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Electromagnetic corrections to πN scattering

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Numerical results are presented for the electromagnetic corrections to the *S*- and *P*-wave phase shifts and inelasticities in π^+p and π^-p scattering. A discussion is given of how to apply the corrections in practical data analysis.

INTRODUCTION

This paper presents our results on the electromagnetic corrections to the *S*- and *P*-waves in π^+p scattering and π^-p elastic and charge-exchange scattering. The corrections to the phase shifts are obtained by the use of a dispersion-relation method which has been developed in a series of papers.[1-5] The corrections to the inelasticities are given by the unitarity equation. We give the corrections due to the $n\gamma$ channel in π^-p scattering.

The present paper gives the results in tabular form. For details concerning the derivation and significance of these results we refer to Ref. 5. In order to establish a "user's manual" for our corrections we include detailed expressions for the differential and total cross sections to which the corrections are to be applied.

The simplest way to deal with the corrections in a phase-shift analysis perhaps is to correct the data before the analysis is carried out.[6] This can be done by using the approximately known phase shifts and inelasticities to calculate, e.g., the differential cross section $d\sigma/d\omega$ twice, first *with* the electromagnetic corrections (including the Coulomb scattering and Coulomb-nuclear interference terms) and then *without* any of these corrections. The difference gives the correction to the data. The corrected data can then be used in a partial-wave analysis where all electromagnetic effects are absent.

Our corrections are determined in such a way that the amplitudes derived from the corrected data have the analyticity and unitarity properties assumed for pure hadronic amplitudes. If our corrections were complete, one would in addition expect the amplitudes to be charge independent. In particular, the isospin-$\frac{3}{2}$ phase shifts in π^+p and π^-p scattering should come out equal. However, as discussed in Ref. 5 there may be significant contributions from short-range electromagnetic effects which have not so far been taken into account in our calculations. We therefore suggest that one should still distinguish between the P_{33} phase shifts in π^+p and π^-p scattering. One may here use the fact that any remaining difference (due to short-range effects) between these two phase shifts can be represented[5] by the form $\sin^2\delta$ times a slowly varying function (δ being the P_{33} phase shift).

NOTATION

We use the following notation.

s, t, u = usual Mandelstam variables for $\pi N \to \pi N$.

M, μ = nucleon mass (proton) and pion mass $(\pi^\pm)$.

q, θ = c.m. momentum and scattering angle.

$W = s^{1/2}$ = total c.m. energy.

$E = (M^2+q^2)^{1/2}$ = nucleon c.m. energy.

J, l = total and orbital angular momenta in πN.

$$\gamma = \alpha\frac{s-M^2-\mu^2}{2qW} = \frac{\alpha}{v}, \quad \alpha = \frac{1}{137},$$

(v = lab relative velocity).

$F_\pi(t)$ = pion form factor $[F_\pi(0)=1]$.

F_1^p, F_2^p = Dirac and Pauli proton form factors $[F_1^p(0)=1,\ 1+2MF_2^p(0)=2.79]$.

f, g = no-flip and spin-flip amplitudes.

$f_{l\pm}$ = partial-wave amplitude $(J = l \pm \frac{1}{2})$.

DIFFERENTIAL CROSS SECTION AND POLARIZATION

The general expressions for the $\pi N \to \pi N$ differential cross section and polarization are

$$\frac{d\sigma}{d\omega}=Z(s,t,\Delta E)(|f_{CH}|^2+|g_{CH}|^2)\,\frac{q_f}{q_i} \tag{1}$$

and

$$P=\frac{2\,\mathrm{Re}(f_{CH}g_{CH}^*)}{|f_{CH}|^2+|g_{CH}|^2}, \tag{2}$$

where

$$f_{CH}=f_C+\sum_{l=0}^{\infty}[(l+1)f_{l+}+lf_{l-}]P_l(\cos\theta)\,, \tag{3a}$$

$$g_{CH}=g_C+i\sum_{l=1}^{\infty}(f_{l+}-f_{l-})P_l'(\cos\theta)\sin\theta\,. \tag{3b}$$

Here f_C, g_C are the Coulomb amplitudes. The factor $Z(s,t,\Delta E)$ in Eq. (1) comes from the emission of undetected soft photons, ΔE being the energy resolution in the experiment.

In most low-energy experiments Z will differ from unity only by fractions of a percent. However, this does not hold if ΔE becomes small. Note that $Z=1$ for forward π^+p and π^-p elastic scattering, while for $\pi^-p\to\pi^0n$ $Z=1$ only at threshold. The detailed form of Z is discussed in the Appendix, where we give approximate formulas which can be used to estimate $Z-1$.

COULOMB AMPLITUDES

For $\pi^+p\to\pi^+p$ the Coulomb amplitudes which correspond to our electromagnetic corrections are given by

$$f_C=\left\{\left(\frac{2q\gamma}{t}+\frac{\alpha}{2W}\,\frac{W+M}{E+M}\right)F_1^p+\left[W-M+\frac{t}{4(E+M)}\right]\frac{\alpha F_2^p}{W}\right\}F_\pi e^{i\phi_C}\,, \tag{4a}$$

$$g_C=\frac{i\alpha F_\pi}{2W\tan(\frac{1}{2}\theta)}\left\{\frac{W+M}{E+M}F_1^p+\left[W+\frac{t}{4(E+M)}\right]2F_2^p\right\} \tag{4b}$$

with[7]

$$\phi_C=-\gamma\ln\left[\sin^2\left(\frac{\theta}{2}\right)\right]+\gamma\int_{-4q^2}^{0}[1-F_\pi(t')F_1^p(t')]\,\frac{dt'}{t'}\,. \tag{4c}$$

[We use the form factors given in Eqs. (142) and (143) of Ref. 2.] The Coulomb amplitudes for $\pi^-p\to\pi^-p$ are $-f_C^*, -g_C$, with f_C, g_C given by Eqs. (4). For $\pi^-p\to\pi^0n$ we have $f_C=g_C=0$.

COULOMB PHASE SHIFTS

The partial-wave projections of f_C, g_C give the Coulomb phase shifts. To order α they are

$$\Sigma_{l\pm}=\frac{q}{2}\int_{-1}^{1}F_\pi\left[\frac{2q\gamma}{t}F_1^p(P_l-1)-\frac{\alpha M}{W}F_2^pP_l\right]dz \pm\frac{\alpha q}{4W(J+\frac{1}{2})}\times\int_{-1}^{1}F_\pi\left[\frac{W+M}{E+M}F_1^p+\left(W+\frac{t}{4(E+M)}\right)2F_2^p\right]\times(P_l'+P_{l\pm1}')dz\,, \tag{5}$$

where $t=-2q^2(1-z)$. Table I gives Σ_{0+} and $\Sigma_{1\pm}$. For $q\to0$, $\Sigma_{1\pm}=\gamma+O(1)$.

ELECTROMAGNETIC CORRECTIONS TO $\pi^+p\to\pi^+p$

For $\pi^+p\to\pi^+p$ the partial-wave projections $f_{l\pm}$ are written as

$$f_{l\pm}=\frac{\eta_{l\pm}e^{2i\delta_{l\pm}}-1}{2iq}\exp(2i\Sigma_{l\pm})\,, \tag{6}$$

where $\Sigma_{l\pm}$ are the Coulomb phase shifts and $\delta_{l\pm}$, $\eta_{l\pm}$ are the so-called nuclear phase shifts and inelasticities. These are related to the pure hadronic quantities $(\delta_H)_{l\pm}$ and $(\eta_H)_{l\pm}$ by

$$\delta_{l\pm}=(\delta_H)_{l\pm}+\Delta_{l\pm}\,, \tag{7a}$$

$$\eta_{l\pm}=(\eta_H)_{l\pm}-\bar{\eta}_{l\pm}\,, \tag{7b}$$

where $\Delta, \bar{\eta}$ are the electromagnetic corrections. Our results for $\Delta_{l\pm}$ are given in Table II. The $\pi^+p\to\pi^+p$ inelasticity corrections $\bar{\eta}_{l\pm}$ are (at low and medium energies) due only to bremsstrahlung and are typically of the order 10^{-4}–10^{-3}, i.e., they can safely be ignored (cf. Table I in Ref. 4).

ELECTROMAGNETIC CORRECTIONS TO π^-p SCATTERING

For $\pi^-p\to\pi^-p$ we write

$$f_{l\pm}=\tfrac{1}{3}(2f_1+f_3-2\sqrt{2}\,f_{13})\,\frac{\exp(-2i\Sigma_{l\pm})}{2iq_-}\,, \tag{8}$$

TABLE I. The S- and P-wave Coulomb phase shifts (in radians).

q/μ	Σ_{0+}	Σ_{1-}	Σ_{1+}
0.5	3.08×10^{-4}	1.32×10^{-2}	1.43×10^{-2}
1.0	5.66×10^{-4}	6.85×10^{-3}	8.80×10^{-3}
1.5	7.38×10^{-4}	4.38×10^{-3}	6.93×10^{-3}
2.0	8.23×10^{-4}	2.90×10^{-3}	5.76×10^{-3}
2.5	8.41×10^{-4}	1.90×10^{-3}	4.87×10^{-3}
3.0	8.20×10^{-4}	1.22×10^{-3}	4.17×10^{-3}
3.5	7.79×10^{-4}	7.63×10^{-4}	3.60×10^{-3}
4.0	7.30×10^{-4}	4.51×10^{-4}	3.14×10^{-3}
5.0	6.33×10^{-4}	9.45×10^{-5}	2.46×10^{-3}
6.0	5.50×10^{-4}	-7.33×10^{-5}	2.00×10^{-3}
8.0	4.29×10^{-4}	-1.89×10^{-4}	1.41×10^{-3}
10.0	3.49×10^{-4}	-2.08×10^{-4}	1.07×10^{-3}

TABLE II. Phase-shift corrections Δ (in degrees) to $\pi^+ p$ scattering.

q/μ	T_L (MeV)	$\Delta(S_{1/2})$	$\Delta(P_{1/2})$	$\Delta(P_{3/2})$
0.5	22.1	0.10	0.01	−0.07
0.8	53.7	0.09	0.02	−0.23
1.0	80.7	0.10	0.04	−0.47
1.1	95.8	0.10	0.04	−0.65
1.2	111.8	0.10	0.05	−0.90
1.3	128.7	0.11	0.06	−1.19
1.35	137.4	0.11	0.07	−1.32
1.4	146.4	0.11	0.07	−1.42
1.45	155.6	0.12	0.07	−1.46
1.5	164.9	0.12	0.08	−1.42
1.6	184.3	0.12	0.09	−1.13
1.7	204.4	0.13	0.09	−0.72
1.8	225.2	0.13	0.10	−0.36
1.9	246.8	0.13	0.11	−0.10
2.0	269.1	0.13	0.12	0.07
2.2	315.8	0.13	0.13	0.25
2.5	391.1	0.14	0.15	0.32
3.0	530.6	0.14	0.20	0.27

TABLE III. Phase-shift corrections (in degrees) to $\pi^- p$ scattering.

	$S_{1/2}$			$P_{3/2}$		
q/μ	Δ_1	Δ_3	Δ_{13}	Δ_1	Δ_3	Δ_{13}
0.5	−0.31	0.52	0.06	0.02	−0.74	0.11
0.8	−0.21	0.35	0.06	0.05	−1.48	0.15
1.0	−0.15	0.30	0.07	0.08	−2.13	0.15
1.1	−0.13	0.28	0.07	0.10	−2.51	0.14
1.2	−0.10	0.27	0.08	0.11	−2.87	0.12
1.3	−0.07	0.26	0.09	0.13	−3.09	0.09
1.35	−0.06	0.25	0.09	0.14	−3.03	0.07
1.4	−0.05	0.25	0.10	0.14	−2.82	0.05
1.45	−0.03	0.25	0.10	0.15	−2.44	0.01
1.5	−0.02	0.24	0.11	0.16	−1.93	−0.02
1.6	0	0.24	0.12	0.17	−0.73	−0.08
1.7	0.01	0.23	0.14	0.19	0.26	−0.13
1.8	0.03	0.22	0.15	0.20	0.87	−0.16
1.9	0.04	0.22	0.16	0.22	1.15	−0.17
2.0	0.05	0.21	0.17	0.23	1.23	−0.18
2.2	0.09	0.20	0.20	0.27	1.14	−0.17
2.5	0.14	0.18	0.24	0.36	0.83	−0.16
3.0	0.20	0.13	0.35	0.47	0.37	−0.13

and for $\pi^- p \to \pi^0 n$

$$f_{l\pm} = \tfrac{1}{3}[\sqrt{2}\,(f_3 - f_1) - f_{13}]\ \frac{\exp(-i\Sigma_{l\pm})}{2i(q_0 q_-)^{1/2}}\ , \tag{9}$$

where in both cases

$$f_1 = \eta^1 e^{2i\delta^1} - 1\ , \tag{10a}$$

$$f_3 = \eta^3 e^{2i\delta^3} - 1\ , \tag{10b}$$

$$f_{13} = \tfrac{2}{3}\sqrt{2}\ (\eta_{13} + i\Delta_{13}) e^{i(\delta^1+\delta^3)}\ . \tag{10c}$$

The Coulomb phase shift $\Sigma_{l\pm}$ is still given by Eq. (5), q_- and q_0 are the c.m. momenta of the $\pi^- p$ and $\pi^0 n$ systems, and δ^1, δ^3 are the nuclear isospin-$\frac{1}{2}$ and isospin-$\frac{3}{2}$ phase shifts. The electromagnetic corrections are here defined by

$$\delta^1 = \delta^1_H - \tfrac{2}{3}\Delta_1\ , \tag{11a}$$

$$\delta^3 = \delta^3_H - \tfrac{1}{3}\Delta_3\ , \tag{11b}$$

where δ^1_H, δ^3_H are the pure hadronic phase shifts. We have assumed that the pure hadronic amplitude obeys SU(2). Our results for Δ_1, Δ_3, and the mixing parameter Δ_{13} are given in Table III. The $P_{1/2}$ corrections are negligible and are not included in the table.[5]

The inelasticity corrections η_{13},

$$\bar{\eta}_1 = \eta^1_H - \eta^1\ , \tag{12a}$$

and

$$\bar{\eta}_3 = \eta^3_H - \eta^3 \tag{12b}$$

come almost entirely from the $n\gamma$ channel. (The bremsstrahlung contributions are negligible as in the $\pi^+ p$ case.[5]) Table IV gives the contributions to the $S_{1/2}$ and $P_{3/2}$ corrections, which are seen to be small except for P_{33}. The $P_{1/2}$ corrections are negligible, of the order 10^{-4}.

THE TOTAL CROSS SECTION

The total nuclear cross section is obtained by extrapolation in the solid angle Ω:

$$\sigma_{\text{tot}} = \lim_{\Omega\to 4\pi}\Big[\sigma(\Omega) - \int_\Omega (|f_C|^2 + |g_C|^2) d\omega - 2\,\mathrm{Re}\int_\Omega (f_C^* f' + g_C^* g') d\omega\Big]\ , \tag{13}$$

TABLE IV. Inelasticity corrections to $\pi^- p$ scattering. All values are to be multiplied by 10^{-4}.

	$S_{1/2}$			$P_{3/2}$		
q/μ	$\bar{\eta}_1$	$\bar{\eta}_3$	η_{13}	$\bar{\eta}_1$	$\bar{\eta}_3$	η_{13}
0.8	17	6	11	1	4	2
1.0	22	8	14	2	14	4
1.2	26	9	16	3	49	10
1.3	27	9	17	3	69	14
1.4	28	10	18	4	89	16
1.5	29	10	18	4	109	19
1.6	30	10	19	5	113	20
1.7	31	11	19	5	105	20
1.8	31	11	20	6	91	20
2.0	32	12	21	7	61	18
2.2	32	12	21	7	39	16
2.5	31	14	22	9	20	13
3.0	26	21	25	9	6	8

with

$$f' = f_{CH} - f_C\,, \quad g' = g_{CH} - g_C\,.$$

The three terms on the right-hand side of (13) are the measured cross section, the Coulomb cross section, and the interference term.

For $\pi^+ p$ scattering we have

$$\frac{q}{4\pi}\sigma_{\text{tot}} = \operatorname{Im}\left(\sum_{l=0}^{\infty}\left[(l+1)(f_N)_{l+} + l(f_N)_{l-}\right]\right), \tag{14}$$

where $(f_N)_{l\pm}$ is the $f_{l\pm}$ given by Eq. (6) but *without* the Coulomb phase factor $\exp(2i\Sigma_{l\pm})$. The $\pi^- p$ total cross section is given by the same expression where now $(f_N)_{l\pm}$ is the $f_{l\pm}$ of Eq. (8) *without* the factor $\exp(-2i\Sigma_{l\pm})$.

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APPENDIX

To arrive at formula (1) for $d\sigma/d\omega$ one has to consider soft-photon emission. If the energy resolution in the experiment is ΔE, one actually measures the bremsstrahlung process

$$\pi + N \to \pi + N + (\text{photons})\,, \tag{A1}$$

where the total energy of the photons is less than ΔE. For sufficiently small ΔE the differential cross section for this process can be written as

$$\frac{d\sigma}{d\omega} = \lim_{\lambda\to 0} h(s,t,\Delta E,\lambda)\left(|f_{CH\lambda}|^2 + |g_{CH\lambda}|^2\right), \tag{A2}$$

where the function $h(s,t,\Delta E,\lambda)$ describes the emission of soft photons of mass λ. This function diverges in the limit $\lambda\to 0$. On the other hand, the amplitudes $f_{CH\lambda}$ and $g_{CH\lambda}$, which describe *elastic* πN scattering in a world of mass-λ photons, are zero in the limit $\lambda\to 0$. The finite amplitudes f_{CH}, g_{CH} of Eqs. (3) are obtained by factoring out the singular λ dependence.

$\pi^+ p \to \pi^+ p$ scattering

We define (cf. Sec. IIC of Ref. 4)

$$f_{CH} = \lim_{\lambda\to 0}\frac{D(s,t)}{|D(s,0)|} f_{CH\lambda}\,, \quad g_{CH} = \lim_{\lambda\to 0}\frac{D(s,t)}{|D(s,0)|} g_{CH\lambda}\,, \tag{A3}$$

where $D(s,t)$ is the function given in Sec. IIIA of Ref. 4. Thus the finite factor Z appearing in Eq. (1) becomes in this case

$$Z = \lim_{\lambda\to 0} h(s,t,\Delta E,\lambda)\left|\frac{D(s,0)}{D(s,t)}\right|^2. \tag{A4}$$

To order α

$$D = e^{-Y}\,, \tag{A5}$$

where

$$Y(s,t) = \sum_{i<j} Y_{ij}(s_{ij})\,. \tag{A6}$$

The sum is over all pairs of the four external particles, and s_{ij} is the square of the c.m. energy in the channel where particles i and j are both incoming or both outgoing (cf. pp. 494–495 and Appendix C of Ref. 2). The functions Y_{ij} depend on the electromagnetic form factors and are given explicitly in Sec. IIIA of Ref. 4. For small λ (cf. Appendix C of Ref. 2),

$$Y_{ij} = \hat{Y}_{ij} + L_{ij}\ln\left(\frac{C_{ij}}{\lambda}\right), \tag{A7}$$

where $\hat{Y}_{ij}$ is independent of λ and goes to a constant as $|s_{ij}|\to\infty$. The function $L_{ij}(s_{ij})$ is given below, and C_{ij} is a constant.

Using the indices s, u, μ, M to indicate the s and u channels and the $\pi^+\pi^-$ and $p\bar{p}$ t channels we may write

$$Y(s,t) = 2Y_s(s) + 2Y_u(u) + Y_\mu(t) + Y_M(t)\,, \tag{A8}$$

where[2]

$$Y_s(s) = -Y_u(s)\,. \tag{A9}$$

In the same notation as above we write

$$L(s,t) \equiv \sum_{i<j} L_{ij} = 2L_s(s) + 2L_u(u) + L_\mu(t) + L_M(t)\,, \tag{A10}$$

where

$$L_s = -\alpha\mathcal{L}\left(\frac{s-s_1}{4M\mu}\right), \quad L_u = \alpha\mathcal{L}\left(\frac{u-s_1}{4M\mu}\right), \quad L_\mu = \alpha\mathcal{L}\left(\frac{t}{4\mu^2}\right), \quad L_M = \alpha\mathcal{L}\left(\frac{t}{4M^2}\right), \tag{A11}$$

with $s_1 = (M-\mu)^2$. The function $\mathcal{L}$ is given by (cf. Ref. 2, p. 490, and Ref. 4, p. 16)

$$\mathcal{L}(z) = \frac{1}{\pi}\left[1 + \frac{z-\frac{1}{2}}{(1-z)^{1/2}(-z)^{1/2}} \times \ln\frac{(1-z)^{1/2}+(-z)^{1/2}}{(1-z)^{1/2}-(-z)^{1/2}}\right]. \tag{A12}$$

The function h in (A2) depends on the details of the experimental arrangement.[8] However, it can in general be approximated by a simple form,[2,9] which for $\pi^+ p \to \pi^+ p$ is

$$h=\left(\frac{\lambda}{2\Delta E}\right)^{2\,\mathrm{Re}L(s,t)} . \quad \text{(A13)}$$

Thus we find

$$Z=\lim_{\lambda\to 0}\exp\left\{2\,\mathrm{Re}\left[Y(s,t)-Y(s,0)+L(s,t)\ln\left(\frac{\lambda}{2\Delta E}\right)\right]\right\}. \quad \text{(A14)}$$

Since $\hat{Y}_\mu$, $\hat{Y}_M$, L_μ, L_M, and ReL are all zero for $t=0$, (A14) can be written as

$$Z=\exp\{2[2V_u(u)-2V_u(u_0)+V_\mu(t)+V_M(t)]\}, \quad \text{(A15)}$$

where

$$V_i(x)\equiv\hat{Y}_i(x)+L_i(x)\ln(C_i/2\Delta E) \quad (i=u,\mu,M), \quad \text{(A16)}$$

and $u_0=2M^2+2\mu^2-s$. The constants C are (in pion-mass units)

$$C_u=2.90, \quad C_\mu=3.13, \quad C_M=2.72, \quad \text{(A17)}$$

and $\hat{Y}_u(u)$ and $\hat{Y}_\mu(t)+\hat{Y}_M(t)$ are given in Table V.

Note that the function Z given by Eq. (78a) in Ref. 2 has an extra factor $[\Gamma(1-2\,\mathrm{Re}L)]^{-1}$. To first order in α, this equation becomes identical with (A14) if in the former we replace ΔE by $0.64\Delta E$. This also applies to Fig. 5 of Ref. 2 which shows $Z(s,t,\Delta E)$ as a function of $\cos\theta$ for $s=80\mu^2$ and $300\mu^2$ for various ΔE.

$\pi^-p\to\pi^-p$ scattering

The finite amplitudes f_{CH}, g_{CH} are still defined by (A3), but with D replaced by D_-, where

$$D_-(s,t,u)=D(u,t,s). \quad \text{(A18)}$$

A similar substitution must be made in the function h in (A13). This implies that (A15) is still valid, if one interchanges the arguments u and u_0.

TABLE V. The functions $\hat{Y}_u$ and $\hat{Y}_\mu+\hat{Y}_M$.

x (μ^2)	$\hat{Y}_u(s_1+x)$	$\hat{Y}_\mu(x)+\hat{Y}_M(x)$
-0	0	0
-1	7.27×10^{-5}	7.06×10^{-4}
-2	1.41×10^{-4}	1.21×10^{-3}
-4	2.65×10^{-4}	1.88×10^{-3}
-6	3.75×10^{-4}	2.31×10^{-3}
-10	5.62×10^{-4}	2.86×10^{-3}
-15	7.48×10^{-4}	3.25×10^{-3}
-25	1.02×10^{-3}	3.67×10^{-3}
-35	1.21×10^{-3}	3.90×10^{-3}
-50	1.40×10^{-3}	4.11×10^{-3}
-90	1.69×10^{-3}	4.39×10^{-3}
-180	1.94×10^{-3}	4.64×10^{-3}
-400	2.11×10^{-3}	4.84×10^{-3}
$-\infty$	2.27×10^{-3}	5.10×10^{-3}

$\pi^-p\to\pi^0n$ scattering

For this case we define

$$f_{CH}=\lim_{\lambda\to 0}\frac{e^{Y_s(s)}}{|D_-(s,0)|^{1/2}}f_{CH\lambda}, \quad \text{(A19)}$$

and similarly for g_{CH}. For the function h we may use the approximation

$$h=\left(\frac{\lambda}{2\Delta E}\right)^{-2\,\mathrm{Re}L_s(s)} . \quad \text{(A20)}$$

The function Z then becomes

$$Z=\exp[2V_u(u_0)], \quad \text{(A21)}$$

where $V_u(u)$ is given in (A16). We note that Z is independent of t. Furthermore, it equals 1 only at threshold.

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