \begin{matrix} \dagger\ \Lambda\pi, \Sigma\pi \\ \dagger\ \Sigma\eta \\ \\ \end{matrix} ,$$

whose top row[4] may be compared with the Ref. 2 results

$$(-0.19 \quad 0.45 \quad 0.87)$$

and

$$\begin{pmatrix} \Sigma(1940, \frac{3}{2}^-) \\ \Sigma(\text{missing}) \\ \Sigma(1670, \frac{3}{2}^-) \end{pmatrix} = \begin{pmatrix} 0 & (\frac{10}{14})^{1/2} & (\frac{4}{14})^{1/2} \\ (\frac{9}{44})^{1/2} & (\frac{10}{44})^{1/2} & -(\frac{25}{44})^{1/2} \\ (\frac{245}{308})^{1/2} & -(\frac{18}{308})^{1/2} & (\frac{45}{308})^{1/2} \end{pmatrix} \begin{pmatrix} (8,2) \\ (8,4) \\ (10,2) \end{pmatrix} \begin{matrix} \dagger\ \Lambda\pi, \Sigma\pi \\ \dagger\ \Sigma\eta \\ \\ \end{matrix} ,$$

which should be compared with the Ref. 2 results[5]

$$\begin{pmatrix} 0.35 & 0.92 & 0.18 \\ 0.31 & 0.07 & -0.95 \\ 0.89 & -0.38 & 0.26 \end{pmatrix}.$$

D. Ξ^* mixing

Owing to the dearth of experimental Ξ^* data it would be pure speculation to guess at the decoupling conditions that presumably govern cascade mixing. However, it is not hard to find patterns among the above elaborated mixing angles whose generalization to the Ξ^* states might contain some physical content. For the present, however, we prefer not to speculate along such lines since the models to be discussed below will make definite predictions.

III. DISCUSSION

A careful study of Sec. II together with the decay systematics[1,2] will reveal that the strength of our case for ideal mixing rests with the Λ^* resonances. For on the one hand the $J^P = \frac{3}{2}^-$ mixing angles are very well established [this is especially true of $\Lambda(1520, \frac{3}{2}^-)$] and on the other hand the decoupling conditions which approximate these $J^P = \frac{3}{2}^-$ mixing angles were formulated[3] for the $J^P = \frac{1}{2}^-$ resonances. After all, merely to formulate a set of decoupling conditions for a single set of (I, Y, J, P) states is not hard given the phenomenologically determined mixing angles. Some couplings must be small, so all we have to do is assume they should be zero in some undefined limit. What tempts us to believe that there is some physics behind ideal mixing is that the *same* set of decoupling conditions governs the (very different) mixing angles of both spinparity sets of resonances.

The N^* situation lends some support to this belief, but here the argument is less compelling because only a single decoupling condition fixes the mixing angles. Phenomenologically,[1,2] $J^P = \frac{3}{2}^- N^*$ mixing is small and $J^P = \frac{1}{2}^-$ mixing large. It so happens that Petersen's and Rosner's[3] $N\eta$ decoupling condition allows this, but so too do other conditions (depending upon how well we think the mixing angles are really known). For example, the decoupling of $N(1535, \frac{1}{2}^-)$ and $N(1700, \frac{3}{2}^-)$ from the $N\pi$ channel would also yield small D_{13} mixing ($-9°$) and large S_{11} mixing ($63°$). But how acceptable is total decoupling compared with an experimentally observed elasticity[6] of ~35% for $S_{11}(1535)$? [Recall in this context that $\Lambda(1670, \frac{1}{2}^-)$ is supposed to decouple from $N\bar{K}$ and $\Sigma\pi$ whereas its observed branching fractions into these channels are, respectively,[6] (15–35)% and (30–50)%. Of course the rationale here[3] is that both of these couplings are small compared with $\Lambda\eta$.]

Clearly then, in order to strengthen our case for ideal mixing among baryons we need a model that predicts such mixing.

IV. MODELS FOR IDEAL MIXING

In our search for a possible explanation for ideal mixing among baryons it may prove useful to reexamine the corresponding situation among the mesons where the phenomenon is well-established and where a number of different theoretical explanations exist. Specifically, for the three known "nonets" the $J^P = 1^-$ and 2^+ states exhibit ideal mixing. The $J^P = 0^-$ states on the other hand do not appear to have a mixing angle related to any decoupling condition.

A. Transition mixing

Katz and Lipkin[7] were the first to attempt an explanation of the ω-ϕ mixing angle. They performed a dynamical calculation in which mixing comes about because of the exchange of VP (vector, pseudoscalar) or PP meson pairs. They argued that on account of the radically smaller mass of the pion compared to those of all the other P and V mesons the self-energy matrix is dominated by the $\rho\pi$ intermediate state. In the ideal limit in which only $\rho\pi$ contributes, this matrix is of rank 1, and one of its eigenstates must decouple from $\rho\pi$. Interest in this model apparently dwindled owing to the appearance shortly thereafter of the quark model,[8] but in retrospect the model of Katz and Lipkin is interesting in that it also offers a natural explanation as to why the tensor nonet (which was not to be completed for another two years) is ideally mixed—dominance of the $\pi\pi$ channel—and why the pseudoscalar nonet is not ideally mixed—no kinematically preferred two-body channel.

This picture of mixing being due to kinematically preferred intermediate states can be applied directly to the baryon spectrum for two-particle mixing situations. For example, our N^* mixing angles in such a picture would presumably be governed by the $N\pi$ intermediate state. For three-particle mixtures a single intermediate state is not sufficient to lift the degeneracy; the ensuing mass matrix would be of rank 1 with two states decoupling from this channel. If, however, we insert the *two* lowest-lying intermediate states then the degeneracy is fully lifted and only one linear combination will decouple from these channels. This would appear to be just what we need since (i) the two lowest-lying channels for Λ^* and Σ^* are indeed $\Sigma\pi, N\bar{K}$ and $\Lambda\pi, \Sigma\pi$ respectively, and (ii) these same channels would determine the mixing for both $J^P = \frac{1}{2}^-$ and $J^P = \frac{3}{2}^-$ triplets. Furthermore, in such a model the relative importance of higher-lying intermediate states would indicate how close to ideal mixing we should expect the physical particles to approach. We shall report separately[9] on the results of such an investigation.

B. The quark model

In the quark model[8] the natural isoscalar eigenstates of the mass matrix are the pure strange-quark-antiquark and nonstrange-quark-antiquark composites rather than the SU(3) singlet and octet states. This is because the strange and unstrange quarks are supposed to have intrinsically different masses. Unfortunately, this teaches us nothing of relevance to baryon mixing since on account of baryon number conservation we are not free to alter the quark content of our composites. Instead one has to invoke specific interquark forces in the manner of nuclear physics to effect the mixing. Much work has been carried out along these lines[10-13] but the mixing angles so derived do not in general appear to reflect the decay data in any simple manner, and there is certainly no hint of ideal mixing.

C. Group theory

In group-theoretical language the ideally mixed ω and ϕ mesons are best classified[14] according to the SU(4)×SU(2) subgroup of SU(6) where they belong, respectively, to unmixed (15, 1) and (1, 3) irreducible representations. The η and η' on the other hand [whatever their SU(3) mixing angle] belong to independent (1, 1) representations.

The SU(3)×SU(2) decomposition of SU(6) is more familiar largely owing to the historical accident that SU(6) grew out of SU(3). In general large mixing angles indicate a poor choice of basis states and in the situation where the strange quark is given a large mass the SU(4)×SU(2) decomposition is clearly the natural one.

Among our baryon ideal mixing angles the $J^P = \frac{1}{2}^-$ Λ^* mixing matrix (especially) displays a simplicity that might reflect the existence of a more sensible group decomposition of SU(6)×O(3) than our inherited SU(3)×SU(2)×O(3) chain. If it could be found, such a group decomposition would presumably have some important implications for the mass operators that arise in the quark model.

D. Duality

Yet another novel way of deducing ideal mixing for the vector and tensor mesons was discovered by Chiu and Finkelstein.[15] They pointed out that owing to the exchange degeneracy of the ρ-ω-f-A_2 trajectories certain nonvanishing two-body cross sections would have to vanish unless there existed a separate pair of exchange-degenerate trajectories with ω-f quantum numbers but having no coupling to pions.

The difficulty in obtaining duality constraints on baryon mixing lies, however, in the fact that among the baryons it is only the nonleading trajectories that mix. Indeed, exchange degeneracy of the leading $\underline{56}, 0^+$ and $\underline{70}, 1^-$ trajectories (respectively, the δ decuplet and the β octet) has led[16] to the requirement of $F/D = -\frac{1}{3}$ for the β octet, but this coupling constraint arises naturally within the SU(6) classification and there is no need (or indeed possibility) to invoke mixing. Chiu and Mitra[17] have explored the possibility that exchange degeneracy among the nonleading trajectories fixes mixing angles. For example, only one linear combination of the (8,2) and (8,4) $J^P = \frac{3}{2}^-$ N^* members of the $\underline{70}, 1^-$ would form the N_γ trajectory that is supposed to be exchange degenerate with the N_α (nucleon) trajectory. Unfortunately a systematic study[18] of this idea reveals that a baryon spectrum comprising only $\underline{56}$, L-even and $\underline{70}$, L-odd trajectories is not rich enough to yield sensible mixing angles among the $\underline{70}$-plet states {specifically, the γ trajectories that are needed in order to produce exchange degeneracy with the unmixed α-octet states all turn out to be pure octet states [i.e., linear combinations of (8,2) and (8,4) only] having $F/D = \frac{2}{3}$; this is a totally unacceptable classification[1,2] of, say, $\Lambda(1520, \frac{3}{2}^-)$, the most famous[19] of the exchange-degenerate baryon trajectories}. To date, studies of duality constraints on baryon trajectories within the context of a richer spectrum[20] have not discussed the mixing aspect.

V. SUMMARY AND CONCLUSIONS

We have argued that configuration mixing among baryons of the $\underline{70}, 1^-$ multiplet that share the same strong-interaction quantum numbers appears to be linked to certain decoupling conditions common to both the $J^P = \frac{1}{2}^-$ and $\frac{3}{2}^-$ sets of resonances. We have inferred a tentative set of ideal mixing angles from the existing experimental data and have suggested several lines of theoretical investigation that could prove profitable for the understanding of such ideal mixing. We have ourselves made a certain amount of progress along one of these directions and shall report on the details elsewhere.[9]

Clearly, an understanding of ideal mixing would not be an end in itself since such mixing is only a gross caricature of the physical world. Nevertheless, the possibility exists that the baryon fine structure exhibits a similar degree of order as the gross structure. For example the lowest-lying states of both parities exhibit a not-yet-understood SU(6)×O(3) structure which is most readily investigated through those states which cannot mix[21] (for example the $J^P = \frac{5}{2}^-$ octet). Even these unmixed states, however, exhibit a certain degree of departure from the strict SU(6) structure as evidenced for example by the incomplete decoupling of $\Lambda(1830, \frac{5}{2}^-)$ from the elastic ($N\bar{K}$) channel. What we are suggesting is that there may exist a "reference frame" from which *all* physical members of the $\underline{70}, 1^-$ are as pure as $\Lambda(1830, \frac{5}{2}^-)$. Such a situation would presumably supply valuable information in the quest for that mythical object, the strong-interaction Hamiltonian.

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