# Combined effects of internal quark motion and SU(6) breaking on the properties of the baryon ground state

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A sizeable departure from naive SU(6) results has been shown to come, within the quark-model approach, from the relativistic character of the internal quark motion: axial-vector couplings and chiral configuration mixing. On the other hand, an interband mixing $(\underline{56},0^+)_{N=0}+(\underline{70},0^+)_{N=2}$ of the harmonic-oscillator levels correctly describes the large-$x$ behavior of the ratio $F_2^{en}/F_2^{ep}$. This paper is devoted to further consequences of both effects which result from their interference. Keeping the same basic parameters already used in the previous calculations, we compute mass differences within the $\frac{1}{2}^+$ octet and axial-vector and magnetic couplings. The most striking results are (i) the survival of the good SU(6) predictions: $\mu_p^{\text{tot}}/\mu_n \simeq -3/2$, $(F/D)_{\text{axial-vector}} \simeq 2/3$; (ii) the explanation in sign and order of magnitude of the $\Sigma$-$\Lambda$ splitting; (iii) the explanation of the sizeable discrepancy between experiment and SU(6) for the various $\Delta$-$N$ couplings, which are systematically underestimated: the magnetic dipole transition $\mu^*/\mu_p^{\text{tot}} \simeq (2\sqrt{2}/3)(1+30\%)$ and the $\Delta \to N\pi$ width. The new parameters introduced are the $\lambda$-$\mathcal{P}$ quark mass difference [fixed by the $(\Sigma+\Lambda)/2$-$N$ splitting], the anomalous quark magnetic moment $\kappa$ (fixed by $\mu_p^{\text{tot}}$), and $(g_A)_q$, the renormalized quark axial-vector coupling (fixed by $|G_A/G_V|$). The conclusions are that—in spite of an unwanted problem for the neutron charge form factor—a good description can be obtained for fine details of the baryon ground state, and that, as suspected, the quarks must be given a structure.

## I. INTRODUCTION

In the course of the study of the various hadronic phenomena made within the framework of the quark model, it has been observed that the naive quark model is only an approximation for at least four reasons:

(i) The hadron center-of-mass motion is often quite relativistic.

(ii) The internal quark motion inside the hadron at rest is relativistic and one should take it into account by introducing Dirac spinors.

(iii) The nucleon at rest is not a pure $\underline{56}$, but SU(6) is broken; a mixing with a $(\underline{70},0^+)$ seems to be required.

(iv) The quarks are dressed by virtual particles: They must acquire form factors, or, in the deep-inelastic region, they appear to be dressed by gluons and $q\bar{q}$ pairs.

(i) In our mind, point (i) has two main effects: First is the Lorentz contraction of the spatial wave function, already considered by Licht and Pagnamenta.[1] It is manifested in the behavior of elastic[1] as well as transition form factors as $q^2$ increases,[2,3] and it ensures scaling of the spatial wave function in the deep-inelastic lepton-hadron scattering.[4,5] Second is that there are spin boost matrices which affect the quark spinors, leading to Wigner rotations. Striking effects of these boosts appear in high-energy two-body reactions [6-8] and also in combination with point (ii).[9]

(ii) In Refs. 9 and 10 we have emphasized that the features of the hadron excitation spectrum (mean level spacing, hadron radius) strongly suggest that the quarks have a relativistic mean velocity inside the hadron. This could be accounted for by solving a relativistic (Dirac) wave equation[11] or, more simply, by at least understanding the spin part of the SU(6) wave function in terms of quasifree Dirac spinors instead of Pauli spinors.[9,12] Bogoliubov[11] has shown that there are large corrections to axial-vector couplings and magnetic moments. We showed (Ref. 9) that the chiral-SU(3) $\otimes$ SU(3) configuration mixing is obtained by boosting at $P_z=\infty$ the modified wave functions. The induced Wigner rotations are large and they generate the wanted SU(3)$\otimes$SU(3) representations. In the photoproduction calculation of Copley, Karl, and Obryk,[13] the same reasons lead to a large convective term. Higher-order corrections have been considered by Close, Copley, and Bowler.[14]

(iii) The $(\underline{56},0^+)_{N=0}+(\underline{70},0^+)_{N=2}$ mixing has been suggested to us[15] by the problem of the large-$x$ behavior of the ratio $F_2^{en}/F_2^{ep}$ in the deep-inelastic scattering.[16] We have discussed[15] alternative interpretations of the ratio $F_2^{en}/F_2^{ep}$ (due to Close,[17] and to Altarelli, Cabibbo, Maiani, and Petronzio[18]).

(iv) Since quarks are strongly interacting particles, there is no *a priori* reason to consider them pointlike. This point of view has been strongly emphasized by, for instance, Morpurgo.[19] More concretely, the quark currents should be dominated by the meson $q\bar{q}$ states bearing the corresponding quantum numbers. Then the form factors at the hadron level should be a product of the (Lorentz-contracted) wave-function form factor and a quark pole-type form factor.[1,2,3] Also, one expects the presence of an anomalous quark magnetic moment and a renormalization of the axial-vector quark current $(g_A)_q$. A large $\kappa_q$ is evidently needed in heavy-quark approaches. With quark effective masses ($m_q \simeq m_N/3$) a certain amount of $\kappa_q$ is also needed when one includes the above relativistic corrections (ii).[3,12,15] Up to now, *for the sake of simplicity*, as we have discussed,[20] we have taken $(g_A)_q = 1$, but there is no compelling reason for it. The presence of a quark structure is also clearly manifested in the deep-inelastic scattering region by the parton $q\bar{q}$ sea and the gluons which dress the valence partons.[18,15]

We have adopted[15] the point of view of Altarelli *et al.*[18] in order to reconcile the three-quark wave function (which manifests itself at small momentum transfer) with the need of a $q\bar{q}$ sea in order to reproduce the Regge behavior ($\sim 1/\sqrt{x}$ for valence quarks, $\sim 1/x$ for $q\bar{q}$ pairs) of the structure functions at small $x$; each of the three valence quarks is dressed by gluons and $q\bar{q}$ pairs. However, much work is needed to help us understand the link between the structure of quarks which appears at small momentum transfer (form factors, anomalous magnetic moments, etc.) and these gluons and $q\bar{q}$ pairs. In this paper, we work in the $\vec{P}=0$ frame with wave functions of hadrons made up of three quarks which are not pointlike.

A more detailed discussion of the various effects involved is given in our papers, and particularly in Refs. 3 and 20. In Ref. 15 we included in our treatment the four effects mentioned above and in particular (ii) and (iii). But in the considered phenomena the interference between the last two effects was not crucial; (ii) was mainly responsible for the $SU(3)\otimes SU(3)$ mixing, and (iii) was mainly responsible for the large-$x$ behavior of the ratio $F_2^{en}/F_2^{ep}$. In this paper we consider more subtle effects which come from the interference between the internal relativistic quark motion and the SU(6) breaking.

In Sec. II we recall the wave functions of the baryon ground state which result from our previous works. In Sec. III we study the $\Sigma$-$\Lambda$ mass difference. In Sec. IV we compute the axial-vector couplings. Section V is devoted to the electromagnetic current. In Sec. VI we compute the magnetic moments and magnetic dipole transition moment $N$-$\Delta$. In Sec. VII we discuss the numerical predictions and the values of our parameters. We conclude in Sec. VIII, where, in particular, we discuss the relevance of a quark model which is certainly becoming a bit complex, but whose complications are unavoidable, and which furthermore gives a description of rather detailed and unexplained phenomena.

## II. THE BARYON GROUND-STATE WAVE FUNCTIONS

### A. The wave functions at rest

According to Ref. 15, the ground-state octet is described by the wave function at rest

$$\Psi_{\vec{P}=0}(8,\tfrac{1}{2}^+) = \cos\varphi\,|(\underline{56},0^+)_{N=0},8,\tfrac{1}{2}^+\rangle + \sin\varphi\,|(\underline{70},0^+)_{N=2},8,\tfrac{1}{2}^+\rangle\,, \tag{2.1}$$

where $N$ is the number of harmonic-oscillator excitations. The motivation for this interband mixing between the quark harmonic-oscillator levels has been discussed at length.[15] For the decuplet $(10,\frac{3}{2}^+)$, we assume that there is *no mixing*, and this is a natural assumption in this scheme because the $(\underline{70},0^+)_{N=2}$ does not contain a $\frac{3}{2}^+$ decuplet. Then we have simply

$$\Psi_{\vec{P}=0}(10,\tfrac{3}{2}^+) = |(\underline{56},0^+)_{N=0},10,\tfrac{3}{2}^+\rangle\,. \tag{2.2}$$

Let us now express these wave functions in terms of spin, SU(3), and spatial parts $\chi,\phi,\psi$, respectively:

$$\Psi_{\vec{P}=0}(8,\tfrac{1}{2}^+) = \cos\varphi\frac{1}{\sqrt{2}}(\phi'\chi'+\phi''\chi'')\psi^s + \sin\varphi\,\tfrac{1}{2}[(\phi'\chi'-\phi''\chi'')\psi'' + (\phi'\chi''+\phi''\chi')\psi']\,, \tag{2.3}$$

$$\Psi_{\vec{P}=0}(10,\tfrac{3}{2}^+) = \phi^s\chi^s\psi^s\,. \tag{2.4}$$

The notation is taken from Mitra and Ross[21]; $\phi'$, $\chi'$, $\psi'$ are antisymmetric relative to the first two quarks, $\phi''$, $\chi''$, $\psi''$ are symmetric, and $\phi^s$, $\chi^s$, $\psi^s$ are fully symmetric with respect to the three quarks. The $\phi$'s and $\chi$'s are given explicitly in Ref. 22.

As for the spatial parts, we take[23]

$$\psi^s = N_s\exp[-\tfrac{1}{2}R^2(\vec{p}_\rho^{\,2}+\vec{p}_\lambda^{\,2})]\,, \tag{2.5}$$

$$\psi' = N2\vec{p}_\rho\cdot\vec{p}_\lambda\exp[-\tfrac{1}{2}R^2(\vec{p}_\rho^{\,2}+\vec{p}_\lambda^{\,2})]\,, \tag{2.6}$$

$$\psi'' = N(\vec{p}_\rho^{\,2}-\vec{p}_\lambda^{\,2})\exp[-\tfrac{1}{2}R^2(\vec{p}_\rho^{\,2}+\vec{p}_\lambda^{\,2})]\,, \tag{2.7}$$

where $\vec{p}_\rho$ and $\vec{p}_\lambda$ are relative momenta:

$$\vec{p}_\rho = \frac{1}{\sqrt{2}}(\vec{p}_1-\vec{p}_2),\quad \vec{p}_\lambda = \frac{1}{\sqrt{6}}(\vec{p}_1+\vec{p}_2-2\vec{p}_3)\,. \tag{2.8}$$

The wave functions $\psi$ are normalized to unity with respect to the measure

$$\prod_{i=1}^{3} d_3\vec{p}_i\,\delta^{(3)}\left(\sum_{j=1}^{3}\vec{p}_j\right)\,. \tag{2.9}$$

The spin wave functions are understood as combinations of quark-free Dirac spinors.[9,15] For the $i$th quark

$$u(\vec{p}(i)) = \left[\frac{\epsilon(i)+m}{2\epsilon(i)}\right]^{1/2}\begin{bmatrix}\chi(i)\\ \dfrac{\vec{\sigma}(i)\cdot\vec{p}(i)}{\epsilon(i)+m}\chi(i)\end{bmatrix}\,, \tag{2.10}$$

where $m$ is the quark mass and $\epsilon(i)=[\vec{p}^2(i)+m^2]^{1/2}$. Then

$$u^\dagger(\vec{p}(i))u(\vec{p}(i))=1\,. \tag{2.11}$$

### B. The wave functions in motion

We now proceed to the internal wave function in motion, by a pure Lorentz transformation along the $z$ axis, which incorporates two effects, the transformation of the coordinates and the boost matrices affecting the Dirac spinors. Since $\vec{P}$ is the hadron center-of-mass momentum, the internal wave functions in motion $\Psi_{\vec{P}}(\{\vec{p}_i\})$ for the nucleon and the $\Delta$ are given by the expressions (2.3) and (2.4), where the arguments of the spatial wave functions are now $\{\tilde{\vec{p}}_i\}$:

$$\begin{aligned}
&\tilde{\vec{p}}_\perp(i)=\vec{p}_\perp(i)\,,\\
&\tilde{p}_z(i)=\frac{E}{M}p_z(i)-\frac{P}{M}\epsilon(i)\,,\\
&\tilde{\epsilon}(i)=\frac{E}{M}\epsilon(i)-\frac{P}{M}p_z(i)\,,\\
&\sum_i \vec{p}(i)=\vec{P}\,,\\
&\tilde{\epsilon}(i)=[\tilde{\vec{p}}(i)^2+m^2]^{1/2}\,,\quad \epsilon(i)=[\vec{p}(i)^2+m^2]^{1/2}\,,
\end{aligned} \tag{2.12}$$

and where the spinors are given by

$$u_{\vec{P}}(\vec{p}(i))=\left[\frac{\tilde{\epsilon}(i)}{\epsilon(i)}\right]^{1/2}\left(\frac{E+M}{2M}\right)^{1/2}\left[1+\frac{P\alpha_z(i)}{E+M}\right]\left[\frac{\tilde{\epsilon}(i)+m}{2\tilde{\epsilon}(i)}\right]^{1/2}\begin{bmatrix}\chi(i)\\ \dfrac{\tilde{\vec{p}}(i)\cdot\vec{\sigma}(i)}{\tilde{\epsilon}(i)+m}\chi(i)\end{bmatrix}. \tag{2.13}$$

The normalization of the spinors $u_{\vec{P}}(\vec{p}(i))$ is such that

$$u^\dagger_{\vec{P}}(\vec{p}(i))\,u_{\vec{P}}(\vec{p}(i))=1\,. \tag{2.14}$$

The spatial wave functions are normalized in such a way that for the complete wave functions $\Psi_{\vec{P}}$

$$\int\prod_{i=1}^{3}d\vec{p}_i\Psi^\dagger_{\vec{P}_f}(\vec{p}_1,\vec{p}_2,\vec{p}_3)\Psi_{\vec{P}_i}(\vec{p}_1,\vec{p}_2,\vec{p}_3)\delta\Big(\sum_{j=1}^{3}\vec{p}_j-\vec{P}_i\Big)\,\delta\Big(\sum_{k=1}^{3}\vec{p}_k-\vec{P}_f\Big)=\delta(\vec{P}_f-\vec{P}_i)\,. \tag{2.15}$$

[In (2.15) and subsequently $\vec{p}(i)$ is replaced by $\vec{p}_i$ for easier notation.]

We then calculate the matrix element of a one-body operator operating on the third quark by the expression

$$\int d\vec{p}_1d\vec{p}_2d\vec{p}_3d\vec{p}_3{}'\Psi^\dagger_{\vec{P}_f}(\vec{p}_1,\vec{p}_2,\vec{p}_3{}')O(\vec{p}_1,\vec{p}_2,\vec{p}_3,\vec{p}_3{}')\Psi_{\vec{P}_i}(\vec{p}_1,\vec{p}_2,\vec{p}_3)\delta(\vec{p}_1+\vec{p}_2+\vec{p}_3{}'-\vec{P}_f)\delta(\vec{p}_1+\vec{p}_2+\vec{p}_3-\vec{P}_i)\,. \tag{2.16}$$

To recover exactly the prescription given in Ref. 15, one must make the approximation

$$\tilde{\epsilon}(i)\simeq m\simeq M/3;$$

then

$$\tilde{p}_z(i)=\frac{M}{E}[p_z(i)-\tfrac{1}{3}P]=(1-\beta^2)^{1/2}[p_z(i)-\tfrac{1}{3}P]\,,$$

where $\beta$ is the hadron velocity.

## III. Σ-Λ MASS DIFFERENCE

One well-known difficulty of the SU(6) scheme is the large mass splitting of about 80 MeV observed between the $\Sigma$ and the $\Lambda$. This splitting cannot be explained by the consideration of the SU(3) breaking under its usual quark-model form, which consists of the introduction of a simple mass difference between the $\lambda$ and the $\mathcal{P}$ quarks. In fact, the $\Sigma$ and the $\Lambda$ have just the same quark content, so that their masses are lifted by just the same amount.

However, as soon as (i) one introduces an SU(6) breaking such as (2.1) and (ii) one considers the relativistic corrections to the mass energy

$$\epsilon(i)\simeq m_i\rightarrow\epsilon(i)\simeq m_i+\frac{\vec{p}^2(i)}{2m_i}+\cdots\,,$$

then the introduction of a mass difference between the $\lambda$ and the $\mathcal{P}$ quark leads to an $\Sigma$-$\Lambda$ splitting.

The above procedure amounts to using the SU(6)-broken [but SU(3)-symmetric] wave functions (2.1) with Dirac spinors (2.10) and introducing the quark-mass perturbation expressed in terms of Dirac operators,

$$H_{\delta m}=\sum_i\beta(i)(m_i-m)\,, \tag{3.1}$$

where $m$ is the nonstrange-quark mass. If we return to Pauli spinors we have, taking into account (2.11),

$$\sum_i u^\dagger(i)\beta(i)(m_i-m)u(i)=\sum_i\chi^\dagger(i)(m_i-m)\frac{m}{[\vec{p}^2(i)+m^2]^{1/2}}\chi(i)\,. \tag{3.2}$$

In this section and in all the subsequent calculations we expand the expressions up to the third power in the momenta, either internal or external. So we write the operator to be computed between the wave functions (2.1) (with Pauli spinors):

$$\hat{\theta} = \left(\frac{m_\lambda - m}{3}\right) \sum_i [1 - \sqrt{3}\lambda^8(i)]\left[1 - \frac{\vec{p}^2(i)}{2m^2}\right], \qquad (3.3)$$

where $\lambda^8$ is the usual Gell-Mann matrix. Using

$$\langle \phi'_{\Sigma^0} | \lambda^8(3) | \phi'_{\Sigma^0}\rangle = -\langle \phi''_{\Sigma^0} | \lambda^8(3) | \phi''_{\Sigma^0}\rangle = 1/\sqrt{3},$$
$$\langle \phi'_{\Lambda^0} | \lambda^8(3) | \phi'_{\Lambda^0}\rangle = -\langle \phi''_{\Lambda^0} | \lambda^8(3) | \phi''_{\Lambda^0}\rangle = -1/\sqrt{3}, \qquad (3.4)$$
$$\langle \phi'_{P} | \lambda^8(3) | \phi'_{P}\rangle = \langle \phi''_{P} | \lambda^8(3) | \phi''_{P}\rangle = 1/\sqrt{3},$$

and

$$\langle \psi'' | \vec{p}^2(3) | \psi^s\rangle = -\frac{1}{\sqrt{3}}\frac{1}{R^2},$$
$$\langle \psi^s | \vec{p}^2(3) | \psi^s\rangle = \frac{1}{R^2}, \qquad (3.5)$$

and keeping the interference terms of the order

$$\tan\varphi \frac{\vec{p}^2(3)}{2m^2},$$

$$\tan^2\varphi,$$

but omitting the terms

$$\tan^2\varphi \frac{\vec{p}^2(3)}{2m^2},$$

we get, setting $x = 1/(2mR)^2$,

$$M_\Sigma - M_\Lambda = -\frac{8}{\sqrt{6}}(m_\lambda - m)\sin\varphi\cos\varphi x, \qquad (3.6)$$

$$\tfrac{1}{2}(M_\Sigma + M_\Lambda) - M_N = (m_\lambda - m)(1 - 2x), \qquad (3.7)$$

$$M_\Xi - M_N = 2(m_\lambda - m)\left[(1 - 2x) + \frac{2}{\sqrt{6}}\sin\varphi\cos\varphi x\right]. \qquad (3.8)$$

One notes that the $\Sigma$-$\Lambda$ splitting comes from the interference between the SU(6) mixing and the kinetic-energy effect. The sign of the splitting is fixed by the sign of $\varphi$, well determined from the deep-inelastic behavior.[15] On the contrary, one notes that (3.7) is already given by the $m_\lambda$-$m$ splitting, as expected; however, it is corrected by the kinetic-energy effect, which is not negligible. Of course, the three relations (3.6), (3.7), and (3.8) are not independent; the masses are related by the Gell-Mann–Okubo formula $2(M_N + M_\Xi) = 3M_\Lambda + M_\Sigma$.

Since $\varphi$ has been found negative, one gets the right sign for $M_\Sigma - M_\Lambda$. The order of magnitude, taking $m \simeq 0.4$ GeV, $\varphi \simeq -20°$ (Ref. 15), and extracting $m_\lambda$-$m$ from (3.7) ($m_\lambda - m \simeq 350$ MeV), comes out as

$$M_\Sigma - M_\Lambda \simeq 70 \text{ MeV}.$$

We postpone a numerical discussion of all the effects under study to Sec. VII. The $\Sigma$-$\Lambda$ splitting has also been studied by Cabibbo and Testa[24] and by De Rújula, Georgi, and Glashow.[25]

Cabibbo and Testa have introduced the idea that $\Sigma$-$\Lambda$ splitting (or equivalently, the $D/F$ ratio for the mass difference within the octet) may be related to the breaking of SU(6) in deep-inelastic electroproduction, as described by Altarelli *et al.*[18] They work in the $p_z = \infty$ frame, where the relevant quantity is the mean value $\langle 1/x_i\rangle$, where $x_i$ is the longitudinal-momentum fraction carried by the $i$th parton. However, we think it useful to work in the more usual and perhaps better defined $\vec{P} = 0$ frame, where, for instance, we can define parity and orbital angular momentum in a much more straightforward way.

The calculation of De Rújula, Georgi, and Glashow is based on an explicit SU(6)-breaking potential (Fermi-Breit) derived from gauge theories with colored gluons. It has the merit of relating the various mass splittings in the ground state. The connection with our own approach, which offers the interesting aspect of describing at the same time the large-$x$ behavior of the deep-inelastic structure functions, is not yet clear.

## IV. AXIAL-VECTOR COUPLINGS

We have already considered[15] the $\frac{1}{2}^+$ axial-vector couplings along the line of this paper. But there is yet to calculate the $\Delta$-$N$ coupling, which is of interest; although not very directly measurable in neutrino experiments, it can be deduced from partial conservation of axial-vector current (PCAC) and the $\Delta$ width. On the other hand, we have assumed[9,15] for sake of simplicity that there was no renormalization of the quark axial-vector current $(g_A)_q = 1$. Theoretically speaking, the quark axial-vector current should be renormalized. For instance, in the Nambu and Jona-Lasinio model[26] one sees that the axial-vector coupling of the fundamental fermions (which could now be identified with the quarks rather than the nucleons) is renormalized by "radiative" (pion) corrections. Practically, we had assumed,[9] following Bogoliubov,[11] that the entire correction to the static-SU(6) result $|G_A/G_V| = \frac{5}{3}$ was due to the relativistic corrections due to the Dirac small components. However, taking into account the SU(6) mixing and the parameter adopted in Sec. III $x = 1/(2mR)^2 \simeq 0.2$ which measures the mean squared velocity of the quarks, we are led to consider $(g_A)_q \neq 1$. This common ratio will not modify the predictions concerning ratios of axial-vector coupling such as $F/D$ or $G^*/G_A$. Note that at the quark level there should also be a pseudoscalar term[3] induced by the pion pole; in the elastic case, it does not contribute to the hadron axial-vector coupling and therefore, in this case, we are left with a quark current

$$(g_A)_q \bar{u}(i)\gamma^\mu(i)\gamma^5(i)\tau^+(i)u(i)\,. \qquad (4.1)$$

Within the approximations considered in Sec. III, we get

$$|G_A/G_V| = F + D = \tfrac{5}{3}(g_A)_q\left[(1-\tfrac{4}{3}x) + \frac{32}{15\sqrt{6}}\,x\tan\varphi - \tfrac{4}{5}\tan^2\varphi\right], \qquad (4.2)$$

$$F/D \simeq \tfrac{2}{3}[1 + (\tfrac{2}{3})^{1/2}x\tan\varphi + \tfrac{1}{2}\tan^2\varphi]\,. \qquad (4.3)$$

With $x \simeq 0.2$ and $\varphi \simeq -20°$, we find that

$$F/D \simeq \tfrac{2}{3}, \qquad (4.4)$$

which is very close to the old SU(6) result, and from $|G_A/G_V| = 1.25$ (experiment) we obtain

$$(g_A)_q \simeq 1.30\,. \qquad (4.5)$$

The experimental value of $F/D$ within the Cabibbo theory is[27]

$$F/D = 0.58 \pm 0.03\,. \qquad (4.6)$$

However, there is some controversy about this value. Considering only the $\Delta S = 0$ transitions, $F/D$ is compatible with $\frac{2}{3}$ (see Close[17]). On the other hand, the theoretical prediction (4.3) is very sensitive to the value of $x$ and $\varphi$, since there is a delicate cancellation between the two corrective terms, so that a value such as (4.6) cannot be excluded in our scheme.

Let us now consider the $\Delta$-$N$ axial-vector transition. In this case, as for the $\Delta$-$N$ electromagnetic transition (Sec. VI), the presence of a sizable difference introduces delicate problems to which we think it useful to pay some attention. We start with an expansion of all the expressions to the third power in the momenta either external or internal (Sec. III).

The various dimensionless quantities which appear in this expansion are, denoting the $\Delta$-$N$ mass difference by $\alpha$, the harmonic-oscillator mean level spacing by $\omega$, and the momentum transfer by $\vec{k}$ (in the $\Delta$ rest frame),

$$R^2\vec{k}^2 \sim \frac{\alpha^2}{\omega m} = \frac{\alpha^2}{\omega^2}\frac{\omega}{m}, \qquad (4.7)$$

$$\frac{1}{m^2R^2} \sim \frac{\omega}{m}, \qquad (4.8)$$

$$\frac{\vec{k}^2}{m^2} \sim \frac{\alpha^2}{m^2} = \frac{\alpha^2}{\omega^2}\left(\frac{\omega}{m}\right)^2. \qquad (4.9)$$

In practice, all these quantities are sizable, since $\alpha, \omega, m$ are of the same order of magnitude. However, as one is considering a power expansion, one must be careful about the theoretical order of magnitude of the various parameters involved: $\omega$ must be considered small compared to $m$, and $\alpha$, which is a breaking of the harmonic spectrum, must be considered small compared to $\omega$, $\alpha \ll \omega \ll m$. Then (4.9) is negligible relative to (4.8) and (4.7) and we shall neglect it. We retain (4.7) together with (4.8).

There is still another independent small quantity, the mixing angle $\varphi$. By perturbation theory, since $\varphi$ represents the SU(6)-symmetry breaking, we could estimate it to be of order

$$\varphi \sim \frac{\alpha}{\omega}\,. \qquad (4.10)$$

Then it is coherent to retain, as we have done, the orders

$$x\varphi \sim \frac{\alpha}{\omega}\frac{\omega}{m}, \qquad (4.11)$$

$$\varphi^2 \sim \frac{\alpha^2}{\omega^2}\,. \qquad (4.12)$$

For the sake of simplicity, we neglect the smaller quantity $\chi\varphi^2$.

In principle, we should consider the two components of the axial-vector current $\langle\Delta|A^z|N\rangle$ and $\langle\Delta|A^0|N\rangle$ which lead to a series of four distinct phenomenological couplings in the isobaric-model scheme.[29] In the limit $k_\mu \to 0$, only one coupling, $C_A{}^5(k^2=0)$, survives and it is given by the $A^z$ matrix element, on which we concentrate. [However, if one wanted to estimate $G^*$, i.e., the $p_z = \infty$ matrix element, we should consider the combination[28] $\langle\Delta|(A^0+A^z)|N\rangle$, where $\langle\Delta|A^0|N\rangle \neq 0$ when $\vec{k} \neq 0$.] Within the approximations indicated above

$$\langle\Delta^{++}\!\uparrow|A_z^{(+)}|P\!\uparrow\rangle = (g_A)_q\left\langle\Delta^{++}\!\uparrow\left|\sum_i \sigma_z(i)\tau^+(i)\left[1 - 2\frac{\vec{p}_\perp{}^2(i)}{(2m)^2}\right]\right|P\!\uparrow\right\rangle$$
$$= -\frac{4}{\sqrt{3}}(g_A)_q\frac{1}{\cos\varphi}\left[\cos^2\varphi(1-\tfrac{4}{3}x) - \frac{4}{3\sqrt{6}}x\sin\varphi\cos\varphi\right]\exp\left(-\frac{\vec{k}^2R^2}{6}\right). \qquad (4.13)$$

Numerically,

$$\langle\Delta^{++}\uparrow|A_z^{(+)}|P\uparrow\rangle=\left(-\frac{4}{\sqrt{3}}\right)\times 0.95\times\exp\left(-\frac{\vec{k}^2R^2}{6}\right)$$
$$=\left(-\frac{4}{\sqrt{3}}\right)(1-14\%)\,, \qquad (4.14)$$

where $-4/\sqrt{3}$ is the exact-SU(6) result. One notes that the departure from SU(6) is less than for $|G_A/G_V|$, where it is about $-25\%$. This is the direction indicated by Llewellyn-Smith[29] using PCAC. However, there is as yet no direct measurement of the weak-interaction quantity because of the large experimental uncertainties in neutrino production. We can directly consider, in the same spirit, the strong-interaction process $\Delta^{++}\to p\pi^+$ where there is indeed an appreciable disagreement with the naive-quark-model prediction for $\Gamma(\Delta\to N\pi)$, as fixed from the $NN\pi$ coupling $f$ (Ref. 30):

$$\Gamma(\Delta\to N\pi)=\frac{48}{25}\frac{f^2}{4\pi}\frac{1}{M_\pi^2}k^3\frac{E_N}{M_\Delta}=78\text{ MeV},$$
$$f^2/4\pi=0.082\text{ or }g_{\pi NN}{}^2/4\pi=14.6\ . \qquad (4.15)$$

In the frame of the oscillator quark model,[23] one has also a squared form factor $\exp(-\vec{k}^2\frac{1}{3}R^2)$ and, in terms, of $qq\pi$ coupling, one gets

$$\Gamma(\Delta\to N\pi)=\frac{16}{3}\frac{f_q^2}{4\pi}\frac{1}{M_\pi^2}k^3\frac{E_N}{M_\Delta}\exp\left(-\frac{\vec{k}^2R^2}{3}\right). \qquad (4.16)$$

Up to the exponential factor, (4.16) reduces to (4.15) if one assumes that $f_q=\frac{3}{5}f$ as given by SU(6), $f_q^2/4\pi=0.03$, but to fit the width one needs rather $f_q^2/4\pi=0.055$ as pointed out by Faiman and Hendry.[23] Using the same $\gamma_5$ coupling, but introducing our various corrections, we get

$$f=\tfrac{5}{3}f_q\left[(1-2x)\cos^2\varphi+\frac{16}{5\sqrt{6}}x\sin\varphi\cos\varphi-\tfrac{4}{5}\sin^2\varphi\right], \qquad (4.17)$$

and for the transition $N$-$\Delta$

$$\left\langle\Delta^{++}\uparrow\left|f_q\sum_i\gamma_0(i)\gamma_5(i)\tau^+(i)\right|P\uparrow\right\rangle=\frac{|\vec{k}|}{2m}\left(-\frac{4}{\sqrt{3}}\right)f_q\frac{1}{\cos\varphi}\left[(1-2x)\cos^2\varphi-\frac{2}{\sqrt{6}}x\sin\varphi\cos\varphi\right]\exp\left(-\frac{\vec{k}^2R^2}{6}\right), \qquad (4.18)$$

whence, instead of (4.16), we obtain

$$\Gamma(\Delta\to N\pi)=\frac{48}{25}(1+\eta)^2\frac{f^2}{4\pi}\frac{1}{M_\pi^2}k^3\frac{E_N}{M_\Delta}\exp\left(-\frac{\vec{k}^2R^2}{3}\right), \qquad (4.19)$$

where

$$1+\eta\simeq\frac{1}{\cos\varphi}\left[1-\frac{26}{5\sqrt{6}}x\tan\varphi+\tfrac{4}{5}\tan^2\varphi\right]\simeq 1+33\%, \qquad (4.20)$$

i.e., we get the right magnitude,

$$\Gamma(\Delta\to N\pi)=120\text{ MeV}. \qquad (4.21)$$

## V. GENERAL EXPRESSION OF THE ELECTROMAGNETIC-CURRENT MATRIX ELEMENTS

Here we only consider the transverse components of the current, which are sufficient to calculate the magnetic moments and the radiative transitions. We shall not calculate the radiative transitions in this paper; however, we think it useful to give the general expression to display the effect of the internal relativistic motion and of the hadron center-of-mass motion.

The matrix element of $J_\pm=(J_x\pm iJ_y)/2$ between two baryon states is given by

$$3\langle\Psi_{\vec{P}_f}|I(1)\otimes I(2)\otimes J_\pm(3)|\Psi_{\vec{P}_i}\rangle, \qquad (5.1)$$

where $\Psi$ are the total wave functions in motion considered in Sec. II, in terms of Dirac spinors (2.13). $I(1)$ and $I(2)$ are the identity operators corresponding to the spectator quarks and $J_\pm(3)$ is the current operator acting on the active quark (in terms of Dirac operators):

$$J_\pm(3)=\alpha_\pm(3)+\kappa_q\{q_z\gamma_0(3)[-i\hat{q}\times\vec{\sigma}(3)]_\pm$$
$$+q_0\gamma_0(3)\alpha_\pm(3)\}, \qquad (5.2)$$

where $q=P_f-P_i$, and $\pm$ denotes the combination $(x\pm iy)/2$. We now come to the effective operator between Pauli spinors,

$$3\times S(1)\otimes S(2)\otimes O(3), \qquad (5.3)$$

where $S$ refers to spectator quarks and $O$ refers to the active quark. Using (2.13) and keeping the third order in all the momenta, and furthermore assuming $\vec{P}_i$ and $\vec{P}_f$ to be collinear, and the $Oz$ axis to lie along $\vec{q}=\vec{P}_f-\vec{P}_i$, we have for the operator $O$

$$O_\pm=A(L_z=\pm1,S=0,S_z=0)$$
$$+B(L_z=0,S=1,S_z=\pm1)$$
$$+C(L_z=\pm1,S=1,S_z=0)$$
$$+D(L_z=\pm2,S=1,S_z=\mp1). \qquad (5.4)$$

The expressions of the operators $A,B,C,D$ are given in Table I. The operator $O$ presents the general SU(6)$\otimes$O(3) algebraic structure proposed in the Melosh approach to the phenomenology of the electromagnetic current.[31]

However, the $S$ operators are not present in the

TABLE I. SU(6)⊗O(3) structure of the electromagnetic-current operator for the active quark up to third order in the momenta (internal or external) in a frame in which $\vec{P}_i$ and $\vec{P}_f$ are collinear.

| | |
|---|---|
| $A(L_z=\pm1, S=0, S_z=0)$ | $\mp\sqrt{2}\left\{\left[2+\left(\frac{P_f-P_i}{2m}\right)\left(\frac{P_f}{2M_f}-\frac{P_i}{2M_i}\right)\right]+2m\kappa_q\left(\frac{P_f-P_i}{2m}\right)\left[\left(\frac{P_f}{2M_f}-\frac{P_i}{2M_i}\right)-\left(\frac{P_f-P_i}{2m}\right)\right]\right\}\frac{p_\pm}{2m}$ |
| $B(L_z=0, S=1, S_z=\pm1)$ | $\pm\left\{\left[\left(\frac{P_f-P_i}{2m}\right)\left(1-\frac{1}{2}\frac{(p_z'+p_z)^2}{(2m)^2}\right)-\left(\frac{P_f}{2M_f}-\frac{P_i}{2M_i}\right)\frac{\vec{p}_\perp^{\,2}}{(2m)^2}\right] + 2m\kappa_q\left[\left(\frac{P_f-P_i}{2m}\right)\left(1+\frac{\vec{p}_\perp^{\,2}}{(2m)^2}\right)-\frac{1}{2}\frac{(p_z+p_z')^2}{(2m)^2}\right]\right\}\sigma_\pm$ |
| $C(L_z=\pm1, S=1, S_z=0)$ | $+\sqrt{2}(1+2m\kappa_q)\left(\frac{P_f-P_i}{2m}\right)\left(\frac{P_f}{2M_f}+\frac{P_i}{2M_i}-\frac{p_z+p_z'}{2m}\right)\frac{p_\pm}{2m}\sigma_z$ |
| $D(L_z=\pm2, S=1, S=\mp1)$ | $\pm4\left[\left(\frac{P_f}{2M_f}-\frac{P_i}{2M_i}\right)-2m\kappa_q\left(\frac{P_f-P_i}{2m}\right)\right]\frac{p_\pm}{2m}\frac{p_\pm}{2m}\sigma_\mp$ |

Melosh approach, and their interference with $O$ breaks the additivity. However, note that we have started from exact additivity at the Dirac-spinor level. Similar nonadditive terms have been considered by Brodsky and Primack.[32] For the $S$ operator we have

$$S=1-\sqrt{2}\left(\frac{\vec{P}_f}{2M_f}-\frac{\vec{P}_i}{2M_i}\right)\left(\frac{p_+}{2m}\sigma_-+\frac{p_-}{2m}\sigma_+\right). \quad (5.5)$$

In Table I and in (5.5)

$$p_\pm=\mp\frac{1}{\sqrt{2}}(p_x\pm ip_y), \quad (5.6)$$

while

$$\sigma_\pm=\tfrac{1}{2}(\sigma_x\pm i\sigma_y).$$

## VI. NUCLEON MAGNETIC MOMENTS AND $N$-$\Delta$ MAGNETIC DIPOLE MOMENT

In this section we apply the general formulas of Sec. V to the nucleon magnetic moments and to the $N$-$\Delta$ magnetic dipole moment. It has always been a difficulty of the quark model to get the right magnitude for the $N$-$\Delta$-$\gamma$ coupling, in contrast with the success of the prediction for the ratio $\mu_p^{\text{tot}}/\mu_n$. Different phenomenological determinations by Dalitz and Sutherland, and by Gourdin and Salin,[33] of the quantity

$$\mu^*=\langle\Delta^+\uparrow|J_z^{\text{em}}|P\uparrow\rangle \quad (6.1)$$

lead to

$$\mu^*/\mu_p^{\text{tot}}\simeq\frac{2\sqrt{2}}{3}(1+30\%), \quad (6.2)$$

where $2\sqrt{2}/3$ is the SU(6) prediction.[34] Dalitz and Sutherland[33] have pointed out that if the mass difference between the nucleon and the $\Delta$ resonance is taken into account in a quark-model calculation, the disagreement is worse. We shall discuss this point later.

As to the explanation of the discrepancy (6.2), some attempts have been made. Kobayashi and Konno[35] have considered the effect of exchange currents; their work shows that if one tries to explain $\mu^*$ in that way, then the nucleon magnetic moment tends to be in disagreement with experiment by the same amount. The effect of the Dirac-small-component corrections is much too small.[36] We shall show now that the SU(6)-breaking scheme described above correctly explains the discrepancy (6.2), while maintaining the ratio $\mu_p^{\text{tot}}/\mu_n$ close to the experimental value.

The operator with which we are concerned in this ground-state transition corresponds to $\Delta L_z=0$. Within the approximations considered in Sec. V, the operator to be considered is

$$\hat{M}=3\frac{1}{2m}\hat{e}(3)\left(\left\{\left[1-\frac{26}{9}\frac{\vec{p}^{\,2}(3)}{(2m)^2}\right]+2m\kappa_q\left[1-2\frac{\vec{p}^{\,2}(3)}{(2m)^2}\right]\right\}\sigma_+(3)+\frac{2}{3}\frac{p_+(3)}{2m}\left[\frac{p_-(1)}{2m}\sigma_+(1)+\frac{p_-(2)}{2m}\sigma_+(2)\right]\right), \quad (6.3)$$

where, furthermore, we have taken $M_\Delta\simeq M_N\simeq3m$. The final expressions, within the approximations considered and setting $x=1/(2mR)^2$, are

$$\mu_p^{\text{tot}}=\frac{1}{2m}\left\{\cos^2\varphi\,[(1-\tfrac{26}{9}x)+2m\kappa_q(1-2x)]+\frac{2}{\sqrt{6}}\sin\varphi\cos\varphi\,x(\tfrac{8}{3}+4m\kappa_q)+\tfrac{1}{3}\sin^2\varphi(1+2m\kappa_q)\right\}, \quad (6.4)$$

$$\mu_n=-\frac{2}{3}\frac{1}{2m}\left\{\cos^2\varphi[(1-3x)+2m\kappa_q(1-2x)]+\frac{1}{\sqrt{6}}\sin\varphi\cos\varphi\,x(\tfrac{8}{3}+4m\kappa_q)\right\}, \quad (6.5)$$

$$\mu^* = \frac{2\sqrt{2}}{3}\frac{1}{2m}\frac{1}{\cos\varphi}\left\{\cos^2\varphi[(1-3x)+2m\kappa_q(1-2x)] - \frac{1}{\sqrt{6}}\sin\varphi\cos\varphi\, x(\tfrac{10}{3}+4m\kappa_q)\right\}\exp\left(-\frac{\vec{k}^2R^2}{6}\right). \tag{6.6}$$

To emphasize the departure from the naive SU(6) results, we express the ratios $\mu_n/\mu_p^{\text{tot}}$ and $\mu^*/\mu_p^{\text{tot}}$ by retaining the orders 1, $x$, $x\varphi$, and $\varphi^2$:

$$\mu_n/\mu_p^{\text{tot}} = -\frac{2}{3}\left[\left(1-\frac{x}{9g}-\tfrac{1}{3}\tan^2\varphi\right)-\frac{2x}{\sqrt{6}}\tan\varphi\left(1+\frac{1}{3g}\right)\right], \tag{6.7}$$

$$\mu^*/\mu_p^{\text{tot}} = \frac{2\sqrt{2}}{3}\frac{1}{\cos\varphi}\left[\left(1-\frac{x}{9g}-\tfrac{1}{3}\tan^2\varphi\right)-\frac{6x}{\sqrt{6}}\tan\varphi\left(1+\frac{4}{9g}\right)\right]\exp\left(-\frac{\vec{k}^2R^2}{6}\right), \tag{6.8}$$

where $g = 1+2m\kappa_q$.

One sees that the two expressions are corrected in the same direction by the interference term, but its contribution to $\mu^*/\mu_p^{\text{tot}}$ is about three times larger. On the other hand, $1/\cos\varphi$ still enhances $\mu^*/\mu_p^{\text{tot}}$. This is just what is desired from the experimental situation. Using the parameters already used in the $\Sigma$-$\Lambda$ splitting calculation ($R^2 \simeq 8$ GeV$^{-2}$, $m \simeq 0.4$ GeV, hence $x \simeq 0.2$; $\varphi \simeq -20°$), we determine $2m\kappa_q \simeq 1.90$ from $\mu_p^{\text{tot}} \simeq 2.79/2M_N$ and then

$$\mu_n = -1.91/2M_N, \tag{6.9}$$

$$\mu^*/\mu_p^{\text{tot}} = \frac{2\sqrt{2}}{3}(1+40\%)\exp\left(-\frac{\vec{k}^2R^2}{6}\right). \tag{6.10}$$

The correction for $\mu_n/\mu_p^{\text{tot}}$ is small and of the right sign, and the correction for $\mu^*/\mu_p^{\text{tot}}$ is large and also of the right sign.

Dalitz and Sutherland[33] noticed that if one takes into account an overlap factor which accounts for the mass difference $\Delta$-$N$ [which they take to be the dipole form factor $G(\vec{k}^2)$] then, since $\vec{k}^2$ is sizable, the experimental disagreement with the quark-model prediction for $\mu^*$ (6.1) is still worse. Here, the overlap factor is given by the exponential in (6.10). It is closer to 1, since $R^2$ is not the whole proton charge radius squared $\langle r_p^2\rangle \approx 16$ GeV$^{-2}$ but almost the half of it.[1-3,13] Moreover, this reduction factor is now desirable since it reduces the 40% correction to about 30%, which is better. In fact, with $\vec{k}^2$ calculated in the $N^*$ rest frame,

$$\vec{k}^2 = \left(\frac{M_\Delta{}^2 - M_N{}^2}{2M_\Delta}\right)^2,$$

$$\exp\left(-\frac{\vec{k}^2R^2}{6}\right) = 0.90.$$

We end with

$$\mu^*/\mu_p^{\text{tot}} = \frac{2\sqrt{2}}{3}(1+28\%). \tag{6.11}$$

## VII. DISCUSSION OF THE NUMERICAL RESULTS

We think it useful to discuss at this point the accuracy of the predictions obtained for the various phenomena. There is first to consider how far the parameters can be trusted. We have retained $R^2 \simeq 8$ GeV$^{-2}$ and $m \simeq 0.4$ GeV, which seems reasonable in the frame of the Faiman and Hendry harmonic-oscillator quark model.[23] (See also our discussion.[10,15]) We must notice that the results are sometimes sensitive to the precise value of $x = 1/(2mR)^2$. For instance, once $R^2$ is fixed, the $\frac{1}{2}(\Sigma+\Lambda)-N$ and $\Sigma-\Lambda$ mass differences are rather sensitive to the value of $m$. However, what can be retained is that, with a value of the effective mass $m \simeq 0.4$ GeV, we find satisfactory magnitudes for the $\Sigma$-$\Lambda$ splitting, the $\Gamma(\Delta \to N\pi)$ width, and $\mu^*/\mu_p^{\text{tot}}$. The new parameters introduced, $m_\lambda - m$, $(g_A)_q$, and $\kappa_q$, are affected by the same type of uncertainties. However, there is a definite indication that $(g_A)_q \neq 1$ and that $\kappa_q \neq 0$, contrary to the simplicity assumptions of Ref. 9 and of other authors.

On the other hand, one must not forget that, although we have been led to perform power expansions, the parameters of these expansions are not actually very small, so that our calculations clearly show only a trend to agree with the data. We think that this is not peculiar to our specific model or to the quark model, but that it is in general the best we can do in strong-interaction physics.

## VIII. CONCLUSION

While trying to extend the naive-quark-model description to deep-inelastic scattering phenomena, we have been led to introduce a *rest frame* broken-SU(6) wave function for the nucleon. In this paper, as well as in Ref. 15, we have investigated the consequences of our mixing hypothesis for the low-momentum-transfer properties of the octet and decuplet ground state, which were formerly assigned to a pure $\underline{56}$. It is shown that the good results of SU(6), $\mu_p^{\text{tot}}/\mu_2 = -\frac{3}{2}$ and $F/D = \frac{2}{3}$, are practically unaltered. There are, on the other hand, sizable departures from SU(6) for a number of quantities. In this paper we have shown that there appears a $\Sigma$-$\Lambda$ mass splitting, as well as a departure from the SU(6) prediction $\mu^*/\mu_p^{\text{tot}} = 2\sqrt{2}/3$ of the right sign and order of magnitude. There are also large corrections to $G^*/G_A$ which seem to be in the right direction. The prediction for the $\Gamma(\Delta \to N\pi)$ width is now in good agreement with experiment. In fact, the various $\Delta$-$N$ couplings were underestimated by SU(6), and our model affords a general explanation for this phenomenon, due to the interference term. But there is also an unwanted consequency for the neutron charge radius. The sign predicted in Ref. 15 was a mistake.

The mixing angle, unambiguously fixed in sign by the large-$x$ behavior of $F_2^{en}/F_2^{ep}$, leads in fact to a negative slope of the neutron electric form factor, i.e., to a *positive* charge squared radius in contradiction with experiment. Indeed a naive argument based on the approximation of only three valence quarks seems to show a rather general difficulty. The large-$x$ behavior of the ratio $F_2^{en}/F_2^{ep}$ implies that $p(x) \gg 2n(x)$ for $x \to 1$. This means that in a proton there is an excess of $\mathcal{P}$ quarks at small distances, or by charge symmetry an excess of $\mathfrak{N}$ quarks in the neutron at small distances. Therefore, assuming that the neutron charge distribution presents one node only (which is the case in our model), one concludes that the charge is negative in the neutron center and positive in the peripheral region, i.e., the charge radius squared is positive.

However, we have not taken into account the important contribution of the $q\bar{q}$ sea at small $x$, i.e., at large distances. The form of the sea could have important consequences on the sign and magnitude of the neutron charge radius. We have not considered the contribution of the pairs since we have computed the neutron charge form factor at $\vec{P}=0$ using a three-quark wave function; it is at large $x$, on which our argument is based, where we can have information on the three valence quarks. The role on the neutron charge radius of the $q\bar{q}$ sea at $P\to\infty$ or of the quark structure at $\vec{P}=0$ is for the moment an open problem.

In spite of this difficulty, we think that the situation is encouraging. The $\Sigma$-$\Lambda$ splitting and the $\mu^*/\mu_p^{\rm tot}$ ratio were known difficulties to the naive quark model, and De Rújula, Georgi, and Glashow[25] only afford a possible explanation for the $\Sigma$-$\Lambda$ splitting.

A byproduct of the discussion is the need for a quark strong-interaction structure. This was not unexpected on theoretical grounds, but was neglected in early works. The effects are of the same order of magnitude as that for the nucleon itself.

We think that the present work must now be pursued into two main directions: understanding the dynamical origin of the SU(6) mixing and understanding the quark structure. As to the SU(6) breaking, the gauge theories have made a proposal based on the Breit-Fermi interaction to describe the hadron spectrum[25]; on the other hand, Carlitz,[37] using phenomenological Regge arguments, has related the large-$x$ behavior of $F_2^{en}/F_2^{ep}$ to $\Delta$-$N$ mass difference. These are interesting suggestions. We are trying to understand the possible links or contradictions between the various approaches to get a dynamical insight into the various manifestations of the SU(6) breaking.

As to the quark structure, we have already suggested that it may be due to the mediation of the photon or weak interaction by $q\bar{q}$ bound states—the low-lying mesons. But we still lack a quantitative treatment of these effects as well as an understanding of the connection between this structure and the one considered in the quark-parton model through a sea of $q\bar{q}$ pairs and gluons.

Finally, one could ask if our treatment of the quark model is not getting too complicated, since it is no longer nonrelativistic, the original SU(6) functions are no longer sufficient, and the quarks are no longer simple pointlike entities. The answer is that these complications seem needed by experiment and ones does not know any simple way of escaping them. In fact these complications afford very simple interpretations of certain phenomena such as the chiral configuration mixing, the large-$x$ behavior of deep-inelastic structure functions, and the ones considered in this paper: $\Sigma$-$\Lambda$ splitting, $\mu^*$ departure from SU(6), etc. Moreover, the new effects introduced, although quite sizable, are sufficiently small to allow for a step-by-step approach.

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