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Current algebra and the low-energy πN scattering amplitudes

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We discuss low-energy pion-nucleon scattering using the current-algebra constraints on the scattering amplitudes. In order to estimate the on-mass-shell amplitudes, the "remainder terms" are calculated as a sum over the contributions from Δ(1236), $N^*(1525)$, and $N^*(1470)$ with particular emphasis on the correct evaluation of the off-mass-shell effects of spin-3/2 resonances. The invariant amplitudes are expanded in powers of ν and ν_B for small values of these variables. The expansion coefficients are calculated explicitly and compared with experiment.

I. INTRODUCTION

Recently the accuracy of the low-energy cross-section data of the πN scattering has been considerably improved due to the work of Bussey *et al.*,[1] and the parameters, such as the πN coupling constant f, and the scattering lengths a_1 and a_3, are now known with far better accuracy from the analysis of Bugg, Carter, and Carter.[2,3] These new data were also analyzed in terms of the πN partial waves by Carter, Bugg, and Carter.[3] Nielsen and Oades (NO)[4] recalculated these partial waves using theoretical predictions for d and f waves and including Coulomb corrections in a nonrelativistic approximation. The πN partial-wave phase shifts so obtained by Nielsen and Oades are expected to be quite reliable and supplement those given by Carter, Bugg, and Carter[3] and also the earlier ones obtained by Almehed and Lovelace.[5]

The availability of these new low-energy πN scattering cross-section data and phase shifts made it possible for Nielsen and Oades to obtain the expansion coefficients of the πN scattering amplitudes about the point $\nu=0$, $t=0$, improving the accuracy of similar results obtained by Höhler *et al.*[6] in an earlier analysis.

It is therefore worthwhile to use these experimental results to test the various theoretical predictions about the low-energy behavior of the πN scattering amplitudes. We shall critically reexamine here the current-algebra constraints on the πN scattering amplitudes written in terms of pole terms and a "remainder term" that receives contributions only from the resonances.[7] The predictions about the πN scattering lengths and effective-range parameters are discussed in the Appendix.

There is considerable controversy in the literature about the various theoretical predictions obtained from the application of current algebra to the low-energy πN scattering. The most notable one is about the estimate of the so-called σ term.[8,9] From a theoretical point of view, the significant contributions were made by Brown, Pardee, and Peccei (BPP),[10] and Osypowski[11] and Wray[12] in resolving some of these controversies. These authors have written the current-algebra constraints on the amplitudes as identities keeping the pions on the mass shell and separating out the "remainders" or the "nonpole terms" that receive contributions from the resonances. However, the translation of the current-algebra results on πN scattering valid for mathematical pions with zero mass and zero momenta to the physical situation involving real pions is ambiguous and doubtful unless one can estimate with reasonable certainty the contributions of the Δ(1236) and higher resonances to the πN scattering amplitudes.

We point out that the BPP and Osypowski calculations for the Δ(1236) contribution did not fully take into account the off-mass-shell effects in the spin-$\frac{3}{2}$ propagator, nor did these authors take into consideration the most general form of the interaction involving a spin-$\frac{3}{2}$ particle, a nucleon and the derivative of the pion field. While Osypowski has taken the usual $\pi N\Delta$ interaction Lagrangian and the Δ(1236) on the mass shell, BPP have used a modified interaction Lagrangian and have neglected the contact terms. No account of the effect due to the presence of the surface terms in the interaction Hamiltonian was taken into consideration.

We have reexamined the calculations of the low-energy πN scattering amplitudes in the spirit of BPP and Osypowski and have evaluated the contributions of Δ(1236), $N^*(1520)$, and $N^*(1470)$ resonances to the "remainder terms" of the amplitudes. These are the three resonances that make significant contributions to the amplitudes; the contributions of other resonances are negligibly small because their masses are higher and also their coupling to the πN elastic channel is small. We have taken into consideration the off-mass-shell effects of Δ(1236) and $N^*(1520)$ in a Lorentz-convariant way following the earlier work of Nath, Etemadi, and Kimel[13] (NEK). To take into account the off-mass-shell effect of the Δ(1236) in a general way, the $\pi N\Delta$ interaction Lagrangian with derivative coupling depends on two arbitrary parameters, A and Z, first introduced by NEK. The parameter A drops out in the final expressions for the amplitudes and the off-mass-shell effect of $\Delta(1236)$ is then given by the parameter Z only. The pure on-shell calculation corresponds to $Z=-\frac{1}{2}$, while the calculations of BPP correspond to $Z=-\frac{1}{4}$.

From purely theoretical considerations NEK were able to fix the value of Z to be $\frac{1}{2}$. However, in the following we shall keep Z as a free parameter and try to determine it from experiments. We shall also find later that many of the predicted results are independent of the values of Z. These are, therefore, expected to be the least controversial and should agree with experiments if the assumption is made that the low-energy πN scattering amplitudes are adequately described by the current-algebra constraints with the "remainder terms" given by a few well-known low-energy resonances only.[7]

Some of our predictions do depend rather appreciably on the parameter Z. However, because of the absence of any experimental information on the t dependence of the $\sigma(t)$ term or the contribution from the weak axial-vector nucleon scattering to the πN scattering, the value of Z cannot be fixed from experiments at present. However, it is observed that a determination of only one of these quantities fixes the value of the others conclusively.

The plan of this paper is as follows: Section II contains the kinematics and the current-algebra constraints on the πN scattering amplitudes. The evaluation of the remainder term as a sum over the contributions from the resonances is presented in Sec. III. Theoretical predictions from our work are compared with experimental results in Sec. IV.

II. KINEMATICS AND THE CURRENT-ALGEBRA CONSTRAINTS

Following the convention, the T matrix is written in terms of the invariant amplitudes $A(s,t)$ and $B(s,t)$,

$$T^{\alpha\beta}(s,t)=\bar{u}(p_2)[A^{\alpha\beta}(s,t)+\tfrac{1}{2}i\gamma\cdot(q_1+q_2)B^{\alpha\beta}(s,t)]u(p_1)\,, \tag{2.1}$$

where q_1 and q_2 are, respectively, the four-momenta of the incoming pion and the outgoing pion in the s-channel c.m. frame. Similarly, p_1 and p_2 represent the four-momenta of the incoming nucleon and the outgoing nucleon, respectively. [We are using the metric where $x_\mu{}^2=\vec{x}^2+x_4{}^2=\vec{x}^2-x_0{}^2$, $s=-(p_1+q_1)^2$, $t=-(p_1-p_2)^2$, $u=-(p_1-q_2)^2$, $\gamma_1\cdots\gamma_5$ are all Hermitian.] In the isospin space, the amplitudes can be further decomposed as

$$A^{\alpha\beta}=\delta_{\alpha\beta}A^{(+)}+\tfrac{1}{2}[\tau_\alpha,\tau_\beta]A^{(-)}\,, \tag{2.2}$$

and there is a similar decomposition for the amplitude $B^{\alpha\beta}$. We define the variables

$$\begin{aligned}\nu&=-q_1\cdot(p_1+p_2)/2m\\&=-q_2\cdot(p_1+p_2)/2m\,,\\&=(s-u)/4m\,,\\ \nu_B&=q_1\cdot q_2/2m\\&=(t-2\mu^2)/4m\,,\end{aligned} \tag{2.3}$$

where m is the nucleon mass.

Current algebra and partial conservation of axial-vector current (PCAC) impose certain constraints on the on-mass-shell pion-nucleon scattering amplitudes.[10] These are

$$A^{(+)}(\nu,\nu_B)=\frac{\sigma_{NN}(\nu_B)}{F_\pi{}^2}+\frac{g^2}{m}+A_R^{(+)}(\nu,\nu_B)\,, \tag{2.4}$$

$$\begin{aligned}G^{(-)}(\nu,\nu_B)&\equiv\nu^{-1}[A^{(-)}(\nu,\nu_B)-\nu\tilde{B}^{(-)}(\nu,\nu_B)]\\&=-\frac{g^2}{2m^2}+\frac{F_1^V(t)}{F_\pi{}^2}+G_R^{(-)}(\nu,\nu_B)\,,\end{aligned} \tag{2.5}$$

where $\sigma_{NN}(\nu_B)$ is the usual σ term proportional to the nucleon matrix element of the equal-time commutator between the axial charge density and the divergence of the axial-vector current. $F_1^V(t)$ is the nucleon isovector electromagnetic form factor, and $F_\pi=92.03$ MeV is the pion decay constant. The tilde means that the nucleon pole terms have been subtracted out. The pion-nucleon coupling constant g^2 equals $(16\pi f^2m^2)/\mu^2$ with $f^2=0.079$.[2] The terms with subscript R are what BPP[10] called the "remainder terms" which are to be saturated with contributions from $\Delta(1236)$ and higher resonances. The remainders in (2.4) and (2.5) are expected to be small for small values of the variables ν and ν_B. One of our aims in this paper is to evaluate explicitly the values of the correction terms $A_R^{(+)}$ and $G_R^{(-)}$, and then to compare the predictions from (2.4) and (2.5) with the experiment. Strictly speaking, current algebra does not constrain the amplitude $\nu^{-1}B^{(+)}(\nu,\nu_B)$ or the combination $\nu^{-1}[A^{(-)}(\nu,\nu_B)+\nu B^{(-)}(\nu,\nu_B)]$.[10] However, in the Ward-identity approach,[11,12] it is possible to approximate the πN scattering amplitudes in terms of the N,ρ,σ contributions and an additional term representing the sum of the contributions from resonances such as $\Delta(1236)$, $N^*(1525)$, and $N^*(1470)$. More explicitly, for small values of the pion momenta,

$$\nu^{-1}A^{(-)}(\nu,\nu_B)=-\frac{F_2^V(t)}{F_\pi{}^2}+C+A_{np}^{(-)}(\nu,\nu_B)\,, \tag{2.6}$$

$$\tilde{B}^{(-)}(\nu,\nu_B)=\frac{g^2}{2m^2}-\frac{G_M^V(t)}{F_\pi{}^2}+C+B_{np}^{(-)}(\nu,\nu_B)\,, \tag{2.7}$$

$$\nu^{-1}\tilde{B}^{(+)}(\nu,\nu_B)=C_1+B_{np}^{(+)}(\nu,\nu_B)\,, \tag{2.8}$$

where the quantities with the subscripts (np), the so-called nonpole parts, will receive contributions from the $\Delta(1236)$ and higher resonances. In the Ward-identity approach, if the propagators and the vertex functions are expanded in powers of the pion momenta, the nucleon contribution reproduces the usual results of the gradient-coupling theory and, in addition to this, gives certain terms which lead to the introduction of the parameters C and C_1 in our scheme. These unknown quantities, to be determined from experiment, can also be related to the weak axial-vector nucleon scattering in the soft-pion limit, following Goldberg and Gross.[14] The experimental values of the nucleon electromagnetic isovector form factors are[15]

$$\begin{aligned} F_1^V(t)&=0.5+0.023t\,,\\ F_2^V(t)&=1.85+0.11t\,,\\ G_M^V(t)&=2.35+0.13t\,. \end{aligned} \tag{2.9}$$

To facilitate the comparison between theoretical predictions and experiment, the amplitudes are expanded as

$$\begin{aligned}\tilde{x}(\nu,\nu_B)=&\,x_1+x_2(4m\nu_B)+x_3\nu^2+x_4(4m\nu_B\nu^2)\\ &+x_5\nu^4+x_6(4m\nu_B\nu^4)+x_7(4m\nu_B)^2\\ &+x_8(4m\nu_B)^2\nu^2\,.\end{aligned} \tag{2.10}$$

III. EVALUATION OF THE REMAINDERS

In this section we calculate the remainders as a sum over the contributions from $\Delta(1236)$, $N^*(1470)$, and $N^*(1520)$. Higher resonances do not make any significant contribution.

The propagator and the interaction Lagrangian for a spin-$\frac{3}{2}$ field coupled to a nucleon field and a pion field are given by

$$\langle 0|\,T(\psi_\mu(x)\bar{\psi}_\nu(y))\,|0\rangle=id_{\mu\nu}(\partial)\Delta_F(x-y)\,, \tag{3.1}$$

where

$$d_{\mu\nu}(\partial)=-(\gamma_\lambda\partial_\lambda-M)\left[\delta_{\mu\nu}-\tfrac{1}{3}\gamma_\mu\gamma_\nu+\frac{1}{3M}(\gamma_\mu\partial_\nu-\gamma_\nu\partial_\mu)-\frac{2}{3M^2}\partial_\mu\partial_\nu\right]$$
$$-\frac{1}{3M^2}\left(\frac{A+1}{2A+1}\right)\left\{\left[\frac{1}{2}\left(\frac{A+1}{2A+1}\right)\gamma_\lambda\partial_\lambda+\left(\frac{A}{2A+1}\right)M\right]\gamma_\mu\gamma_\nu+\gamma_\mu\partial_\nu+\left(\frac{A}{2A+1}\right)\gamma_\nu\partial_\mu\right\}(\Box-M^2)\,,$$

$$\Delta_F(x-y)=-(2\pi)^{-4}\int e^{ik(x-y)}(k^2+M^2-i\epsilon)^{-1}d^4k\,,$$

and

$$\mathcal{L}_I=\frac{1}{\sqrt{2}}g^*\bar{\psi}_\mu\theta_{\mu\nu}T^\alpha\psi\partial_\nu\varphi^\alpha+\text{H.c.}\,,\qquad \theta_{\mu\nu}=\{\delta_{\mu\nu}+[\tfrac{1}{2}(1+4Z)A+Z]\gamma_\mu\gamma_\nu\}\,, \tag{3.2}$$

where A and Z are two arbitrary parameters, T's are a set of (3×2) matrices corresponding to isospin $\frac{3}{2}$ (see Ref. 16), and M denotes the mass of the $\frac{3}{2}$ $(\frac{3}{2}^+)$ resonance. Expression (3.2) shows the most general form of the interaction Lagrangian containing the first-order derivative of the pion field, which is invariant under a point transformation of the interacting fields.[13] As expected, the contribution of $\Delta(1236)$ to the pion-nucleon scattering amplitudes does not depend on A, but it does depend on Z. This parameter A determines the contribution of the spin-$\frac{3}{2}$ resonance to the pion-nucleon scattering when the spin-$\frac{3}{2}$ particle is not on the mass shell. From purely theoretical considerations, NEK[13] showed that Z must equal $\frac{1}{2}$. In this paper we keep Z as a free parameter and examine the possibility of determining it from experiment. The exact expressions for $\Delta(1236)$ contributions to the pion-nucleon scattering are given in Ref. 13. The following are the expansions in powers of ν and ν_B of the Δ contribution to $A^{(\pm)}$ and $B^{(\pm)}$:

$$\begin{aligned}A^{(+)}(\nu,\nu_B)=\frac{g^{*2}}{9M^2}\Bigg\{&\frac{2(2M+m)}{(M^2-m^2)}\,\mu^4+(4m\nu_B)[(1-\xi)m-2\eta M]+\frac{(4m\nu_B)}{(M^2-m^2)^2}(6R_2-R_1)+\frac{(8m^2R_1)\nu^2}{(M^2-m^2)^3}\\ &+\frac{(32m^4R_1)\nu^4}{(M^2-m^2)^5}+\frac{(4m\nu_B\nu^2)12m^2}{(M^2-m^2)^4}(2R_2-R_1)+\frac{(4m\nu_B)^2}{(M^2-m^2)^3}(\tfrac{1}{2}R_1-3R_2)\\ &+\frac{(12m^2)(4m\nu_B)^2\nu^2}{(M^2-m^2)^5}(R_1-3R_2)+\frac{16m^4(4m\nu_B\nu^4)}{(M^2-m^2)^6}(-5R_1+6R_2)\Bigg\}\,,\end{aligned} \tag{3.3}$$

$$\nu^{-1}A^{(-)}(\nu,\nu_B)=\frac{g^{*2}}{18M^2}\left\{4m[(1-\xi)m-2\eta M]-\frac{4mR_1}{(M^2-m^2)^2}+\frac{(4m\nu_B)4m}{(M^2-m^2)^3}(R_1-3R_2)-\frac{(16m^3R_1)\nu^2}{(M^2-m^2)^4}\right.$$
$$-\frac{(64m^5R_1)\nu^4}{(M^2-m^2)^6}-\frac{16m^3(4m\nu_B\nu^2)}{(M^2-m^2)^5}(2R_1-3R_2)-\frac{3m(4m\nu_B)^2}{(M^2-m^2)^4}(R_1-4R_2)$$
$$\left.+\frac{192m^5(4m\nu_B\nu^4)}{(M^2-m^2)^7}(R_1-R_2)-\frac{(8m^3)(4m\nu_B)^2\nu^2}{(M^2-m^2)^6}(5R_1-12R_2)\right\}, \tag{3.4}$$

$$\nu^{-1}B^{(+)}(\nu,\nu_B)=\frac{g^{*2}}{9M^2}\left[(4m)(1+\eta-\sqrt{\xi})+\frac{4mR_3}{(M^2-m^2)^2}-\frac{4m(4m\nu_B)}{(M^2-m^2)^3}(R_3+3R_4)+\frac{(16m^3R_3)\nu^2}{(M^2-m^2)^4}\right.$$
$$+\frac{(64m^5R_3)\nu^4}{(M^2-m^2)^6}-\frac{32m^3(4m\nu_B\nu^2)}{(M^2-m^2)^5}(R_3+\tfrac{3}{2}R_4)+\frac{3m(4m\nu_B)^2}{(M^2-m^2)^4}(R_3+4R_4)$$
$$\left.+\frac{(8m^3)(4m\nu_B)^2\nu^2}{(M^2-m^2)^6}(5R_3+12R_4)-\frac{192m^5(4m\nu_B\nu^4)}{(M^2-m^2)^7}(R_3+R_4)\right], \tag{3.5}$$

$$B^{(-)}(\nu,\nu_B)=\frac{g^{*2}}{18M^2}\left[R_5-\frac{2R_3}{(M^2-m^2)}+(1+\eta-\sqrt{\xi})(4m\nu_B)+\frac{(4m\nu_B)}{(M^2-m^2)^2}(R_3+6R_4)-\frac{(8m^2R_3)\nu^2}{(M^2-m^2)^3}\right.$$
$$-\frac{(32m^4R_3)\nu^4}{(M^2-m^2)^5}+\frac{12m^2(4m\nu_B\nu^2)}{(M^2-m^2)^4}(R_3+2R_4)-\frac{(4m\nu_B)^2}{(M^2-m^2)^3}(\tfrac{1}{2}R_3+3R_4)$$
$$\left.+\frac{16m^4(4m\nu_B\nu^4)}{(M^2-m^2)^6}(5R_3+6R_4)-\frac{12m^2(4m\nu_B)^2\nu^2}{(M^2-m^2)^5}(R_3+3R_4)\right], \tag{3.6}$$

where

$$\begin{aligned}
R_1&=(2M+m)\mu^4+(M^2-m^2)[2(M+m)\mu^2+(2M^3+3M^2m-m^3)],\\
R_2&=M^2(M^2-m^2)(M+m),\\
R_3&=[(-M^4+2M^3m+6M^2m^2+2Mm^3-m^4)+2\mu^2(m^2-mM-2M^2)-\mu^4],\\
R_4&=M^2(M^2-m^2),\\
R_5&=2\{[-M^2-(2+4\eta)Mm+m^2(1-2\xi)-\mu^2(2+\eta-\sqrt{\xi}-\xi)]\},\\
\eta&=2Z(2Z+1),\quad \sqrt{\xi}=(2Z+1).
\end{aligned} \tag{3.7}$$

The interaction Lagrangian for a system consisting of a pion field, a nucleon field, and a spin-$\frac{3}{2}$ particle with odd parity and isospin $\frac{1}{2}$ is given by

$$\mathcal{L}_1=g_1^*\bar{\psi}_\mu\theta_{\mu\nu}\gamma_5\tau^\alpha\psi\partial_\nu\varphi^\alpha+\text{H.c.}, \tag{3.8}$$

where $\theta_{\mu\nu}$ is defined as in (3.2). So the contribution from $(\frac{3}{2}^-)N^*$ to the pion-nucleon scattering can easily be evaluated from the expressions (3.3)–(3.7) with the replacement $M\to -M_1$, $g^*\to g_1^*$, where M_1 is the mass of the $(\frac{3}{2}^-)$ resonance. To take into account the difference in isospins, the even and odd amplitudes are to be multiplied by $\frac{3}{2}$ and -3, respectively.

Finally, using the pseudovector coupling, the interaction Lagrangian for the $(\frac{1}{2}^+)$ N^* field coupled to a pion field and a nucleon field can be written as

$$\mathcal{L}_1=\frac{1}{\sqrt{2}}\frac{f'}{\mu}\bar{\psi}_{N^*}\gamma_\mu\gamma_5\tau^\alpha\psi_N\partial_\mu\varphi^\alpha+\text{H.c.} \tag{3.9}$$

The $N^*(1470)$ contributions to the pion-nucleon scattering, calculated from (3.9) and expanded in powers of ν and ν_B, are

$$A^{(+)}(\nu,\nu_B)=g'^2(M_2-m)Y_1, \tag{3.10}$$

$$\nu^{-1}A^{(-)}(\nu,\nu_B)=-g'^2\frac{4m(M_2-m)}{(M_2^2-m^2)^2}Y_2, \tag{3.11}$$

$$\nu^{-1}B^{(+)}(\nu,\nu_B)=-g'^2\frac{4m}{(M_2^2-m^2)^2}Y_2, \tag{3.12}$$

$$B^{(-)}(\nu,\nu_B)=g'^2\left[Y_1-\frac{4m(M_2-m)}{(M_2^2-m^2)}\right], \tag{3.13}$$

where

$$g'^2=f'^2\frac{(M_2+m)^2}{\mu^2},$$

$$Y_1=\left[\frac{(4m\nu_B)}{(M_2^2-m^2)^2}-\frac{(8m^2)\nu^2}{(M_2^2-m^2)^3}-\frac{(32m^4)\nu^4}{(M_2^2-m^2)^5}+\frac{(12m^2)(4m\nu_B\nu^2)}{(M_2^2-m^2)^4}\right.$$
$$\left.-\frac{1}{2}\frac{(4m\nu_B)^2}{(M_2^2-m^2)^3}-\frac{(12m^2)(4m\nu_B)^2\nu^2}{(M_2^2-m^2)^5}+\frac{(80m^4)(4m\nu_B\nu^4)}{(M_2^2-m^2)^6}\right],$$
$$Y_2=\left[1-\frac{4m\nu_B}{(M_2^2-m^2)}+\frac{(4m^2)\nu^2}{(M_2^2-m^2)^2}+\frac{(16m^4)\nu^4}{(M_2^2-m^2)^4}-\frac{(8m^2)(4m\nu_B\nu^2)}{(M_2^2-m^2)^3}\right.$$
$$\left.+\frac{3}{4}\frac{(4m\nu_B)^2}{(M_2^2-m^2)^2}-\frac{(48m^4)4m\nu_B\nu^4}{(M_2^2-m^2)^5}+\frac{(10m^2)(4m\nu_B)^2\nu^2}{(M_2^2-m^2)^4}\right], \tag{3.14}$$

and M_2 is the mass of the $(\frac{1}{2}^+)$ N^* resonance.

Using the experimental values of the decay widths and resonance masses[17] we find that

$$\frac{g^{*2}}{4\pi}=0.3359\ \mu^{-2},$$
$$\frac{g_1^{*2}}{4\pi}=0.1968\ \mu^{-2}, \tag{3.15}$$
$$f'^2=0.1321.$$

IV. RESULTS AND CONCLUSIONS

We have attempted in this paper to determine the πN scattering amplitudes as functions of ν and ν_B for small values of these variables, using the current-algebra constraints. The coefficients in the expansion (2.10) have been calculated explicitly and are given in Tables I and II. Using the formulas given in Sec. III, the contributions from the resonances $\Delta(1236)$, $N^*(1520)$, and $N^*(1470)$ have been evaluated numerically and the contributions from the pole terms have been added to this in order to get the total value of the expansion coefficients. The $\Delta(1236)$ and $N^*(1520)$ contributions shown in Tables I and II correspond to the spin-$\frac{3}{2}$ off-mass-shell parameter $Z=\frac{1}{2}$. The parameters C and C_1 have been fixed by making use of the experimental values of $\nu^{-1}A^{(-)}$ and $\nu^{-1}B^{(+)}$ at $\nu=\nu_B=0$. These quantities are related, as we have already noted, to the contribution from the weak axial-vector nucleon scattering to the πN scattering and, within the framework of the current algebra as it is applied to the low-energy πN scattering, the role of the weak axial-vector nucleon scattering cannot be ignored even in the soft-pion limit.[14,18] The experimental results from the work of HJS and NO are displayed alongside our theoretical predictions.

We first note that the off-mass-shell parameter Z of the $\Delta(1236)$ and $N^*(1520)$ occurs only in five different coefficients x_i, e.g., x_2 for A^+, x_1 for $\nu^{-1}A^{(-)}$, x_1 for $\nu^{-1}B^{(+)}$, and x_1 and x_2 for $B^{(-)}$. We shall try to determine the value of Z from the known experimental values of these coefficients. But before we do so let us try to resolve in our work the controversy regarding the value of $\sigma_{NN}(t=2\mu^2)$ and make predictions on the t-slope parameter of the σ commutator. As the expression (3.3) shows, the contributions from the spin-$\frac{3}{2}$ resonances $\Delta(1236)$ and $N^*(1520)$ to $A^{(+)}$ at $\nu=\nu_B=0$ are independent of the spin-$\frac{3}{2}$ off-mass-shell parameter Z, and the contribution from the $N^*(1470)$ to $A^{(+)}$ at the Cheng-Dashen point is zero if we use gradient coupling for the $N^*(\frac{1}{2}^{\pm})\,N\pi$ interaction. Now, using Eq. (2.4) and the experimental value of $A^{(+)}(\nu=\nu_B=0)$ as given by NO, we find that

$$\sigma_{NN}(t=2\mu^2)=63.5\pm 8.6\text{ MeV}. \tag{4.1}$$

Further, if we write $\sigma_{NN}(t)$ as

$$\sigma_{NN}(t)=\sigma_{NN}(t=2\mu^2)[1+\sigma'(t-2\mu^2)], \tag{4.2}$$

and again make use of the experimental information on $A^{(+)}$,[4] we obtain from Eq. (2.4), after taking into consideration the contribution from the resonances,

$$\sigma'=\begin{cases}0.79\pm 0.10\ \mu^{-2} & \text{for } Z=\frac{1}{2},\\ 0.49\pm 0.10\ \mu^{-2} & \text{for } Z=-\frac{1}{4},\\ 0.48\pm 0.10\ \mu^{-2}, & \text{for } Z=-\frac{1}{2}.\end{cases} \tag{4.3}$$

The t-slope parameter of σ changes appreciably with Z, unlike the value of σ at $t=2\mu^2$.

There are then four coefficients x_i for the determination of Z. Because of the occurrence of the unknown term C_1 along with Z in the coefficient x_1 for $\nu^{-1}\tilde{B}^{(+)}$, the parameter Z cannot be determined

TABLE I. Comparison of theoretical predictions of $A^{(+)}$ and $G^{(-)}$ with experiment. $x_1, \ldots, x_8$ are the coefficients of an expansion of the amplitudes given by (2.10).

Amplitude	Contribution	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
$A^{(+)}$	σ	1.034	0.817						
	$\Delta(1236)$	0.009	0.503	5.314	−0.023	0.875	−0.030	−0.013	−0.003
	$N^*(1525)$	−0.001	−0.035	−0.117	−0.002	−0.004	0	0.001	0
	$N^*(1470)$	0	0.035	−0.191	0.004	−0.008	0	0	0
	Total	1.042	1.320	5.006	−0.021	0.863	−0.030	−0.012	−0.003
	Exp $-g^2/m$ (HJS)	0.66	1.13	4.40	0	1.12			
		±0.32	± 0.10						
	Exp $-g^2/m$ (NO)	1.04	1.32	4.63	0.00	1.081	−0.050	0.035	
		± 0.14	± 0.05	± 0.11	± 0.03			± 0.007	
$G^{(-)}$	ρ	1.26	0.05						
	$\Delta(1236)$	−0.151	−0.191	−0.262	−0.023	−0.044	−0.002	0.006	0.001
	$N^*(1520)$	0.025	0.015	0.005	0.001	0	0	0	0
	$N^*(1470)$	0	0.005	0.011	0	0	0	0	0
	Total	1.134	−0.121	−0.246	−0.022	−0.044	−0.002	0.006	0.001
	Exp $+g^2/2m^2$ (HJS)	1.30	−0.15	−0.24	−0.04	−0.04			
	Exp $+g^2/2m^2$ (NO)	1.204	−0.115	−0.261	−0.042	−0.065	−0.012	0.008	
		± 0.032	± 0.009					± 0.002	

TABLE II. Comparison of theoretical predictions of $\nu^{-1}\tilde{B}^{(+)}$, $\nu^{-1}A^{(-)}$, and $\tilde{B}^{(-)}$ with experiment. The expansion coefficients are defined by (2.10).

Amplitude	Contribution	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
$\nu^{-1}\tilde{B}^{(+)}$	$\Delta(1236)$	4.184	−0.156	0.662	−0.046	0.109	−0.011	0.004	0.002
	$N^*(1520)$	−0.048	−0.004	−0.005	0	0	0	0	0
	$N^*(1470)$	−0.246	0.004	−0.010	0	0	0	0	0
	C_1	−0.712							
	Total	3.178	−0.156	0.647	−0.046	0.109	−0.011	0.004	0.002
	Exp (HJS)	2.90	−0.19	0.74	−0.09	0.28			
	Exp (NO)	3.178	−0.142	0.802	−0.089	0.212	−0.044	0.007	
		± 0.116	± 0.03					± 0.002	
$\nu^{-1}A^{(-)}$	ρ	−4.78	−0.25						
	$\Delta(1236)$	−11.022	−0.071	−1.078	0.021	−0.178	0.009	0.004	0
	$N^*(1520)$	−2.829	−0.017	−0.021	0	−0.001	0	0	0
	$N^*(1470)$	−0.934	0.014	−0.039	0.001	−0.002	0	0	0
	C	9.988							
	Total	−9.577	−0.324	−1.138	0.022	−0.181	0.009	0.004	0
	Exp (HJS)	−9.30	−0.45	−1.11	0.02	−0.29			
	Exp (NO)	−9.577	−0.371	−1.175	0.021	−0.275	0.019	−0.006	
		± 0.208	± 0.032					± 0.003	
$\tilde{B}^{(-)}$	ρ	−6.04	−0.30						
	$\Delta(1236)$	−10.871	0.120	−0.816	0.044	−0.134	0.011	−0.002	−0.001
	$N^*(1520)$	−2.854	−0.032	−0.026	−0.001	−0.001	0	0	0
	$N^*(1470)$	−0.934	0.009	−0.050	0.001	−0.002	0	0	0
	C	9.988							
	Total	−10.711	−0.203	−0.892	0.044	−0.137	0.011	−0.002	−0.001
	Exp $-g^2/2m^2$ (HJS)	−10.60	−0.30	−0.87	0.06	−0.25			
		± 0.57	± 0.20						
	Exp $-g^2/2m^2$ (NO)	−10.782	−0.256	−0.914	0.063	−0.210	0.030	−0.014	
		± 0.208	± 0.031					± 0.002	

TABLE III. $\Delta(1236)$ and $N^*(1520)$ contributions to the scattering amplitudes for different value of Z.

	Z	$\Delta(1236)$ contribution x_1	x_2	$N^*(1520)$ contribution x_1	x_2
$A^{(+)}$	$\frac{1}{2}$	0.009	0.503	−0.001	−0.035
	$-\frac{1}{4}$	0.009	0.892	−0.001	−0.118
	$-\frac{1}{2}$	0.009	0.876	−0.001	−0.093
$\nu^{-1}A^{(-)}$	$\frac{1}{2}$	−11.022	−0.071	−2.829	−0.017
	$-\frac{1}{4}$	−5.787	−0.071	−0.604	−0.017
	$-\frac{1}{2}$	−6.008	−0.071	−1.268	−0.017
$\nu^{-1}B^{(+)}$	$\frac{1}{2}$	4.184	−0.156	−0.048	−0.004
	$-\frac{1}{4}$	4.063	−0.156	−0.118	−0.004
	$-\frac{1}{2}$	4.184	−0.156	−0.048	−0.004
$B^{(-)}$	$\frac{1}{2}$	−10.871	0.120	−2.854	−0.032
	$-\frac{1}{4}$	−5.653	0.118	−0.609	−0.029
	$-\frac{1}{2}$	−5.880	0.120	−1.266	−0.032

from there, and we also note from Table III that the Z dependence of x_2 in $B^{(-)}$ is rather very weak. We are therefore left with the two coefficients x_1 of the amplitudes $\nu^{-1}A^{(-)}$ and $B^{(-)}$, which are functions of two parameters C and Z, and also very appreciably with Z. The C dependence of these two x_1's is the same, so we find that for their linear combination $G^{(-)}(\nu, \nu_B)$ from NO

$$G^{(-)}(\nu,\nu_B)_{\nu=\nu_B=0}=[\nu^{-1}A^{(-)}(\nu,\nu_B)-\tilde{B}^{(-)}(\nu,\nu_B)]^{\text{exp}}_{\nu=\nu_B=0}$$

$$=-\frac{g^2}{2m^2}+(1.204\pm0.032)\mu^{-2}. \qquad (4.4)$$

From Tables III and II we have

$$[\nu^{-1}A^{(-)}(\nu,\nu_B)-\tilde{B}^{(-)}(\nu,\nu_B)]^{\text{theor}}_{\nu=\nu_B=0}$$

$$=-\frac{g^2}{2m^2}+\begin{cases}1.134\ \mu^{-2} & \text{for}\quad Z=\frac{1}{2}\ ,\\ 1.131\ \mu^{-2} & \text{for}\quad Z=-\frac{1}{4}\,,\\ 1.130\ \mu^{-2} & \text{for}\quad Z=-\frac{1}{2}\ .\end{cases} \qquad (4.5)$$

The variation of (4.5) with Z is again small, so no definite conclusion about the value of Z can be made from the comparison of (4.5) with (4.4). We also mention that Olsson *et al.*[19] have discussed low-energy πN scattering using a model which is also based on the current-algebra constraints on the amplitudes and have found that, for $Z=-\frac{1}{2}$, the experimental data are compatible with the predictions from their model. However, these authors have included contributions from a ρ exchange and a σ exchange in addition to those from the nucleon isovector form factors and the σ commutator which follow from the current algebra, and have neglected the contribution from $N^*(1520)$ which is nearly 25% of the $\Delta(1236)$ contribution for some of the amplitudes. We should point out here that the generalized Matthews theorem, which states that the S matrix calculated from the canonical interaction Hamiltonian and the noncovariant propagator should be the same as that evaluated from the invariant interaction Hamiltonian (i.e., $-\mathcal{L}_1$) and the covariant propagator, is true for the interaction (3.2) only if the parameter Z equals $\frac{1}{2}$.[20] For any value of Z other than $\frac{1}{2}$, the effect of the surface terms in the interaction Hamiltonian must be taken into consideration. Of course, the violation of the Matthews theorem would also imply that the S matrix is not Lorentz-invariant. In addition to this, there are other theoretical justifications for taking $Z=\frac{1}{2}$.[13] From a practical point of view, for $Z=\frac{1}{2}$, the $\Delta(1236)$ contributions to the S-wave scattering lengths $a_S^{(+)}$ and $a_S^{(-)}$ are identically zero. On the other hand, for $Z=-\frac{1}{2}$, the $\Delta(1236)$ contribution to $a_S^{(+)}$ is −0.055.

To sum up, we find that the parameters C, C_1, $\sigma'(t)$, and Z cannot be fixed unambiguously from the experiment. However, if one of them is assigned a value, the remaining ones are determined uniquely by the experimental data. Therefore, we chose the theoretically preferred value $\frac{1}{2}$ for Z (see Ref. 21), and then fixed the quantities C, C_1 and $\sigma'(t)$ by using the experimental information on x_1 for $A^{(-)}$ and $B^{(+)}$, and x_2 for $A^{(+)}$. We observe that for the rest of the coefficients the agreement between theory and experiment is quite good except for x_7 in the expansion of $A^{(+)}$ where the predicted value is −0.012 and the experimental number is 0.035 ±0.007. At present the only source of experimental information for this coefficient is the work of NO. We suggest that this experimental number be reexamined.[22]

It should be mentioned here that this problem has also been investigated by the dispersion-theoretic method.[23,24] The controversial quantities in this approach are the subtraction constants. However, Scadron and Thebaud[24] have used unsubtracted dispersion relations to calculate the axial-vector nucleon scattering amplitudes and have obtained good agreement with experiment. The parameters C and C_1 in our scheme can be related, in principle, to the subtraction constants in the dispersion-theoretic approach. However, such a correlation will not be unique unless the values of the resonance contributions are known accurately and unambiguously. In other words, the relations between the subtraction constants and the parameters C and C_1 will depend on Z, too.

Finally, in our calculations the mass and the coupling constant for the $\Delta(\frac{3}{2}^-)$ resonance are taken as $M = 1236$ MeV and $g^{*2}/4\pi = 0.3359\ \mu^{-2}$. This value for the coupling constant is obtained by using the narrow-resonance approximation. Höhler, Jacob, and Strauss[6] have calculated the correction to the narrow-resonance approximation and have found that a pole model for the $\Delta(\frac{3}{2}^-)$ is compatible with the experimental data for $M = 1219$ MeV and $g^{*2}/4\pi = 0.264\ \mu^{-2}$. If the latter values of the mass and the coupling constant are used as inputs in our calculation, the Δ contributions to the expansion coefficient x_2 for $A^{(+)}$ and x_1 for other amplitudes will be reduced in magnitude by 11–17%, but not by 40% as is usually thought.[6] The remaining coefficients will not be affected appreciably. In any case, our predictions on the scattering lengths and effective ranges will hardly change, as a reduction in the Δ contributions will be taken care of by the consequent changes in the value of the parameters $\sigma'(t)$, C_1, and C.

Since a preliminary version of this paper has been issued as a report,[25] Olsson and Osypowski[26] have reexamined this problem. They have found, as in their earlier work,[19] that the value $Z = -0.45 \pm 0.20$ is compatible with the experimental data. In the Olsson-Osypowski model, there are two free parameters, in addition to those in our scheme, in the isospin-even amplitudes, whereas in the isospin-odd amplitudes there is no quantity corresponding to the parameter C of our model. Secondly, these authors have not taken into account the contributions from the $N^*(1520)$ and $N^*(1470)$, and there is no way to obtain agreement between the experiment and the predictions from their model if these contributions are included. Duality does not help to resolve this problem, as these authors seem to suggest. If the contributions from the resonances $N^*(1520)$ and $N^*(1470)$ are to be excluded because of duality, the Δ contribution should also be ignored. These are the reasons which led Olsson and Osypowski to suggest from the phenomenology a value for Z which is unacceptable from theoretical considerations.

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TABLE IV. Isospin-even scattering lengths and the S-wave effective range.

Contribution from	$a_S^{(+)}$	$a_{P_{3/2}}^{(+)}$	$a_{P_{1/2}}^{(+)}$	$b^{(+)}$
N	−0.011	0.054	−0.105	0.005
$\Delta(1236)$	0	0.051	0.017	0.005
$N^*(1520)$	0	−0.002	−0.002	0
$N^*(1470)$	−0.001	0.001	0.003	0
σ	−0.042	0.037	0.037	−0.110
C_1	0.049	0.001	0.005	0.053
Total	−0.005	0.142	−0.045	−0.047
Exp (HJS), Ref. 6	−0.02± 0.02	0.133	−0.056	−0.05± 0.02
Exp, Ref. 27	−0.007± 0.02	0.133	−0.056	−0.06± 0.02

TABLE V. Isospin-odd scattering lengths and the S-wave effective range.

Contribution from	$a_S^{(-)}$	$a_{P_{3/2}}^{(-)}$	$a_{P_{1/2}}^{(-)}$	$b^{(-)}$
N	0.001	−0.054	−0.053	−0.022
Δ(1236)	0	−0.036	0.041	0
N*(1520)	0	−0.004	0.013	0
N*(1470)	0	−0.001	0.004	0
ρ	0.080	−0.005	0.026	0.030
C	0	0.017	−0.042	0.007
Total	0.081	−0.083	−0.011	0.015
Exp (HJS), Ref. 6	0.100	−0.081	−0.013	0.009± 0.005
Exp, Ref. 27	0.096± 0.004	−0.081± 0.002	−0.013± 0.002	0.009± 0.005

APPENDIX: CALCULATION OF SCATTERING LENGTHS AND EFFECTIVE RANGES

The S- and P-wave scattering lengths are defined by

$$a_S=[\mu f_{0^+}]_{q^2=0},$$
$$a_{P_{3/2}}=[\mu^3 f_{1^+}/q^2]_{q^2=0}, \qquad \text{(A1)}$$
$$a_{P_{1/2}}=[\mu^3 f_{1^-}/q^2]_{q^2=0},$$

where q is the magnitude of the pion momentum in the pion-nucleon c.m. frame and

$$f_{l^\pm}=\tfrac{1}{2}\int_{-1}^{+1}[F_1P_l(x)+F_2P_{l\pm1}(x)]dx\ ,$$
$$F_1=\frac{E+m}{8\pi W}[A-(W-m)B]\ ,$$
$$F_2=-\frac{E-m}{8\pi W}[A+(W+m)B]\ , \qquad \text{(A2)}$$
$$W=\sqrt{s}\ ,\quad E=(q^2+m^2)^{1/2}\ ,$$
$$x=\cos\theta\ .$$

The S-wave effective range can be calculated from the formulas

$$b=\frac{\mu}{8\pi(m+\mu)}\left\{-\mu(A+mB)+\frac{\mu^2}{2m}(A-\mu B)+\left[2\mu(m+\mu)^2\frac{\partial}{\partial s}-4m\mu^2\frac{\partial}{\partial t}\right]\times(A-\mu B)\right\}_{t=0,\ s=(m+\mu)^2}. \qquad \text{(A3)}$$

The scattering amplitudes are known as functions of ν and ν_B from Tables I and II. Using this knowledge of the scattering amplitudes and the definitions (A1)–(A3), the scattering lengths and the effective ranges can be calculated. Our results are shown in Tables IV and V beside the experimental determinations of these parameters. The predictions from this model are in good agreement with experiment, except for $a_S^{(-)}$. However, the experimental values of $a_S^{(-)}$ quoted in Table IV are rather large compared with a more recent evaluation of this quantity by Bugg, Carter, and Carter,[2] who found that $a_S^{(-)}=0.087\pm0.004$, which is closer to our prediction.

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