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Decay of the neutral intermediate boson*

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The decay of the neutral intermediate Z boson into various channels—leptonic and hadronic—is calculated in a general way. Specification to various models (e.g., the Salam-Weinberg model) is done. The leptonic decay of the Z boson is of the same order of magnitude as the hadronic decay.

I. INTRODUCTION

The question whether weak interactions are mediated by an intermediate boson is of fundamental importance. Hence a great effort has already been devoted toward establishing at least lower bounds to the mass of this particle. This can easily be confirmed from an inspection of the proceedings of any recent conference pertinent to this subject. The best way to study this question is by a direct production experiment. Since only the decay products can be observed, a knowledge of the decay process of intermediate bosons is of utmost importance. In particular, leptonic branching ratios should be known because most experiments aim for a detection of the leptonic decay mode of those particles.

For the charged bosons, these questions have been studied in the literature.[1] After the discovery of neutral weak currents, the same problem arose for the neutral intermediate boson[8]. An experimental search should not be biased by theoretical ideas. While this request cannot possibly be completely fulfilled, we shall try to stay away from particular model assumptions as much as possible. In the case of leptonic decay channels, which we shall discuss in Sec. II, the result will depend on the particular gauge model. Hence we shall compare various of these models. Hadronic decays are dealt with in Sec. III. We shall leave the neutral hadronic weak current in a general parametrized form and again compare the results of various specifications of these parameters. The numerical discussion of our results is presented in Sec. IV.

II. LEPTONIC DECAYS

Consider a general two-body decay of a neutral intermediate boson Z,

$$Z(k)\to a(q_1)+\bar{b}(q_2), \qquad (1)$$

where the four-momenta of the particles have been given in the parentheses. The decay rate for reaction (1) can be written as[2]

$$\Gamma(W,a\bar{b})=\frac{g^2}{48\pi M_Z^{3}}\left[\frac{A}{M_Z^{2}}\,|w(M_Z^{2},m_a^{2},m_b^{2})|^2 - 3Bw(M_Z^{2},m_a^{2},m_b^{2})\right], \qquad (2)$$

where w is the familiar kinematical function

$$w(a,b,c)=[a^2+b^2+c^2 - 2(ab+ac+bc)]^{1/2}, \qquad (3)$$

and A and B are coefficients in the following decomposition of the T-matrix element:

$$\langle a\bar{b}|T|Z\rangle = g\epsilon_\lambda M^\lambda, \qquad (4)$$

$$\sum_{\text{spins}} M_\lambda M_\nu^\dagger = (q_1-q_2)_\lambda (q_1-q_2)_\nu A + g_{\lambda\nu}B + \text{other terms}. \qquad (5)$$

ϵ_λ is the polarization vector of the Z boson.

The part of the Lagrangian describing the weak neutral-current interaction of the "classical" leptons μ, e, ν_μ, and ν_e shall be written in the form

$$\mathfrak{L}_1 = gZ^\lambda \sum_l \left[\bar{\nu}_l\gamma_\lambda(1+\gamma_5)\nu_l + \bar{l}\gamma_\lambda(C_V+C_A\gamma_5)l\right]. \qquad (6)$$

(Notation and definition follow Ref. 2.) Notice in passing that we have assumed μ-e universality.

From (6), the decomposition (5) is readily achieved and yields

$$A(Z\to\nu\bar{\nu})=-4, \qquad (7a)$$

$$B(Z\to\nu\bar{\nu})=-4M_Z^{2}, \qquad (7b)$$

and

$$A(Z\to l\bar{l})=-2(C_V^{2}+C_A^{2}), \qquad (8a)$$

$$B(Z\to l\bar{l})=-2(C_V^{2}+C_A^{2})(M_Z^{2}-2m_l^{2}) - 4(C_V^{2}-C_A^{2})m_l^{2}. \qquad (8b)$$

Insertion in Eq. (2) gives the decay rates

$$\Gamma(Z\to\nu\bar{\nu})=\frac{g^2M_Z}{6\pi} \tag{9}$$

and

$$\Gamma(Z\to l\bar{l})=\frac{g^2M_Z}{24\pi}\left[3\left(C_V{}^2+C_A{}^2-\frac{4m_l{}^2}{M_Z{}^2}C_A{}^2\right)\left(1-\frac{4m_l{}^2}{M_Z{}^2}\right)^{1/2} -(C_V{}^2+C_A{}^2)\left(1-\frac{4m_l{}^2}{M_Z{}^2}\right)^{3/2}\right]. \tag{10}$$

In Eq. (10), we have kept the lepton mass because the same result obviously also holds for possible heavy sequential leptons with a suitable adjustment of C_V and C_A.

In order to estimate leptonic decay widths numerically, we define

$$\frac{G}{\sqrt{2}}=\frac{g^2}{M_Z{}^2}c, \tag{11}$$

where c is to be specified in each particular model. In the Salam-Weinberg model,[3] we have

$c=2$ (Salam-Weinberg).

Numerically, the decay width of the Z boson into a particular type of neutrino pair is

$$\Gamma(Z\to\nu\bar{\nu})\ (\text{MeV})=\frac{0.44}{c}\times10^{-3}M_Z{}^3, \tag{12}$$

where M_Z is given in GeV. Thus for a neutral intermediate boson of 75 GeV in the Salam-Weinberg model, we obtain a partial width of about 93 MeV for each kind of neutrino pair. Since this is a rather appreciable width, it opens a possibility—at least in principle—to obtain an indication of the total number of different types of neutrinos (plus light neutral leptons) by subtracting the total hadronic width and the width for decay into charged lepton pairs plus heavy neutral leptons from the total decay width. The difference is the product of Eq. (13) with the total number of different types of neutrinos (plus light neutral leptons) if we suppose that all of them are coupled to the Z boson with the same strength.

The decay width into charged leptons also depends on the relative values of the coupling constants. For various choices, numerical results are collected in Table I. We see that these widths are considerably large, owing to the third power of M_Z, which implies that the integrated cross section for the process $e^+e^-\to$ all is proportional to M_Z,

$$A=\int_{Z\text{ peak}}\sigma(e^+e^-\to\text{all})\,dE_{\text{c.m.}} =\frac{6\pi^2}{M_Z{}^2}\Gamma(Z\to e^+e^-) =\frac{G}{\sqrt{2}}\frac{\pi}{c}M_Z\tfrac{1}{2}(C_V{}^2+C_A{}^2). \tag{13}$$

For a Salam-Weinberg Z boson we get an area under the Z peak of 220 nb GeV. This is by a factor 22 more than the area under the $\psi(3.1)$ peak. Thus the Z boson will show up in e^+e^- collisions very spectacularly (if it exists) and cannot be overlooked.[7]

III. THE TOTAL HADRONIC WIDTH

The total hadronic decay width of the Z boson can be written as

$$\Gamma(Z\to\text{hadrons})=\frac{\pi g^2}{3M_Z}\left(g^{\lambda\nu}-\frac{k^\lambda k^\nu}{M_Z{}^2}\right)\rho_{\lambda\nu}(k), \tag{14}$$

with

$$\rho_{\lambda\nu}(k)=-(2\pi)^3\sum_n\delta^{(4)}(k-k')\langle0|j_\lambda(0)|n(k')\rangle\times\langle n(k')|j_\nu(0)|0\rangle =\left(g_{\lambda\nu}-\frac{k_\lambda k_\nu}{k^2}\right)\rho(k^2)+\frac{k_\lambda k_\nu}{k^2}\rho'(k^2), \tag{15}$$

where k_λ is again the four-momentum of the Z boson. $j_\lambda(x)$ is the neutral hadronic current coupled to the Z boson with strength g.

Insertion of Eq. (15) into Eq. (14) gives the total hadronic decay width in compact form:

$$\Gamma(Z\to\text{hadrons})=\frac{\pi g^2}{M_Z}\rho(M_Z{}^2). \tag{16}$$

In order to arrive at a relation to experimentally accessible quantities, we have to decompose the neutral hadronic current[4]

TABLE I. Decay width of Z boson into various channels, for $M_Z=75$ GeV, $m_E=5$ GeV.

	Γ (MeV) "Naive IVB theory", $c=\frac{1}{2}$	Γ (MeV) "Salam-Weinberg", $c=2$
$Z\to\nu\bar{\nu}$	371	93
$Z\to l^-l^+$	371	54
$Z\to E^-E^+, E^0\bar{E}^0$	366	...

$$j_\mu = c_1 V_\mu^1 + c_2 V_\mu^0 + c_3 A_\mu^1 + c_4 A_\mu^0, \tag{17}$$

where the superscripts on vector currents V_μ and axial-vector currents A_μ denote their isospin properties. We shall assume henceforth that the vector currents can be identified with their electromagnetic counterparts, and that the isovector axial-vector current is the isospin-rotated partner of the charged hadronic axial-vector current.

In a large class of models, the number of parameters can be reduced[4] by assuming that

$$\begin{aligned} c_1 &= \kappa + \lambda, \quad c_2 = \kappa \\ c_3 &= \lambda, \quad c_4 = 0. \end{aligned} \tag{18}$$

In this case, the neutral hadronic current can be written as

$$j_\mu = \kappa j_\mu^{\text{em}} + \lambda j_\mu^{\text{weak}}, \tag{19}$$

where j_μ^{weak} is the isospin-rotated charged hadronic current.

Let us first consider the simpler case of Eq. (19). In this case, the spectral function of Eq. (16) becomes

$$\rho(s) = \kappa^2 \rho^{\text{em}}(s) + \lambda^2 \rho^{\text{weak}}(s) + 2\kappa\lambda\rho^{\text{int}}(s). \tag{20}$$

The first term can be related to the familiar ratio R by

$$\begin{aligned} R &= \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \\ &= \frac{12\pi^2}{s}\,\rho^{\text{em}}(s) + O\left(\frac{m_\mu^{\,2}}{s}\right), \end{aligned} \tag{21}$$

where s is—as usual—the square of the total center-of-mass energy.

By an isospin rotation, the second term in Eq. (20) can be related to the hadronic decay width of the charged intermediate boson. In so doing, we have to assume that $\rho(s)$ goes linearly with s at large s; this is equivalent to assuming constant R at large s, which is also borne out from current-algebra considerations.[1]

The third term in Eq. (20) is the "interference" term where both j_μ^{em} and j_μ^{weak} enter in the matrix elements of Eq. (15). Obviously, only V_μ^1 will survive and contribute to ρ^{int}. Hence it can be related to the cross section of electron-positron annihilation into hadrons of total isospin 1.

Let us define

$$R^{I=1} = \frac{\sigma(e^+e^- \to \text{hadrons},\ I=1)}{\sigma(e^+e^- \to \mu^+\mu^-)}\ . \tag{22}$$

We can now write the hadronic width of the Z boson as

$$\begin{aligned} \Gamma(Z^0 \to \text{hadrons}) &= \lambda^2\left(\frac{M_Z}{M_W}\right)^3 \Gamma(W \to \text{hadrons}) \\ &\quad + \frac{G}{\sqrt{2}}\,\frac{M_Z^{\,3}}{6\pi}\left[\kappa^2 R(M_Z) + 2\kappa\lambda R^{I=1}(M_Z)\right]. \end{aligned} \tag{23}$$

In the Salam-Weinberg model[3] ($\kappa = -2\sin^2\theta_W$, $\lambda = 1$), Eq. (23) becomes

$$\begin{aligned} \Gamma_{\text{SW}}(Z^0 \to \text{hadrons}) &= \frac{1}{\cos^3\theta_W}\,\Gamma(W \to \text{hadrons}) \\ &\quad + \frac{G}{\sqrt{2}}\,\frac{2M_Z^{\,3}}{3\pi}\,\sin^2\theta_W\left[\sin^2\theta_W R(M_Z) - R^{I=1}(M_Z)\right]. \end{aligned} \tag{24}$$

In order to facilitate our expression (23) further, let us assume that the difference between axial-vector and vector spectral functions can be neglected at high energy.[1,5] Under this assumption, the total hadronic width of the Z boson becomes

$$\Gamma(Z^0 \to \text{hadrons}) = \frac{G}{\sqrt{2}}\,\frac{M_Z^{\,3}}{6\pi}\,kR(M_Z), \tag{25}$$

with

$$k = \kappa^2 + \frac{2(\kappa\lambda + \lambda^2)}{1 + a(M_Z)}. \tag{26}$$

a is the ratio of isospin-0 to isospin-1 final states of electron-positron annihilation into hadrons, i.e.,

$$a = \frac{R^{I=0}}{R^{I=1}}\ . \tag{27}$$

In the Salam-Weinberg model, k becomes

$$k_{\text{SW}} = \frac{2 - 4\sin^2\theta_W}{1 + a(M_Z)} + 4\sin^4\theta_W. \tag{28}$$

If we retain all four parameters in the neutral current as in Eq. (17), the width can be expressed again by Eq. (25) if we replace k by

$$k \to \frac{c_1^{\,2} + c_3^{\,2} + a(c_2^{\,2} + c_4^{\,2})}{1 + a}. \tag{29}$$

To compare our results with the numerical prediction of Eq. (13), let us write

$$\Gamma(Z^0 \to \text{hadrons})\ (\text{MeV}) = 0.44 \times 10^{-3} \times kR(M_Z)M_Z^{\,3} \tag{30}$$

where M_Z is in GeV. It is seen that the total hadronic decay width is of the same order of magnitude as the partial width for decay into neutrino pairs.

Several exclusive hadronic decay channels can easily be calculated with the formalism outlined in Sec. III (for the explicit formulas see the Appendix). However, owing to form-factor effects and to the large mass of the Z boson, each of these channels contributes a very small partial width and is therefore of no practical interest.

IV. DISCUSSION OF RESULTS

For practical purposes, branching ratios are very important. Let us define

$$\frac{\Gamma(Z\to A)}{\Gamma(Z\to A')}=B\left(\frac{A}{A'}\right), \tag{31}$$

so that, for example,

$$B\left(\frac{\nu\bar{\nu}}{\text{hadrons}}\right)=\frac{1}{ckR(M_Z)}, \tag{32a}$$

$$B\left(\frac{l\bar{l}}{\nu\bar{\nu}}\right)=\frac{C_V{}^2+C_A{}^2}{2}+O\left(\frac{m_l{}^2}{M_Z{}^2}\right), \tag{32b}$$

$$B\left(\frac{l\bar{l}}{\text{hadrons}}\right)=\frac{C_V{}^2+C_A{}^2}{2ckR(M_Z)}+O\left(\frac{m_l{}^2}{M_Z{}^2}\right), \tag{32c}$$

$$B\left(\frac{\text{leptons}}{\text{hadrons}}\right)=\frac{C_V{}^2+C_A{}^2+2}{ckR(M_Z)}+O\left(\frac{m_l{}^2}{M_Z{}^2}\right). \tag{32d}$$

"Leptons" in Eq. (32d) refers to the known leptons, i.e. electrons, muons, and the two kinds of neutrinos.

One obstacle to a numerical prediction of branching ratios is the uncertainty of the value of R at large energies. It may very well be that this ratio exhibits one or more new thresholds, thus rising above its present value of 5.2 ± 0.7. Such a threshold may be either due to the appearance of new heavy leptons or the opening of a channel with a new quantum number. In the former case, we are justified to use the value below this threshold because R as defined in Eq. (21) should not include a heavy-lepton channel. In the latter case, we can also use the value of R below the new threshold if we exclude from the total hadronic width all hadrons carrying the new quantum number. We are aware of the fact that this choice is not satisfactory, but in the present situation it is the only way to obtain numerical predictions.

In choosing

$$R_{\text{exp}}=5.2\pm0.7, \tag{33}$$

we should also remember that the possibility of the threshold at about 4 GeV, due to heavy leptons, is not excluded. Thus our predictions are tentative pending an experimental clarification of the situation. However, if changes are necessary, they are trivial.

Let us first turn to the Salam-Weinberg model. The most interesting branching ratio, Eq. (32c), becomes

$$B_{\text{SW}}\left(\frac{l\bar{l}}{\text{hadrons}}\right)=\frac{[1+a(M_Z)](1-4\sin^2\theta_W+8\sin^4\theta_W)}{2R(M_Z)\{1-2\sin^2\theta_W+2\sin^4\theta_W[1+a(M_Z)]\}}. \tag{34}$$

For various values of $a(M_Z)$ and for R given by Eq. (33), this branching ratio is plotted in Fig. 1.

Next, let us try to start from a general, phenomenological level. The analysis of Ref. 4 leads to the following value:

$$k_{\text{exp}}=\frac{0.74}{1+a(M_Z)}+0.21. \tag{35}$$

It is more difficult to restrict C_A and C_V in Eq. (6). By taking the experimental values[6] numerical

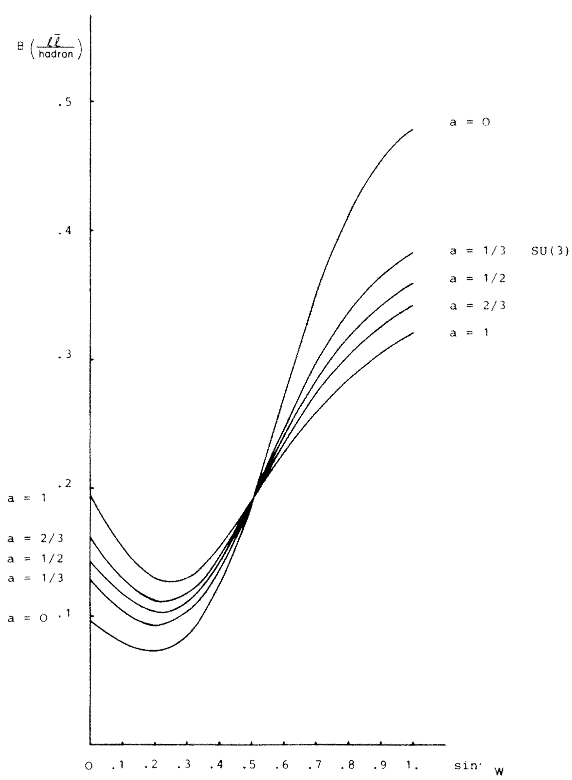

FIG. 1. The branching ratio $B(l\bar{l}/\text{hadron})$ in the Salam-Weinberg model (34) is plotted for several values of $a=R^{I=0}/R^{I=1}$.

TABLE II. Numerical predictions of the branching ratio (32c) for various values of $a(M_Z)$.

	$a=\frac{R^{I=0}}{R^{I=1}}$	$B\left(\frac{l\bar{l}}{\text{hadrons}}\right)$
	0	$0.145\le B\le 0.405$
SU(3)	$\frac{1}{3}$	$0.180\le B\le 0.503$
	$\frac{1}{2}$	$0.195\le B\le 0.547$
	$\frac{2}{3}$	$0.210\le B\le 0.588$
	1	$0.237\le B\le 0.663$

predictions of the branching ratio (32c) for various values of $a(M_Z)$ can be made, and they are collected in Table II.

As a general rule we can say that the decay of the Z boson into muon or electron pairs contributes a considerable branching ratio in all cases. Therefore, searching for the Z boson by looking for charged-lepton pairs is a sensible method.

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APPENDIX

We have

$$\Gamma(Z\to\pi^-\pi^+)=\frac{g^2(\kappa+\lambda)^2}{48\pi}M_Z|F_\pi(M_Z^2)|^2\left(1-\frac{4m_\pi^2}{M_Z^2}\right)^{3/2}, \tag{A1}$$

$$\Gamma(Z\to N\bar{N})=\frac{g^2(\kappa+\lambda)^2}{24\pi}\times M_Z\left\{3[F_1(M_Z^2)+2m_N F_2(M_Z^2)]^2\left(1-\frac{4m_N^2}{M_Z^2}\right)^{1/2}-\left[F_1^2(M_Z^2)-M_Z^2F_2^2(M_Z^2)-\frac{2\lambda^2}{(\kappa+\lambda)^2}G_1^2(M_Z^2)\right]\left(1-\frac{4m_N^2}{M_Z^2}\right)^{3/2}\right\}, \tag{A2}$$

$$\Gamma(Z\to\pi^0\gamma)=\frac{g^2M_Z^3}{24\pi}\left(\frac{\kappa^2a^2}{4}+\lambda^2b^2\right)\left(1-\frac{m_\pi^2}{M_Z^2}\right)^3,$$

where

$$a^2=\frac{64\pi\Gamma(\pi^0\to\gamma\gamma)}{m_\pi^3}\left|\frac{F_{\pi^0\gamma\gamma}(M_Z^2)}{F_{\pi^0\gamma\gamma}(0)}\right|^2,$$

$$a^2\simeq\frac{64\pi\Gamma(\pi^0\to\gamma\gamma)}{m_\pi^3}\frac{m_\rho^4}{M_Z^4},$$

$$b^2=\frac{\pi\alpha f_\pi^2}{2M_Z^4}, \tag{A3}$$

$$f_\pi=0.13\text{ GeV}.$$

We also have

$$\Gamma(Z\to\epsilon\gamma)=\frac{g^2m_\epsilon\Gamma(\epsilon\to\gamma\gamma)}{3M_Z}\left(\kappa+\frac{9\lambda}{9+d}\right)^2\left(2-\frac{m_\epsilon^2}{M_Z^2}-\frac{m_\epsilon^4}{M_Z^4}\right),$$

where

$$d=\frac{g_{\epsilon\omega\omega}}{g_{\epsilon\rho\rho}}. \tag{A4}$$

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