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Measurement of $K_L^0 \to \pi^+\pi^-\pi^0$ decay parameters*

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The decay $K_L^0 \to \pi^+\pi^-\pi^0$ has been studied in an experiment using the Argonne 12-ft hydrogen bubble chamber exposed to a K_L^0 beam of momentum 550 ± 35 MeV/c. After radiative corrections to the data, a fit using a matrix element of the form $|M|^2 \propto 1 + g(s_3 - s_0)/m_{\pi^+}{}^2 + h[(s_3 - s_0)/m_{\pi^+}{}^2]^2 + k[(s_1 - s_2)/m_{\pi^+}{}^2]^2$ gives $g = 0.681 \pm 0.24$, $h = 0.095 \pm 0.032$, and $k = 0.024 \pm 0.010$. The branching ratio $\Gamma(K_L^0 \to \pi^+\pi^-\pi^0)/\Gamma(K_L^0 \to \text{charged})$ was measured to be 0.163 ± 0.003. These results, when compared to those obtained in other K_L^0 and $K^\pm$ decay experiments, indicate a substantial $\Delta I > 1/2$ contribution to the K_L^0 decay amplitude.

I. INTRODUCTION

Studies of the decay $K \to 3\pi$ can be used to measure the isospin change in the decay amplitudes. Specifically, the existence of $\Delta I > \frac{1}{2}$ amplitudes can be detected by a comparison of K_L^0 and $K^\pm$ branching ratios and the energy distribution of the odd pion in the decay. There is, however, disagreement among recent K_L^0 experiments on the magnitude of the parameters which describe the K_L^0 decay matrix element.[1] The latest measurements of the $K_L^0 \to \pi^+\pi^-\pi^0$ branching ratio are also inconsistent.

Previous K_L^0 experiments have either had relatively low detection efficiency which required large corrections based on Monte Carlo calculations or have been limited by statistical accuracy. All experiments have used broad-band K_L^0 beams resulting in a nonnegligible background from leptonic decays in the $K_{3\pi}$ sample used for the analysis. Some experiments also have ambiguous solutions for the π^0 momentum. These problems have been reduced in our experiment by the use of a large-volume bubble chamber exposed to a monoenergetic K_L^0 beam. This paper reports results on both the analysis of the $K_{3\pi}$ decay Dalitz plot and the $K_{3\pi}$ branching ratio.

II. EXPERIMENTAL PROCEDURE

The data were obtained from two exposures of the Argonne 12-ft hydrogen bubble chamber[2] to a monoenergetic K_L^0 beam. The K_L^0 beam was produced by the reaction $\pi^- p \to \Lambda K^0$ in a liquid hydrogen target located 8 m upstream from the bubble chamber. The π^- momentum was chosen to be about 1 GeV/c, below the threshold for $\pi^- p \to \Sigma^0 K^0$, so that the K_L^0 momentum spectrum was directly related to the π^- momentum spectrum and the beam geometry. The beam-transport system is shown in Fig. 1. To reduce the background from low-energy photons produced by bremsstrahlung of electrons in the beam, the K_L^0 production angle was selected to be $4° \pm 3°$. High-energy photons present in the K_L^0 beam were removed by about 10 radiation lengths of lead absorber placed in the sweeping magnet located in front of the chamber. A negligible number of K_S^0 mesons and Λ's reached the chamber because of the long distance from the H_2 target or lead filter to the bubble chamber. The ratio of neutron interactions to K_L^0 decays or interactions in the bubble chamber was about 60 : 1, but this presented no problem in the scanning of the film since the K_L^0 decay has a distinctive V topology in the bubble chamber.

In Fig. 2 we show the beam spectrum for the two exposures. The exposures consisted of 60 000 pictures with a K_L^0 momentum of 510 ± 35 MeV/c and 250 000 pictures with a K_L^0 momentum of 550 ± 35 MeV/c. All the film has been scanned once for V^0 topologies and a second scan has been completed on about one-third of the film. We have determined the single-scan efficiency for finding $K_L^0 \to 3\pi$ decays to be $(93.6 \pm 0.9)\%$. The fiducial volume was chosen to match the beam entrance window in the dimensions transverse to the beam and was restricted to 3.2 m along the beam direction.

All events found were measured and kinematically fitted to the following hypotheses:

$$K_L^0 \to \pi^+\pi^- , \quad (1)$$

$$K_L^0 \to \pi^+\pi^-\pi^0 , \quad (2)$$

$$K_L^0 \to \pi^\pm\mu^\mp\nu , \quad (3)$$

$$K_L^0 \to \pi^\pm e^\mp\nu , \quad (4)$$

$$K_L^0 p \to K_S^0(p) , \text{ unseen proton} \quad (5)$$

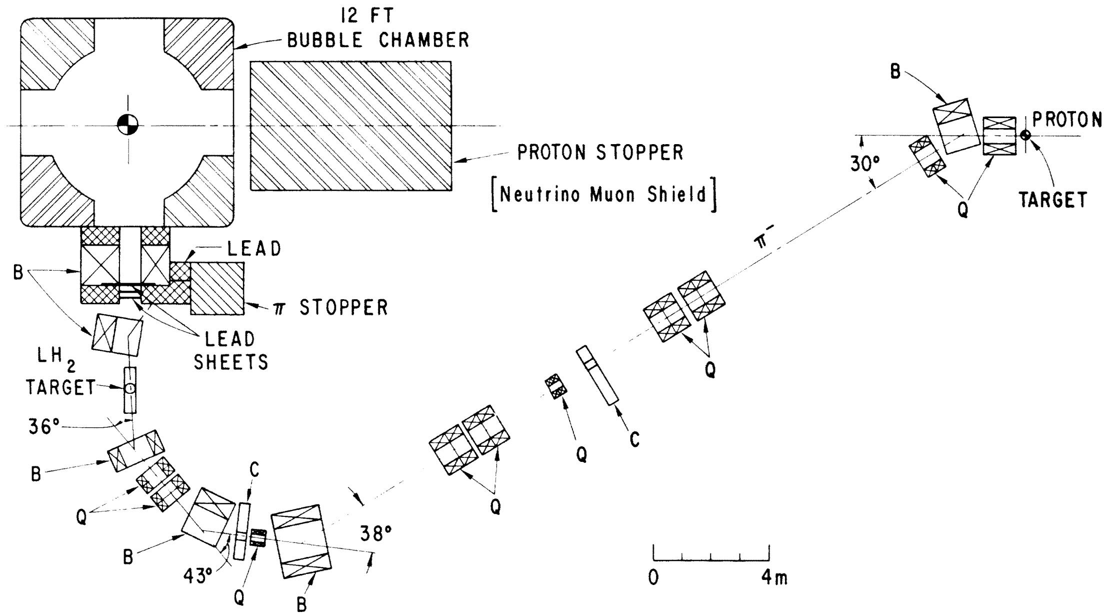

FIG. 1. A schematic diagram of the π^- beam line from the primary proton beam to the hydrogen target and the K_L^0 beam from the hydrogen target to the 12-ft bubble chamber. B, Q, and C designate bending magnets, quadrupole magnets, and collimators, respectively. Distances between elements are approximately to scale.

$$\Lambda \to p\pi^- , \quad (6)$$

$$\gamma \to e^+e^- . \quad (7)$$

The K_L^0 momentum in the fits was assigned a nominal value and error which was determined from events fitting the reactions $K_L^0 p \to K_S^0 p$ and $K_L^0 p \to \Lambda\pi^+$ without constraints on the K_L^0 momentum. The K_L^0 angles were constrained by using the measured coordinates of the K_L^0 decay and the profile of the π^- beam at the hydrogen target which had been measured during the beam tuning. All fits were required to have a probability of χ^2, $P(\chi^2)$, greater than 10^{-4} and to be within a factor of 10 of the $P(\chi^2)$ for the best hypothesis for the event. Events which failed geometric reconstruction or kinematic fitting were edited and remeasured. Approximately 7% of the events do not fit any of the hypotheses (1)-(7). This number is consistent with the fraction of K_L^0 which has scattered in the lead filter as determined from the fits to the $K_S^0 p$ and $\Lambda\pi^+$ final states.

After applying the $P(\chi^2)$ criteria, a total of 6250 events fitted the $K_L^0 \to \pi^+\pi^-\pi^0$ hypothesis uniquely and another 550 events were ambiguous with one or more of the semileptonic hypotheses (3) and (4). All of these candidates were subjected to additional tests. Because of the large volume of the 12-ft bubble chamber, ~75% of the charged π's interact or decay and so can be visually identified. If a particle interacted or decayed in flight or had two consecutive decays, it was assumed to be a pion. In addition, the range of any stopping particle was required to be consistent with the range calculated using the measured momentum and assumed mass. These tests resolve 70% of the ambiguous events. The tracks of particles which left the chamber were measured in two segments, and the mass hypothesis for the track was required to be consistent with the measured energy loss. This check resolved a further 5% of the ambiguous events.

The remaining ambiguities (147 events) were resolved using the variable $(P_0')^2$, which depends only on the transverse momenta of the charged

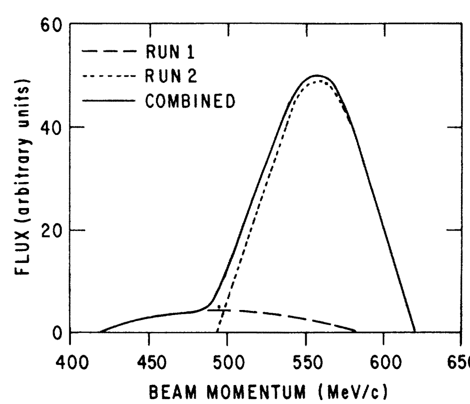

FIG. 2. The K_L^0 momentum spectrum for the two runs shown separately and combined.

particles:

$$(P_0')^2 = \frac{(M_K{}^2 - M_0{}^2 - M^2)^2 - 4(M_0{}^2 M^2 + P_T{}^2 M_K{}^2)}{4(P_T{}^2 + M^2)}, \tag{8}$$

where M_0 is the mass of π^0, M_K is the mass of K_L^0, M is the invariant mass of the charged-particle system assuming the particles are $\pi^\pm$, and P_T is the component of $\vec{P}_{\pi^+} + \vec{P}_{\pi^-}$ perpendicular to the K_L^0 line of flight. The $(P_0')^2$ distribution for all events satisfying the above selection criteria is shown in Fig. 3. The shaded region corresponds to the ambiguous events. Ambiguous events with $(P_0')^2 > -6 \times 10^3$ $(\text{MeV}/c)^2$ are taken to be $K_{3\pi}$ (80 events), and those with $(P_0')^2 < -6 \times 10^3$ $(\text{MeV}/c)^2$ are assumed to be leptonic decays. This selection procedure yielded 6499 decays $K_L^0 \to \pi^+\pi^-\pi^0$.

As a check on the selection of $\pi^+\pi^-\pi^0$ decays, we generated Monte Carlo samples of K_{e3}, $K_{\mu 3}$, and $K_{3\pi}$ events. These were then passed through our fitting and selection programs and given weights proportional to their branching fractions. Less than 0.5% of the generated semileptonic events were selected as $K_{3\pi}$ and less than 0.25% of the $K_{3\pi}$ were classified as semileptonic.

Events with an outgoing charged pion which interacts or decays close to the K_L^0 decay vertex will, in general, have a poor or unobtainable momentum measurement for that track. We correct for this bias by removing all events containing a charged pion with $P_{\text{lab}} \leq 70$ MeV/c. The remaining events were given weights equal to their probability of having either charged pion with a momentum less than 70 MeV/c. This correction, which is independent of the K_L^0 decay matrix element, is easily calculated using the fact that the pion angular distribution is isotropic in the K_L^0 rest frame. In addition, all events having either track shorter than 8 cm were removed and the remaining events weighted by the probability that either charged pion interacts or decays within 8 cm. After these cuts, a total of 6133 $K_L^0 \to \pi^+\pi^0\pi^-$ events remain for analysis.

Figure 4 shows the average weight as a function of the variables V and W defined in Sec. III. The cut data sample represents about 84% of the total number of $K_L^0 \to \pi^+\pi^-\pi^0$ decays which occurred within the accepted fiducial volume, including events lost by scanning and measuring inefficiencies. This combined efficiency changes by less than 10% over the entire kinematic range in the K_L^0 rest frame.

Radiative corrections for $K_{3\pi}$ decay are expected to be small, but not negligible compared to our statistical precision. We have used the formalism developed by Neveu and Scherk[3] for the contribution from virtual photons and photons with momentum less than 1 MeV/c, and applied the correction by reweighting each event. This correction depends on T_3, the π^0 kinetic energy in the K_L^0 rest system, and ranges from 2% at $T_3 = 0$, through 3% at $T_3 = 30$, to 10% at $T_3 = 53$ MeV. The correction diverges for $T_3 = T_{\max}$. To correct for photons with momentum greater than 1 MeV/c, we have generated a sample of $K_L^0 \to \pi\pi\pi\gamma$ events

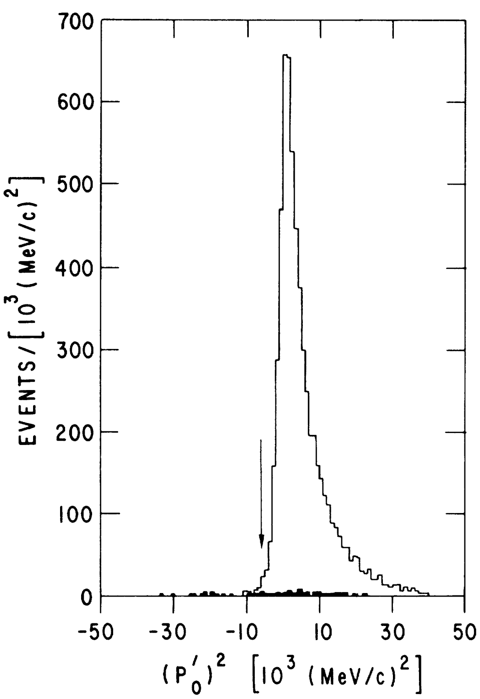

FIG. 3. The distribution of the variable $(P_0')^2$, defined in the text, for events fitting $K_L^0 \to \pi^+\pi^-\pi^0$ and satisfying all other criteria. The shaded events also fit semileptonic decay hypotheses. The arrow indicates the cut used to resolve these ambiguous events.

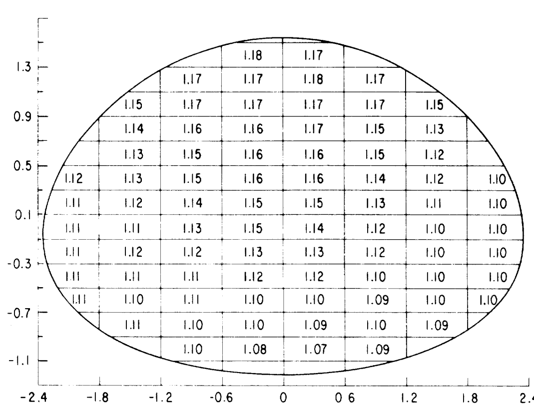

FIG. 4. The average weight used to correct for π interactions and decays as a function of the analysis variables V and W.

according to the matrix element given by Ferrari and Rosa-Clot.[4] The generated events were passed through the kinematic fitting programs and, after appropriate normalization, events satisfying the $K_{3\pi}$ decay selections were subtracted from our sample. The magnitude of this subtraction is less than 1% of the events.

III. ANALYSIS OF $K_{3\pi}$ DECAYS

The matrix element for $K_{3\pi}$ decay can be expressed as

$$|M|^2 \propto 1 + gV + hV^2 + jW + kW^2, \qquad (9)$$

where

$$V = (s_3 - s_0)/m_{\pi^+}^2,$$
$$W = (s_2 - s_1)/m_{\pi^+}^2,$$
$$s_0 = \tfrac{1}{3}(M_K^2 + M_1^2 + M_2^2 + M_3^2),$$
$$s_i = (M_K - M_i)^2 - 2M_K T_i, \quad i = 1, 2, 3.$$

The subscripts 1, 2, and 3 refer to π^+, π^-, and π^0, respectively, and T_i is the kinetic energy of the pion in the K_L^0 rest frame.

The weighted numbers of events for bins of V and W are displayed in Fig. 5. The density of events including radiative corrections is plotted in Fig. 6(a) and (b) as functions of T_3 and W, respectively. There is a clear gradient in the concentration of events along the $T_3 \propto V$ axis and a suggestion of a W^2 dependence.

Equation (9) has been fitted to our data sample using the maximum likelihood technique. Different fits were made with the g, h, k, and j parameters successively allowed to vary from zero. Each fit was initially done without radiative corrections and then repeated with those corrections applied. The radiative corrections increase g by about 0.02 and have negligible effects on h, j, and k. The curves on Fig. 6 are Eq. (9) with the g, h, and k parameters fitted to the data including radiative corrections. As a measure of the quality of the fit, the V vs W Dalitz plot has been divided into approximately 100 bins and a χ^2 has been calculated from the differences between Eq. (9) and the event densities. Table I summarizes the results of the fits, all of which have a satisfactory overall probability.

The errors quoted in Table I are statistical uncertainties, including the contributions from the radiative corrections and the weighting. To check for the presence of systematic uncertainties in our data, we generated a sample of 6000 τ decays, passed it through our kinematic-fitting program, and applied the same cuts and weights as were applied to the data. We then fitted for the parameters g, h, j, and k and found them to be zero as

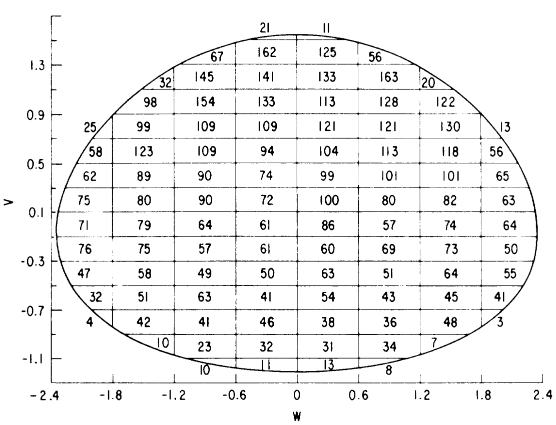

FIG. 5. Number of weighted events without radiative corrections as a function of V and W.

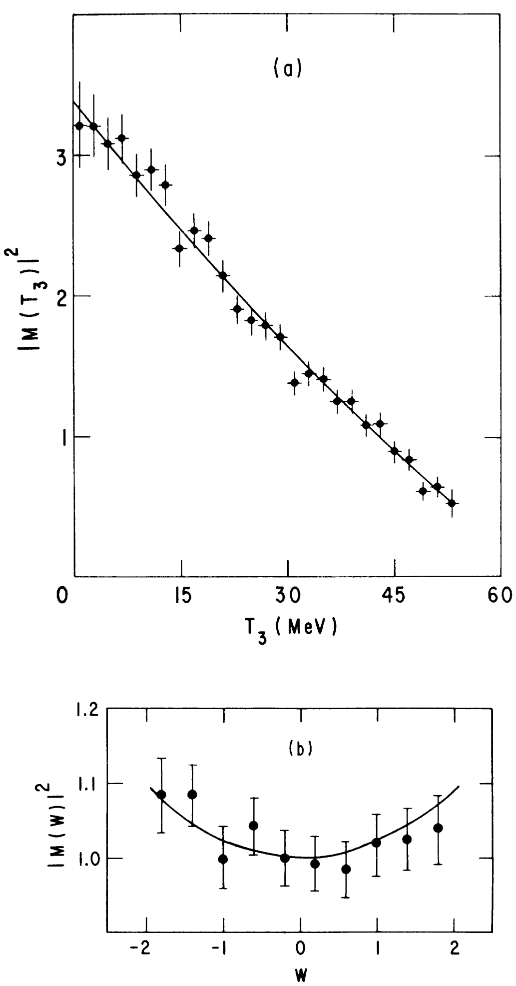

FIG. 6. The density of corrected events, including radiative corrections, projected onto (a) the T_3 axis and (b) the W axis. The curves are Eq. (9) with the g, h, and k parameters from fit number 6 of Table I.

TABLE I. The results of fits of Eq. (9) to our data.

Fit number	Fitted parameters[a] g	h	j	k	Radiative corrections	Probability of χ^2
1	0.614 ± 0.017	0	0	0	No	0.26
2	0.629 ± 0.017	0	0	0	Yes	0.24
3	0.638 ± 0.021	0.067 ± 0.029	0	0	No	0.35
4	0.655 ± 0.021	0.068 ± 0.029	0	0	Yes	0.33
5	0.664 ± 0.024	0.095 ± 0.032	0	0.024 ± 0.010	No	0.44
6	0.681 ± 0.024	0.095 ± 0.032	0	0.024 ± 0.010	Yes	0.41
7	0.664 ± 0.024	0.095 ± 0.032	0.001 ± 0.011	0.024 ± 0.010	No	0.41
8	0.681 ± 0.024	0.095 ± 0.032	0.001 ± 0.011	0.024 ± 0.010	Yes	0.39

[a] 0 denotes that the parameter was constrained to be zero in the fit.

in the generating matrix element. The scanning and measurement efficiencies have been calculated as a function of both V and W, and these corrections to g, h, j, and k were found to be negligible. The possibility of a systematic loss of events near the kinematic limits has been checked by repeating the fits, excluding all events having any T_i within 5 MeV of the boundary of the Dalitz plot. The parameters changed by amounts less than the errors in Table I. We are thus satisfied that any systematic errors introduced by our experimental procedure are smaller than the statistical errors on the parameters.

The normalized error-correlation matrix,

$$\rho_{ij} = \frac{\epsilon_{ij}}{(\epsilon_{ii}\epsilon_{jj})^{1/2}},$$

where the ϵ_{ij} are the elements of the error matrix of the fitted parameters, is given in Table II for fit number 8. It is clear that while the j parameter is nearly independent of g, h, and k, all the other parameters are strongly correlated. Because of these correlations, the results of different experiments can be strictly compared only if the same set of parameters has been used to fit the experimental data.

Our result from the one-parameter fit, $g = 0.629 \pm 0.017$, is consistent with the average of the results obtained in other experiments using the linear term alone,[1] as seen in Fig. 7. We obtain a term proportional to V^2, which is in agreement

TABLE II. Normalized error-correlation matrix for the four-parameter fit.

	g	h	j	k
g	1			
h	0.54	1		
j	−0.02	−0.00	1	
k	0.47	0.37	−0.012	1

with the recent experiment of Messner *et al.*,[5] although our fit with only the linear term has a reasonable confidence level. The combination $\frac{1}{4}g^2 - h$ is equal to 0.021 ± 0.028, consistent with a matrix element which is linear in π^0 kinetic energy. The presence of the term proportional to W^2 was first observed by Ford *et al.*[6] for $K^\pm \to \pi^\pm\pi^+\pi^-$ decay and by Messner *et al.*[5] for $K_L^0 \to \pi^+\pi^-\pi^0$ decay. Our data also support the existence of this term with a magnitude in agreement with the latter experiment. *CP* conservation requires j to be zero, consistent with our data.

IV. $K_{3\pi}$ BRANCHING RATIO

To determine the branching ratio, $B = \Gamma(K_L^0 \to \pi^+\pi^-\pi^0)/\Gamma(K_L^0 \to \text{charged})$, we must select a sample of τ events and a corresponding sample of decays into the semileptonic $K_L^0 \to \pi^\pm e^\mp \nu$ and $K_L^0 \to \pi^\pm \mu^\mp \nu$. As we have discussed, the use of a monoenergetic beam and a large-volume bubble

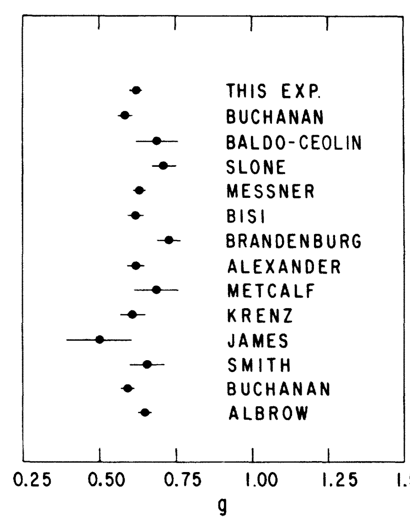

FIG. 7. A comparison of measurements of the slope parameter g. Numerical values and detailed references can be found in Ref. 1.

chamber gives us a clean sample of events satisfying the τ hypothesis. The semileptonic decays are also identified by their one-constraint kinematic fits. For the measurement of B, we do not have to resolve the ambiguities within the semileptonic decay modes. A small correction was introduced to account for the different scanning and measuring efficiencies in each decay mode. The dominant effects are: decays and interactions close to the K_L^0 decay vertex which prevent a good momentum measurement for the outgoing particle; invisible $K_L^0 \to \pi e \nu$ decays where the electron has $P \le 5$ MeV/c; events with a Dalitz decay of the π^0. In making these corrections, which amount to 8% for the $K_{3\pi}$ decay, 12% for the K_{e3} decay, and 9% for $K_{\mu 3}$ decay, we have used the established $K_{e3}/K_{\mu 3}$ branching ratio.[1] The resulting correction to B amounts to 2.1%, giving a final result of $B = 0.163 \pm 0.003$.

Our result is compared in Fig. 8 with other recent measurements of the branching ratio. We disagree with the value of 0.146 ± 0.004 reported by Brandenburg *et al.*,[7] but are in good agreement with the average of all other measurements,[1] 0.1610 ± 0.0027. We emphasize that none of the previous experiments had a monoenergetic K_L^0 beam, thus no kinematic fit was possible and their results depend on their estimates of the background from K_{e3} and $K_{\mu 3}$ decays.

Combining our result for B with the average of measurements of $K_L^0 \to$ neutrals[1] gives $\Gamma(K_L^0 \to \pi^+\pi^-\pi^0)/\Gamma(K_L^0 \to \text{All}) = 0.128 \pm 0.003$.

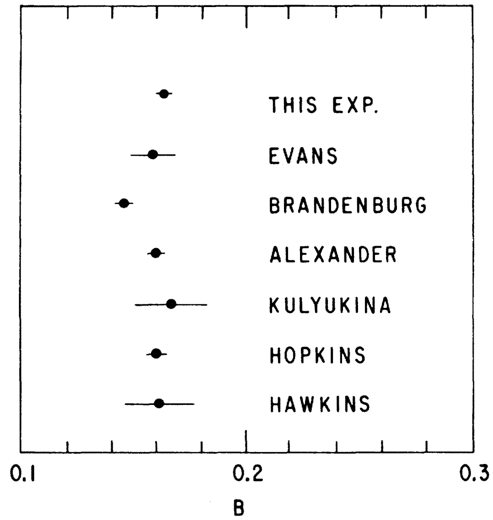

FIG. 8. A comparison of recent measurements of the branching ratio $\Gamma(K_L^0 \to \pi^+\pi^-\pi^0)/\Gamma(K_L^0 \to \text{charged})$. Numerical values and detailed references can be found in Ref. 1.

V. DISCUSSION

The isospin structure of the $K_{3\pi}$ decay amplitudes can be investigated using relations between the K_L^0 and $K^\pm$ decay rates into 3π states and between the slope parameters for the odd-pion energy distribution.[8,9] These relations assume that the matrix element has only a linear dependence on the odd-pion energy and that final-state interactions can be ignored.

The $K_{3\pi}$ decays are predominantly $\Delta I = \frac{1}{2}$ but the presence of $\Delta I > \frac{1}{2}$ amplitudes has been established.[9,1] The $\Delta I = \frac{3}{2}$ transition to an $I = 1$ three-pion state is indicated by a nonzero value for T_1 defined by

$$T_1 = \frac{1}{2} \frac{\Gamma_{++-}}{\phi_{++-}} \left(\frac{\Gamma_{+-0}}{\phi_{+-0}}\right)^{-1} - 1, \tag{10}$$

where Γ is the decay rate and ϕ is a phase-space factor. The subscripts indicate the charge of the three pions. Our result for B yields $T_1 = 0.19 \pm 0.02$.[10] We note that the calculation made by Holstein[11] using a current-algebra model (including a nonzero $\Delta I = \frac{3}{2}$) gives $T_1 = 0.23$, which is consistent with this result. Our value is somewhat higher than the result of a calculation by Hara[12] using a generalized Veneziano model, which gives $T_1 = 0.13$.

The absence of $\Delta I \ge \frac{5}{2}$ amplitudes would imply a zero value for T_2 defined by

$$T_2 = \frac{2}{3} \frac{\Gamma_{000}}{\phi_{000}} \left(\frac{\Gamma_{+-0}}{\phi_{+-0}}\right)^{-1} - 1. \tag{11}$$

Using our value of B, we calculate $T_2 = -0.007 \pm 0.040$. We therefore see no indication of $\Delta I \ge \frac{5}{2}$ amplitudes.

The slope parameters can also be related to the $\Delta I = \frac{1}{2}$ rule. Assuming only $\Delta I = \frac{1}{2}$ amplitudes, the following quantities are expected to be zero:

$$G_1 = g_{+-0} - g_{+00}, \tag{12}$$

$$G_2 = g_{++-} + \tfrac{1}{2} g_{+-0}. \tag{13}$$

Using our value for g_{+-0}, we obtain $G_1 = 0.079 \pm 0.026$ and $G_2 = 0.100 \pm 0.010$, indicating the presence of $\Delta I > \frac{1}{2}$ amplitudes.

VI. CONCLUSION

We have measured the $K_{3\pi}^0$ decay parameters in an experiment which has low systematic errors. Our result is consistent with the average of previous measurements of the parameter g, which multiplies the term linear in π^0 energy. The introduction of nonlinear terms improves the fit to our data and has a significant effect on the value obtained for g. We measured the branching ratio, $B = \Gamma(K_L^0 \to \pi^+\pi^-\pi^0)/\Gamma(K_L^0 \to \text{charged})$ to be 0.163 ± 0.003. A comparison of our results with other K-decay experiments shows a substantial amplitude for transitions with $\Delta I > \frac{1}{2}$; no evidence for $\Delta I \geq \frac{5}{2}$ transition is observed.

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†On leave from Tel-Aviv University, Tel-Aviv, Israel.

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