Neutrino oscillations and the number of neutrino types*

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A brief treatment of neutrino oscillations, generalized to an arbitrary number of neutrino types, is given as the basis for design of a feasible experiment to search for neutrino oscillations using the neutrino beam produced at a high-energy proton accelerator.

INTRODUCTION

It is generally assumed that the known neutrinos ν_e and ν_μ (and $\bar{\nu}_e$, $\bar{\nu}_\mu$) have zero mass, although the present experimental limits[1] are only $m(\nu_e)<60$ eV and $m(\nu_\mu)<650$ keV. This physical assumption corresponds to the mathematical assumption that all terms involving neutrino fields ψ_ν in the world Hamiltonian are invariant under the transformation $\psi_\nu \to \gamma_5\psi_\nu$.

There are a number of reasons to question the validity of rigorous γ_5 invariance in addition to the loose experimental limits on $m(\nu_e)$ and $m(\nu_\mu)$. Even in the purely leptonic reaction $\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$, the γ_5 invariance of the leptonic weak currents is experimentally verified (e.g., by helicity measurements) to no better than 1%, i.e.,

$$l_\alpha = \psi_\mu^\dagger \gamma_4\gamma_\alpha[(1+\gamma_5)+\epsilon(1-\gamma_5)]\psi_{\nu_\mu}$$

with $|\epsilon|^2<0.01$. In addition, there is a hint in the analysis of inelastic neutrino-nucleon scattering[2] that the hadronic weak current may have a right-handed part, i.e., $\psi_{q_2}^\dagger\gamma_4\gamma_\alpha(1-\gamma_5)\psi_{q_1}$, where ψ_{q_1} and ψ_{q_2} are quark fields; if this is true, the introduction of the $\epsilon\psi_\mu^\dagger\gamma_4\gamma_\alpha(1-\gamma_5)\psi_{\nu_\mu}$ term into l_α is perhaps more plausible. More generally, the principle of γ_5 invariance is at best significantly more restricted than the analogous principle of electromagnetic gauge invariance. Thus, electromagnetic gauge invariance, which ensures both zero mass of the photon and conservation of electric charge, applies to all parts of the world Hamiltonian. But, as we have noted, γ_5 invariance is thought to apply only to those parts involving neutrinos, and is in fact further implemented by the assumption that no terms exist in the world Hamiltonian which fail to preserve *separately* the conservation of muon lepton number and electron lepton number. In the same vein, since there are at least two nominally independent types of neutrinos, ν_e and ν_μ, with associated charged leptons of very different mass, a situation in which both types of neutrinos have zero mass (and are therefore distinguished from each other only by their leptonic quantum numbers) suggests either a remarkable coincidence or some much higher symmetry in nature than we have heretofore encountered.[3] In this conjectured "supersymmetry," zero-mass particles might form a unique family, independent of their boson or fermion properties, distinct from the conventional separate families of bosons and fermions all with nonzero mass.

Apart from the implications of nonzero neutrino mass for particle physics and field theory, there would also be significant repercussions in astrophysics and cosmology, if only because the total number of neutrinos in the universe is expected to be large.[4] There is no reliable theoretical guide to the region in which to look for nonzero neutrino mass, but the importance of the question leads us to speculate on how to measure very small neutrino masses, say, $\lesssim 10^{-3}$ times the present limit on $m(\nu_e)$. Even a mass as small as 10^{-1} to 10^{-2} eV would be significant because it would point to an interaction responsible for that mass value and would possibly also signal a violation of the separate conservation of muon lepton number and electron lepton number.

One possibility with regard to such a measurement lies in the suggestion of Pontecorvo[5] that neutrinos of different types may exhibit oscillations of type as a function of time, similar to the oscillations between K^0 (strangeness $=+1$) and $\bar{K}^0$ (strangeness $=-1$). Neutrino oscillations were discussed initially by Pontecorvo,[5] and by Gribov and Pontecorvo,[6] and more recently by Pakvasa and Tennakone,[7] by Eliezer and Swift,[8] and by Fritzsch and Minkowski[9]; in addition, a general review of the subject has just been given by Bilenky and Pontecorvo.[10] Here we wish to consider a realistic experiment to search for neutrino oscillations using a neutrino beam produced by the protons from a high-energy proton accelerator. This experiment, if it were to observe neutrino oscillations, would lead immediately to three important results: (i) The mass of at least one of the neutrino types would be nonzero, (ii) the separate conservation of muon lepton number and electron lepton number would not hold, and (iii) the total number of neutrino types would be determined.

It is perhaps worth mentioning again[5] that any actual neutrino-oscillation phenomenon, e.g., $\nu_\mu \leftrightarrow \nu_e$, while undoubtedly associated with an oscillation length much longer than that between K^0 and $\bar{K}^0$, might conceivably provide another means of observing a CP violation and thus allow a new attack on that fundamental problem. Furthermore, the question of a neutrino with relatively large mass, capable of decaying to another (lighter) neutrino[11] and (for example) a photon, will be addressed by the experiment considered here.

THEORY

In this section we give a brief account of the theory of neutrino oscillations. Our treatment, while generally similar to that previously developed in Refs. 5–10, does introduce several modifications and also discusses the case of three neutrino types explicitly. In addition, we present some speculations on the lower limit for the oscillation length based on the experimental upper limit on the branching ratio for the decay $\mu^+ \to e^+ + \gamma$.

Let $|\nu_\xi\rangle$ denote a state with definite momentum $\vec{p}_\nu$ occupied by a single neutrino of type ξ; here $\nu_\xi = \nu_e$ or ν_μ or ν_L or $\cdots$, so that the states $|\nu_\xi\rangle$ are produced in the various known weak-interaction processes, e.g., $\pi^+ \to \mu^+ + \nu_\mu$. Further, let the states $|\nu_j\rangle$ $(j = 1, 2, 3, \ldots)$ be such linear combinations of the states $|\nu_\xi\rangle$ that

$$H|\nu_j\rangle = (H^{(0)} + H^{(1)})|\nu_j\rangle = E_j|\nu_j\rangle , \qquad (1)$$

where H is the Hamiltonian of the world and $H^{(1)}$ is the part of H which is responsible for the neutrino oscillations. Since $H^{(0)} = H - H^{(1)}$ is invariant under $\psi_{\nu_\xi} \to \gamma_5 \psi_{\nu_\xi}$ and conserves muon lepton number and electron lepton number *separately*, we have $\langle\nu_\xi|H^{(0)}|\nu_\xi\rangle = p_\nu$ and $\langle\nu_\xi|H^{(0)}|\nu_\eta\rangle = 0$. On the other hand, we shall assume that $\langle\nu_\xi|H^{(1)}|\nu_\xi\rangle$ is greater than zero and $\langle\nu_\xi|H^{(1)}|\nu_\eta\rangle$ is different from zero so that we shall consider situations where $\nu_e, \nu_\mu, \nu_L, \ldots$ are endowed with mass and where *separate* conservation of muon and electron lepton number does not hold. As we shall see, both of these conditions must be satisfied if neutrino oscillations are to occur.

To proceed, we write

$$\begin{aligned}
|\nu_j\rangle &= \sum_\xi |\nu_\xi\rangle\langle\nu_\xi|\nu_j\rangle , \\
|\nu_\eta\rangle &= \sum_k |\nu_k\rangle\langle\nu_k|\nu_\eta\rangle \qquad (2) \\
&= \sum_k |\nu_k\rangle\langle\nu_\eta|\nu_k\rangle^* ,
\end{aligned}$$

whence

$$\begin{aligned}
\langle E\rangle_\xi &= \langle\nu_\xi|(H^{(0)} + H^{(1)})|\nu_\xi\rangle \\
&= \sum_j |\langle\nu_j|\nu_\xi\rangle|^2 E_j , \\
\langle E\rangle_\xi &= p_\nu + \langle\nu_\xi|H^{(1)}|\nu_\xi\rangle , \\
m(\nu_\xi) &\equiv \{\langle E\rangle_\xi\}_{p_\nu=0} \\
&= \langle\nu_\xi|H^{(1)}|\nu_\xi\rangle_{p_\nu=0} , \qquad (3a) \\
E_j &= \langle\nu_j|(H^{(0)} + H^{(1)})|\nu_j\rangle \\
&= \sum_\xi |\langle\nu_\xi|\nu_j\rangle|^2\langle E\rangle_\xi \\
&\quad + \sum_{\xi,\eta}{}' \langle\nu_\xi|\nu_j\rangle^*\langle\nu_\eta|\nu_j\rangle\langle\nu_\xi|H^{(1)}|\nu_\eta\rangle , \\
E_j &= \{p_\nu^2 + [m(\nu_j)]^2\}^{1/2} \\
&= p_\nu + \langle\nu_j|H^{(1)}|\nu_j\rangle , \qquad (3b)
\end{aligned}$$

$$\sum_\xi \langle E\rangle_\xi = \sum_j E_j , \qquad (3c)$$

and

$$\begin{aligned}
P(\nu_\eta; t|\nu_\xi; 0) &= |\langle\nu_\eta|e^{-itH}\nu_\xi\rangle|^2 \\
&= \sum_j |\langle\nu_\eta|\nu_j\rangle|^2 |\langle\nu_j|\nu_\xi\rangle|^2 \\
&\quad + \sum_{j,k}{}' \langle\nu_\eta|\nu_j\rangle\langle\nu_\eta|\nu_k\rangle^*\langle\nu_j|\nu_\xi\rangle \\
&\quad \times\langle\nu_k|\nu_\xi\rangle^* e^{i(E_k - E_j)t} ,
\end{aligned}$$

$$\langle P(\nu_\eta; t|\nu_\xi; 0)\rangle_{\text{time average}} = \sum_j |\langle\nu_\eta|\nu_j\rangle|^2 |\langle\nu_j|\nu_\xi\rangle|^2 , \qquad (4)$$

where $P(\nu_\eta; t|\nu_\xi; 0)$ is the probability of finding a neutrino of type η at time t in a physical situation where a neutrino of type ξ is present initially. We see from Eq. (3c) that if each of the $m(\nu_\xi)$ vanishes, each of the $m(\nu_j)$ will vanish as well; under these circumstances each of the $E_k - E_j$ must also vanish and, as seen from Eq. (4), neutrino oscillations will not occur. Conversely, if neutrino oscillations do occur, at least one of the oscillation frequencies

$$(E_k - E_j) \cong (1/2p_\nu)[m(\nu_k) + m(\nu_j)][m(\nu_k) - m(\nu_j)]$$

must be nonzero, and this implies [again according to Eq. (3c)] that one or more of the $m(\nu_\xi)$ are nonzero. Similarly, if each of the $\langle\nu_\xi|H^{(1)}|\nu_\eta\rangle$ in Eq. (3b) vanishes, $|\nu_\xi\rangle$, $|\nu_\eta\rangle, \ldots$ can be identified with $|\nu_j\rangle$, $|\nu_k\rangle, \ldots$, and again [as seen from Eq. (4)] neutrino oscillations will not occur. Thus, the other necessary condition for the existence of neutrino oscillations is the nonvanishing of at least one of the $\langle \nu_\xi | H^{(1)} | \nu_\eta \rangle$.

We now specialize our results to the cases of two and three neutrino types: $\nu_\xi = \nu_e$ or ν_μ and $\nu_\xi = \nu_e$ or ν_μ or ν_L. Considering first the case of two neutrino types and parametrizing the $\langle \nu_\xi | \nu_j \rangle$ as

$$\begin{pmatrix} \langle \nu_e | \nu_1 \rangle & \langle \nu_e | \nu_2 \rangle \\ \langle \nu_\mu | \nu_1 \rangle & \langle \nu_\mu | \nu_2 \rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}, \tag{5}$$

we have, substituting Eq. (5) into Eqs. (2), (3a), and (3b),

$$\begin{aligned} |\nu_1\rangle &= |\nu_e\rangle \cos\theta + |\nu_\mu\rangle (-\sin\theta), \\ |\nu_2\rangle &= |\nu_e\rangle \sin\theta + |\nu_\mu\rangle \cos\theta, \\ |\nu_e\rangle &= |\nu_1\rangle \cos\theta + |\nu_2\rangle \sin\theta, \\ |\nu_\mu\rangle &= |\nu_1\rangle (-\sin\theta) + |\nu_2\rangle \cos\theta, \end{aligned} \tag{6}$$

$$\begin{aligned} m(\nu_e) &= \tfrac{1}{2}[m(\nu_1) + m(\nu_2)] + \tfrac{1}{2}\cos 2\theta[m(\nu_1) - m(\nu_2)], \\ m(\nu_\mu) &= \tfrac{1}{2}[m(\nu_1) + m(\nu_2)] - \tfrac{1}{2}\cos 2\theta[m(\nu_1) - m(\nu_2)], \\ m(\nu_1) &= \tfrac{1}{2}[m(\nu_e) + m(\nu_\mu)] + \tfrac{1}{2}\cos 2\theta[m(\nu_e) - m(\nu_\mu)] \\ &\quad - \tfrac{1}{2}\sin 2\theta\, m_{e\mu}, \end{aligned} \tag{7a}$$

$$\begin{aligned} m(\nu_2) &= \tfrac{1}{2}[m(\nu_e) + m(\nu_\mu)] - \tfrac{1}{2}\cos 2\theta[m(\nu_e) - m(\nu_\mu)] \\ &\quad + \tfrac{1}{2}\sin 2\theta\, m_{e\mu}, \\ m_{e\mu} &\equiv 2\,\mathrm{Re}\langle \nu_e | H^{(1)} | \nu_\mu \rangle_{p_\nu = 0}, \end{aligned} \tag{7b}$$

while substitution of Eq. (7b) into Eq. (7a) [corresponding in the general case to substitution of Eq. (3b) into Eq. (3a)] yields

$$\tan 2\theta = \frac{m_{e\mu}}{m(\nu_\mu) - m(\nu_e)}. \tag{8}$$

Further, substituting Eq. (5) into Eq. (4), and using Eq. (8), we obtain

$$\begin{aligned} P(\nu_e; t | \nu_\mu; 0) &= \frac{1}{2}\left(\frac{m_{e\mu}^2}{[m(\nu_\mu) - m(\nu_e)]^2 + m_{e\mu}^2}\right) \\ &\quad \times [1 - \cos(E_2 - E_1)t], \\ P(\nu_\mu; t | \nu_\mu; 0) &= 1 - \frac{1}{2}\left(\frac{m_{e\mu}^2}{[m(\nu_\mu) - m(\nu_e)]^2 + m_{e\mu}^2}\right) \\ &\quad \times [1 - \cos(E_2 - E_1)t], \end{aligned} \tag{9}$$

$$\langle P(\nu_e; t | \nu_\mu; 0)\rangle_{\text{time average}} = \frac{1}{2}\left(\frac{m_{e\mu}^2}{[m(\nu_\mu) - m(\nu_e)]^2 + m_{e\mu}^2}\right),$$

$$\langle P(\nu_\mu; t | \nu_\mu; 0\rangle_{\text{time average}} = 1 - \frac{1}{2}\left(\frac{m_{e\mu}^2}{[m(\nu_\mu) - m(\nu_e)]^2 + m_{e\mu}^2}\right),$$

with the oscillation frequency given by Eqs. (7b) and (8), as

$$\begin{aligned} E_2 - E_1 &\cong \frac{1}{2p_\nu}[m(\nu_2) + m(\nu_1)][m(\nu_2) - m(\nu_1)] \\ &= \frac{1}{2p_\nu}[m(\nu_\mu) + m(\nu_e)] \\ &\quad \times \{[m(\nu_\mu) - m(\nu_e)]^2 + m_{e\mu}^2\}^{1/2}. \end{aligned} \tag{10}$$

Equations (9) and (10) show explicitly that $P(\nu_e; t | \nu_\mu; 0) = 0$ and no oscillations occur if $m(\nu_\mu) = m(\nu_e) = 0$, or if $m_{e\mu} = 0$ [with $m(\nu_\mu) \neq 0$ and $m(\nu_e) \neq 0$].

Again, consider the particular values: $\cos\theta = \sin\theta = 1\sqrt{2}$. In this case Eqs. (7a), (7b), (9), and (10) become

$$\begin{aligned} m(\nu_e) &= \tfrac{1}{2}[m(\nu_1) + m(\nu_2)] = m(\nu_\mu), \\ m(\nu_1) &= m(\nu_\mu) - \tfrac{1}{2} m_{e\mu}, \\ m(\nu_2) &= m(\nu_\mu) + \tfrac{1}{2} m_{e\mu}, \\ P(\nu_e; t | \nu_\mu; 0) &= \tfrac{1}{2}[1 - \cos(E_2 - E_1)t], \\ P(\nu_\mu; t | \nu_\mu; 0) &= \tfrac{1}{2}[1 + \cos(E_2 - E_1)t], \\ \langle P(\nu_e; t | \nu_\mu; 0)\rangle_{\text{time average}} &= \langle P(\nu_\mu; t | \nu_\mu; 0)\rangle_{\text{time average}} \\ &= \tfrac{1}{2}, \\ E_2 - E_1 &\simeq \frac{1}{2p_\nu}[m(\nu_2) + m(\nu_1)][m(\nu_2) - m(\nu_1)] \\ &= \frac{1}{2p_\nu}[2m(\nu_\mu)](m_{e\mu}). \end{aligned} \tag{11}$$

Equations (11) describe what might be called the "maximal" oscillation situation in the case of two neutrino types, i. e., the situation in which $\langle P(\nu_e; t | \nu_\mu; 0)\rangle_{\text{time average}}$ has its maximum possible value of $\frac{1}{2}$, i.e., 1/(number of neutrino types).

We consider next the case of three neutrino type types. As an example, we assign numerical values to the $\langle \nu_\xi | \nu_j \rangle$ as follows:

$$\begin{bmatrix} \langle \nu_e | \nu_1 \rangle & \langle \nu_e | \nu_2 \rangle & \langle \nu_e | \nu_3 \rangle \\ \langle \nu_\mu | \nu_1 \rangle & \langle \nu_\mu | \nu_2 \rangle & \langle \nu_\mu | \nu_3 \rangle \\ \langle \nu_L | \nu_1 \rangle & \langle \nu_L | \nu_2 \rangle & \langle \nu_L | \nu_3 \rangle \end{bmatrix} = \begin{bmatrix} -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}} \end{bmatrix}. \tag{12}$$

Then, substituting Eq. (12) into Eqs. (2), (3a), (3b), and (4) we have

$$|\nu_1\rangle = |\nu_e\rangle\left(-\frac{1}{\sqrt{2}}\right) + |\nu_\mu\rangle\frac{1}{\sqrt{3}} + |\nu_L\rangle\frac{1}{\sqrt{6}},$$
$$|\nu_2\rangle = |\nu_\mu\rangle\frac{1}{\sqrt{3}} + |\nu_L\rangle\left(-\frac{2}{\sqrt{6}}\right),$$
$$|\nu_3\rangle = |\nu_e\rangle\frac{1}{\sqrt{2}} + |\nu_\mu\rangle\frac{1}{\sqrt{3}} + |\nu_L\rangle\frac{1}{\sqrt{6}}, \quad (13)$$
$$|\nu_e\rangle = |\nu_1\rangle\left(-\frac{1}{\sqrt{2}}\right) + |\nu_3\rangle\frac{1}{\sqrt{2}},$$
$$|\nu_\mu\rangle = |\nu_1\rangle\frac{1}{\sqrt{3}} + |\nu_2\rangle\frac{1}{\sqrt{3}} + |\nu_3\rangle\frac{1}{\sqrt{3}},$$
$$|\nu_L\rangle = |\nu_1\rangle\frac{1}{\sqrt{6}} + |\nu_2\rangle\left(-\frac{2}{\sqrt{6}}\right) + |\nu_3\rangle\frac{1}{\sqrt{6}},$$

$$m(\nu_e) = \tfrac{1}{2}[m(\nu_1) + m(\nu_3)],$$
$$m(\nu_\mu) = \tfrac{1}{3}[m(\nu_1) + m(\nu_2) + m(\nu_3)],$$
$$m(\nu_L) = \tfrac{1}{6}[m(\nu_1) + 4m(\nu_2) + m(\nu_3)], \quad (14a)$$

$$m(\nu_1) = \tfrac{1}{2}m(\nu_e) + \tfrac{1}{3}m(\nu_\mu) + \tfrac{1}{6}m(\nu_L) - \frac{1}{\sqrt{6}}m_{e\mu} + \frac{1}{\sqrt{18}}m_{\mu L} - \frac{1}{\sqrt{12}}m_{Le},$$
$$m(\nu_2) = \tfrac{1}{3}m(\nu_\mu) + \tfrac{2}{3}m(\nu_L) - \frac{2}{\sqrt{18}}m_{\mu L}, \quad (14b)$$
$$m(\nu_3) = \tfrac{1}{2}m(\nu_e) + \tfrac{1}{3}m(\nu_\mu) + \tfrac{1}{6}m(\nu_L) + \frac{1}{\sqrt{6}}m_{e\mu} + \frac{1}{\sqrt{18}}m_{\mu L} + \frac{1}{\sqrt{12}}m_{Le},$$

while substitution of Eq. (14b) into Eq. (14a) yields

$$m(\nu_e) = m(\nu_\mu) = m(\nu_L), \quad m_{\mu L} = 0, \quad (15)$$

so that

$$m(\nu_1) = m(\nu_\mu) - \tfrac{1}{2}M,$$
$$m(\nu_2) = m(\nu_\mu), \quad (16)$$
$$m(\nu_3) = m(\nu_\mu) + \tfrac{1}{2}M$$

where

$$M \equiv 2\left(\frac{m_{e\mu}}{\sqrt{6}} + \frac{m_{Le}}{\sqrt{12}}\right).$$

Further, substituting Eq. (12) into Eq. (4) yields

$$P(\nu_e; t\,|\,\nu_\mu; 0) = \tfrac{1}{3}[1 - \cos(E_3 - E_1)t],$$
$$P(\nu_L; t\,|\,\nu_\mu; 0) = \tfrac{1}{3}[1 - \tfrac{2}{3}\cos(E_2 - E_1)t - \tfrac{2}{3}\cos(E_3 - E_2)t + \tfrac{1}{3}\cos(E_3 - E_1)t],$$
$$P(\nu_\mu; t\,|\,\nu_\mu; 0) = \tfrac{1}{3}[1 + \tfrac{2}{3}\cos(E_2 - E_1)t + \tfrac{2}{3}\cos(E_3 - E_2)t + \tfrac{2}{3}\cos(E_3 - E_1)t],$$

$$\langle P(\nu_e; t\,|\,\nu_\mu; 0)\rangle_{\text{time average}} = \langle P(\nu_L; t\,|\,\nu_\mu; 0)\rangle_{\text{time average}} = \langle P(\nu_\mu; t\,|\,\nu_\mu; 0)\rangle_{\text{time average}} = \tfrac{1}{3}, \quad (17)$$

$$E_2 - E_1 \cong \frac{1}{2p_\nu}[m(\nu_2) + m(\nu_1)][m(\nu_2) - m(\nu_1)] = \frac{1}{2p_\nu}[2m(\nu_\mu) - \tfrac{1}{2}M](\tfrac{1}{2}M),$$
$$E_3 - E_2 \cong \frac{1}{2p_\nu}[m(\nu_3) + m(\nu_2)][m(\nu_3) - m(\nu_2)] = \frac{1}{2p_\nu}[2m(\nu_\mu) + \tfrac{1}{2}M](\tfrac{1}{2}M),$$
$$E_3 - E_1 \cong \frac{1}{2p_\nu}[m(\nu_3) + m(\nu_1)][m(\nu_3) - m(\nu_1)] = \frac{1}{2p_\nu}[2m(\nu_\mu)](M).$$

Equations (12)–(17) describe a "maximal" oscillation situation in the case of three neutrino types since

$$\langle P(\nu_e; t\,|\,\nu_\mu; 0)\rangle_{\text{time average}} = \langle P(\nu_L; t\,|\,\nu_\mu; 0)\rangle_{\text{time average}} = \langle P(\nu_\mu; t\,|\,\nu_\mu; 0)\rangle_{\text{time average}} = 1/(\text{number of neutrino types})$$

Also, Eqs. (15)–(17) show explicitly that $P(\nu_e; t\,|\,\nu_\mu; 0) = P(\nu_L; t\,|\,\nu_\mu; 0) = 0$, and no oscillations occur if $m(\nu_e) = m(\nu_\mu) = m(\nu_L) = 0$ [remember that $m(\nu_1), m(\nu_2), m(\nu_3)$ are each ≥ 0] or if $M = 0$ [with $m(\nu_e) = m(\nu_\mu) = m(\nu_L) \neq 0$].

Finally, we present some conjectures on the lower limit for oscillation length. Confining ourselves for the sake of simplicity to the "maximal" oscillation situation in the case of two neutrino types, we attempt to estimate an upper limit on the quantity $\{[m(\nu_2)]^2 - [m(\nu_1)]^2\}$ which enters into the expression for the oscillation frequency $E_2 - E_1$ [Eq. (11)]. We do this by consideration of the radiative weak decay $\mu^+ \to e^+ + \gamma$. Thus, if ν_μ and ν_e were identical ($\nu_\mu \equiv \nu_e \equiv \nu$), this decay might be expected to proceed at a rate

$$\Gamma(\mu^+ \to e^+ + \gamma;\ \nu_\mu \equiv \nu_e) \approx \frac{\alpha}{\pi}(\tau_\mu)^{-1},$$

where $\tau_\mu = 2.2 \times 10^{-6}$ sec is the muon lifetime.[12] On the other hand, if muon lepton number and electron lepton number are separately conserved, $\Gamma(\mu^+ \to e^+ + \gamma)$ must vanish. However, if $\nu_\mu \leftrightarrow \nu_e$ oscillations take place, $\mu^+ \to e^+ + \gamma$ might also be expected to proceed at a nonvanishing rate, $\Gamma(\mu^+ \to e^+ + \gamma;\ \nu_\mu \leftrightarrow \nu_e)$, with the relation between $\Gamma(\mu^+ \to e^+ + \gamma;\ \nu_\mu \leftrightarrow \nu_e)$ and $\Gamma(\mu^+ \to e^+ + \gamma;\ \nu_\mu \equiv \nu_e)$ given by

$$\Gamma(\mu^+ \to e^+ + \gamma;\ \nu_\mu \to \nu_e) \approx \Gamma(\mu^+ \to e^+ + \gamma;\ \nu_\mu \equiv \nu_e) \int_0^\infty P(\nu_e; t\,|\,\nu_\mu; 0) e^{-t/\tau^*} \frac{dt}{\tau^*}\ . \tag{18}$$

Here τ^* is the average time that the neutrinos, which we assume mediate the $\mu \to e$ transition, are present, and the form of $P(\nu_e; t|\nu_\mu; 0)$ is supposed to be specified by Eq. (11). Evaluation of the integral yields approximately

$$\begin{aligned}\tfrac{1}{2}[(E_2 - E_1)\tau^*]^2 &\approx \frac{\Gamma(\mu^+ \to e^+ + \gamma;\ \nu_\mu \to \nu_e)}{\Gamma(\mu^+ \to e^+ + \gamma;\ \nu_\mu \equiv \nu_e)} \\ &\lesssim \frac{\Gamma(\mu^+ \to e^+ + \gamma;\ \text{exper. limit})}{(\alpha/\pi)(\tau_\mu)^{-1}} = \frac{1.5 \times 10^{-8} (\tau_\mu)^{-1}}{(\alpha/\pi)(\tau_\mu)^{-1}} = 6.5 \times 10^{-6}\,,\end{aligned} \tag{19}$$

where

$$\begin{aligned}E_2 - E_1 &\cong \frac{1}{2p^*}\ \{[m(\nu_2)]^2 - [m(\nu_1)]^2\} \\ &\cong \frac{l^*}{2}\ \{m(\nu_2)]^2 - [m(\nu_1)]^2\}\ ,\end{aligned} \tag{20}$$

with p^* the average momentum of the mediating neutrinos and l^* the average linear dimension of the region within which these neutrinos are confined. Equations (19) and (20) give (restoring the appropriate powers of $\hbar$ and c)

$$[m(\nu_2)]^2 - [m(\nu_1)]^2 \lesssim 7 \times 10^{-3} \left(\frac{\hbar}{cl^*}\right)\left(\frac{\hbar}{c^2\tau^*}\right), \tag{21}$$

and if $\mathrm{Re}\langle \nu_e | H^{(1)} | \nu_\mu \rangle_{p_\nu = 0} \equiv \frac{1}{2} m_{e\mu} \cong m(\nu_\mu) = \langle \nu_\mu | H^{(1)} | \nu_\mu \rangle_{p_\nu = 0}$ [which corresponds to $m(\nu_1) \cong 0$], Eqs. (21) and (11) yield in addition

$$m(\nu_\mu) = m(\nu_e) \cong \tfrac{1}{2}\{[m(\nu_2)]^2 - [m(\nu_1)]^2\}^{1/2} \lesssim 4 \times 10^{-2} \left(\frac{\hbar^2/c^3}{l^*\tau^*}\right)^{1/2}. \tag{22}$$

From Eq. (21) we obtain at once the upper limits on the $\nu_\mu \to \nu_e$ oscillation period and oscillation length. We have from Eqs. (11) and (21)

$$c\tau_{\mathrm{osc}}(p_\nu) = l_{\mathrm{osc}}(p_\nu) = \frac{2\pi\hbar c}{E_2 - E_1} \cong \frac{4\pi p_\nu \hbar/c^2}{[m(\nu_2)]^2 - [m(\nu_1)]^2} \gtrsim 1.8 \times 10^3 \left(\frac{p_\nu l^*}{\hbar}\right)(c\tau^*)\ , \tag{23}$$

and we implement this last equation by considering various possible values of τ^* and l^*. One possibility is suggested by a lowest-order perturbation treatment of a model where the neutrinos (and the vector boson W^+) which mediate $\mu^+ \to e^+ + \gamma$ undergo the sequence

$$\mu^+ \to \bar\nu_\mu + W^+ \to \bar\nu_\mu + W^+ + \gamma \xrightarrow[\substack{\text{via } \nu_\mu \to \nu_e \\ \text{oscillations}}]{} \bar\nu_e + W^+ + \gamma \to e^+ + \gamma\,.$$

This yields

$$\tau^* \approx \hbar / \{[(m_W c^2)^2 + (c\hbar/l^*)^2]^{1/2} + (c\hbar/l^*)\}$$

and $(\hbar/l^*) \approx m_W c$, i.e., $\tau^* \approx (l^*/c) \approx (\hbar/m_W c^2) = 1.3 \times 10^{-26}$ sec for $m_W = 50$ GeV/c^2,[13] whence, using Eqs. (21) and (23), we have $\{[m(\nu_2)]^2 - [m(\nu_1)]^2\}^{1/2} \lesssim 4$ GeV/c^2 and $l_{\mathrm{osc}}(p_\nu) \gtrsim 3 \times 10^{-13}$ cm for $p_\nu = 20$ GeV/c. Obviously, these limits are not very helpful since we already know from experiment that $l_{\mathrm{osc}}(p_\nu \approx 20\ \mathrm{GeV}/c) \gtrsim 2$ km (Ref. 14) and thus $\{[m(\nu_2)]^2 - [m(\nu_1)]^2\}^{1/2} \lesssim 5$ eV/c^2. A possibility of a very different kind is suggested by the idea that dynamical constraints may exist which permit the neutrinos that mediate $\mu^+ \to e^+ + \gamma$ to remain confined in the region of linear dimension l^* for a time $\tau^* \gg l^*/c$. This inequality holds, for example, in the case that τ^* and l^* are assumed to have their maximum "reasonable" values, i.e., $\tau^* \approx \tau_\mu = 2 \times 10^{-6}$ sec and $l^* \approx l_\mu \equiv$ upper limit on the radius of the muon $\approx 10^{-2}(\hbar/m_\mu c) = 2 \times 10^{-15}$ cm.[15] (Here l_μ is the length which determines the deviation of muonic QED behavior from that of a point.) These values for τ^* and l^* lead to a very small upper limit, $\{[m(\nu_2)]^2 - [m(\nu_1)]^2\}^{1/2} \lesssim 0.15$ eV/c^2, and a very large lower limit, $l_{\mathrm{osc}}(p_\nu) \gtrsim 2000$ km for $p_\nu = 20$ GeV/c. We also mention that a lower limit some 30 times smaller than this, i.e., $l_{\mathrm{osc}}(p_\nu) \gtrsim 70$ km for $p_\nu = 20$ GeV/c, is obtained if one assumes $\tau^* \approx \tau_\mu$ and $l^* \approx$ length characteristic of the weak interactions $= (G/\hbar c)^{1/2}$. Thus, none of our speculations on the lower limit of the oscillation length (at $p_\nu \leq 20$ GeV/c) appears to be significantly larger than the distance between the accelerator and the (distant) detector (≅1000 km) contemplated in the next section.

We conclude this account of the theory of neutrino oscillations by emphasizing that the time-averaged oscillation probability for a fixed neutrino momentum p_ν [Eqs. (4), (9), (11), (17)] is equal to the average of the oscillation probability over a broad neutrino momentum spectrum $N(p_\nu)$ at a fixed time t, i.e., at a fixed distance x between the neutrino detector and the accelerator.[16] More precisely, this equality holds if

$$\left[\frac{\langle (p_\nu - \langle p_\nu\rangle)^2\rangle}{(\langle p_\nu\rangle)^2}\right]^{1/2} \equiv \frac{\Delta p_\nu}{\langle p_\nu\rangle} \gtrsim \frac{l_{\mathrm{osc}}(\langle p_\nu\rangle)}{x},$$

so that, under these circumstances,

$$\langle P(\nu_\eta; t \,|\, \nu_\xi; 0)\rangle_{\substack{\text{time average}\\ \text{fixed } p_\nu}} = \sum_j |\langle \nu_\eta | \nu_j\rangle|^2 |\langle \nu_j | \nu_\xi\rangle|^2$$

$$= \langle P(\nu_\eta; t \,|\, \nu_\xi; 0|\rangle_{\substack{p_\nu \text{ average}\\ \text{fixed } t \text{ or } x}}$$

i.e. $(T \gg 2\pi[E_k(p_\nu) - E_j(p_\nu)]^{-1})$, (24)

$$\frac{1}{T}\int_0^T \sum_{j,k}{}' \langle \nu_\eta | \nu_j\rangle\langle \nu_\eta | \nu_k\rangle^* \langle \nu_j | \nu_\xi\rangle\langle \nu_k | \nu_\xi\rangle^* e^{i[E_k(p_\nu)-E_j(p_\nu)]t} dt$$

$$= 0$$

$$= \int_0^\infty \sum_{j,k}{}' \langle \nu_\eta | \nu_j\rangle\langle \nu_\eta | \nu_k\rangle^* \langle \nu_j | \nu_\xi\rangle\langle \nu_k | \nu_\xi\rangle^* e^{i[E_k(p_\nu)-E_j(p_\nu)]t} N(p_\nu) dp_\nu . \quad (25)$$

Accordingly, for a broad-band neutrino spectrum, the neutrino-momentum-averaged oscillation probability will not vanish at any detector-accelerator distance $>l_{\mathrm{osc}}(\langle p_\nu\rangle)$ as long as the neutrino oscillations actually occur.

EXPERIMENT

Arrangement of neutrino beam and detectors

To be concrete, consider an experimental arrangement as in Fig. 1 with detectors I and II separated by $x = 10^3$ km, approximately $\frac{1}{6}$ of the earth's radius. To reach detector II the neutrino beam must initially be directed about 78 mrad

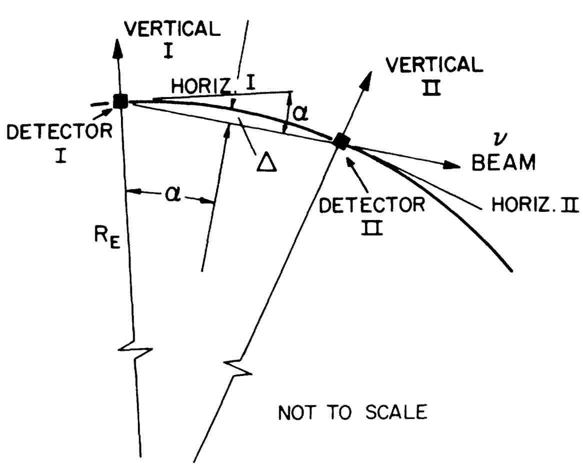

FIG. 1. Geometry of a feasible experiment. If the distance between detectors I and II is 1000 km, then $\alpha = 0.078$ rad and $\Delta = 19$ km. R_E is the radius of the earth $= 6.4 \times 10^3$ km.

below the horizontal. It will emerge at detector II directed 78 mrad above the horizontal. As shown in Fig. 1, the sagitta between the earth's surface and the beam path is 19 km. To form the neutrino beam from the decays of secondary mesons produced by a high-energy proton beam requires a free region about 1 m in diameter and roughly 200 m long, which would therefore have its downstream end 15 m below the horizontal. A typical neutrino beam produced by, say, 300-GeV protons has an angular divergence of about 1 mrad. Hence, at a distance of 10^3 km from its source, the beam has a radius of 1 km.

There is no special requirement on the neutrino-beam energy, although lower energy is somewhat favored. The total cross section for ν-nucleon interactions rises linearly with laboratory energy E_ν, which accordingly increases the observed interaction rate at detector II as higher-energy neutrinos are employed. On the other hand, the oscillation length [Eq. (23)] also increases linearly with E_ν, which leads to the possibility of violating the condition $\Delta E_\nu / \langle E_\nu\rangle \gtrsim l_{\mathrm{osc}}(\langle E_\nu\rangle)/x$ if too-high-energy neutrinos are employed. In addition, the "on" time of the beam should be as short as possible to minimize the background at detector II. This will be discussed in more detail later.

Interaction rates at detector II

Each of the detectors I and II must be capable of detecting electrons from ν_e-induced reactions, $\nu_e + N \rightarrow e^- +$ hadrons, and muons from ν_μ-induced reactions, $\nu_\mu + N \rightarrow \mu^- +$ hadrons, where N is a nucleon. Weak neutral-current reactions, that may in some detectors appear as ν_e interactions, will be discussed in the section on backgrounds. A realistic area for detector II is 10 m × 10 m, with about 220 metric tons of ν_e-detecting capacity and 220 metric tons of ν_μ-detecting capacity. Hence detector II subtends 0.32×10^{-4} of the neutrino beam area at detector I.

(*a*) *If only ν_e and ν_μ exist and "maximally" oscillate one into another* [*Eq.* (*11*). At a detector (the equivalent of detector I) in the Fermilab ν beam, at present, one observes about 30 muons from ν_μ-induced events per metric ton of detector per hour. Thus at detector II one might expect 30 muons/(metric ton × hour) × 220 metric tons $\times 0.32 \times 10^{-4} \approx 0.2$ muon/hr, and $\lesssim 0.005$ electron/hr, if $\tau_{\rm osc} \gg t_{\rm II}$, because the number of ν_e's initially in the beam[17] is $\lesssim 2 \times 10^{-2}$ times the number of ν_μ's. For $\tau_{\rm osc} \ll t_{\rm II}$, we should observe 0.10 muon/hr and 0.10 electron/hr. In a 500-hour run, we obtain 51 muon events and 51 electron events, if oscillations are present, to be contrasted with 100 muon events and less than 3 electron events if no oscillations take place.

(*b*) *If neutrinos other than ν_e and ν_μ also exist and ν_μ maximally oscillates into all the others* [*Eq.* (*17*)]. As discussed earlier, if there exist a total of n different but communicating neutrino types, all of comparable mass, the expected muon and electron count rates in detector II are more complicated but still quite distinctive. We assume the additional neutrinos $\nu_L, \nu_{L'}, \ldots$ to be associated with charged heavy leptons of mass $m_L, m_{L'}, \ldots \gtrsim 1.5$ GeV, which will decay both leptonically and semileptonically with a lifetime $<10^{-11}$ sec. These leptons will be produced by the interactions of $\nu_L, \nu_{L'}, \ldots$ in detector II through $\nu_L + N \to L^- +$ hadrons, etc. The leptonic decay modes, $L^- \to \mu^- + \bar{\nu}_\mu + \nu_L$ and $L^- \to e^- + \bar{\nu}_e + \nu_L$, will occur with approximately equal probability and therefore contribute to the observed muon and electron events in detector II in the same ratio as the muons and electrons that come directly from ν_μ and ν_e interactions in detector II. The semileptonic decay modes, $L^- \to \nu_L +$ hadrons, will make negligible contribution to the observed muon and electron count rates. Thus, if the purely leptonic branching fraction is f, we again expect the observed number of muons and electrons to be equal to each other but given by $N_\mu = N_e = (N_0/n)[1 + (n-2)f/2]$, where N_0 is the total number of muons that would be observed in detector II in the absence of oscillations. We plot in Fig. 2 the ratio N_μ/N_0 against the number of different neutrino types n for different values of f. We conclude from Fig. 2 that, if oscillations are observed, and if $f \lesssim 0.5$ as expected for the leptonic decays of heavy leptons,[18] it is possible to determine n if n is large, say $\gtrsim 5$, and to

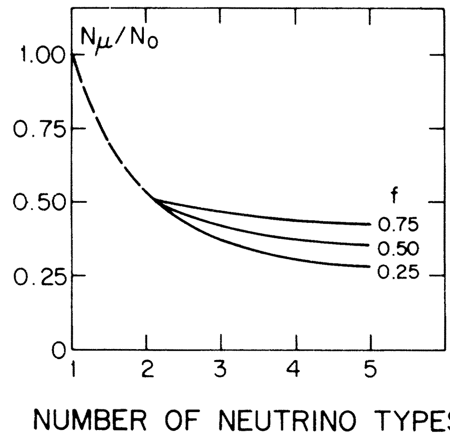

FIG. 2. Plot of N_μ, the number of observed muons if oscillations occur, divided by N_0, the number of observed muons if no oscillations take place, against number of neutrino types n for different values of the leptonic branching ratio f. The value of N_μ/N_0 at $n = 1$ corresponds to no neutrino oscillations.

set an upper limit on n if n is smaller than 5. The accuracy with which this is done depends on the uncertainty in N_0, which would be directly determined by measurement in detector I.

Apparatus

There are a number of possible detector arrangements that would satisfy the relatively simple requirements of this search for a signal of neutrino oscillations. The apparatus described here has been constructed for and in part tested in previous neutrino experiments.[19] It provides a definite design of a feasible experiment that incorporates adequate reliability and redundancy in its performance.

The basic idea is to use a target-detector that serves as the target for both ν_e and ν_μ interactions. It is also the specific detector for the ν_e interactions. Immediately downstream of the target is a 1-m-thick iron wall, the rear surface of which is covered with large-area liquid scintillation counters (see Fig. 3). The iron wall absorbs all hadrons and electromagnetic radiation produced in the target but permits muons of energy greater than about 1 GeV to penetrate to the scintillation counters behind it. Thus a count in the downstream counters in time coincidence with both a count in the target-detector and a gate from the accelerator serves to identify muons from ν_μ reactions, $\nu_\mu + N \to \mu^- +$ hadrons, in the target. In contrast, a count in the target-detector representing the deposition therein of more than a few GeV of energy in time coincidence with a

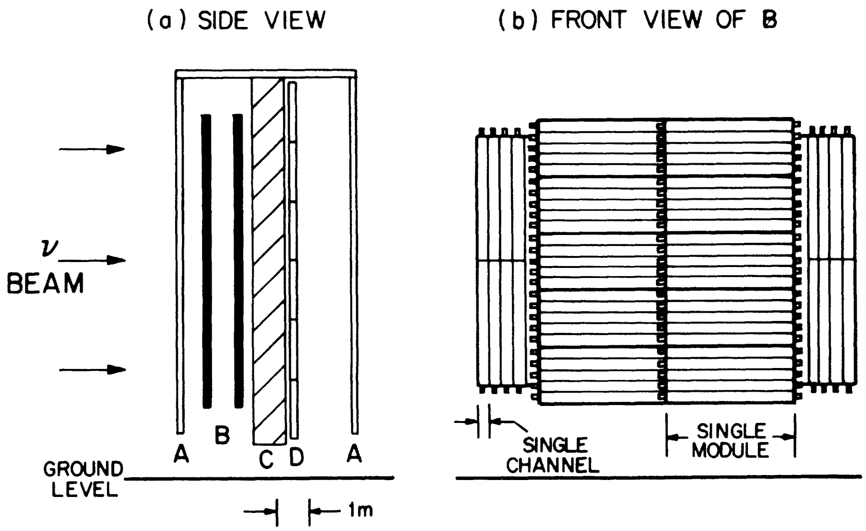

FIG. 3. Schematic diagram of the apparatus. (a) Here A is a liquid scintallator anticounter (if necessary), B is the target-detector, C is a 1-meter thickness of iron, and D is a liquid scintillator counter to record muons that pass through C. (b) Front view of the target-detector showing a possible geometric arrangement of individual modules. Many but not all photomultiplier tubes are also shown. Observe that the fiducial area can be varied horizontally and vertically by including more or fewer of the individual channels in each module.

gate from the accelerator, and the absence of a count in the muon identifying counters, is the signal of a ν_e interaction, $\nu_e + N \to e^- +$ hadrons, in the target-detector.

The 220-metric-ton modular target-detector might consist of 28 modules in two arrays of 14 modules as shown in Fig. 3. Each module has an area of 4 m × 2 m and is approximately 8 metric tons in weight. A module contains alternating thicknesses of lead sheets and liquid scintillator to form a very efficient ($\cong 14\lambda_{\rm rad}$) electromagnetic shower detector. The total thickness of the double array in Fig. 3 is, however, about one absorption length for strongly interacting particles, and will certainly be penetrated by muons with energy greater than about 0.5 GeV.

A ν_μ-induced reaction in the target-detector will give rise to an average deposition of energy in the target-detector $\langle E_D\rangle_\mu$ about equal to the energy carried by the neutral-pion component of the hadronic cascade, i.e., $\langle E_D\rangle_\mu \approx 0.25\ E_\nu$. If ν_e interactions with nucleons have the same essential nature as ν_μ interactions, then $\langle E_D\rangle_e \approx 0.75\ E_\nu$, since the energy spectrum of the ν_e induced by oscillations should be the same as that of the original ν_μ, and the energy of the outgoing electron will on average equal $0.5\ E_\nu$. Thus an additional signature of a statistically significant sample of ν_e-induced events due to oscillations would be the ratio $\langle E_D\rangle_e/\langle E_D\rangle_\mu \approx 3$. This is a useful check on the experiment, and is particularly important in discriminating against the background of muonless (i.e., weak-neutral-current) events that will be produced by ν_μ interacting in the target-detector, as discussed in the next section.

Backgrounds

(*a*) *Cosmic rays*. It will be convenient and relatively inexpensive if detector II can be located at the earth's surface, even if it is necessary to provide active (anticounter) shielding against cosmic rays for the entire detector. This depends on the magnitude and nature of the cosmic-ray background at the surface, the time duration and structure of the ν_μ beam pulse from the accelerator, and the precision with which a timing signal from the accelerator can be brought to detector II. In the Harvard-Pennsylvania-Wisconsin-Fermilab neutrino experiment[19] at Fermilab, the observed time-averaged cosmic-ray rate in a part of the apparatus with a geometrical arrangement similar to, and an area $\frac{1}{8}$, that of the experimental arrangement shown in Fig. 3 is about 10^2 total cosmic-ray counts per second. To extract protons from over the entire circumference of the Fermilab proton synchrotron requires 20 μsec, which is therefore the shortest-duration accelerator pulse, apart from the rf structure of the beam pulse. Correcting for the ratio of the areas gives about 0.02 cosmic-ray count per shortest accelerator pulse for the apparatus in

Fig. 3, which indicates the feasibility of locating detector II at the earth's surface, if only a minimal amount of active shielding against cosmic rays is provided. The actual background rate is further reduced by discrimination against the very low energy (< 100 MeV) deposition in the target-detector of cosmic-ray muons traversing it in favor of the larger deposition ($\gtrsim$500 MeV) due to ν-induced reactions.

It is, nevertheless, worth noting that almost total active shielding against all types of cosmic-ray events (showers as well as single muons) can be accomplished with the anticounter arrangement shown in Fig. 3, which will reduce the cosmic-ray background of all kinds by more than two orders of magnitude. The upstream and overhead counters in Fig. 3 serve directly as anticounters in the usual way. The downstream counters must be used to veto cosmic rays by a time-of-flight discrimination against particles directed from downstream toward upstream. Since the shortest flight path is 4 m, or about 13 nsec as shown in Fig. 3, it is easily possible with present timing techniques to distinguish upstream-going cosmic rays. Further reduction by another factor of 100 at a small sacrifice of useful target tonnage can be obtained by using the outer sections of each target-detector module as a veto counter against charged particles incident from the outside. Thus the counts due to cosmic rays in detector II located at the earth's surface may certainly be made negligible with active shielding, compared with the expected neutrino-induced count rate of >0.2 counts/hr.

(*b*) *Neutrino-beam-induced backgrounds.* These are events produced by ν_μ coming from the accelerator for which a muon is not observed in the muon identifier. They arise primarily from weak neutral-current interactions, although some will be the result of geometric detection inefficiency. This type of event will simulate a ν_e-induced event in the target-detector of Fig. 3. Observe, however, that for these events $\langle E_D \rangle \lesssim 0.25\,E_\nu$, as mentioned earlier, which will aid in identifying them as spurious. Furthermore, it is known[20] that at Fermilab energies

$$R^\nu = \frac{\sigma(\nu_\mu + N \to \nu_\mu + \text{hadrons})}{\sigma(\nu_\mu + N \to \mu^- + \text{hadrons})} = 0.30 \pm 0.03\,.$$

In the 500-hr sample experiment given above, we might then expect $100 \times (0.30 \pm 0.03) = 30 \pm 4$ muonless or spurious ν_e events, even in the absence of neutrino oscillations. If oscillations and only two neutrinos are present, we would again obtain [assuming $\sigma(\nu_e + N \to \nu_e + \text{hadrons}) = \sigma(\nu_\mu + N \to \nu_\mu + \text{hadrons})$] $50 \times (0.30 \pm 0.03) \times 2 = 30 \pm 5$ spurious ν_e events. Hence, with no oscillations, $N_e/N_\mu \approx 33/100 = 0.33$, i.e., $\approx R^\nu$, while with oscillations $N_e/N_\mu \approx 84/51 = 1.6$, and the raw N_e/N_μ signal again clearly distinguishes the presence of real ν_e events even without energy discrimination.

Another source of error may arise from a difference in the angular distributions of ν_e and ν_μ produced at the accelerator, which would modify the ratio with no oscillations, but this is a small effect.

Unstable neutrinos

We conclude this discussion of the particulars of the proposed experiment by noting that if $m(\nu_\mu)$ is several times larger than $m(\nu_e)$ and *separate* conservation of muon and electron lepton number does not hold, ν_μ will be unstable and should decay into $\nu_e + \gamma$. Theoretical estimates of $[\Gamma(\nu_\mu \to \nu_e + \gamma)]^{-1}$, e.g.,

$$[\Gamma(\nu_\mu \to \nu_e + \gamma)]^{-1} \approx [\Gamma(\mu^+ \to e^+ + \gamma)]^{-1} \frac{m_\mu}{m(\nu_\mu)} > \left(\frac{2 \times 10^{-6}\ \text{sec}}{1.5 \times 10^{-8}}\right)\left(\frac{0.1\ \text{GeV}}{m(\nu_\mu)}\right) = (1.3 \times 10^{10}\ \text{sec}) \times [m(\nu_\mu)\ \text{in eV}]^{-1}\,,$$

indicate that the ν_μ lifetime in its rest frame is enormously greater than the value

$$\left(\frac{x}{c}\right)\left(\frac{m(\nu_\mu)}{E_\nu}\right) = \left(\frac{10^3\ \text{km}}{3 \times 10^{10}\ \text{cm/sec}}\right)\left(\frac{m(\nu_\mu)}{20\ \text{GeV}}\right) = (1.6 \times 10^{-13}\ \text{sec}) \times [m(\nu_\mu)\ \text{in eV}]$$

which is set by the scale of our experiment. However, if the theoretical estimate above is entirely wrong, the $\nu_\mu \to \nu_e + \gamma$ decay might significantly modify the ratio N_e/N_μ and provide an experimental value for the ν_μ lifetime which would be $\approx 10^3$ times longer than any existing direct limit on the instability of the ν_μ.[21]

SUMMARY AND CONCLUSIONS

The essential content of this paper is the assertion that a realistic experimental search for neutrino oscillations may at present be made over a distance of approximately 10^3 km. The experiment would utilize a high-energy ν_μ beam from a proton accelerator such as that at Fermilab, and existing neutrino-detecting apparatus. The approximate geography of an experiment with a neutrino beam originating at Fermilab is shown in Fig. 4.

If neutrino oscillations of the kind $\nu_\mu \leftrightarrow \nu_e$ do occur, they would significantly modify the ratio

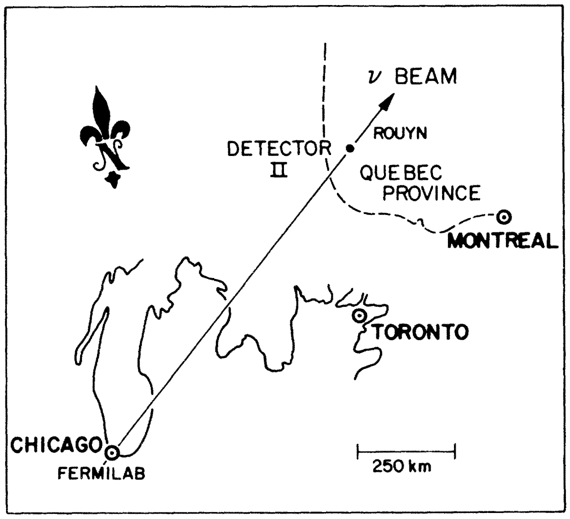

FIG. 4. Approximate geography of the proposed experiment. The present ν beam at Fermilab is directed 38°13′52″ east of north as indicated roughly.

of ν_e-induced electrons to ν_μ-induced muons observed in the distant detector relative to the same ratio observed in the detector much nearer to the origin of the neutrino beam. Thus, for example, an experiment of 500 hours' duration using a Fermilab neutrino beam of average energy ≈20 GeV with a detector separation of 10^3 km would yield a clear signal of neutrino oscillations if the oscillation length is less than 10^3 km.

If neutrino oscillations are observed, it would necessarily imply that the mass of at least one of the neutrino types, ν_μ or ν_e, is nonzero, and that the separate conservation of muon and electron lepton number does not hold. Subsequent experiments to determine the actual oscillation length (by moving the distant detector closer to the accelerator) would then explicitly measure the quantity $[m(\nu_2)]^2 - [m(\nu_1)]^2$ in Eq. (11) or the quantities $[m(\nu_2)]^2 - [m(\nu_1)]^2$ and $[m(\nu_3)]^2 - [m(\nu_2)]^2$ in Eqs. (15)–(17).

Furthermore, if neutrino oscillations do occur, it should be possible in the same experiment to determine the total number of "communicating" neutrino types by a direct comparison of the absolute number of ν_μ-induced muons in the distant detector with the absolute number predicted from measurement in the near detector of the ν_μ flux and the ν_μ-beam divergence. This is the only method known to us which holds forth at least the promise of specifying the total number of neutrino types in a single terrestrial experiment.

If neutrino oscillations are not observed, but if one still assumes that separate conservation of muon and electron number does not hold (i.e., $\mathrm{Re}\langle\nu_e|H^{(1)}|\nu_\mu\rangle_{p_\nu=0} \equiv \frac{1}{2}m_{e\mu} \neq 0$) and that, for the sake of simplicity, $m(\nu_e) = m(\nu_\mu)$, the principal result of the 10^3-km experiment with 20-GeV neutrinos would be an upper limit on the quantity $\{[m(\nu_2)]^2 - [m(\nu_1)]^2\}^{1/2} = \{2m(\nu_\mu)m_{e\mu}\}^{1/2}$, namely ≤ 0.2 eV/c^2. Thus, granting in addition the approximate equality $\frac{1}{2}m_{e\mu} \cong m(\nu_\mu)$ [as in the discussion following Eq. (21)], a negative result at 10^3 km with 20-GeV neutrinos would yield an upper limit on $m(\nu_\mu)$ or $m(\nu_e)$ of about 0.1 eV/c^2. Note that extension of the 10^3-km experiment at 20 GeV to the maximum possible terrestrial distance of about 10^4 km would reduce this upper limit on $\{[m(\nu_2)]^2 - [m(\nu_1)]^2\}^{1/2}$ by only a factor of about $\sqrt{10}$.

If no increase is observed in the ν_e signal and no decrease of the ν_μ signal is noted relative to the expected ν_μ and ν_e signals at the distant detector, a second result of interest is a limit on the instability of ν_μ. For a ν_μ with a mass equal to 1 eV, the mean life in the neutrino rest frame would be greater than about 10^{-13} sec.

Finally, there are interesting dividends, apart from those mentioned above, to be expected from any terrestrial neutrino experiment using an accelerator-produced neutrino beam over a distance of 10^3 km. The problem of correlated timing over large distances is under attack in long-baseline radio astronomy[22] and in terrestrial neutrino astronomy,[23] but certain aspects of the timing problem in an accelerator-based terrestrial neutrino experiment present a significant additional challenge, as, for example, in the attempt to correlate the time at the distant detector with the rf structure of the accelerator pulse. Furthermore, relatively high precision is required of the survey necessary to locate the distant detector close to the center of the accelerator neutrino beam, and good control of the direction of the extracted proton beam from the accelerator is necessary to maintain a constant neutrino beam direction. Partial solutions of these technical problems are available that are sufficient to permit the experiment described here to be carried out, but further development of these solutions would be advantageous to this experiment and, possibly, to other areas of physics.

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[12]See, for example, S. Frankel, in *Muon Physics*, Vol. II, edited by V. W. Hughes and C. S. Wu (Academic, New York, 1975).

[13]Upon substitution into Eqs. (19) and (20), the values $\tau^* \approx \hbar/m_W c^2$ and $l^* \approx \hbar/m_W c$ yield the result

$$\frac{\Gamma(\mu^+ \to e^+ + \gamma;\ \nu_\mu \leftrightarrow \nu_e)}{\tau_\mu^{-1}} \approx \frac{\alpha}{8\pi}\left(\frac{[m(\nu_2)]^2 - [m(\nu_1)]^2}{m_W^2}\right)^2 .$$

in essential agreement with a formula derived directly by Fritzsch (private communication).

[14]In the Harvard-Pennsylvania-Wisconsin-Fermilab experiments so far performed at Fermilab the distance between the source of roughly 20-GeV neutrinos and the detector is ≈ 2 km. No indication of neutrino oscillations is found in these experiments.

[15]As an illustration, admittedly in a completely different context, the lifetime of a positive pion, τ_π, could be viewed as the average time that the confined quarks, which mediate the pionic weak decay, are present within a region of linear dimension comparable with the radius of the pion, l_π. In this case, the mediation occurs via $\pi^+ \to u + \bar{d}$ and $u + \bar{d} \to \mu^+ + \nu_\mu$ with the u and $\bar{d}$ in strong mutual dynamical interaction. One has $\tau_\pi \gg l_\pi/c$ since $\tau_\pi = 2.6\times 10^{-8}$ sec and $l_\pi \cong 8\times 10^{-14}$ cm.

[16]Averaging the oscillation probability over the neutrino momentum spectrum has been discussed by J. Bahcall and S. Frautschi [Phys. Lett. 29B, 623 (1969)] in connection with the effect of any $\nu_e \leftrightarrow \nu_\mu$ oscillations on the terrestrially detected ν_e flux from the sun. As may be seen, e.g., from our Eq. (11), the averaging in question produces a decrease in the flux by a factor of 2. The appropriate decrease in the flux in the case of more than two neutrino types which can oscillate one into another, i.e., in the case of $\nu_e \leftrightarrow \nu_\mu, \nu_e \leftrightarrow \nu_L$, ... oscillations, is discussed by S. Nussinov [Phys. Lett. 63B, 201 (1976)].

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