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Radiative corrections to leptonic decays of charged pseudoscalar mesons*

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We show that weak-interaction gauge-theory renormalization corrections are negligible, and that explicit model dependences are small for the ratio $\Gamma(\pi\to e\nu$ or $e\nu\gamma)/\Gamma(\pi\to\mu\nu$ or $\mu\nu\gamma)$. We use the postulate that this continues to hold for other pseudoscalar mesons to justify examining these radiative corrections for π, K, and D in a phenomenological fashion.

I. INTRODUCTION

Since the original measurements[1] of the muonic $(\pi\to\mu\nu)$ and electronic $(\pi\to e\nu)$ decay rates of the pion there has been considerable interest in the radiative corrections to these decays. This interest was for the most part motivated by the relatively high precision of the measurements and by the fact that the ratio of these rates constitutes an important test of muon-electron universality.[2,3] In general, the problem consists of calculating the ratio[4]

$$R_\pi=\frac{\Gamma(\pi\to e\nu)+\Gamma(\pi\to e\nu\gamma)}{\Gamma(\pi\to\mu\nu)+\Gamma(\pi\to\mu\nu\gamma)}\,, \tag{1.1}$$

which can be written as

$$R_\pi=R_\pi^0(1+\Delta_\pi)\,, \tag{1.1'}$$

where

$$R_\pi^0=\frac{\Gamma(\pi\to e\nu)}{\Gamma(\pi\to\mu\nu)}=\left(\frac{m_e}{m_\mu}\right)^2\left(\frac{m_\pi^{\,2}-m_e^{\,2}}{m_\pi^{\,2}-m_\mu^{\,2}}\right)^2 \tag{1.2}$$

is the uncorrected ratio calculated in lowest order.

There are three broad components to the problem of calculating Δ: (1) cancellation of infrared divergences associated with soft photons; (2) removal of ultraviolet divergences associated with higher-order (loop) corrections, i.e., renormalization; and (3) the description of dynamical strong-interaction effects. Using a field-theoretical point pion, Berman[2] and Kinoshita[3] treated the first component by means of electromagnetic gauge invariance and this continues to be the standard approach. They ignored the third component and solved the second by making the reasonable ansatz that the field-theory ultraviolet cutoff is the same for electron and muon.

Their result, which is frequently quoted in the literature, is most easily expressed as $\Delta_\pi=\Delta_\pi^{\rm IB}+\Delta_\pi^{\rm loop}$, where

$$\begin{aligned}\Delta_\pi^{\rm IB}=\frac{\alpha}{\pi}\Bigg\{&2\left[\left(\frac{1+\mu_e^{\,2}}{1-\mu_e^{\,2}}\right)\ln\mu_e+1\right]\left[\ln\frac{\lambda}{m_\pi}-\ln(1-\mu_e^{\,2})-\tfrac{1}{2}\ln\mu_e+\tfrac{3}{4}\right]\\ &-\frac{\mu_e^{\,2}(10-7\mu_e^{\,2})}{2(1-\mu_e^{\,2})^2}\ln\mu_e+\frac{2(1+\mu_e^{\,2})}{(1-\mu_e^{\,2})}L(1-\mu_e^{\,2})+\frac{15-21\mu_e^{\,2}}{8(1-\mu_e^{\,2})}\Bigg\}\\ &-(\text{same with }\mu_e\to\mu_\mu)\,,\end{aligned} \tag{1.3}$$

with

$$\mu_e=m_e/m_\pi\,,\quad \mu_\mu=m_\mu/m_\pi\,,\quad\text{and}\quad L(x)=\int_0^x\left(\frac{dt}{t}\right)\ln(1-t)\,.$$

IB refers to inner bremsstrahlung—the real photon emission prescribed by electromagnetic gauge invariance; λ is a small photon mass used to handle the infrared divergences. In addition, there are loop corrections due to higher-order graphs:

$$\begin{aligned}\Delta_\pi^{\rm loop}=\frac{\alpha}{\pi}&\left\{-2\left[\left(\frac{1+\mu_e^{\,2}}{1-\mu_e^{\,2}}\right)\ln\mu_e+1\right]\left(\ln\frac{\lambda}{m_\pi}-\tfrac{1}{2}\ln\mu_e+\tfrac{3}{4}\right)+\frac{\mu_e^{\,2}}{1-\mu_e^{\,2}}\ln\mu_e+\tfrac{1}{2}\right\}\\ &-(\text{same with }\mu_e\to\mu_\mu)+\frac{3\alpha}{\pi}\ln\left(\frac{m_e}{m_\mu}\right)+\frac{3\alpha}{2\pi}\ln\left(\frac{\Lambda_\mu}{\Lambda_e}\right),\end{aligned} \tag{1.4}$$

where Λ_e and Λ_μ are the ultraviolet cutoffs for electron and muon, respectively. Under the assumption that $\Lambda_e = \Lambda_\mu$, Kinoshita concluded that

$$R_\pi = R_\pi^0 \left\{ 1 + \frac{\alpha}{\pi} [-0.92 + 3\ln(m_e/m_\mu)] \right\}$$
$$= 1.233 \times 10^{-4} \qquad (1.5)$$

to be compared with

$$R_\pi^0 = 1.2835 \times 10^{-4} . \qquad (1.6)$$

Since this pioneering work, most efforts have been directed at the third component—by means of current-algebra techniques,[5] phenomenological Lagrangians, or specific structural models of the pion.[6] In these approaches the second component is ignored. (The first is, as always, handled by gauge invariance in the manner of Berman and Kinoshita.) The principal effect of the strong interactions is to introduce additional (gauge-invariant) terms in the real-photon-emission amplitude. These terms are generally called structure-dependent (SD) terms.

More recently, we have reported[7] that in a gauge theory of weak and electromagnetic interactions with a point-pion field the second (renormalization) component can be treated unambiguously. The result agrees with the ansatz of Berman and Kinoshita up to corrections less than or the order of $G_F m_\pi^2$. Thus, although nonzero, the contributions to Δ_π associated with subtleties in renormalization are experimentally negligible. In another paper,[8] we handled all three components within a field-theory framework, again using a gauge theory of the weak interactions and the static quark model[9] (SQM) for the pions. There it was very important to properly include the effects of strong-interaction dynamics as the SD corrections are potentially large. In the end, however, the value calculated for R_π was very close to that of Berman[2] and Kinoshita.[3]

Since that time Marciano and Sirlin[10] have shown that the largest contribution to Δ_π $[(3\alpha/\pi)\ln(m_e/m_\mu)]$ is essentially fixed by gauge invariance and that most strong-interaction dynamical effects cancel out in the total rates (although not in the lepton spectrum). Only the pure SD contributions are not handled by their argument, but these seem to be small in the pion case.[11]

We conclude from all of the above that reliable estimates for R_π can be arrived at more simply by using phenomenological-Lagrangian techniques. The plan of attack is as follows: (1) Calculate the sum of the IB corrections and the loop corrections. By the argument of Marciano and Sirlin this will always give a contribution to Δ_π equal to $(3\alpha/\pi)\ln(m_e/m_\mu)$ plus small corrections. By our argument, renormalization effects can only introduce corrections to this of order $G_F m_\pi^2$. (2) Calculate the pure SD part using a phenomenological Lagrangian in the tree (no loop) approximation. The various coupling constants and mass parameters will provide the essential information on the effects of strong-interaction dynamics, thus taking care of the third component.

This last method of calculating may seem superfluous since Δ_π has already been calculated in some detail. However, the phenomenological-Lagrangian method may prove indispensable in calculating radiative corrections to kaon decay (or charmed-meson decay) where suitable gauge-theory models have not yet been constructed. It also has the virtues of ease and simplicity.

The plan of this paper is as follows: In Sec. II, we present our point-pion model in expanded detail. In Sec. III we discuss the static quark model for the pion with a gauge weak interaction, also in greater detail than previously. In Sec. IV we calculate R_π in the phenomenological Lagrangian approach described above—very easily reproducing the results of Secs. II and III. In addition, we calculate the analogous result for kaon decay. The (rather interesting) kaon results are improved over previous estimates[5] in two ways: The results of Secs. II and III put the calculations on a firmer theoretical footing, and we use more recent data to determine our parameters. We also make a few remarks about the application of this phenomenological-Lagrangian technique to the purely leptonic decays of charmed mesons.[12] In Sec. V, we summarize and present our conclusions.

II. GAUGE-THEORY MODEL

As we pointed out in the previous section the original work on $\pi \to l\nu\gamma$ by Berman and Kinoshita suffered from two shortcomings. First, the renormalization procedure is not clearly defined; and second, strong-interaction corrections are ignored. In this section we address the first problem, and treat the second in subsequent sections. The most natural way to attack the renormalization problem is within the context of a spontaneously broken gauge theory of weak and electromagnetic interactions such as that of Weinberg and Salam[13] (WS). The advantages of such a theory are that μ-e universality appears naturally as a lowest-order symmetry and the renormalization procedure is well defined. Our only problem is to include the pion field in the WS Lagrangian.

Including the pions in the WS Lagrangian is actually quite simple. Our notation and methods will be quite similar to those used by Appelquist, Primack, and Quinn (APQ) in treating W decay.[14] As in APQ all our calculations will be done in the U-gauge formalism. Our Lagrangian is just

$$\mathcal{L}=-\tfrac{1}{4}(\partial_\mu\vec{A}_\nu-\partial_\nu\vec{A}_\mu+g\vec{A}_\mu\times\vec{A}_\nu)^2-\tfrac{1}{4}(\partial_\mu B_\nu-\partial_\nu B_\mu)^2+\sum_l \bar{L}_l(i\not\partial+\tfrac{1}{2}g\vec{\tau}\cdot\vec{\not A}+\tfrac{1}{2}g'\not B)L_l+\sum_l \bar{R}_l(i\not\partial+g'\not B)R_l$$

$$+\tfrac{1}{2}\left|(\partial_\mu+\tfrac{1}{2}ig\vec{\tau}\cdot\vec{A}_\mu-\tfrac{1}{2}ig'B_\mu)\Phi\right|^2-\sum_l G_l(\bar{L}_l\Phi R_l+\bar{R}_l\Phi^\dagger L_l)+\tfrac{1}{2}\left|(\partial_\mu+\tfrac{1}{2}ig\vec{\tau}\cdot\vec{A}_\mu-\tfrac{1}{2}ig'B_\mu)\Sigma\right|^2-V(\Sigma,\Phi)\,, \tag{2.1}$$

where

$$L_l=\frac{1-\gamma_5}{2}\binom{\nu_l}{l^-}\,,\quad R_l=\frac{1+\gamma_5}{2}l^-\,.$$

The first six terms of this expression are just the standard Lagrangian of the Weinberg-Salam $SU(2)\times U(1)$ model,[13] where the Φ field is defined to be

$$\Phi=\binom{i\sqrt{2}\eta^+}{\varphi-i\eta^0}\,,$$

with φ and η^0 real and η^+ complex. The field Σ is defined to be

$$\Sigma=\binom{i\sqrt{2}\eta^+}{\sigma-i\tau^0}\,,$$

with σ and τ^0 real and τ^+ complex. The τ's will eventually be associated with the π's and the σ is an additional scalar field. $V(\Sigma,\Phi)$ is the "potential" in our model. It includes all possible Σ-Φ interactions which are $SU(2)\times U(1)$ symmetric and at most quartic in the fields. We write it as

$$\begin{aligned}V(\Sigma,\Phi)=&-A(\Sigma^\dagger\Sigma)+B(\Sigma^\dagger\Sigma)^2-C(\Phi^\dagger\Phi)+D(\Phi^\dagger\Phi)\\&+E(\Sigma^\dagger\Sigma)(\Phi^\dagger\Phi)\\&-F\left(\frac{\Sigma^\dagger\Phi+\Phi^\dagger\Sigma}{2}\right)^2+H\left(\frac{\Sigma^\dagger\Phi-\Phi^\dagger\Sigma}{2}\right)^2.\end{aligned} \tag{2.2}$$

Notice that we have eliminated a possible $\Sigma^\dagger\Phi+\Phi^\dagger\Sigma$ term by imposing an additional symmetry: $\Sigma\to-\Sigma$. To insure stability we require B, $D>0$ and $E-F>-2\sqrt{BD}$.

We now find the vacuum expectation values of σ and φ by minimizing V. The minimum occurs at

$$\begin{aligned}\varphi_0&=\left[\frac{2BC-A(E-F)}{4BD-(E-F)^2}\right]^{1/2}=\lambda\\ \sigma_0&=\left[\frac{2AD-C(E-F)}{4BD-(E-F)^2}\right]^{1/2}=\delta\,.\end{aligned} \tag{2.3}$$

The φ and σ fields must be shifted by these amounts. The shifted Lagrangian contains, among other things, η-τ mass-mixing terms which can be eliminated by rotation in η-τ space. The fields with definite masses are just

$$\begin{aligned}\eta'&=\eta\cos\gamma+\tau\sin\gamma\,,\\ \pi&=-\eta\sin\gamma+\tau\cos\gamma\,,\end{aligned} \tag{2.4}$$

with $\tan\gamma=\delta/\lambda$. The masses of these new states are then

$$\begin{aligned}m_{\eta'}{}^2&=0\,,\\ m_{\pi^\pm}{}^2&=2F(\lambda^2+\delta^2)\,,\\ m_{\pi^0}{}^2&=2(F+H)(\lambda^2+\delta^2)\,.\end{aligned} \tag{2.5}$$

The factor H just represents an isospin breaking in the pion mass spectrum, so that $H\ll F$.

In the U-gauge formalism the massless η' fields are absorbed into the definitions of the massive vectors W and Z and do not appear explicitly in the Lagrangian. The photon (A) and the neutral intermediate vector boson (Z) are identified as the mass eigenstates

$$\begin{aligned}A_\mu&=(g^2+g'^2)^{-1/2}(gB_\mu-g'A_\mu^3)\,,\\ Z_\mu&=(g^2+g'^2)^{-1/2}(g'B_\mu+gA_\mu^3)\,.\end{aligned}$$

The charged intermediate vector boson is, of course, $W_\mu^\pm=(A_\mu^1\pm iA_\mu^2)/\sqrt{2}$; and $G_F/\sqrt{2}=g^2/8M_W{}^2$ provides the usual connection to the Fermi theory. The Lagrangian becomes

$$\begin{aligned}\mathcal{L}=&\text{kinetic terms}-F(\lambda^2+\delta^2)\vec{\pi}^2-H(\lambda^2+\delta^2)(\pi^0)^2-4B\delta^2\sigma^2-4(E-F)\delta\lambda\sigma\varphi-4D\lambda^2\varphi^2+\tfrac{1}{4}g^2(\delta^2+\lambda^2)|W_\mu|^2\\&+\frac{1}{8}\frac{g^2}{\cos^2\theta}(\delta^2+\lambda^2)Z_\mu{}^2-\sum_l G_l\lambda\bar{l}l-\sum_l G_l\varphi\bar{l}l+\sum_l\frac{iG_l\sin\gamma}{\sqrt{2}}\{\bar{\nu}_l(1+\gamma_5)l\pi^+-\sqrt{2}\,\bar{l}\gamma_5 l\pi^0-\bar{l}(1-\gamma_5)\nu_l\pi^-\}\\&+\tfrac{1}{2}(g^2+g'^2)^{1/2}\{\pi^+ i\overleftrightarrow{\partial}_\mu\pi^-[\cos(2\theta)Z^\mu-\sin(2\theta)A^\mu]+\pi^0 i\overleftrightarrow{\partial}_\mu\pi^+W^{-\mu}\cos\theta+\pi^- i\overleftrightarrow{\partial}_\mu\pi^0W^{+\mu}\cos\theta\}\\&+\text{ many additional interaction terms}\,,\end{aligned} \tag{2.6}$$

where we have introduced the Weinberg angle,[13] $\tan\theta = g'/g$. Notice that $\frac{1}{2}(g^2+g'^2)^{1/2}\sin(2\theta)=e$ so that the charged pion has the correct coupling to the photon. Among the additional terms are the usual weak and electromagnetic interaction of the leptons and a multitude of couplings involving the $\vec{\pi}$, σ, and φ fields. However, there are no additional pion-vector couplings like $\partial_\mu \pi V^\mu$ or $\pi V_\mu V^\mu$. Notice that there remains in the Lagrangian a σ-φ mixing term proportional to $E-F$. In order to simplify our calculations we will ignore the σ-φ mixing induced by this term, which is equivalent to setting $E=F$. This assumption will not alter the validity of our results. We can now read off the masses of the remaining particles directly from the Lagrangian:

$$\begin{aligned} M_W^2 &= \tfrac{1}{4} g^2(\lambda^2+\delta^2) \\ &= \cos^2\theta\, M_Z^2\,, \\ M_l &= \lambda G_l\,, \\ m_\sigma^2 &= 8B\delta^2\,, \\ m_\varphi^2 &= 8D\lambda^2\,. \end{aligned} \tag{2.7}$$

The key term in our Lagrangian is the first term in braces: The pion has a direct, nonderivative coupling to the leptons of strength $G_l \sin\gamma/\sqrt{2}$. (The $\pi^0 \to e^+e^-$ decay generated by the middle term there causes no problem since it is a weak decay. The rate calculated for $\pi^0\to e^+e^-$ is orders of magnitude below even the $\pi^0 \to e^+e^-\gamma$ rate.) By equating this with the known strength of the pion-lepton coupling, $G_F m_l f_\pi/\sqrt{2}$, we find that

$$\tan^2\gamma = G_F f_\pi^2/\sqrt{2} \tag{2.8}$$

so that the angle γ and, hence, the η-τ mixing are very small indeed.

We now have our Lagrangian nearly into its final form. All that remains is to replace the unrenormalized fields, coupling constants, and masses in Eq. (2.6) by properly renormalized ones. We henceforth denote all unrenormalized quantities by a subscript zero and all renormalized quantities will have no subscript zero. [Thus, all the quantities in Eq. (2.6) should be understood to have zero subscripts.] We then proceed to define the renormalization constants

$$\begin{aligned} \vec{\pi}_0 &= \sqrt{Z_\pi}\,\vec{\pi}\,, \\ \sigma_0 &= \sqrt{Z_\sigma}\,\sigma\,, \\ \sin\gamma_0 &= \frac{Z_s}{Z_{G_\mu}}\left(\frac{Z_\lambda Z_{\mu_L}}{Z_\pi Z_{\nu_\mu}}\right)^{1/2}\sin\gamma\,; \end{aligned} \tag{2.9}$$

the remaining coupling constants are defined as in Table I of APQ.[14] Notice that there is not a separate renormalization for δ since it is defined in terms of λ and $\sin\gamma$ which already have renormalization constants.

With these definitions we find that the renormalization counterterms for the $\pi\mu\nu$ and $\pi e\nu$ couplings are

$$(Z_s-1)\sin\gamma\, G_\mu \pi^+ \bar{\nu}_\mu(1+\gamma_5)\mu^- \tag{2.10}$$

and

$$\left[Z_s\left(\frac{Z_{G_e}{}^2 Z_{\nu_e} Z_{\mu_L}}{Z_{G_\mu}{}^2 Z_{\nu_\mu} Z_{e_L}}\right)^{1/2} - 1\right]\sin\gamma\, G_e \pi^+\bar{\nu}_e(1+\gamma_5)e^-\,, \tag{2.11}$$

where Z_{ν_e}, Z_{e_L}, Z_{ν_μ}, and Z_{μ_L} are the wave-function renormalization constant for the electron neutrino, the left-handed electron, the muon neutrino, and the left-handed muon. (The right- and left-handed leptons are separately renormalized since the weak interactions do not conserve parity.) Z_{G_e} and Z_{G_μ} are essentially mass-renormalization constants for the electron and muon. These expressions are really the key results of this section. The electronic and muonic vertices cannot be separately renormalized. The two counterterms are explicitly related by calculable factors. We now proceed to determine what that relation is.

We will calculate corrections to the one-loop level. We need only consider those graphs which contain a lepton propagator within a loop since only these can contribute to a difference between μ and e decay.

In Fig. 1 we list the lowest-order graph and all

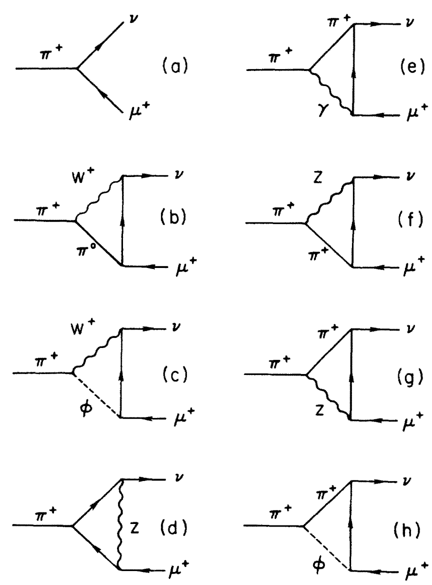

FIG. 1. Some low-order graphs for pion decay in the point-pion model: (a) lowest-order and (b)–(h) next-higher-order one-loop graphs, contributing to $\pi\mu\nu$ vertex renormalization (Z_s).

relevant one-loop graphs contributing to Z_s. (If we had allowed σ, φ mixing there would be additional graphs with σ replacing φ. In that case the sum of the σ and φ graphs would give the same infinities as the φ graphs alone do without mixing.) Doing the integrals, we find three types of ultraviolet divergences: (1) quadratic divergences which are independent of the lepton mass, (2) logarithmic divergences which are independent of the lepton mass, and (3) logarithmic divergences which are preceded by a factor of m_l^2/M_W^2. Thus, we can write each renormalization constant in the form

$$Z_i^l - 1 = A_i \Gamma\left(1-\frac{n}{2}\right) + B_i \Gamma\left(2-\frac{n}{2}\right) + \frac{m_l^2}{M_W^2} C_i \Gamma\left(2-\frac{n}{2}\right)$$
$$+ \text{ultraviolet-finite terms},$$

where we have written the divergences as Γ functions (to be evaluated at $n=4$) to remind the reader that a simple and proper way to calculate is to use dimensional regularization.[15] The A_i, B_i, and C_i are independent of the particular lepton involved. Thus, the only constants that concern us are the C_i's. A necessary and sufficient condition that the counterterm of Eq. (2.11) will render $\pi \to e\nu$ finite is that

$$C_s = C_{G_\mu} + \tfrac{1}{2} C_{\nu_\mu} - \tfrac{1}{2} C_{\mu_L} . \tag{2.12}$$

We have explicitly calculated these C_i's and find that

$$\begin{aligned} C_s &= \frac{g^2}{16\pi^2}\left(\tfrac{3}{4} - \tfrac{1}{4}\cos^2\theta\right), \\ C_{G_\mu} &= \frac{g^2}{16\pi^2}\left(-\frac{1}{8}\right), \\ C_{\nu_\mu} &= \frac{g^2}{16\pi^2}\left(\tfrac{3}{4} - \tfrac{1}{4}\tan^2\gamma\right), \\ C_{\mu_L} &= \frac{g^2}{16\pi^2}\left(-1 + \tfrac{1}{2}\cos^2\theta - \tfrac{1}{4}\tan^2\gamma\right). \end{aligned} \tag{2.13}$$

In Fig. 2 we show explicitly the graphs which contribute to Z_{μ_L}, Z_{G_μ}, and Z_{ν_μ}. It is a trivial exercise to see that Eqs. (2.13) satisfy relation (2.12), i.e., the renormalization procedure works. Having seen that there are no ultraviolet divergence problems we can now find the breakdown of μ-e universality in the remaining ultraviolet-finite pieces.

We can now divide the graphs into two classes—those with virtual photons [such as Fig. 1(e)] and those without. Those without photons can be separated into a part independent of the lepton mass and a part proportional to $G_F m_l^2$. They make a negligible contribution to universality breakdown. Aside from these small corrections the entire effect comes from the photon graphs. However,

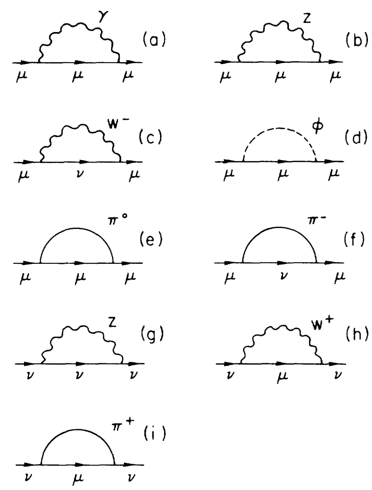

FIG. 2. Lowest-order graphs which contribute to muon [(a)–(f)] and muon-neutrino [(g)–(i)] wave-function renormalization (Z_{μ_L}, Z_{ν_μ}) and to muon-scalar coupling-constant renormalization (Z_{G_μ}), by means of the relation to muon mass renormalization.

these are exactly the same graphs considered in the original calculations of Berman[2] and Kinoshita.[3] To be more explicit, the graphs they considered are those of Figs. 1(e), and 2(a), and the additional graph of Fig. 3. When these loop graphs are combined with the appropriate real photon-emission graphs (to handle infrared problems) a finite result is achieved—that of Berman and Kinoshita.

Having expended a lot of energy to recreate an old result, we are led to ask what we have gained. What we have gained is that we have replaced the intuitively appealing ansatz that the electronic and muonic cutoffs are the same by an explicit calculation that essentially says they *must* be the same (with only very small finite corrections). Of course, we have only established this result in one particular model of pion decay. However, it would seem reasonable at this point to extrapolate from our model and postulate that it will also be true in *any* other renormalizable theory of pion decay. It is difficult to see how corrections bigger than $O(G_F m_l^2)$ could appear in any sensible model of the weak interactions.

Along with Berman and Kinoshita we have still

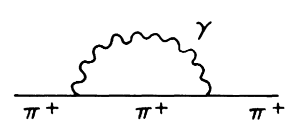

FIG. 3. The lowest-order one-loop pion-propagator graph which contributes to the cancellation of infrared divergences in radiatively corrected pion decay.

failed to take into account possible effects due to strong-interaction dynamics. It is useless to expand in more loops since the strong-coupling constants are presumably large. In any case, a model with only pions cannot hope to give a realistic picture of the strong interactions. In the next section we will attempt to fill this void by considering a model of the pion as a quark-antiquark composite particle.

III. THE STATIC QUARK MODEL

The static quark model[9] (SQM) is a very simplistic model which has had unexpectedly wide success in describing static quantities and has recently been applied, with some success, to dynamical properties of hadrons.[16] In the SQM, a pion in its rest frame consists of a quark and an antiquark, each of mass one-half the pion mass (m_π) and at rest. The binding energy is zero, and the momentum-space wave function is a δ function. Our motivation for using this model is twofold: (1) Because the quarks are on-shell, we can unambiguously use an on-shell amplitude for quark-antiquark scattering into a lepton pair, or into a lepton pair plus a photon. In a gauge theory of weak and electromagnetic interactions such an amplitude is well defined and, in addition, has no net ultraviolet divergence,[17] so that no problem arises from the second component described in the Introduction. (2) The wave function of the pion is spread over all of configuration space; this is an extremely soft, extended model of the pion. It is difficult to imagine a model more different from the point model of the previous section.

Considering the radically different pictures of the pion adopted in the two models, we might expect the value of R_π obtained from the SQM to be maximally different from the point model—provided R_π is monotonic in the pion size (or binding strength). As we shall see, the difference is quite small, in agreement with the theorem of Marciano and Sirlin,[10] once the strong interactions are correctly handled. The major problem with the SQM is that $m_\rho = m_\pi$—no spin-spin strong couplings are included. [The SQM is a very simplistic realization of SU(6) symmetry.] This (bad) approximation leads to an exceptionally large pure SD correction, which we correct by changing the effective mass in a propagator in the pure SD amplitude. The result is then similar to what appears in a more natural way in the analysis of Sec. IV.

We use the same Lagrangian as in Sec II for the leptons and the weak and electromagnetic interactions [Eq. (2.1)], but delete all terms containing the scalar Σ field (which provided the point-pion field). Instead, we introduce a quark doublet (u,d) as follows:

$$\begin{aligned}\mathcal{L}_q &= \overline{L}_q\left(i\partial\!\!\!/ + \frac{g}{2}\vec{\tau}\cdot\vec{A\!\!\!/} - \frac{g'}{6}B\!\!\!/\right)L_q \\ &\quad + \overline{R}_u\left(i\partial\!\!\!/ - \frac{2g'}{3}B\!\!\!/\right)R_u + \overline{R}_d\left(i\partial\!\!\!/ + \frac{g'}{3}B\!\!\!/\right)R_d \\ &\quad - G_u(\overline{L}_q\Phi R_u + \text{H.c.}) - G_d(\overline{L}_q\tilde{\Phi}R_d + \text{H.c.})\,, \end{aligned}\tag{3.1}$$

where

$$L_q = \frac{1-\gamma_5}{2}\begin{pmatrix} u \\ d\end{pmatrix},\quad R_u = \frac{1+\gamma_5}{2}u\,,$$

$$R_d = \frac{1+\gamma_5}{2}d\,,$$

and

$$\tilde{\Phi} = e^{-i(\pi/2)\tau_2}(\Phi^*)\,.$$

Note that we have set the Cabibbo angle equal to zero, as it is irrelevant here. After shifting the Φ field by its vacuum expectation value, φ_0, we again find

$$G_F/\sqrt{2} = g^2/8M_W^2,\quad e = gg'/(g^2+g'^2)^{1/2}$$

and, in addition,

$$m_u = G_u\varphi_0,\quad m_d = G_d\varphi_0\,,$$

$$Q_u = \tfrac{2}{3},\quad Q_d = -\tfrac{1}{3}\,.$$

Neglecting isospin breaking we choose $G_u = G_d$, so that $m_u = m_d = \frac{1}{2}m_\pi$ to conform to the SQM picture.

We are now ready to begin calculating. Before we do, however, we should recall that we are not interested in the entire amplitude for quark-antiquark scattering. We only need the part corresponding to our SQM pion. For an amplitude of the form $\bar{u}\mathcal{O}d$, we can "project" out the pion by taking

$$\mathfrak{C}\,\mathrm{tr}\sum \bar{d}\gamma_5 u\bar{u}\mathcal{O}d\,,$$

where the sum is over all u and d polarizations, and $\mathfrak{C}$ is a normalization factor. It is convenient to do this projection before any other manipulation of an amplitude, to avoid complications and extraneous information on channels other than $J^P = 0^-$. Thus, the lowest-order amplitude, as

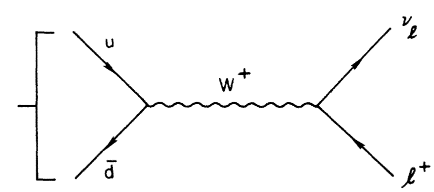

FIG. 4. Lowest-order graph for π^+ decay in the SQM. The square bracket at left represents here, and in the subsequent figures, the projection into the $J^P = 0^-$ sector which is described in the text.

shown in Fig. 4, is just

$$\mathfrak{M}_0(\pi \to l\nu) = \frac{G_F}{\sqrt{2}} m_l (2m_\pi \mathfrak{C}) \bar{l}(1-\gamma_5)\nu \tag{3.2}$$

so that

$$\mathfrak{C} = f_\pi / 2m_\pi \tag{3.3}$$

and f_π is the usual pion-decay constant.

As in Sec. II, the radiative corrections (to lowest nontrivial order) may be divided into two classes: (1) those where a real photon appears in the final state, as shown in Fig. 5, and (2) loop graphs of the types shown in Fig. 6.

When the real-photon-emission graphs are added together, they yield the following amplitude:

$$\mathfrak{M}(\pi \to l\nu\gamma) = \frac{G_F}{\sqrt{2}} f_\pi \left\{ \left[\left(\frac{\epsilon_\gamma \cdot P_\pi}{P_\gamma \cdot P_\pi} - \frac{\epsilon_\gamma \cdot P_l}{P_\gamma \cdot P_l} \right) m_l \bar{l}(1-\gamma_5)\nu - \frac{m_l}{2P_\gamma \cdot P_l} \bar{l} \not{\epsilon}_\gamma \not{P}_\gamma (1-\gamma_5)\nu \right] + \left(\frac{2}{3} + \left(-\frac{1}{3} \right) \right) \frac{2i\epsilon_{\mu\nu\alpha\beta} \epsilon_\gamma^\nu P_\gamma^\alpha P_\pi^\beta}{2P_\gamma \cdot P_\pi} \bar{l} \gamma_\mu (1-\gamma_5)\nu \right\}, \tag{3.4}$$

where P_l is the charged-lepton momentum, P_π is the pion momentum, and ϵ_γ and P_γ are the photon polarization and momentum, respectively. In the last term we have made the approximation $M_W^2 - (P_\pi - P_\gamma)^2 \simeq M_W^2$. The amplitude (3.4) is the sum of two parts. First, there is the IB part (enclosed in square brackets) which is equal to that found by Berman and Kinoshita; it is essentially fixed by electromagnetic gauge invariance and includes an infrared divergence. It gives a contribution to Δ_π given by Eq. (1.3). The remainder of the amplitude is an infrared-finite SD part, which was absent in the point-pion calculations.

As we will see in Sec. IV, this is very similar in structure to what is obtained by calculating the amplitude for the virtual decay $\pi^+ \to \rho^+\gamma$ followed by the weak decay of the off-shell ρ^+: $\rho^+ \to l^+\nu_l$. The difference lies in the denominator, which is essentially the quark propagator in Figs. 5(a) and 5(b). In the phenomenological-Lagrangian approach of Sec. IV the corresponding propagator denominator is $m_\rho^2 - (P_\pi - P_\gamma)^2 = m_\rho^2 - m_\pi^2 + 2P_\pi \cdot P_\gamma$. Note that the two denominators are the same if $m_\rho = m_\pi$, as is true in the SQM. The SQM, by ignoring spin-spin interactions, fails to recognize that when the emitted photon flips the quark spin, the resulting ρ-like state is far off-shell.

This is a serious defect in the applicability of the SQM to this process, for these SD terms do not have the standard lepton-mass factor m_l, the

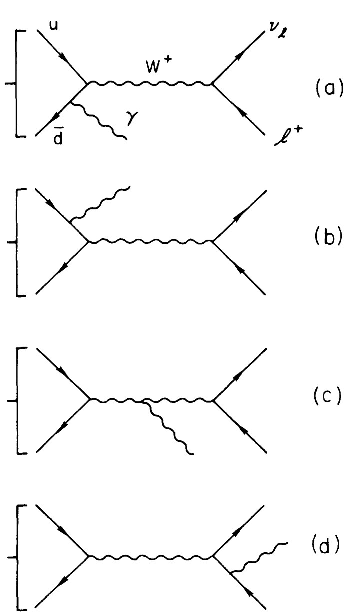

FIG. 5. Real-photon-emission graphs for π^+ decay in the SQM.

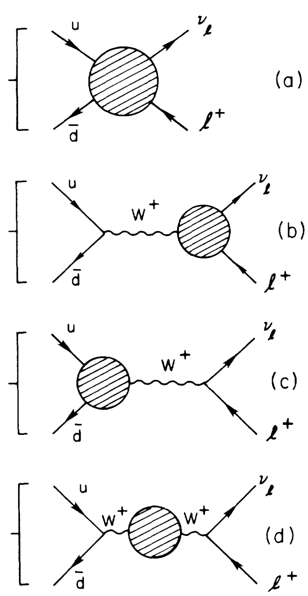

FIG. 6. Schematic of higher-order corrections to π^+ decay in the SQM. The shaded blobs are 1PI diagrams including (a) "box" graphs, (b) lepton-W-boson vertex corrections, (c) quark-W-boson vertex corrections, and (d) W-boson propagator corrections.

factor which suppresses $\pi \to e\nu$ relative to $\pi \to \mu\nu$. This factor is absent since the photon emission leaves a spin-1 intermediate hadronic state. The usual helicity argument for the spin-0 pseudoscalar case is avoided; the weak interaction is able to proceed with full strength to produce a lepton and its (anti-) neutrino with opposite helicity. Thus, this $O(\alpha)$ SD contribution is enhanced relative to the lowest-order (no photon emitted) decay by a factor of $(m_\pi/m_l)^2$ in the rate. Explicit calculations show that this supposedly $O(\alpha)$ contribution to R_π is actually $O(1)$ if $m_\pi = m_\rho$ [i.e., using Eq. (3.4) uncorrected]. Thus, we find it necessary to modify the SQM, in this SD part only, by the replacement[18] $2P_\pi\cdot P_\gamma \to 2P_\pi\cdot P_\gamma + M^2$, where we expect $M^2 \simeq m_\rho^2 - m_\pi^2$.

This more or less *ad hoc* procedure can be partially tested by using the SQM to calculate the decay rate for $\pi_0 \to \gamma\gamma$. The decay amplitude for this decay is quite similar to the SD part of $\pi^+ \to l^+\nu\gamma$ [they are related by CVC (conservation of vector current)]. An adequate fit to the data[19] can be made by retaining the M^2 correction of the last paragraph and setting $M \simeq 685$ MeV.[20] (Notice that $\{[M^2 - (m_\rho^2 - m_\pi^2)]/m_\rho^2\} \simeq 18\%$.) An additional test can be found in the mass distribution of the e^+e^- in the decay $\pi^0 \to e^+e^-\gamma$. To the extent that this shows no variation beyond that expected from kinematics[21] it can be concluded that the mass parameter of a form-factor correction (which is the net effect of the $2P_\pi\cdot P_\gamma + M^2$ denominator) must be greater than a few hundred MeV—which is consistent with the value of M above.

Returning to our expression for the real-photon decay amplitude, Eq. (3.4), we find that it should now read

$$\mathfrak{M}' \simeq \frac{G_F}{\sqrt{2}} f_\pi \left\{ \left[\left(\frac{\epsilon_\gamma\cdot P_\pi}{P_\gamma\cdot P_\pi} - \frac{\epsilon_\gamma\cdot P_l}{P_\gamma\cdot P_l} \right) m_l \bar{l}(1-\gamma_5)\nu - \frac{m_l}{2P_\gamma\cdot P_l}\bar{l}\,\not\epsilon_\gamma \not P_\gamma(1-\gamma_5)\nu \right] + \frac{2i}{3M^2}\epsilon_{\mu\nu\alpha\beta}\epsilon_\gamma^\nu P_\gamma^\alpha P_\pi^\beta \bar{l}\gamma_\mu(1-\gamma_5)\nu \right\}. \qquad (3.4')$$

Squaring this amplitude and integrating over phase space, we find three contributions to the $\pi \to l\nu\gamma$ rate. First, there is the pure IB contribution as previously calculated. This contains infrared divergences which will be cancelled by infrared divergences in loop graphs. Next, there is interference between the IB and SD parts. This makes a negligible contribution to the rate since it is again proportional to m_l^2. Finally, there is a pure SD term given by[22]

$$\Gamma^{SD}(\pi^+ \to e^+\nu\gamma) = G_F^2 \frac{\alpha m_\pi^7}{1920\pi^2}|v_\pi|^2, \qquad (3.5)$$

where

$$v_\pi = \frac{2}{3}\frac{f_\pi}{M^2}.$$

In Eq. (3.5) we have neglected $O(m_e^2/m_\pi^2)$ corrections. We do not quote the form of $\Gamma^{SD}(\pi \to \mu\nu\gamma)$ since this makes only a negligible contribution to the muonic decay modes [$(m_\pi/m_e)^2$ is large, but $(m_\pi/m_\mu)^2$ is not].

Turning now to the loop graphs, we note that these are of four classes: (1) "box" graphs as in Fig. 6(a), (2) lepton-vertex corrections as in Fig. 6(b), (3) quark-vertex corrections as in Fig. 6(c), and (4) W-boson-propagator corrections as in Fig. 6(d). This last class can produce no detectable correction since it is symmetric between the electron and muon cases and contains no infrared divergences. (Since the W is off-shell there is an effective infrared cutoff.) The quark-vertex corrections (partially displayed in Fig. 7) are also symmetric between electronic and muon decay; and so only the parts that contribute to infrared-divergence cancellation are important. In fact, these infrared vertex corrections need not be calculated in detail because such infrared divergences cannot be sensitive to the pion structure—they must add up to the same correction as that obtained in the point-pion case, i.e., the infrared part of Fig. 3, which is just

$$\mathfrak{M}_{e2}^q = -\frac{\alpha}{2\pi}\left(\ln\frac{\lambda}{m_\pi}\right)\mathfrak{M}_0, \qquad (3.6)$$

where λ is a small photon mass, and $\mathfrak{M}_0$ is the zeroth-order amplitude given by Eq. (3.2).

Next we consider the lepton-vertex corrections (detailed in Fig. 8). Because of the zeroth-order symmetry between the muon and electron in their

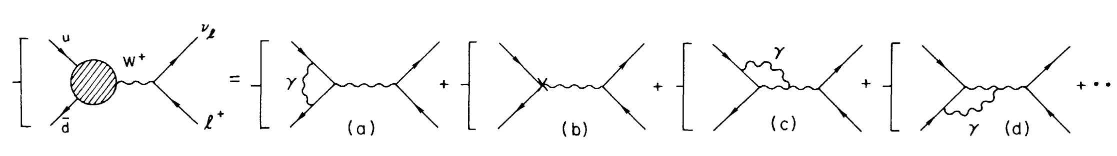

FIG. 7. Expansion of quark–W-boson vertex correction showing some lowest-order one-loop graphs explicitly.

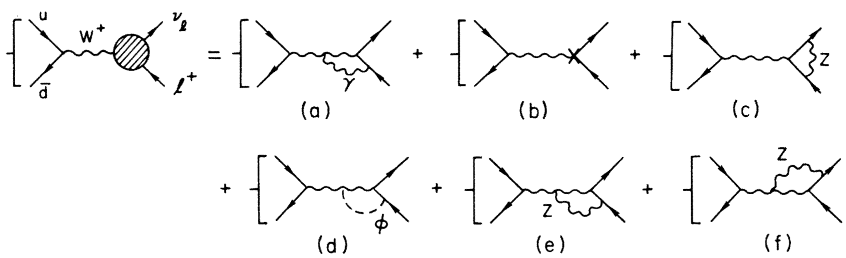

FIG. 8. Expansion of lepton–W-boson vertex correction showing lowest-order one-loop graphs explicitly.

gauge coupling to the W boson, the sum of the ultraviolet divergences must be essentially the same for the two, leaving only an ultraviolet-finite difference. This difference includes an infrared divergence arising from the lepton wave-function renormalization. It is given by

$$\mathfrak{M}^{l}_{e2}=\left(\frac{3\alpha}{4\pi}\ln\mu_l-\frac{\alpha}{2\pi}\ln\frac{\lambda}{m_\pi}\right)\mathfrak{M}_0\,, \tag{3.7}$$

where $\mu_l=m_l/m_\pi$. The details of this lepton-vertex-correction calculation are analogous to those of APQ in their paper[14] on W decay, so we will not discuss them further in this work.

One final class of graphs—the box graphs detailed in Fig. 9—remains to be calculated. Of these, only the ones involving a photon [Figs. 9(a) and 9(b)] are of interest. The others [Figs. 9(c)–9(h)] are all infrared-finite and second-order weak[23] and so are negligible.[24] Evaluating the box graphs involving photons, we find

$$\mathfrak{M}^{\text{box}}_{e2}=f_\pi\frac{G_F M_W{}^2}{\sqrt{2}}\frac{e^2}{3(2\pi)^4}\int d^4k\,\{(k^2+k\cdot P_\pi)(k^2+2k\cdot P_l)(k^2-\lambda^2)[(k+P_\pi)^2-M_W{}^2]\}^{-1}$$
$$\times[3(P_\pi^\mu k^\nu+P_\pi^\nu k^\mu+P_\pi^\mu P_\pi^\nu-2k\cdot P_\pi g^{\mu\nu})+i\epsilon^{\alpha\mu\beta\nu}P_{\pi\alpha}k_\beta]\,\bar{l}[m_l\gamma_\mu\gamma_\nu+\gamma_\mu(\not k+\not P_l)\gamma_\nu](1-\gamma_5)\nu_l\,, \tag{3.8}$$

where λ is again a photon mass needed to handle the infrared divergences. The term proportional to 3 inside the second set of square brackets contains all the infrared divergences. This term is related to the IB part of the real-photon-emission amplitude. Its infrared structure is insensitive to the pion structure; hence, we will find that it has the same infrared divergences as the graph of Fig. 1(e).

The term with the ϵ tensor is related to the SD part of the real-photon-emission amplitude. We presumably should make the substitution $(k^2+k\cdot P_\pi)\to(k^2+k\cdot P_\pi-M^2/2)$ in the denominator of this term in order to be consistent with the changes made previously.[25] In practice, this loop SD correction is entirely negligible. This comes about for two reasons. First, the lepton mass factor, which is absent in the pure SD part of the real-photon emission, reappears owing to the connection of the virtual photon to the lepton line. (This is the same effect that made the IB-SD interference term negligible in the real photon emission rate.) Second, the mass factor, M, reduces the SD contribution by a factor of order $m_\pi{}^2/M^2$ compared to the result we would have gotten by using Eq. (3.8) as it stands.

Integrating the remainder of Eq. (3.8) yields the following expression:

$$\mathfrak{M}^{\text{box}}_{e2}=\mathfrak{M}_0\left(\frac{\alpha}{4\pi}\right)\left\{\left(\frac{1+\mu^2}{1-\mu^2}\right)[(\ln 2)^2+2(\ln\mu)^2-2K(\mu)]+\frac{1-2\mu^4}{(2\mu^2-1)^2}\ln 2\mu^2-\frac{\mu^2}{2\mu^2-1}-4\left(\frac{1+\mu^2}{1-\mu^2}\right)\ln\mu\,\ln\frac{\lambda}{m_\pi}\right\}, \tag{3.9}$$

where $\mu=m_l/m_\pi$ and

$$K(\mu)=\int_1^2(dt/t)\ln[(2\mu^2-1)t+2(1-\mu^2)]\,. \tag{3.10}$$

[In obtaining Eq. (3.9), we needed to use the identity

$$(m_\pi{}^2-m_l{}^2)^{-1}\bar{l}\sigma_{\mu\nu}(1-\gamma_5)\nu_l\epsilon^{\xi\mu\eta\nu}(P_l)_\eta(P_\pi-P_l)_\xi=\bar{l}(1-\gamma_5)\nu_l$$

to rewrite some terms in a form proportional to $\mathfrak{M}_0$.]

Combining all of our $O(\alpha)$ loop corrections to $\mathfrak{M}_0$ [Eqs. (3.6), (3.7), and (3.9)] we get

$$\mathfrak{M}^{\text{total}}_{e2}=\mathfrak{M}_0\left(\frac{\alpha}{4\pi}\right)\left\{\left(\frac{1+\mu^2}{1-\mu^2}\right)[(\ln 2)^2+2(\ln\mu)^2-2K(\mu)]+\frac{1-2\mu^4}{(2\mu^2-1)^2}\ln 2\mu^2-\frac{\mu^2}{2\mu^2-1}+3\ln\mu\right\}$$
$$-\mathfrak{M}_0\frac{\alpha}{\pi}\left[\left(\frac{1+\mu^2}{1-\mu^2}\right)\ln\mu+1\right]\ln\frac{\lambda}{m_\pi}\,. \tag{3.11}$$

The main origin of the deviation of this result from that of Kinoshita is that the quark-propagator pole in the "box" graphs Figs. 9(a) and 9(b) is not in the same position for nonzero photon (loop) momentum as the pion pole would be. This is not an error in the SQM, but a *difference* from what a more complete model would give. (It corresponds to an atomic calculation, for instance, where one does not take into account the binding of the electrons to the nucleus in some interaction involving the electrons. As the electron pole and not the atomic pole is important there, up to binding effects, so here the quark pole and not the pion pole is what we concentrate on.) Presumably the effects of strong binding in a more complete model would shift this case closer to the case studied by Kinoshita. This would involve extending the SQM to include momentum correlations, as we have done above for spin correlations through the parameter M. We have avoided this for two reasons: (1) It generates problems of gauge invariance and renormalizability which we are presently unable to handle. (2) We are interested in the maximum (possible) change in R_π; as the preceeding argument suggests, this is likely to be the case when the strong-binding corrections are completely ignored.

Finally, we can combine the expression for the loop corrections with the real-photon-emission contributions [Eqs. (1.3) and (3.5)] to get the total $O(\alpha)$ contribution to Δ_π, the fractional change in the ratio, R_π. This expression, which is of course infrared-finite, is

$$\begin{aligned}\Delta_\pi \simeq \frac{\alpha}{\pi}\Bigg\{&\Bigg[\left(\frac{1+\mu_e^2}{1-\mu_e^2}\right)[2L(1-\mu_e^2)-K(\mu_e)-2\ln\mu_e\ln(1-\mu_e^2)+\tfrac{1}{2}(\ln 2)^2]-2\ln(1-\mu_e^2)\\ &+\frac{(1-2\mu_e^4)}{2(2\mu_e^2-1)^2}\ln 2-\frac{(38\mu_e^4-47\mu_e^2+15)}{8(1-\mu_e^2)(2\mu_e^2-1)}+\mu_e^2\ln\mu_e\left(\frac{(\mu_e^2-4)}{2(1-\mu_e^2)^2}+\frac{2(2-3\mu_e^2)}{(2\mu_e^2-1)^2}\right)\Bigg]\\ &-\Bigg[\text{same terms with } \mu_e\to\mu_\mu\Bigg]\Bigg\}\\ &+\frac{3\alpha}{\pi}\ln\left(\frac{m_e}{m_\mu}\right)+\frac{\alpha}{240\pi}\left(\frac{m_\pi}{m_e}\right)^2\left(\frac{2}{3}\right)^2\left(\frac{m_\pi}{M}\right)^4. \end{aligned} \tag{3.12}$$

We should once again recall that we have neglected terms of order $\alpha G_F m_\pi^2$ and $\alpha(m_\pi/M)^4$ in calculating this expression. Note that the term $(3\alpha/\pi)\ln(m_e/m_\mu)$ is in agreement with the theorem of Marciano and Sirlin.[10] The last term, the pure SD contribution, is numerically quite small (0.05%). The value of R_π calculated from Eq. (3.12) is just

$$\begin{aligned} R_\pi &= R_0\left[1+\frac{\alpha}{\pi}\left(0.84+3\ln\frac{m_e}{m_\mu}\right)+0.0005\right]\\ &=(1.239\pm 0.001)\times 10^{-4}, \end{aligned} \tag{3.13}$$

where the error allows for the uncertainty in the SD part. This is very close to the value obtained by Kinoshita [Eq. (1.5)] and by us in Sec. II. It is also an explicit demonstration of the validity of the Marciano-Sirlin theorem.[10]

IV. PHENOMENOLOGICAL-LAGRANGIAN METHOD

In the previous two sections we have analyzed the second and third components of the pion-decay problem in particular models. The lessons we have learned are: (1) The second component, the ultraviolet-divergence renormalization, is innocuous—up to weak corrections the electronic and

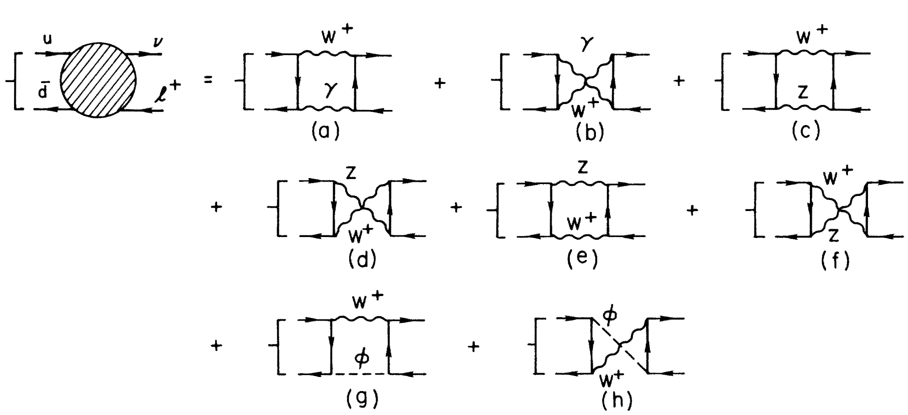

FIG. 9. Lowest-order one-loop "box" graphs.

muonic loops are renormalized the same way—and (2) the effects of the third component, strong-interaction dynamics, are small except possibly for the structure-dependent contributions—this is essentially the Marciano and Sirlin theorem at work. We can therefore short-circuit most of the formalism and employ a phenomenological-Lagrangian approach which will reproduce the IB parts calculated previously and give the SD contributions in a less *ad hoc* manner than in the SQM. We will, of course, find an SD contribution of the same size as we did previously; however, using this technique we can analyze the radiative corrections to any pseudoscalar-meson decay. We will find that the SD contribution in the kaon case can be quite large and may be dominant in the case of the charmed mesons (D,F).

In general our approach is a standard vector (and axial-vector) dominance scheme with certain peculiarities. For instance, our Lagrangian contains a photon–pseudoscalar—(axial-) vector vertex. We could, instead, use vector dominance for the photon and include a vector—pseudoscalar—(axial-) vector vertex; however, since we will *always* examine processes with at least one photon on-shell, we would gain nothing by this added complication. Also, we do not explicitly include the W meson in our model since finite-W-mass effects are in general negligible. Our analysis will be quite similar to that of Carron and Schult[5] who also treated the $K\to\gamma l\nu$ decays. The differences in our results will essentially be due to improved estimates of the input parameters rather than to differences in theoretical approach.

We begin by writing down the relevant parts of our Lagrangian:

$$\mathcal{L}_I=-e\sum_l \bar{l}\gamma_\mu l B^\mu + ie\,\mathrm{Tr}\underline{B}_\mu[\underline{P},\partial^\mu\underline{P}] + e\frac{g_{VP\gamma}}{M}\epsilon_{\mu\nu\alpha\beta}\mathrm{Tr}\{\underline{V}^{\mu\nu},\underline{F}^{\alpha\beta}\}\underline{P} + ie\frac{g_{AP\gamma}}{M}\mathrm{Tr}[\underline{A}^{\mu\nu},\underline{F}_{\mu\nu}]\underline{P} + e(\sqrt{2}g_V M^2)\,\mathrm{Tr}\underline{V}^\mu\underline{B}_\mu$$
$$+\frac{G_F}{\sqrt{2}}(\sqrt{2}g_V M^2)\,\mathrm{Tr}\underline{V}^\mu\underline{W}_\mu + \frac{G_F}{\sqrt{2}}(\sqrt{2}g_A M^2)\,\mathrm{Tr}\underline{A}^\mu\underline{W}_\mu + f_P\frac{G_F}{\sqrt{2}}\sum_l \mathrm{Tr}\,m_l\,\underline{P}\,\underline{W}^l\,, \tag{4.1}$$

where

$$\underline{F}_{\mu\nu}=\partial_\mu\underline{B}_\nu-\partial_\nu\underline{B}_\mu\,,\quad \underline{V}_{\mu\nu}=\partial_\mu\underline{V}_\nu-\partial_\nu\underline{V}_\mu\,,$$

$$\underline{B}_\mu=\begin{pmatrix}\frac{2}{3}B_\mu & 0 & 0\\ 0 & -\frac{1}{3}B_\mu & 0\\ 0 & 0 & -\frac{1}{3}B_\mu\end{pmatrix},$$

and $\underline{W}_\mu=\sum_l\underline{W}^l_\mu$, with ($\theta$ is the Cabibbo angle)

$$\underline{W}^l_\mu=\begin{pmatrix}0 & \cos\theta\,\bar{l}\gamma_\mu(1-\gamma_5)\nu_l & \sin\theta\,\bar{l}\gamma_\mu(1-\gamma_5)\nu_l\\ \cos\theta\,\bar{\nu}_l(1+\gamma_5)\gamma_\mu l & 0 & 0\\ \sin\theta\,\bar{\nu}_l(1+\gamma_5)\gamma_\mu l & 0 & 0\end{pmatrix},$$

and $\underline{W}^l$ has the same form as $\underline{W}^l_\mu$ but without the γ_μ factors. $\underline{P}$ is the 3×3 matrix representing the pseudoscalar octet and $\underline{V}_\mu$ $(\underline{A}_\mu)$ is the 3×3 matrix for the vector (axial-vector) nonet; B_μ is the photon. We treat only the electronic decay below, as the SD corrections in the muonic decay are always smaller by $(m_e/m_\mu)^2$. The factor of M which appears in various places is just an arbitrary mass scale[26] designed to make the various g's dimensionless. It should be clear from the notation that we have asssumed SU(3) and nonet symmetry.

In this model the lowest-order pion decay process proceeds as in Fig. 10(a), whereas the one-photon-emission graphs are shown in Figs. 10(b)–10(e). Figures 10(b) and 10(c) contribute to the IB part. The contribution of Fig. 10(d) is an SD part analogous to the SD part calculated in the previous section. Figure 10(e) is an additional SD contribution which vanishes in the SQM but need not vanish in general; we include it here.

The calculation of the IB part is again identical to that of Berman[2] and Kinoshita,[3] so we will concentrate here on the SD contributions. It is a straightforward but tedious calculation to show that the SD contribution to the pion decay rate is[22]

$$\Gamma_\pi^{\mathrm{SD}}=\frac{G_F{}^2\alpha}{1920\pi^2}m_\pi{}^7|v_\pi|^2(1+\gamma_\pi{}^2)\cos^2\theta\,, \tag{4.2}$$

where

$$v_\pi=\frac{(4g_{VP\gamma}/M)(\sqrt{2}g_V M^2)}{m_\rho{}^2}I\left(\frac{m_\pi{}^2}{m_\rho{}^2}\right) \tag{4.3}$$

and

$$\gamma_\pi v_\pi=\frac{(2g_{AP\gamma}/M)(\sqrt{2}g_A M^2)}{m_{A_1}{}^2}I\left(\frac{m_\pi{}^2}{m_{A_1}{}^2}\right). \tag{4.4}$$

The square of the function $I(y)$ is defined as

$$\begin{aligned} I^2(y) &= 20\int_0^1 dx\, x^3(1-x)[1-y(1-x)]^{-2} \\ &= -20y^{-5}[4y-7y^2+(\tfrac{17}{6})y^3 \\ &\qquad +(1-y)^2(4-y)\ln(1-y)] \\ &= 120\sum_{n=0}^{\infty}(n+1)(n+1)![(n+5)!]^{-1}y^n \end{aligned} \tag{4.5}$$

and includes corrections due to the momentum dependence of the vector (axial-vector) meson propagators. For the pion, $y\sim 0$ and $I\simeq 1$; i.e., these corrections are negligible. However, they will be important in the kaon case. [This is the one place where treatment of the muon differs—its non-negligible mass affects the definition of $I^2(y)$.] Another useful formula is the fractional change in the electronic decay due to the SD part:

$$\frac{\Gamma_\pi^{\mathrm{SD}}}{\Gamma_\pi^0}=\frac{\alpha}{240\pi}\left(\frac{m_\pi}{f_\pi}\right)^2\left(\frac{m_\pi}{m_e}\right)^2|m_\pi v_\pi|^2(1+\gamma_\pi^2)\,. \tag{4.6}$$

In order to estimate the SD contribution we need only ascertain the value of $|v_\pi|$. We can do this in a straightforward manner by first calculating the decay rate for $\pi^0\to\gamma\gamma$, using vector dominance and the Lagrangian (4.1) (which is equivalent to using CVC),

$$\Gamma_{\pi^0\to\gamma\gamma}=\alpha^2 m_\pi^{\,3}\left(\frac{\pi}{2}\right)|v_\pi|^2\,, \tag{4.7}$$

which yields

$$|v_\pi|=\frac{1}{\alpha}\left(\frac{2\Gamma_{\pi^0}}{\pi m_\pi^{\,3}}\right)^{1/2}.$$

Substituting into this equation the known π^0 decay rate[19] yields

$$|v_\pi|=\frac{0.0259}{m_\pi}\,. \tag{4.8}$$

Finally, if we use Eq. (4.8) in Eq. (4.2) we find that

$$\Gamma_\pi^{\mathrm{SD}}/\Gamma_\pi^0=+0.0005(1+\gamma_\pi^{\,2})\,.$$

This is the same result that we got in the previous section except for the additional factor of $(1+\gamma_\pi^{\,2})$, which is due to the axial-vector coupling. [The reason that $\gamma_\pi=0$ in the SQM is that the A_1 is an $l=1$ state in the $\mathrm{SU}(6)\times\mathrm{O}(3)$ classification, which clearly cannot exist in a *static* model.] Experimentally, γ_π is measured to have one of two possible values[27]

$$\gamma_\pi^{\mathrm{exp}}=0.15\pm0.11 \quad\text{or}\quad -2.07\pm0.11\,.$$

In either case the SD contribution to R_π is very small compared to the usual IB part, i.e., our result is essentially the same as it was in the previous section. This, of course, was to be expected since in both cases we used the $\pi^0\to\gamma\gamma$ width to estimate the size of the SD part.

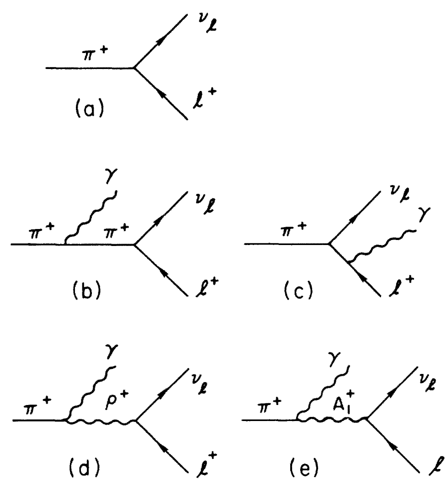

FIG. 10. Low-order tree graphs for π^+ decay in the phenomenological Lagrangian method: (a) lowest-order graph, (b) and (c) real-photon-emission (radiative) corrections which also appear in the point pion model, and (d) and (e) structure-dependent real-photon-emission (radiative) corrections.

Having seen how the phenomenological-Lagrangian approach works in the pion decay case we can now easily estimate the size of the kaon decay corrections. The relevant graphs are shown in Fig. 11. The uncorrected e/μ ratio for kaon decay [from Fig. 11(a)] is just

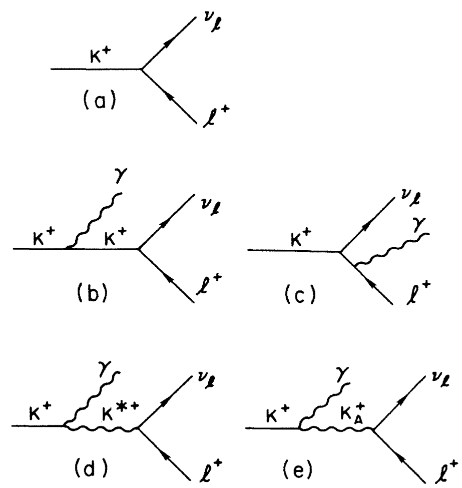

FIG. 11. Low-order tree graphs for K^+ decay in the phenomenological Lagrangian method: (a) lowest-order graph, and (b)–(e) real-photon-emission (radiative) corrections.

$$R_K^0=\left(\frac{m_e}{m_\mu}\right)^2\left(\frac{m_K{}^2-m_e{}^2}{m_K{}^2-m_\mu{}^2}\right)^2$$
$$=2.57\times10^{-5}. \tag{4.9}$$

We must now correct for the effects of real-photon emission and virtual-photon loops. The theorem of Marciano and Sirlin[10] tells us that to a good approximation the IB [Figs. 11(b) and 11(c)] and loop corrections add up to produce a factor of $(3\alpha/\pi)\ln(m_e/m_\mu)+O(\alpha/\pi)$, i.e.,

$$R_K^0\to R_K^0\left[1+\frac{3\alpha}{\pi}\ln\left(\frac{m_e}{m_\mu}\right)+O\left(\frac{\alpha}{\pi}\right)\right]\simeq2.47\times10^{-5}. \tag{4.10}$$

To this we must add the SD contribution [Figs. 11(d) and 11(e)]. (The muon SD part is still negligible.) In a manner completely analogous to the pion decay case we find that[22]

$$\Gamma_K^{\rm SD}=\frac{G_F{}^2\alpha}{1920\pi^2}m_K{}^7|v_K|^2(1+\gamma_K{}^2)\sin^2\theta \tag{4.11}$$

or

$$\frac{\Gamma_K^{\rm SD}}{\Gamma_K^0}=\frac{\alpha}{240\pi}\left(\frac{m_K}{f_K}\right)^2\left(\frac{m_K}{m_e}\right)^2|m_Kv_K|^2(1+\gamma_K{}^2), \tag{4.12}$$

i.e., we need only replace all the π indices by K indices. Once again, everything is known except $|v_K|$ and γ_K; however, if we take our Lagrangian, Eq. (4.1), at face value we can derive all the necessary parameters. In particular, we find

$$f_K=f_\pi, \tag{4.13}$$

$$v_K=\frac{(4g_{VP\gamma}/M)(\sqrt{2}g_VM^2)}{m_{K^*}{}^2}I(m_K{}^2/m_{K^*}{}^2), \tag{4.14}$$

$$\gamma_Kv_K=\frac{(4g_{AP\gamma}/M)(\sqrt{2}g_AM^2)}{m_{K_A}{}^2}I(m_K{}^2/m_{K_A}{}^2), \tag{4.15}$$

where the propagator corrections in $I(y)$ are no longer negligible. (They make a 25% correction to $|v_K|^2$.) Comparing Eqs. (4.14) and (4.15) with (4.3) and (4.4) we find

$$v_K/v_\pi\simeq\left(\frac{m_\rho}{m_{K^*}}\right)^2I(m_K{}^2/m_{K^*}{}^2) \tag{4.16}$$

and

$$\gamma_K/\gamma_\pi\simeq\left(\frac{m_{K^*}m_{A_1}}{m_\rho m_{K_A}}\right)^2\frac{I(m_K{}^2/m_{K_A}{}^2)}{I(m_K{}^2/m_{K^*}{}^2)}. \tag{4.17}$$

We can now compare the SD contribution to kaon decay with the contribution to pion decay by taking the ratio of Eqs. (4.12) and (4.6)

$$\frac{\Gamma_K^{\rm SD}/\Gamma_K^0}{\Gamma_\pi^{\rm SD}/\Gamma_\pi^0}=\left(\frac{m_K}{m_\pi}\right)^6\left(\frac{v_K}{v_\pi}\right)^2\left(\frac{1+\gamma_K{}^2}{1+\gamma_\pi{}^2}\right)$$
$$\simeq\left(\frac{m_K}{m_\pi}\right)^6\left(\frac{m_\rho}{m_{K^*}}\right)^4I(m_K{}^2/m_{K^*}{}^2). \tag{4.18}$$

Substituting the relavant mass factors into Eq. (4.18) we find

$$\frac{\Gamma_K^{\rm SD}}{\Gamma_K^0}=(1360)\times\frac{\Gamma_\pi^{\rm SD}}{\Gamma_\pi^0}$$
$$=(0.68\pm0.08)(1+\gamma_K{}^2) \tag{4.19}$$

which is a very sizable effect, indeed. (The error here is due to the error in our experimental input—the $\pi^0\to\gamma\gamma$ rate.) Experimentally, the value of this ratio is[28]

$$\left(\frac{\Gamma_K^{\rm SD}}{\Gamma_K^0}\right)_{\rm exp}=1.05\pm0.25$$

in reasonable agreement with our theoretical estimate.

Since the theory and experiment are moderately close to each other, it would seem prudent to reexamine the assumptions that went into our calculation. In particular, the SU(3) symmetry of our interaction Lagrangian was a crucial input in our calculation; it gave us the relations $f_K=f_\pi$, $g_{\rho\pi\gamma}=g_{K^*K\gamma}$, $g_\rho=g_{K^*}$, etc. It is possible, however, to test these assumptions by independent means. For instance, by comparing $\pi\to\mu\nu$ and $K\to\mu\nu$ (where radiative corrections are relatively unimportant) we find that

$$f_\pi\lesssim f_K\lesssim1.2f_\pi;$$

the variation indicates uncertainty as to whether or not the Cabibbo angles for the vector and axial-vector currents are equal. The upper limit assumes equality, while the lower limit reflects theoretical prejudices.[29] (Recall we used $f_K=f_\pi$ above.) Next, we can in principle measure $g_{\rho\pi\gamma}$ and $g_{K^*K\gamma}$ from the radiative decays of the vector mesons. Although the branching ratios for such decays are not very well known, a preliminary analysis of all the data[30,31] indicates that a reasonable fit is given by $g_{K^{*+}K^+\gamma}/g_{\rho\pi\gamma}=m_\rho/m_{K^*}$. Finally we can estimate the relation between g_ρ and g_{K^*} by using the first Weinberg sum rule,[32] which says, in effect, that $g_{K^*}/g_\rho=m_{K^*}/m_\rho$. Combining the last two results we again obtain $g_{K^*K\gamma}g_{K^*}=g_{\rho\pi\gamma}g_\rho$; although the individual terms violate SU(3), their product may still exhibit SU(3) symmetry to a large extent since the SU(3)-breaking effects tend to cancel.[33]

As far as the axial vectors are concerned, there is very little experimental evidence on $g_{AP\gamma}$ or g_A. If our Eq. (4.17) holds true, then the factor $\gamma_K{}^2$ equals $\gamma_\pi{}^2$, which is either very small (~0.02) or very large (~4). At present the larger value does not seem to agree with experiment.[28,34] If the smaller value of γ_K is the correct one, then its error is of little consequence at this point.

The discussion of the last two paragraphs is meant to remind us that the error quoted in Eq. (4.19) is probably an underestimate; it is difficult, however, to estimate what the actual error quoted should be since this depends so much on one's theoretical prejudices. If, for example, we assume $f_K/f_\pi = 1.1 \pm 0.1$, attribute a 10% error to v_K/v_π, and assume that γ_K is negligible, then we get finally

$$\Gamma_K^{SD}/\Gamma_K^0 = (57 \pm 15)\%. \qquad (4.19')$$

We believe that our results are better than order-of-magnitude estimates, but we still see little hope at present of improving the theoretical calculations to a 5–10% error level.

As a final comment to end this section, we point out that our phenomenological-Lagrangian method can be expanded to include possible charmed pseudoscalar mesons (D,F) in a very straightforward way.[12,35] We simply replace our 3×3 matrices in Eq. (4.1) by the appropriate 4×4 matrices. A simple order-of-magnitude estimate suggests that the SD part of these decays is enhanced by a factor of $(m_D/m_K)^2(m_{K^*}/m_K)^4 \sim 10^2$ over the kaon case, so that the radiative electronic decay rate is more comparable to the muonic decay rate. Thus, the decays $D(F)\to e\nu$ are completely dominated by the SD parts of $D(F)\to\gamma e\nu$; i.e., calculations of $D(F)\to e\nu$ alone[12] will grossly underestimate the electronic branching ratios of the charged-charmed-pseudoscalar-meson decays. Of course, the $D^*(F^*)$ radiative widths may be exceptionally small and reduce the factor of 10^2 above. The situation is further confused by what may be very many closely spaced D^*'s, observed experimentally.[35] Their constructive or destructive interference could change the order of magnitude of our estimate. Nonetheless, it is apparent that the decay $D(F)\to e\nu\gamma$ may be an excellent area to examine pure SD effects.

V. CONCLUSION

The present status of radiative corrections to leptonic decays of pseudoscalar mesons may be summarized as follows: (1) The problem of infrared-divergence cancellation is completely handled by means of gauge invariance. (2) The problem of ultraviolet-divergence cancellation can be ignored; renormalization effects induced by such cancellations are negligibly small for experiments of the foreseeable future (of order $G_F m_{\text{lepton}}^2$). (3) Sizable ($> 0.5\%$ in R_P) effects of strong-interaction dynamics appear, if at all, only in the pure-SD real-photon-emission part. The size of these SD corrections grows rapidly as the meson mass increases. In Sec. IV, we showed that the size of these effects may be sufficiently reliably estimated using a phenomenological-Lagrangian approach together with (experimental) data on radiative decays of vector and axial-vector mesons. The only improvement that specific models could provide is to include (presumably) small effects such as that of form factors (except possibly in the decay of D (F) mesons)[12,35] at the radiative decay vertices, and the background, or nonresonant, contributions in the vector or axial-vector subchannels.[36]

In the case of the pion, the SD effects are very small. Our analysis restores to the ratio R_π the status of being a strong test of μ-e universality: Even allowing for extreme model dependence, we find that $1.233\times 10^{-4} \lesssim R_\pi \lesssim 1.239\times 10^{-4}$. If R_π lies well outside this range, we believe that such a deviation is unlikely to come from pion structure, and that one should, instead, consider the possibility of a breakdown of μ-e universality.[37] That is, μ-e universality may be tested to the 0.5% level by R_π. We remind the reader that the best present value[38] for R_π is $(1.274\pm 0.024)\times 10^{-4}$, so that the question remains open.[39,40]

In the case of the kaon, the SD effects are much larger, of the same order of magnitude as the lowest-order (electronic) weak decay. Our calculation gives a somewhat smaller effect than is observed experimentally, but still in reasonable agreement (within about two standard deviations) especially when the uncertainties in the theoretical calculation are considered.

Finally, we note that the SD contribution could well dominate the purely electronic decays of the newly discovered charmed meson[35] [$D(1865)$]. These decays of D mesons may provide an excellent area to examine pure SD effects.

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[4]Until we arrive at Sec. IV, our explicit remarks will be limited to the pion. It should be clear, however, that much of what we have to say applies, in form at least, to radiative corrections to the purely leptonic decays of any pseudoscalar meson.

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[18]This must also be done for the corresponding terms in the loop graphs, to be consistent; it does not, however, interfere with the delicate ultraviolet-divergence cancellations as might be feared. Explicitly, this is true because the loop terms proportional to the ϵ tensor are more convergent than the other terms. This can be inferred directly from the real-photon-emission amplitude (3.4) by noting that it is separately gauge invariant; it vanishes when the photon momentum replaces the letpon current.

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[20]This value of M differs by a factor of $\sqrt{2}$ from that in Ref. 8 as the definition of M^2 there differs by a factor of 2.

[21]N. P. Samios, Phys. Rev. 121, 275 (1961); S. Devons *et al.*, *ibid.* 184, 1356 (1969).

[22]This is just the integrated form of Eq. (2) of A. Stetz *et al.* [Phys. Rev. Lett. 33, 1455 (1974)] with their parameters $|a(0)|$ and γ replaced by the specific $|v|$ and γ of our model.

[23]This is true even for graphs involving the neutral scalar φ as the coupling to the lepton is $G_l \sim g m_l/M_W \sim e m_l/M_W$, and similarly for the quark.

[24]Once the cancellation of the ultraviolet divergences is appreciated, this can be seen immediately by estimating the values of these graphs with cut off integrals. For example, for Fig. 9(c), we find an amplitude

$$\sim g^4 \int d^4k\, \bar{l}\, \not{k}(1-\gamma_5)\nu m_\pi P_\pi \cdot k \times\{k^2k^2(k^2-M_W^2)(k^2-M_Z^2)\}^{-1}.$$

In the cutoff integral, $k \sim P_\pi$ in the numerator and

$$\int d^4k\, k^{-4}\{(k^2-M_W^2)(k^2-M_Z^2)\}^{-1} \sim M_W^{-2} M_Z^{-2},$$

so that this amplitude is

$$\sim (g^2 m_l m_\pi/M_W^2)(g^2 m_\pi^2/M_Z^2)\bar{l}(1-\gamma_5)\nu,$$

i.e., reduced from the zeroth-order amplitude by a factor of $\sim G_F m_\pi^2$.

[25]See the discussion of Ref. 18 for the (lack of) effect on divergence cancellations.

[26]It should not be confused with parameter M in the previous two sections.

[27]See Stetz *et al.*, Ref. 22.

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[29]The upper limit conflicts with an application of the first Weinberg sum rule (see Ref. 32 below). For $f_K \gtrsim 1.1 f_\pi$, the solution for $g_{\text{axial-vector}}$, from the equation derived from the sum rule by resonance saturation, is imaginary when the CVC value for $g_{\text{vector}} = g_\rho$ obtained from e^+e^- data is used (see Ref. 33 below). For $f_K \sim f_\pi$, one obtains instead $(g_{\text{axial-vector}}/g_{\text{vector}}) \sim 0.8$.

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[33]We can also measure g_ρ directly from the decay $\rho^0 \to e^+e^-$, using CVC. By comparing this with $\omega \to e^+e^-$, $\varphi \to e^+e^-$, and $\psi \to e^+e^-$, we find the best agreement between a symmetry hypothesis and the data (see Ref. 19) if $g_V \propto (M_V)^{3/2}$. An extrapolation of this result, of the form $(g_{K^*}/g_\rho) = (M_{K^*}/M_\rho)^{3/2}$, tends to increase our estimate of the SD part of kaon decay.

[34]Babcock and Rosner (see Ref. 31) have recently estimated $\Gamma(A \rightarrow P\gamma)$ along with many other radiative widths in a quark model with a free parameter, a/b. We find that γ_K comes out much too large unless $a/b \sim 0$, which minimizes their values for the radiative rates.

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[36]In current-algebra approaches (see Ref. 5), all of this information is contained in the dispersion-relation spectral function. That, however, is usually unknown except to the extent that it can be approximated by resonant contributions; again, background contributions are ignored.

[37]Effects due to possible masses of the electron or muon neutrinos are less than 0.01% using the current experimental upper limits on these masses (see Ref. 19).

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[39]An experiment designed to measure R_π to $\pm 0.5\%$ is presently under way at TRIUMF, Vancouver, Canada (D. Bryman and A. Olin, private communication).

[40]With the recent suggestion of a new heavy lepton, U [M. Perl *et al.*, Phys. Rev. Lett. 35, 1489 (1975)], which may have an associated neutrino of nonzero mass, the possibility of a μ-U Cabibbo-type mixing arises. This would tend to increase R_π above the range of values we have obtained, if the electron does not also participate in the mixing.