PHYSICAL REVIEW D VOLUME 15, NUMBER 3 1 FEBRUARY 1977

π production near threshold and s-wave $\pi\pi$ parameters*

J. J. Brehm
Department of Physics and Astronomy, University of Massachusetts, Amherst, Massachusetts 01002
(Received 24 March 1976)

We show how to construct amplitudes for π production, $\pi N \to \pi\pi N$, which are suitable for energies not far above threshold and which are simple enough to be useful phenomenologically. The $\frac{1}{2}^+$ amplitudes are treated and the construction of their energy dependence is based on the unitarity relations for the coupled channels πN and $\pi\pi N$. An ansatz is made about the $\pi\pi$-mass dependence to permit a parametrization which employs the parameters of the s-wave $\pi\pi$ system. The well-known features of the $\frac{1}{2}^+$ πN elastic amplitudes are cited to fix the values of several of the constants appearing in the construction. Formulas are obtained for the various $\pi^{\pm}p$ production cross sections. Some early production-cross-section data are used to show how the s-wave $\pi\pi$ parameters may be extracted. The formulas are offered for phenomenological application of this sort when more production data in the near-threshold region become available.

INTRODUCTION

Our knowledge of values for the parameters of the low-energy $\pi\pi$ system continues to be very meager. The s-wave scattering lengths in particular are not known despite the obvious intrinsic interest which has been associated with them for many years. The $\pi\pi$ phase shifts have been extracted[1] from Chew-Low analyses of the π production reaction $\pi N \to \pi\pi N$, but not for $\pi\pi$ masses below about 500 MeV. The decay $\psi' \to \psi\pi\pi$ offers a unique new source of isospin $I=0$ $\pi\pi$ information,[2] but, here too, not for very low $\pi\pi$ mass. Extrapolation to the very-low-mass regime requires that the data be very precise and that the validity of the parametrization be secure over the span of the extrapolation. To determine the s-wave $\pi\pi$ scattering lengths we should really have recourse to data near the $\pi\pi$ threshold and we should have an analytical procedure which is reliable for this region. In K_{e4} decay we have such a method[3] for $I=0$, but the data[4] are hard to extract, so that the determination is so far not decisive and not readily improved upon. In this investigation we shall return to π production, $\pi N \to \pi\pi N$, and provide an analysis suitable not far above threshold for direct extraction of the low-energy s-wave $\pi\pi$ parameters. We shall have to await new accurate production data for π lab kinetic energy T_π somewhat below 300 MeV; tentatively, however, we can examine older existing data at sufficiently low energies in order to appraise our method.

The spirit of our procedure is to extend the threshold method suggested long ago by Ansel'm and Gribov[5] to the energy region slightly above the threshold. Following these authors into this regime we shall take the production amplitude to contain the $\pi\pi$ amplitude as an explicit factor, and thus restrict interactions in the final $\pi\pi N$ state to the $\pi\pi$ system. We know that we soon reach energies where πN interactions in the final state become important[6]; our assumption is that at the energies of interest the onset of these effects has not yet been reached. Ideally we would prefer to restrict the range of the method to $T_\pi < 250$ MeV; in fact, present data are so scanty that we shall extend our range almost to 300 MeV. Near the production threshold we can confine our attention, because of centrifugal barriers, to a final state in which only s waves occur. Thus the production amplitude we wish to construct is that for $J^P = \frac{1}{2}^+$. The notation is indicated in Fig. 1(a): The isospin T can be $\frac{1}{2}$ or $\frac{3}{2}$, the c.m. energy is W, the s-wave $\pi\pi$ system can have isospin $I=0$ for $T=\frac{1}{2}$ or $I=2$ for $T=\frac{3}{2}$, the $\pi\pi$ mass-squared is x. It is of course possible to develop a formalism which includes more effects; our purpose here is to exploit the restrictions applicable at low energy so that the end result will be a *simple* formula which can be readily employed phenomenologically. We hope that the result will have some range of corresponding restricted validity.

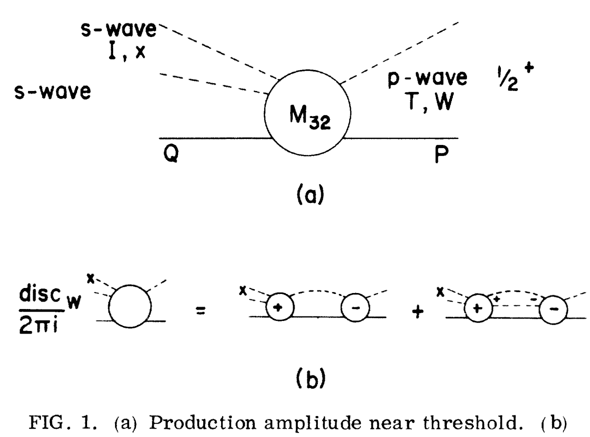

FIG. 1. (a) Production amplitude near threshold. (b) Unitarity relation for the production amplitude.

CONSTRUCTION

We denote the elastic amplitude, for $\pi N \to \pi N$ in the $\frac{1}{2}^+$ state for definite isospin T, by $M_{22} = M_{22}(W)$, and the $\frac{1}{2}^+$ production amplitude for definite T and I by $M_{32} = M_{32}(W, x)$; the labels T and I are suppressed for now. These amplitudes will be constructed in accord with unitarity constraints.[7] At these energies only channels 2 (πN) and 3 ($\pi\pi N$) are open, so we can write

$$\frac{1}{2\pi i}[M_{22}(W+) - M_{22}(W-)] = M_{22}(W+)\rho_2 M_{22}(W-) + \int dz\, M_{23}(W+, z+)\rho_3 M_{32}(W-, z-), \tag{1}$$

$$\frac{1}{2\pi i}[M_{32}(W+, x) - M_{32}(W-, x)] = M_{32}(W+, x)\rho_2 M_{22}(W-) + \int dz\, M_{33}(W+, x, z+)\rho_3 M_{32}(W-, z-). \tag{2}$$

The $\pi\pi N \to \pi\pi N$ amplitude M_{33} occurs in (2); there are two additional discontinuity formulas, for M_{23} and for M_{33}, which we shall not write down. The $\pm$ notation refers to top and bottom of the cuts in question; $W\pm$ means $W \pm i0$ in the c.m. energy and $z\pm$ means $z \pm i0$ in the $\pi\pi$ mass-squared. The factors ρ_2 and ρ_3 are phase-space factors:

$$\rho_2(W) = \frac{4m}{(4\pi)^3}\frac{P}{W} \tag{3}$$

and

$$\rho_3(W, z) = \left[\frac{4m}{(4\pi)^3}\frac{Q}{W}\right]\left[\frac{1}{(4\pi)^3}\frac{1}{2}\left(\frac{z - 4\mu^2}{z}\right)^{1/2}\right] \tag{4}$$

in which P and Q are the nucleon c.m. momenta, as in Fig. 1(a); note that Q depends on the dipion mass $\sqrt{z}$. The nucleon and pion mass are m and μ. The range of integration for z is, for given W, $4\mu^2 \le z \le (W - m)^2$. Equation (2) is represented diagrammatically in Fig. 1(b).

Kinematic factors with known W dependence may be identified; they appear explicitly in the amplitudes: Channel 2 (πN, p wave) carries the factor $[(P_0 - m)/2m]^{1/2}$, and channel 3 ($\pi\pi N$, s waves) carries the factor $[(Q_0 + m)/2m]^{1/2}$. To give an example, a calculation of the nucleon-pole term in M_{22} for $J^P = \frac{1}{2}^+$, $T = \frac{1}{2}$ yields

$$M_{22,\,\text{pole}}^{T=1/2} = \frac{P_0 - m}{2m}\frac{12\pi g^2}{m - W}; \tag{5}$$

we shall need this result later. Our basic assumption stated above is that near threshold the $\pi\pi$ interaction is dominant over the πN interaction in channel 3. This amounts to an assumption about the x dependence of amplitudes connected to channel 3: Channel 3 then carries the s-wave $\pi\pi$ amplitude $t_I(x)$ as an explicit factor corresponding to isospin I. We shall implement these observations by writing (and again suppressing isospin labels)

$$\begin{pmatrix} M_{22}(W) & M_{23}(W, y) \\ M_{32}(W, x) & M_{33}(W, x, y) \end{pmatrix} = \begin{pmatrix} \frac{P_0 - m}{2m}F(W) & \left(\frac{P_0 - m}{2m}\frac{Q_0 + m}{2m}\right)^{1/2} t(y)G(W) \\ \left(\frac{P_0 - m}{2m}\frac{Q_0 + m}{2m}\right)^{1/2} t(x)G(W) & \frac{Q_0 + m}{2m}t(x)H(W)t(y) \end{pmatrix}. \tag{6}$$

Our ansatz serves to exhibit all of the dependence on the $\pi\pi$ mass. This construction is motivated by the proposal,[8] made long ago, that the interaction of the beam pion with a pion in the nucleon cloud should provide the dominant contribution to low-energy π production. The expression given for M_{32} in Eq. (6) is as prescribed in the isobar model, truncated to include only the $\pi\pi$ isobar in the $\pi\pi N$ state. It should then be noted that our ansatz implies that a zero of the $\pi\pi$ s-wave $t(x)$ is consequently a zero of $M_{32}(W, x)$ at the same point x. Morgan and Pennington[2] argue that this is not what one should necessarily expect from partial conservation of axial-vector current (PCAC) and current algebra because the occurrence on-shell of the Adler zero is process-dependent. In the context of the present work this distinction would be provided for by the inclusion of the other small isobar contributions. The existence of such a zero will appear as central to our parametrization. Once expression (6) has been adopted then the unitarity constraints may be applied so that the functions F, G, and H, introduced in Eq. (6), must satisfy

$$\frac{1}{\pi}\text{Im}\begin{pmatrix} F & G \\ G & H \end{pmatrix} = \begin{pmatrix} F & G \\ G & H \end{pmatrix}\begin{pmatrix} \rho & 0 \\ 0 & \bar{\rho} \end{pmatrix}\begin{pmatrix} F & G \\ G & H \end{pmatrix}^*, \tag{7}$$

in which the new phase-space factors are

$$\rho = \frac{2}{(4\pi)^3}\frac{P}{W}(P_0 - m) \tag{8}$$

and

$$\bar{\rho} = \int_{4\mu^2}^{(W-m)^2} dx \left[\frac{2}{(4\pi)^3}\frac{Q}{W}(Q_0+m)\right] \times \left[\frac{1}{(4\pi)^3}\frac{1}{2}\left(\frac{x-4\mu^2}{x}\right)^{1/2}\right]|t(x)|^2 . \quad (9)$$

Note that only W dependence remains in Eqs. (7) to (9).

We want to identify functions F, G, and H which provide adequate approximations in the production threshold region. Thus we need only be concerned about local considerations of analyticity and unitarity. We can even go so far as to neglect those contributions to the unitarity relation (7) associated with $\bar{\rho}$. We could include these effects readily; however, they are estimable effects which prove to be negligible. As empirical evidence for this we cite the πN-phase-shift solutions of Carter *et al.*[9] in which the elasticity factors for P_{11} and P_{31} remain well above 95% up to $T_\pi \simeq 300$ MeV. We therefore need only implement the constraint

$$\frac{1}{\pi}\mathrm{Im}\begin{pmatrix} F & G \\ G & H \end{pmatrix} = \rho\begin{pmatrix} |F|^2 & FG^* \\ GF^* & |G|^2 \end{pmatrix} . \quad (10)$$

In our limited regime of interest it is also an excellent approximation to make the replacement $[(W+m)^2-\mu^2]^{1/2} \to (W+m)$ in ρ, Eq. (8), to obtain

$$\rho = \frac{1}{2}\frac{W+m}{(4\pi W)^3}[(W-m)^2-\mu^2]^{3/2} . \quad (11)$$

The following parametrization satisfies (10) and (11) (here we reinstate the isospin subscript for $T=\frac{1}{2},\frac{3}{2}$):

$$F_{1/2}(W) = \left[X_1 + \frac{Y}{W-W_0} + J(W)\right]^{-1} , \quad (12)$$

$$F_{3/2}(W) = [X_3 + J(W)]^{-1} , \quad (13)$$

$$G_T(W) = \bar{G}_T F_T(W) , \quad (14)$$

where

$$J(W) = \frac{f(W)-p(W)}{2W^3}\frac{W+m}{(4\pi)^3}[(W-m)^2-\mu^2] . \quad (15)$$

X_1, X_3, Y, and W_0 are parameters which we shall be able to fix at the outset. $\bar{G}_T(T=\frac{1}{2},\frac{3}{2})$ are constants about which we have no *a priori* knowledge. Note that a denominator-pole term has been included in (12); we introduce this in order to accommodate the behavior of the P_{11} elastic amplitude.[10] The function $f(W)$ is analytic with the elastic cut,

$$f(W) = [(W-m)^2-\mu^2]^{1/2} \times \left[\ln\frac{(W-m+\mu)^{1/2}+(W-m-\mu)^{1/2}}{(W-m+\mu)^{1/2}-(W-m-\mu)^{1/2}} - i\pi\right] , \quad (16)$$

for $W \gtrsim m+\mu$. The function $p(W)$ is a lengthy second-order polynomial designed to eliminate the apparent third-order pole at $W=0$ in expression (15):

$$p(W) = (m^2-\mu^2)^{-1/2}\left[\frac{\mu^2 W^2}{2(m^2-\mu^2)} + m(W-m) + \mu^2\right]\ln\frac{m+(m^2-\mu^2)^{1/2}}{\mu} + W - \frac{mW^2}{2(m^2-\mu^2)} . \quad (17)$$

The parameters appearing in (12) and (13) we shall regard as known *a priori* by fitting to the well-known features of the elastic $\frac{1}{2}^+$ amplitudes. For $T=\frac{1}{2}$, there are three parameters: X_1, Y, and W_0. We determine these by requiring that $M_{22}^{T=1/2}$ have the correct nucleon pole position and residue, and by requiring a fit to the P_{11} phase-shift solution of Carter *et al.*[9] For $T=\frac{3}{2}$ the single parameter X_3 is determined by fitting to the Carter P_{31} phase shift. The $\frac{1}{2}^+$ phase shifts for isospin T satisfy

$$\cot\delta_T = 2W(P_0+m)(4\pi)^2\,\mathrm{Re}F_T^{-1}/P^3 . \quad (18)$$

Near the nucleon pole, $W=m$, we have from Eq. (5)

$$F_{1/2} = 12\pi g^2/(m-W) , \quad (19)$$

with $g^2/4\pi = 14.5$. This fitting procedure yields

$$X_1 = 2.74\times10^{-4}\mu, \quad X_3 = -13.36\times10^{-4}\mu, \quad Y = 5.84\times10^{-4}\mu^2, \quad W_0 = 8.65\mu; \quad (20)$$

the quality of the resulting fit to the Carter phase shifts up to $T_\pi = 300$ MeV is shown in Fig. 2. In this energy range we can see that *the factor $F_T(W)$, which occurs in M_{32} in this construction, is a major contributor to the W dependence of the production amplitude*, especially for isospin $T=\frac{1}{2}$.

The production amplitude M_{32}^{TI} may be assembled from Eqs. (6) and (14). In it, a unitary expression for the $\pi\pi$ amplitude $t_I(x)$ could be adopted[11]; however, a simpler parametrization is in order. Our construction from Eq. (10) on neglects inelastic cut effects due to the $\pi\pi N$ channel; it therefore should also suffice to employ a real linear form for $t_I(x)$:

$$t_I(x) = \text{constant}\times(x-x_I) . \quad (21)$$

The parameters x_0 and x_2, in (21), are of direct theoretical and phenomenological interest. They are the Weinberg s-wave zeros[12]; the other s-wave $\pi\pi$ parameters may be determined from them. We can now show how these quantities appear in

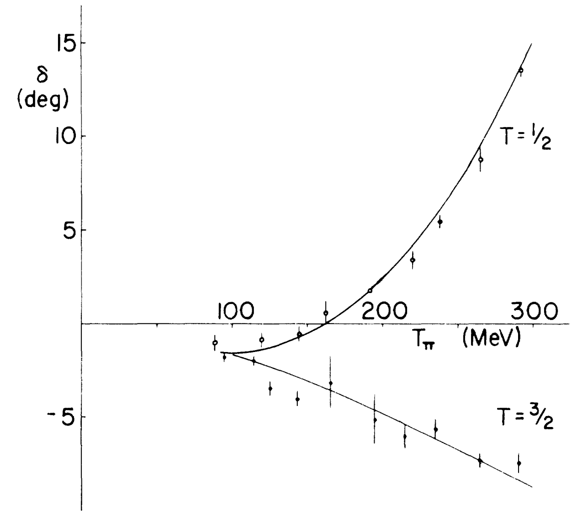

FIG. 2. Fit to the P_{11} and P_{31} phase shifts, δ_1 and δ_3, of Ref. 9.

the production cross section; with that we shall then have a direct means of providing for a determination of their values from production data near threshold.

The production cross section is obtained from the appropriate combination of isospin amplitudes by means of

$$\sigma(2\to 3)=\int dx\,\rho_3\frac{m}{4PW}\left|M(2\to 3)\right|^2; \tag{22}$$

without $I=1$ effects to consider we have

$$\left|M(\pi^+p\to\pi^+\pi^+n)\right|^2=\tfrac{4}{5}\left|M_{32}^{3/2,2}\right|^2, \tag{23a}$$

$$\left|M(\pi^+p\to\pi^+\pi^0p)\right|^2=\tfrac{1}{5}\left|M_{32}^{3/2,2}\right|^2, \tag{23b}$$

$$\left|M(\pi^-p\to\pi^-\pi^0p)\right|^2=\tfrac{1}{5}\left|M_{32}^{3/2,2}\right|^2, \tag{23c}$$

$$\left|M(\pi^-p\to\pi^+\pi^-n)\right|^2=\tfrac{2}{45}\left|M_{32}^{3/2,2}-\sqrt{10}M_{32}^{1/2,0}\right|^2, \tag{23d}$$

$$\left|M(\pi^-p\to\pi^0\pi^0n)\right|^2=\tfrac{2}{45}\left|\sqrt{2}M_{32}^{3/2,2}+\sqrt{5}M_{32}^{1/2,0}\right|^2. \tag{23e}$$

We shall refer to the corresponding processes as reactions (a) to (e). For the ith process the cross section is

$$\sigma_i=\frac{P}{W^4(P_0+m)}\int_{4\mu^2}^{(W-m)^2}dx\left(\frac{x-4\mu^2}{x}\right)^{1/2}\left[(W-m)^2-x\right]^{1/2}\times\left[(W+m)^2-x\right]^{3/2}\times\left|\{g_T(x-x_I)F_T(W)\}_i\right|^2, \tag{24}$$

where $g_{1/2}$ and $g_{3/2}$ are the unknown constants which have accumulated in front of each isospin amplitude, and where the notation $\{\}_i$ instructs us to combine amplitudes for process i as in Eqs. (23).

If we let $\chi=|g_{3/2}|^2$ and $\xi=g_{1/2}/g_{3/2}$ then we can prescribe simple formulas for the W dependence of all the cross sections in terms of the four parameters x_0, x_2, χ, and ξ. This parametrization is a minimal one if we admit no further constraints on the construction. We let

$$Z(W)=\frac{P}{W^4(P_0+m)}$$

and

$$R(W,x)=\left(\frac{x-4\mu^2}{x}\right)^{1/2}\left[(W-m)^2-x\right]^{1/2}\times\left[(W+m)^2-x\right]^{3/2};$$

then we have

$$\sigma_a=\tfrac{4}{5}\chi Z(W)\int dx\,R(W,x)(x-x_2)^2\left|F_{3/2}(W)\right|^2, \tag{25a}$$

$$\sigma_b=\sigma_c \tag{25b}$$

$$=\tfrac{1}{4}\sigma_a, \tag{25c}$$

$$\sigma_d=\tfrac{2}{45}\chi Z(W)\int dx\,R(W,x)\left|(x-x_2)F_{3/2}(W)-\sqrt{10}\,\xi(x-x_0)F_{1/2}(W)\right|^2, \tag{25d}$$

$$\sigma_e=\tfrac{2}{45}\chi Z(W)\int dx\,R(W,x)\left|\sqrt{2}(x-x_2)F_{3/2}(W)+\sqrt{5}\,\xi(x-x_0)F_{1/2}(W)\right|^2. \tag{25e}$$

These equations, together with (12), (13), (15), (16), (17), and (20), comprise our construction of the cross sections; the important amplitude-zero parameters x_0 and x_2 appear directly in the final formulas.

APPLICATION

It is hoped that it will become possible to obtain cross-section data at low energies well below $T_\pi=300$ MeV, with precision sufficient to determine the parameters. To know χ and ξ from such a determination would be of little interest. However, to know the positions of the s-wave zeros, x_0 and x_2, would be of great value. They ought to lie in the real region of the variable x, i.e., in $(0,4\mu^2)$. The linear simplification adopted to $t_I(x)$ in Eq. (21) suggests a means for reducing the number of parameters. If the s waves in (21) correspond to a crossing-symmetric $\pi\pi\to\pi\pi$ amplitude linear in *all* invariant variables, then it can be shown that x_0 and x_2 must satisfy

$$4x_0+5x_2=12\mu^2, \tag{26}$$

and that the constants appearing in (21), call them c_0 and c_2, are also constrained:

$$c_0/c_2 = -2. \tag{27}$$

The s-wave scattering lengths are

$$a_I = (\mu/8\pi)c_I(1 - x_I/4\mu^2), \quad I = 0, 2. \tag{28}$$

By means of these steps, knowledge of x_0 and x_2 leads to a determination of the scattering length ratio, a_0/a_2.

We can go even further and determine both a_I if we adopt the conclusions of soft-pion physics[13] and invoke Weinberg's relation[12] between the scattering lengths,

$$2a_0 - 5a_2 = 6L, \tag{29}$$

where $L = \mu/8\pi F_\pi^2$ in which F_π is the pion decay constant. It follows that $c_2 = -1/F_\pi^2$. To pursue this interpretation, we define the parameters η_0 and η_2 by

$$(2\omega)^{1/2}\langle\pi^a|[Q_5^b, \sigma^{jk}]|0\rangle = iF_\pi\mu^2(\mathcal{P}^{(0)}\eta_0 + \mathcal{P}^{(2)}\eta_2)_{ab,jk}, \tag{30}$$

in which the indices are isovector labels, $\mathcal{P}^{(I)}$ is an isospin projection operator, Q_5 denotes an axial-vector charge, and σ refers to the σ commutator; the notation is that of Ref. 11. An exact relation holds between η_0 and η_2:

$$2\eta_0 - 5\eta_2 = 6. \tag{31}$$

The size of η_2 furnishes a direct measure of the exotic $I=2$ content in the σ commutator; as such, it is of considerable interest in theories of chiral-symmetry breaking. For example, with $\eta_2 = 0$, σ is purely isoscalar and the Weinberg prediction[12] is obtained:

$$a_0^{(W)} = 7L/4 \text{ and } a_2^{(W)} = -L/2 \tag{32}$$

with $x_0 = \mu^2/2$ and $x_2 = 2\mu^2$. To date, we have no information about η_2 from experiment. The phenomenological application of the present analysis, to yield values for x_0 and x_2, would provide the means for its determination; it is obtained[11] from

$$\eta_2 = (x_2 - 2\mu^2)/\mu^2. \tag{33}$$

Cross-section data well below $T_\pi = 300$ MeV scarcely exist so only a preliminary application of our construction is possible. We are limited to some old results of Batusov *et al.*[14] from reaction (d). These authors provide values for σ_d at $T_\pi = 210$, 222, 233, and 246 MeV; they assert that for $T_\pi \leq 245$ MeV each final particle's angular distribution in the c.m. system is isotropic, in support of the s-wave assumption we have made in the construction. We vary x_0 and x_2 subject to constraint (26) and fit (25d) to their values for σ_d

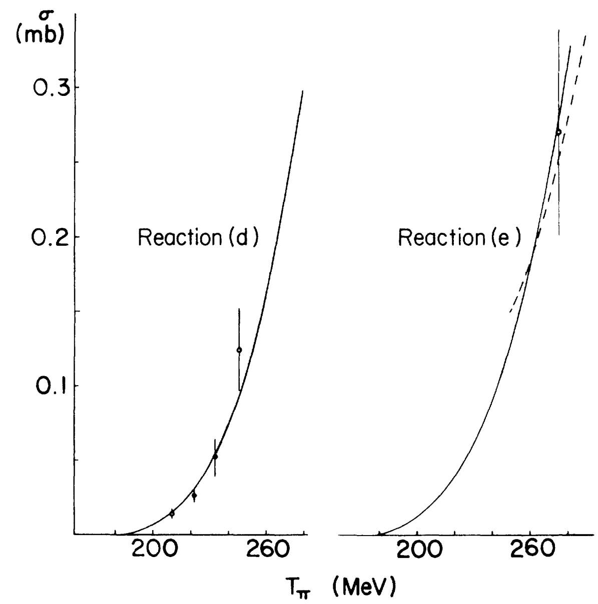

FIG. 3. Cross sections for reaction (d): $\pi^- p \to \pi^+\pi^- n$, and reaction (e): $\pi^- p \to \pi^0\pi^0 n$. The data are from Ref. 14 and Ref. 15, respectively. The dashed curve is the prediction for (e) from Ref. 16.

at 210 and 233 MeV. For each choice of (x_0, x_2) a determination of ξ and χ results; to select an ultimate solution, data from the other reactions at low energies are necessary. Nothing is available below 300 MeV from reactions (a), (b), and (c). One datum from reaction (e) exists,[15] for $T_\pi = 276$ MeV. This result has large uncertainty; however, it is in good agreement with an extrapolation, down to 250 MeV, of a reconstruction[16] which has been made of σ_e, based on available data in the 300–500 MeV range from all of the other reactions. The reconstruction of σ_e in Ref. 16 is based on isospin invariance and amounts to a prediction of a cross section which is difficult to measure. We can obtain general agreement with this prediction, and fit the lone value of σ_e together with the data of Ref. 14, with the parameter solution

$$x_0 = 0.3\mu^2, \quad x_2 = 2.2\mu^2,$$

$$\xi = 0.032, \quad \chi = 1.1 \times 10^{-5} \text{ mb}\mu^{-4}.$$

The fit is shown in Fig. 3, where we have restricted the plots to $T_\pi < 280$ MeV. More recent data[17] exist for σ_d around $T_\pi = 300$ MeV and beyond; even though these energies are judged to be beyond the range of applicability of our ansatz the curve for σ_d shown in Fig. 3 continues to fit these data within their sizable uncertainties. Equations (27) and (28) provide a result for the scattering length ratio:

$$a_0/a_2 = -4.1.$$

If we also adopt (29) and (33) we obtain values for the scattering lengths and the σ-commutator parameter:

$$a_0 = 7.4L/4, \quad a_2 = -0.9L/2, \quad \eta_2 = 0.2;$$

the numbers for a_I are given to invite comparison with the values in (32). Of course in the absence of more solid data these results can only be considered as suggestive at best.

The emphasis in our method has been to develop simple cross-section formulas having reliable W dependence, based on unitarity. The preceding application to obtaining the $\pi\pi$ parameters can be put to real use when more real data at low energies become available for more than one reaction. If the $I = 2$ $\pi\pi$ amplitude is as small as the indication from the value for a_2 would suggest, then reactions (a), (b), and (c) may not be useful for this kind of study; our construction loses its validity when final-state πN interactions cannot be neglected. From this point of view *reaction (e) would be the most promising*, albeit the most challenging, candidate to study: The final-state s-wave scattering $\pi^0 n \to \pi^0 n$ is measured by the πN scattering length combination $\frac{1}{3}a_{1/2} + \frac{2}{3}a_{3/2}$, a quantity which is an order of magnitude smaller than either $a_{1/2}$ or $a_{3/2}$. The πN interaction is correspondingly suppressed relative to the $\pi\pi$ in the final state of reaction (e). We hope that the challenge associated with obtaining the data for both reactions (d) and (e) can be met in the near future.

---

*Work supported in part by the National Science Foundation.

[1]E.g., S. D. Protopopescu *et al.*, Phys. Rev. D 7, 1279 (1973).

[2]D. Morgan and M. R. Pennington, Phys. Rev. D 12, 1283 (1975); R. N. Cahn, *ibid.* 12, 3559 (1975).

[3]A. Pais and S. B. Treiman, Phys. Rev. 168, 1858 (1968).

[4]E. W. Beier *et al.*, Phys. Rev. Lett. 30, 399 (1973).

[5]A. A. Ansel'm and V. N. Gribov, Zh. Eksp. Teor. Fiz. 37, 501 (1959) [Sov. Phys.—JETP 10, 354 (1960)].

[6]E.g., M. G. Olsson and G. B. Yodh, Phys. Rev. 145, 1309 (1966).

[7]R. Blankenbecler, Phys. Rev. 122, 983 (1961); L. F. Cook and B. W. Lee, *ibid.* 127, 283 (1962).

[8]L. S. Rodberg, Phys. Rev. Lett. 3, 58 (1959).

[9]J. R. Carter *et al.*, Nucl. Phys. B58, 378 (1973).

[10]J. J. Brehm and L. F. Cook, Phys. Rev. 170, 1381 (1968); 170, 1387 (1968).

[11]J. J. Brehm and E. Golowich, Phys. Rev. D 9, 2064 (1974).

[12]S. Weinberg, Phys. Rev. Lett. 17, 616 (1966).

[13]Current algebra and PCAC have not been specifically used anywhere up to this point.

[14]Y. A. Batusov *et al.*, Yad. Fiz. 1, 526 (1965) [Sov. J. Nucl. Phys. 1, 374 (1965)].

[15]A. V. Kravtsov *et al.*, Yad. Fiz. 20, 942 (1974) [Sov. J. Nucl. Phys. 20, 500 (1974)].

[16]M. M. Makarov *et al.*, Yad. Fiz. 17, 170 (1973) [Sov. J. Nucl. Phys. 17, 88 (1973)].

[17]D. H. Saxon *et al.*, Phys. Rev. D 2, 1790 (1970); J. A. Jones *et al.*, Nucl. Phys. B83, 93 (1974).