

Spectrum-generating SU(3) and the semileptonic decays of hyperons*

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Semileptonic decays of hyperons are considered within a model with SU(3) as a spectrum-generating group with a precise relation to the Poincaré group. Comparison with experimental data indicates that a reasonable fit may be obtained.

I. INTRODUCTION

The semileptonic decays of hyperons are conventionally analyzed in terms of the Cabibbo model¹ assuming an underlying SU(3) symmetry. Since the SU(3) symmetry is only approximate, as exhibited by the observed mass differences within the multiplets, the question arises as to the symmetry-breaking mechanism. Various *ad hoc* assumptions have been advocated but we still lack a coherent treatment of the problem that affords an adequate description of observed phenomena. As an alternative to the often rather ill-defined approach with SU(3) as an approximate symmetry one of the authors² has suggested that SU(3) be regarded as a spectrum-generating group with a precise relation to the Poincaré group so that exact calculations are made possible as far as "symmetry breaking" is concerned. In this paper we shall apply these ideas to the semileptonic decays of hyperons.

Recent studies of the semileptonic decays of hyperons indicate that rather large pseudotensor contributions may be present.³ Indications of sizable pseudotensor contributions in ordinary beta decay of nuclei⁴ provide for further evidence of second-class currents in semileptonic processes. Within the conventional framework these effects are ascribed to the underlying SU(3) symmetry being broken. Attempts to estimate the size of such effects have been made using dispersion-relation techniques⁵ and current-algebra sum rules,⁶ but the results appear to be too small to account for observations.⁷ Following the suggestion of Bohm² we shall make precise assumptions about the relationship of SU(3), regarded as a spectrum-generating group, and the Poincaré group. From this we obtain the result that there is a pseudotensor term present in the matrix element, and it is proportional to the mass difference

of the baryons. This is true even though we assume that there are no second-class currents with respect to the spectrum-generating group in the theory. A comparison of the predictions of the model with the data from the semileptonic decays of hyperons indicates that a reasonable fit may be obtained.

II. THE CONVENTIONAL MODEL FOR SEMILEPTONIC DECAYS OF BARYONS

The rate Γ for the semileptonic decay process $B \rightarrow B' + l + \nu$, where (B, B') denote baryons and $l = (\mu, e)$, is conventionally written⁸

$$\Gamma = \frac{1}{(2\pi)^5} \frac{1}{2E_B} \times \int \frac{d^3p'}{2E_{B'}} \frac{d^3p_l}{2E_l} \frac{d^3p_\nu}{2E_\nu} \delta^4(p - p' - p_l - p_\nu) \times \frac{1}{2} \sum_{\text{pol}} |M|^2. \quad (1)$$

The transition matrix element M is then given by⁹

$$M = \frac{G}{\sqrt{2}} \bar{u}_l(p_l) \gamma^\mu (1 + \gamma_5) v_\nu(p_\nu) \times \langle p' \sigma' \alpha' | j_\mu(0) | p \sigma \alpha \rangle \quad (2)$$

The baryon states $|p \sigma \alpha\rangle$ are labeled by the four-momentum p , the third component of the spin σ ($s = \frac{1}{2}$), and the SU(3) and other internal quantum numbers $\alpha = (I, I_3, Y, \dots)$. The hadronic transition operator $j_\mu(0)$ in the Cabibbo model¹ is written

$$j_\mu(0) = \cos \theta_C (V_\mu^{+1} + A_\mu^{+1}) + \sin \theta_C (V_\mu^{+2} + A_\mu^{+2}), \quad (3)$$

where θ_C is the Cabibbo angle, V_μ^α and A_μ^α , with $\alpha = (0, \pm 1, \pm 2, \pm 3, 8)$ (Ref. 10), are Lorentz vector and axial-vector operators, respectively, which are assumed to be SU(3)-octet operators in the symmetry limit, that is, they will satisfy the ap-

appropriate commutation relations with the SU(3) generators.

In order to specify the hadronic matrix elements further, we must specify the relationship between the particle-classifying SU(3) and the Poincaré group, which provide the labels of the baryon states. In the conventional approach it is assumed that SU(3) is a symmetry group, i.e., that the momentum operators and, therefore, also the mass operator commute with all the SU(3) generators,

$$[\mathcal{P}_\mu, E_\alpha] = 0. \quad (4)$$

The state space is, therefore, a direct-product space with the SU(3) content separated from the Poincaré content. By the Wigner-Eckart theorem we then find

$$\begin{aligned} M^h &= \frac{G}{\sqrt{2}} \langle p' \sigma' \alpha' | (V_\mu^\beta + A_\mu^\beta) | p \sigma \alpha \rangle \\ &= \frac{G}{\sqrt{2}} \sum_{\gamma=1,2} C(\gamma; \alpha \beta \alpha') \langle p' \sigma' | V_\mu + A_\mu | p \sigma \rangle^{(\gamma)}, \end{aligned} \quad (5)$$

where $C(\gamma=1; \alpha \beta \alpha')$ is the f -type (antisymmetric) Clebsch-Gordan coefficient, and $C(\gamma=2; \alpha \beta \alpha')$ is the (symmetric) d -type coefficient. The reduced matrix elements in (5) define operators $V_\mu^{(\gamma)}$ and $A_\mu^{(\gamma)}$ which act on the Poincaré space, i.e., we may also write (5)

$$\begin{aligned} \langle p' \sigma' \alpha' | V_\mu^\beta + A_\mu^\beta | p \sigma \alpha \rangle \\ = \left\langle p' \sigma' \left| \sum_{\gamma=1,2} C(\gamma; \alpha \beta \alpha') (V_\mu^{(\gamma)} + A_\mu^{(\gamma)}) \right| p \sigma \right\rangle. \end{aligned} \quad (6)$$

By the usual arguments of Lorentz invariance and the use of the Dirac equation this matrix element may be expressed in terms of form factors,

$$\begin{aligned} M^h &= \frac{G}{\sqrt{2}} \bar{u}_B(p', \sigma') [f_1^{\alpha' \beta \alpha}(q^2) \gamma_\mu + f_2^{\alpha' \beta \alpha}(q^2) i \sigma_{\mu\nu} q^\nu \\ &\quad + f_3^{\alpha' \beta \alpha}(q^2) q_\mu + g_1^{\alpha' \beta \alpha}(q^2) \gamma_\mu \gamma_5 \\ &\quad + g_2^{\alpha' \beta \alpha}(q^2) i \sigma_{\mu\nu} \gamma_5 q^\nu + g_3^{\alpha' \beta \alpha}(q^2) \gamma_5 q_\mu] \\ &\quad \times u_B(p, \sigma), \end{aligned} \quad (7)$$

where $q_\mu = p'_\mu - p_\mu$ is the momentum transfer between the baryons and

$$f_i^{\alpha' \beta \alpha}(q^2) = \sum_{\gamma=1,2} C(\gamma; \alpha \beta \alpha') f_i^{(\gamma)}(q^2), \quad i = 1, 2, 3 \quad (8)$$

$$g_i^{\alpha' \beta \alpha}(q^2) = \sum_{\gamma=1,2} C(\gamma; \alpha \beta \alpha') g_i^{(\gamma)}(q^2). \quad (9)$$

However, as previously stated, SU(3) is not a symmetry group, and

$$[P_\mu, E_{\pm 2, \pm 3}] \neq 0. \quad (10)$$

Thus the Wigner-Eckart theorem cannot be used to obtain (5) or alternatively (6), and the form factors $f_i^{(\gamma)}$ and $g_i^{(\gamma)}$ will in general depend upon the SU(3) quantum numbers through the masses m' and m of the baryons and their q^2 dependence. This in turn introduces a dependence upon the SU(3) quantum numbers through the mass formula. The usual procedure is then to assume that (5) may be used as an approximation good to lowest order, and that symmetry-breaking corrections may be introduced at a later stage. However, in view of the large breaking of SU(3), and even larger breaking of SU(4) if that turns out to be the appropriate group, it is not clear that (5) is even approximately correct and that it can serve as a basis for further analysis. Therefore, we shall abandon this line of approach altogether and find an exact substitute for the assumption (4), which will allow us to compute the symmetry breaking in the conventional scheme.

Before turning to the alternative model we note in passing that through (8) and (9) all the form factors and, thereby, all decay parameters for the semileptonic decays within the baryon octet family are uniquely given in terms of the 12 (unknown) "reduced matrix elements" $f_i^{(\gamma)}$ and $g_i^{(\gamma)}$ ($\gamma=1, 2, 3$). This is the starting point for all attempts to fit experimental data.

III. A MODEL BASED ON SU(3) AS A SPECTRUM-GENERATING GROUP

Instead of making the symmetry-group assumption (4), we shall assume that the four-velocity \hat{P}_μ commutes with the SU(3) generators,¹¹ that is,

$$[\hat{P}_\mu, E_\alpha] = 0. \quad (11)$$

This is by no means a unique choice as an alternative to (4), but it is the simplest and most obvious generalization one may attempt. The physical requirement that the mass operator must not commute with the SU(3) raising and lowering operators is, however, rather restrictive and rules out many otherwise possible alternatives, such as $[P_\mu/M^2, E_\alpha] = 0$ to mention one example. An important merit of (11) is that it does not obviously contradict the experimental situation whereas (4) does.

To analyze the consequences of the assumption (11) it is most convenient, although not necessary, to employ the Poincaré group $P \equiv P(\hat{P}_\mu, L_{\mu\nu})$ generated by the four-velocity \hat{P}_μ , rather than the physical Poincaré group $P(P_\mu, L_{\mu\nu})$. The physical mass operator M is now independent of the choice of the representation for $P(\hat{P}_\mu, L_{\mu\nu})$ since we have $\hat{P}_\mu \hat{P}^\mu = 1$. It is instead given by some function of the diagonal SU(3) generators, that is, $M \equiv$

$M(I_3, I, Y)$. Under the assumption (11) we may write the basis vectors of the space of physical states as direct products of the form

$$|\hat{p}\sigma\alpha\rangle \equiv |\hat{p}\sigma\rangle \otimes |\alpha\rangle, \quad (12)$$

where $\{|\hat{p}\sigma\rangle\}$ span the representation space for $P(\hat{P}_\mu, L_{\mu\nu})$ and $\{|\alpha\rangle\}$ are basis vectors in an SU(3)-octet space. The basis vectors $|\rho\sigma\alpha\rangle$ and $|\hat{p}\sigma\alpha\rangle$ are normalized differently:

$$\langle \rho'\sigma'\alpha' | \rho\sigma\alpha \rangle = 2\rho_0 \delta^3(\vec{\rho}' - \vec{\rho}) \delta_{\alpha'\alpha}, \quad (13)$$

$$\langle \hat{p}'\sigma'\alpha' | \hat{p}\sigma\alpha \rangle = 2\hat{p}_0 \delta^3\left(\frac{\vec{p}'}{m'} - \frac{\vec{p}}{m}\right) \delta_{\alpha'\alpha}. \quad (14)$$

If we assume that V_μ^β and A_μ^β are SU(3)-octet operators, then by the Wigner-Eckart theorem we obtain instead of (5)

$$\begin{aligned} \hat{M}^h &= \frac{g}{\sqrt{2}} \langle \hat{p}'\sigma'\alpha' | (V_\mu^\beta + A_\mu^\beta) | \hat{p}\sigma\alpha \rangle \\ &= \frac{g}{\sqrt{2}} \sum_{\gamma=1,2} C(\gamma; \alpha\beta\alpha') \\ &\quad \times [\langle \hat{p}'\sigma' | V_\mu | \hat{p}\sigma \rangle^{(\gamma)} + \langle \hat{p}'\sigma' | A_\mu | \hat{p}\sigma \rangle^{(\gamma)}] \\ &= \frac{g}{\sqrt{2}} \left\langle \hat{p}'\sigma' \left| \sum_{\gamma=1,2} C(\gamma; \alpha\beta\alpha') (V_\mu^{(\gamma)} + A_\mu^{(\gamma)}) \right| \hat{p}\sigma \right\rangle, \end{aligned} \quad (15)$$

where the last equality defines the operators $V^{(\gamma)}$ and $A^{(\gamma)}$. Equation (15), unlike (5) and (6), is then an exact equation under assumption (11) even when the SU(3) is not an exact symmetry group. However, even though it is an appealing assumption

$$\begin{aligned} \hat{M}^h &= \frac{g}{\sqrt{2}} \langle \hat{p}'\sigma'\alpha' | (V_\mu^\beta + A_\mu^\beta) | \hat{p}\sigma\alpha \rangle \\ &= \frac{g}{\sqrt{2}} \left\langle \hat{p}'\sigma' \left| \sum_{\gamma=1,2} C(\gamma; \alpha\beta\alpha') (\phi_V^{\alpha'\alpha} v_\mu^{(\gamma)} + \phi_A^{\alpha'\alpha} a_\mu^{(\gamma)}) \right| \hat{p}\sigma \right\rangle, \end{aligned} \quad (19)$$

where $\phi_{V,A}^{\alpha'\alpha}$ are functions of the masses. As seen above these functions are at least partially determined by the particular choice of basic octet operators in the theory. From Lorentz covariance and the Dirac equation we obtain

$$\begin{aligned} \hat{M}^h &= \frac{g}{\sqrt{2}} \bar{u}_B(\hat{p}', \sigma') \{ \phi_V^{\alpha'\alpha} [F_1^{\alpha\beta\alpha'}(\hat{q}^2) \gamma_\mu + F_2^{\alpha\beta\alpha'}(\hat{q}^2) i\sigma_{\mu\nu} \hat{q}^\nu + F_3^{\alpha\beta\alpha'}(\hat{q}^2) \hat{q}_\mu] \\ &\quad + \phi_A^{\alpha'\alpha} [G_1^{\alpha\beta\alpha'}(\hat{q}^2) \gamma_\mu \gamma_5 + G_2^{\alpha\beta\alpha'}(\hat{q}^2) i\sigma_{\mu\nu} \gamma_5 \hat{q}^\nu + G_3^{\alpha\beta\alpha'}(\hat{q}^2) \gamma_5 \hat{q}_\mu] \} u_B(\hat{p}\sigma), \end{aligned} \quad (20)$$

with $\hat{q} = \hat{p}' - \hat{p}$ and

$$F_i^{\alpha\beta\alpha'}(\hat{q}^2) = \sum_{\gamma=1,2} C(\gamma; \alpha\beta\alpha') F_i^{(\gamma)}(\hat{q}^2), \quad i = 1, 2, 3 \quad (21)$$

$$G_i^{\alpha\beta\alpha'}(\hat{q}^2) = \sum_{\gamma=1,2} C(\gamma; \alpha\beta\alpha') G_i^{(\gamma)}(\hat{q}^2). \quad (22)$$

With the assumptions stated above this is an exact equation. Since data analysis is conventionally

performed in terms of the form factors f_i and g_i of Eq. (7) it is convenient for our purposes to express the f_i 's and the g_i 's in terms of the SU(3)-invariant form factors $F_i^{(\gamma)}$ and $G_i^{(\gamma)}$. This is straightforward but requires that careful attention be paid to normalization conditions, etc. when rewriting (1) in terms of the basis (12) as noted above.¹³ The results for constant form factors are

that V_μ^α and A_μ^α are SU(3)-octet operators this may, in general, not be true, in which case (15) would no longer hold. In seeking generalizations it appears, based on past experience, reasonable to require that V_μ^α and A_μ^α , at least in the limit of exact SU(3) symmetry, behave as octet operators. One may then attempt to require that some suitable functions of V_μ^α and A_μ^α , which in the symmetry limit reduce to V_μ^α and A_μ^α , are octet operators for which the Wigner-Eckart theorem applies. Denoting these generalized octet operators by v_μ^α and a_μ^α , respectively, we may choose

$$v_\mu^\alpha = [M^N, V_\mu^\alpha]_+, \quad a_\mu^\alpha = [M^{N'}, A_\mu^\alpha]_+, \quad N, N' = \pm 1, \pm 2, \dots \quad (16)$$

or

$$v_\mu^\alpha = M^N V_\mu^\alpha M^N, \quad a_\mu^\alpha = M^{N'} A_\mu^\alpha M^{N'}, \quad N, N' = \pm 1, \pm 2, \dots \quad (17)$$

to give two examples. We shall only consider modifications by means of the mass operator. Applying the Wigner-Eckart theorem to the case (17) with $N=N'$ it is easily seen that (15) is replaced by

$$\begin{aligned} \hat{M}^h &= \frac{1}{(m_B m_B')^N} \frac{g}{\sqrt{2}} \\ &\quad \times \left\langle \hat{p}'\sigma' \left| \sum_{\gamma=1,2} C(\gamma; \alpha\beta\alpha') (v_\mu^{(\gamma)} + a_\mu^{(\gamma)}) \right| \hat{p}\sigma \right\rangle. \end{aligned} \quad (18)$$

To account for possibilities such as (18), and other possibilities to be mentioned below, we will write (15) in the more general form

$$f_1^{\alpha'\beta\alpha} = \frac{\phi_V^{\alpha'\alpha}}{(m_\alpha m_{\alpha'})^{3/2}} \sum_{\gamma=1,2} C(\gamma; \alpha\beta\alpha') \left[F_1^{(\gamma)} + \left(2 - \frac{(m_\alpha + m_{\alpha'})^2}{2m_\alpha m_{\alpha'}} \right) F_2^{(\gamma)} + \frac{m_\alpha^2 - m_{\alpha'}^2}{2m_\alpha m_{\alpha'}} F_3^{(\gamma)} \right], \quad (23)$$

$$f_2^{\alpha'\beta\alpha} = \frac{\phi_V^{\alpha'\alpha}}{2(m_\alpha m_{\alpha'})^{5/2}} \sum_{\gamma=1,2} C(\gamma; \alpha\beta\alpha') [(m_\alpha + m_{\alpha'}) F_2^{(\gamma)} - (m_\alpha - m_{\alpha'}) F_3^{(\gamma)}], \quad (24)$$

$$f_3^{\alpha'\beta\alpha} = \frac{\phi_V^{\alpha'\alpha}}{2(m_\alpha m_{\alpha'})^{5/2}} \sum_{\gamma=1,2} C(\gamma; \alpha\beta\alpha') [-(m_\alpha - m_{\alpha'}) F_2^{(\gamma)} + (m_\alpha + m_{\alpha'}) F_3^{(\gamma)}], \quad (25)$$

$$g_1^{\alpha'\beta\alpha} = \frac{\phi_A^{\alpha'\alpha}}{(m_\alpha m_{\alpha'})^{3/2}} \sum_{\gamma=1,2} C(\gamma; \alpha\beta\alpha') \left[G_1^{(\gamma)} + \frac{m_\alpha^2 - m_{\alpha'}^2}{2m_\alpha m_{\alpha'}} G_2^{(\gamma)} - \frac{(m_\alpha - m_{\alpha'})^2}{2m_\alpha m_{\alpha'}} G_3^{(\gamma)} \right], \quad (26)$$

$$g_2^{\alpha'\beta\alpha} = \frac{\phi_A^{\alpha'\alpha}}{2(m_\alpha m_{\alpha'})^{5/2}} \sum_{\gamma=1,2} C(\gamma; \alpha\beta\alpha') [(m_\alpha + m_{\alpha'}) G_2^{(\gamma)} - (m_\alpha - m_{\alpha'}) G_3^{(\gamma)}], \quad (27)$$

$$g_3^{\alpha'\beta\alpha} = \frac{\phi_A^{\alpha'\alpha}}{2(m_\alpha m_{\alpha'})^{5/2}} \sum_{\gamma=1,2} C(\gamma; \alpha\beta\alpha') [-(m_\alpha - m_{\alpha'}) G_2^{(\gamma)} + (m_\alpha + m_{\alpha'}) G_3^{(\gamma)}]. \quad (28)$$

We may think of equations (23) through (28) as giving a parametrization of the form factors $f_i^{\alpha'\beta\alpha}$ and $g_i^{\alpha'\beta\alpha}$ in terms of the corresponding SU(3)-invariant form factors.

In order to reduce the number of free parameters in the expression for the hadronic matrix element we will make two further assumptions, to be made more precise below:

(1) The electromagnetic current density is identified with the component $v_\mu^Q = v_\mu^0 + (1/\sqrt{3})v_\mu^8$ [generalized CVC (conservation of vector current)].

(2) There are no second-class contributions.

The first assumption leads to

$$F_1^{(\gamma=2)}(0) = 0 \quad (29)$$

and, from (21),

$$F_1^{pn}(0) \cong \frac{1}{\sqrt{6}} F_1^{(\gamma=1)}(0), \quad (30)$$

where $F_1^{pn}(\hat{q}^2)$ is the isovector nucleon electromagnetic form factor and the $1/\sqrt{6}$ factor is a Clebsch-Gordan coefficient.¹² Also

$$F_2^{(\gamma=1)}(0) = \left(\frac{\mu_p}{2} + \frac{\mu_n}{4} \right) F_1^{(\gamma=1)}(0), \quad (31)$$

$$F_2^{(\gamma=2)}(0) = \frac{\mu_n \sqrt{5}}{4} F_1^{(\gamma=1)}(0), \quad (32)$$

where μ_p and μ_n are the proton and neutron magnetic moments, respectively. The connection between our coupling constant g and the usual vector coupling constant $G_V \cong 1.0025 \times 10^{-5} m_p^{-2}$ for beta decay is found to be

$$|G_V|^2 = \frac{|\phi_V^{pn}|^2}{(m_n m_p)^3} g^2 |F_1^{pn}(0)|^2. \quad (33)$$

We use the normalization $F_1^{pn}(0) = 1$, so the value and dimension of g^2 will depend upon ϕ_V^{pn} . For example, we shall consider a case when $\phi_V^{pn} = (m_n m_p)^{3/2}$ and then we find $g^2 = |G_V|^2$.

The second assumption, concerning the absence of second-class contributions, leads, by the same arguments as in the conventional derivation of the consequences of only first-class currents, to¹³

$$\begin{aligned} \text{Im} F_1^{(\gamma)}(\hat{q}^2) &= \text{Im} F_2^{(\gamma)}(\hat{q}^2) = 0, \\ F_3^{(\gamma)}(\hat{q}^2) &= 0, \end{aligned} \quad (34)$$

and

$$\begin{aligned} \text{Im} G_1^{(\gamma)}(\hat{q}^2) &= \text{Im} G_3^{(\gamma)}(\hat{q}^2) = 0, \\ G_2^{(\gamma)}(\hat{q}^2) &= 0. \end{aligned} \quad (35)$$

Clearly, in the case that SU(3) is a symmetry group the same line of arguments leads to the vanishing of the second-class-current form factors $f_3^{\alpha'\beta\alpha}$ and $g_2^{\alpha'\beta\alpha}$. For this reason we shall refer to the conditions (34) and (35) as the absence of terms which are "second class with regard to the spectrum-generating SU(3)."

Combining (27) and (35) we find

$$g_2^{\alpha'\beta\alpha} = - \frac{\phi_A^{\alpha'\alpha}}{2(m_\alpha m_{\alpha'})^{5/2}} \sum_{\gamma=1,2} C(\gamma; \alpha\beta\alpha') (m_\alpha - m_{\alpha'}) G_3^{(\gamma)}. \quad (36)$$

Thus we note that an ordinary pseudotensor term appears even when there is no term which is second class with regard to the spectrum generating SU(3). This pseudotensor term is proportional to the mass difference.

Finally, we emphasize that the functions $\phi_V^{\alpha'\alpha}$ and $\phi_A^{\alpha'\alpha}$ are unknown functions which will be determined by comparison with experiments. One contribution to them, as discussed above, could be the SU(3) properties of the transition operators. In addition, for example, the inclusion of a linear \hat{q}^2 dependence of the form factors $F_i(\hat{q}^2)$ and $G_i(\hat{q}^2)$ would induce an overall mass factor which would contribute to the functions $\phi_V^{\alpha'\alpha}$ and $\phi_A^{\alpha'\alpha}$.¹⁴

IV. COMPARISON WITH EXPERIMENTAL DATA

To test the ideas outlined in the previous section we shall undertake to compare the predictions of the model with existing experimental data on semi-leptonic decays of hyperons. One natural choice would be to assume that V_μ^β and A_μ^β are octet operators so that $\phi_V^{\alpha'\alpha} = \phi_A^{\alpha'\alpha} = 1$. However, this choice can be discarded because it gives a very poor fit to the data. Since the usual Cabibbo suppression has given good results in the past, another natural choice would be to use those values of the functions which reproduce the Cabibbo suppression. These are

$$\phi_V^{\alpha'\alpha} = \phi_A^{\alpha'\alpha} = (m_\alpha m_{\alpha'})^{3/2} \times \begin{cases} \cos\theta_C & \text{for } \Delta Y = 0, \\ \sin\theta_C & \text{for } \Delta Y = 1. \end{cases}$$

This is a model in which the symmetry breaking in the "currents" is still described by one Cabibbo angle θ_C , but in which a pseudotensor term, (36), appears as a consequence of assumption (11).

Finally, we have made use of the generalized CVC hypothesis and the assumption that there are no second-class currents with respect to the spectrum-generating SU(3) as discussed in Sec. III. This leaves us with four unknown real form factors, which will be taken to be constants over the range of q^2 values of these decays. Thus we have

θ_C , $G_1^{(1)}$, $G_1^{(2)}$, $G_3^{(1)}$, and $G_3^{(2)}$ as fitting parameters. The results representing the best fit for our model with the above assumption for the ϕ_V and ϕ_A functions are given in Tables I and II where the experimental values for the relevant decay data¹⁵ are also quoted. For comparison we have given the best fits for the conventional Cabibbo model (allowing for q^2 dependence in the form factors) and a modified Cabibbo model with $G_3^{(1)} = G_3^{(2)} = 0$ and constant form factors. In the latter case, the symmetry-breaking corrections enter through the use of the physical masses and an arbitrary choice for the normalization of the weak magnetism and the pseudotensor form factors. As seen, this freedom does not suffice to give a better fit than the conventional Cabibbo model. The confidence level is found to be less than 2%. With the restrictions on $G_3^{(1)}$ and $G_3^{(2)}$ lifted, one achieves a substantial improvement, as in our model reaching a confidence level of about 20%. The main difference lies in the different predictions for the electron asymmetry parameter and the electron-neutrino correlation coefficient in the decay $\Sigma^- \rightarrow ne\nu$.

In Table II we have listed the values obtained for some of the form factors which appear in the analysis. We note that the induced pseudotensor form factors in general are comparable to the corresponding values for g_1 . The decay $\Sigma^- \rightarrow ne\nu$ is an exception with g_2 quite large. For the neutron de-

TABLE I. Experimental data (from Ref. 15) and predictions for the semileptonic decays of hyperons; A: the model of this paper with constant form factors; B: modified Cabibbo model with constant form factors and with $G_3^{(1)} = G_3^{(2)} = 0$; C: conventional Cabibbo model with q^2 -dependent form factors. All transition rates are in 10^6 sec^{-1} except for neutron decay, which is in 10^{-3} sec^{-1} .

Process	Experimental value	A	B	C
$n \rightarrow pe\nu$ (rate)	1.089 ± 0.017	1.066	1.066	1.062
$\Lambda \rightarrow pe\nu$ (rate)	3.169 ± 0.104	3.181	3.150	3.213
$\Sigma^+ \rightarrow \Lambda e\nu$ (rate)	0.252 ± 0.059	0.282	0.296	0.290
$\Sigma^- \rightarrow \Lambda e\nu$ (rate)	0.407 ± 0.040	0.470	0.489	0.481
$\Sigma^- \rightarrow ne\nu$ (rate)	7.301 ± 0.270	7.288	7.283	7.050
$\Xi^- \rightarrow \Lambda e\nu$ (rate)	6.928 ± 5.404	3.171	2.787	2.870
$\Xi^- \rightarrow \Lambda e\nu$ (rate) } $\Xi^- \rightarrow \Sigma^0 e\nu$ (rate) }	$3.735^{+1.206}_{-1.808}$	3.692	3.324	3.394
$\Lambda \rightarrow p\mu\nu$ (rate)	0.643 ± 0.138	0.511	0.506	0.531
$\Sigma^- \rightarrow n\mu\nu$ (rate)	3.012 ± 0.289	2.945	3.233	3.285
$np \alpha_{e\nu}$	-0.095 ± 0.028	-0.110	-0.110	-0.110
$np \alpha_e$	-0.116 ± 0.007	-0.124	-0.124	-0.122
$np \alpha_\nu$	1.001 ± 0.038	0.986	0.986	0.987
$\Sigma^\pm \Lambda \alpha_{e\nu}$	-0.40 ± 0.18	-0.505	-0.427	-0.437
$\Sigma^- n \alpha_{e\nu}$	0.284 ± 0.041	0.302	0.358	0.289
$\Sigma^- n \alpha_e$	0.04 ± 0.27	-0.127	-0.688	-0.736
$\Lambda p \alpha_e$	0.134 ± 0.064	0.012	0.013	0.019
$\Lambda p \alpha_{e\nu}$	0.007 ± 0.037	0.001	0.003	-0.0353
$\Lambda p \alpha_\nu$	0.839 ± 0.064	0.972	0.972	0.994
$\Lambda p \alpha_p$	-0.526 ± 0.070	-0.575	-0.576	-0.602

TABLE II. Values of the parameters and the form factors obtained from the fits of Table I. The confidence level of fit A is 20% and that of fit B is about 1%.

Form factor	process	A	B	C
$G_1^{(F)}$...	1.091	1.093	$g_1^{(F)} = 1.114$
$G_1^{(D)}$...	-1.535	-1.534	$g_1^{(D)} = -1.510$
$G_3^{(F)}$...	-21.864	0	...
$G_3^{(D)}$...	-50.254	0	...
g_1	$n \rightarrow p e \nu$	1.287	1.286	1.282
	$\Lambda \rightarrow p e \nu$	-0.884	-0.889	-0.895
	$\Sigma^- \rightarrow \Lambda e \nu$	0.635	0.686	0.675
	$\Sigma^+ \rightarrow \Lambda e \nu$	0.639	0.686	0.675
	$\Sigma^- \rightarrow n e \nu$	-0.680	0.394	0.372
	$\Xi^- \rightarrow \Lambda e \nu$	0.521	0.204	0.220
	$\Xi^- \rightarrow \Sigma^0 e \nu$	0.840	0.910	0.906
g_2	$n \rightarrow p e \nu$	-0.012	0	0
	$\Lambda \rightarrow p e \nu$	0.029	0	0
	$\Sigma^- \rightarrow \Lambda e \nu$	-0.823	0	0
	$\Sigma^+ \rightarrow \Lambda e \nu$	-0.743	0	0
	$\Sigma^- \rightarrow n e \nu$	-5.002	0	0
	$\Xi^- \rightarrow \Lambda e \nu$	2.045	0	0
	$\Xi^- \rightarrow \Sigma^0 e \nu$	-0.711	0	0
g_3	$\Lambda \rightarrow p e \nu$	0.334	0	-6.473
	$\Sigma^- \rightarrow n e \nu$	-41.457	0	2.802
θ_C	...	0.236	0.234	0.230
n_D	...	14	16	16
χ^2	...	18.5	34.0	31.3

we note that g_2 comes out quite small. It is well below the upper bound given by Wilkinson.¹⁶

The magnitudes obtained for the SU(3)-invariant form factors $G_3^{(1)}$ and $G_3^{(2)}$ seem quite large. How-

ever, there is no *a priori* reason why they could not be large. The resulting values for the induced pseudotensor form factors do not seem unreasonable. The size of g_3 in the Σ decay is striking, but it is a quantity that cannot easily be experimentally determined.

V. CONCLUSION

The treatment of semileptonic hyperon decays proposed here gives a significantly better fit to the available data than does the conventional Cabibbo model. Since the differences in the predictions are most evident in the $\Sigma^- \rightarrow n e \nu$ data, these results should provide strong encouragement for further experimental work on this decay. On the other hand, if the present results are even approximately correct, then the conventional symmetry-breaking calculations are inadequate to explain the data. The present results instead provide encouragement for further study of the consequences of the assumptions made in this investigation.

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¹⁰We use the Cartan notation, which differs from that conventionally used in physics literature. The connection is

$$V_\mu^{\pm 1} \sim v_{1\mu} \pm i v_{2\mu}, \quad A_\mu^{\pm 1} \sim a_{1\mu} \pm i a_{2\mu},$$

$$V_\mu^{\pm 2} \sim v_{4\mu} \pm i v_{5\mu}, \quad A_\mu^{\pm 2} \sim a_{4\mu} \pm i a_{5\mu},$$

etc. The electromagnetic current operator in the Cartan notation is given by

$$V_\mu^Q = V_\mu^0 + \frac{1}{\sqrt{3}} V_\mu^8.$$

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