Spin-zero mesons and current algebras

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Large chiral algebras, using the f and d coefficients of SU(3) can be constructed with spin-$\frac{1}{2}$ baryons. Such algebras have been found useful in some previous investigations. This article examines under what conditions similar or identical current algebras may be realized with spin-0 mesons. A curious lack of analogy emerges between meson and baryon currents. Second-class currents, made of mesons, are required in some algebras. If meson and baryon currents are to satisfy the same extended SU(3) algebra, four meson nonets are needed, in terms of which we give an explicit construction for the currents.

I. INTRODUCTION

A Lagrangian built from hadron fields, rather than from quarks, can serve as the starting point for a large class of low-energy calculations not accessible from quark Lagrangians. This fact has been demonstrated in a number of previous publications.[1] As a guide to the structure of such a quarkless Lagrangian, it is natural to assume the exact validity of certain current algebras based on SU(3), or possibly SU(n) ($n>3$). In the course of such work one comes across an interesting feature of the hadrons: Their currents can consistently be required to satisfy much larger algebras than quark currents (if we discount the color degree of freedom). This fact arises from the existence, in the octet representations, of f and d combinations, which have no counterpart in the quark representations. It has been shown in previous work[1] that these larger algebras, as well as being available, actually lead to reasonable physical predictions, including satisfactory mass formulas. It may one day be possible to show the equivalence of our formulation to one of the more fundamental-looking models using confined quarks. However, such an equivalence will only remain a speculation for the time being.

This paper summarizes the SU(3) algebras available with baryon currents, and attempts to extend these algebras to currents made of spin-0 mesons. The purpose of doing so is threefold: First, the results are needed for the further development of a compensation theory of hadrons[1]; second, the results are interesting in their own right, as they exhibit less analogy between baryon and meson currents than might be expected; third, it is shown that a particularly appealing possibility implies the existence of new meson nonets. The properties of SU(3) used in what follows are standard and have been known for a long time.[2]

The main conclusions of this paper are as follows:

(a) With a nonet of spin-$\frac{1}{2}$ baryons, a 16-dimensional chiral (e.g. right-handed) algebra can be constructed, involving the coefficients f and d of Gell-Mann. The algebra can be factored into $\mathrm{SU}(3)_{if+d}\times \mathrm{SU}(3)_{if-d}$.

(b) Already in order to construct an SU(3) (8-dimensional) chiral[3] algebra with spin-0 mesons one needs two meson nonets (scalar and pseudoscalar, respectively).

(c) Also this algebra can have its dimensions increased to 16, but the enlarged algebra cannot be factored into SU(3)×SU(3), and corresponds instead to a noncompact group. *The enlarged algebra necessarily contains second-class currents.*

(d) In order to reproduce the 16-dimensional algebra of (a) in terms of meson fields, one needs at least four meson nonets. Whether or not second-class currents are present depends on the parity of the two additional nonets. An explicit construction is provided for these currents.

(e) Although presented in terms of SU(3) for definiteness, the results can be extended to the adjoint representation of SU(n) ($n>3$), since they depend only on the availability of f and d coefficients.

II. BARYON CURRENTS

In this section we review the baryon current algebras in order to provide a comparison with the mesonic ones. Consider the vector and axial-vector bilinear combinations

$$V^0_{\alpha\beta}(x)=\psi^\dagger_\alpha(x)\psi_\beta(x),\qquad A^0_{\alpha\beta}(x)=\psi^\dagger_\alpha(x)i\gamma^5\psi_\beta(x) \tag{2.1}$$

($i\gamma^5$ Hermitian). Using the shorthand

$$V^0_{\alpha\beta}(x)\to V_{\alpha\beta},\quad V^0_{\gamma\delta}(y)\to V_{\gamma\delta},\qquad \delta_{\alpha\delta}\delta(\vec{x}-\vec{y})\to\delta_{\alpha\delta},\quad \delta_{\beta\gamma}\delta(\vec{x}-\vec{y})\to\delta_{\beta\gamma}, \tag{2.2}$$

and taking all commutators at equal times, we have

$$
\begin{aligned}
[V_{\alpha\beta}, V_{\gamma\delta}] &= \delta_{\beta\gamma} V_{\alpha\delta} - \delta_{\alpha\beta} V_{\gamma\beta},\\
[V_{\alpha\beta}, A_{\gamma\delta}] &= \delta_{\beta\gamma} A_{\alpha\delta} - \delta_{\alpha\delta} A_{\gamma\beta},\\
[A_{\alpha\beta}, A_{\gamma\delta}] &= \delta_{\beta\gamma} V_{\alpha\delta} - \delta_{\alpha\delta} V_{\gamma\beta}.
\end{aligned}
\tag{2.3}
$$

Digressing for a moment to a general observation, not confined to baryons, we note that relations of this type can readily be converted to SU(3) relations as follows. Let the indices run from 0 to 8; let $f_{\alpha\beta\gamma}$ be completely antisymmetric, $d_{\alpha\beta\gamma}$ be completely symmetric, and

$$
f_{0\alpha\beta} = 0, \quad d_{0\alpha\beta} = (\tfrac{2}{3})^{1/2}\, \delta_{\alpha\beta}. \tag{2.4}
$$

Then (summing over repeated indices) it is known[2] that

$$
\begin{aligned}
f_{\alpha pq} f_{\beta qr} - f_{\beta pq} f_{\alpha qr} &= -f_{\alpha\beta\gamma} f_{\gamma pr},\\
f_{\alpha pq} d_{\beta qr} - d_{\beta pq} f_{\alpha qr} &= -f_{\alpha\beta\gamma} d_{\gamma pr},\\
d_{\alpha pq} d_{\beta qr} - d_{\beta pq} d_{\alpha qr} &= +f_{\alpha\beta\gamma} f_{\gamma pr}.
\end{aligned}
\tag{2.5}
$$

If, for some operators $Q_{\alpha\beta}$, $R_{\alpha\beta}$, $S_{\alpha\beta}$, we have

$$
[Q_{\alpha\beta}, R_{\gamma\delta}] = \delta_{\beta\gamma} S_{\alpha\delta} - \delta_{\alpha\delta} S_{\gamma\beta} \tag{2.6}
$$

(the relative minus sign is essential), and if we define[4]

$$
\begin{aligned}
Q^f_\alpha &= f_{\alpha\beta\gamma} Q_{\beta\gamma},\\
Q^d_\alpha &= d_{\alpha\beta\gamma} Q_{\beta\gamma},
\end{aligned}
\tag{2.7}
$$

and similarly for R, S, then Eqs. (2.5) lead to

$$
\begin{aligned}
[Q^f_\alpha, R^f_\beta] &= -f_{\alpha\beta\gamma} S^f_\gamma,\\
[Q^f_\alpha, R^d_\beta] &= -f_{\alpha\beta\gamma} S^d_\gamma,\\
[Q^d_\alpha, R^f_\beta] &= -f_{\alpha\beta\gamma} S^d_\gamma,\\
[Q^d_\alpha, R^d_\beta] &= +f_{\alpha\beta\gamma} S^f_\gamma
\end{aligned}
\tag{2.8}
$$

(space δ function omitted).

We now go back to the baryon currents (2.1). If all weak currents belong to a Lie algebra and occur in the interaction Lagrangian, they can always be taken in Hermitian combinations. (If any current occurs in the Lagrangian, its Hermitian adjoint must occur also.) The Hermitian baryon currents are V^{if}, A^{if}, V^d, and A^d. Then Table I(a) summarizes the SU(3) algebra involving these currents. The table entries, here and in all subsequent tables, show J'' in the relation

$$
[J_\alpha, J'_\beta] = -i f_{\alpha\beta\gamma} J''_\gamma \tag{2.9}
$$

when J and J' are given. This algebra can be factored, or subalgebras extracted, in several ways. For example, the left-handed algebra, which we assume dominates the weak interaction,[5] involves $(V-A)^{if}$. The right-handed algebra shown in Table I(b) is of particular interest in compensation theory, as well as being remarkable for its size (16 currents). It can further be factored according to Table I(c).

TABLE I. In (a) a baryon current algebra is given, showing J'' in $[J_\alpha, J'_\beta] = -if_{\alpha\beta\gamma} J''_\gamma$ when J and J' are given. (All subsequent tables have the same interpretation.) In (b) a right-handed algebra with 16 currents is given. In (c) the 16-dimensional algebra of (b), factored into two mutually commuting SU(3) algebras, is given.

(a)

J' \ J	V^{if}	A^{if}	V^d	A^d
V^{if}	V^{if}	A^{if}	V^d	A^d
A^{if}	A^{if}	V^{if}	A^d	V^d
V^d	V^d	A^d	V^{if}	A^{if}
A^d	A^d	V^d	A^{if}	V^{if}

(b)

J' \ J	$(V+A)^{if}$	$(V+A)^d$
$(V+A)^{if}$	$2(V+A)^{if}$	$2(V+A)^d$
$(V+A)^d$	$2(V+A)^d$	$2(V+A)^{if}$

(c)

J' \ J	$(V+A)^{if+d}$	$(V+A)^{if-d}$
$(V+A)^{if+d}$	$4(V+A)^{if+d}$	0
$(V+A)^{if-d}$	0	$4(V+A)^{if-d}$

Finally we note that the algebra of Table I(a) is invariant under any permutation of the three sets A^{if}, V^d, A^d, thus leading to corresponding variations on the theme of Table I(b). However, none of these variations seems to be physically applicable.

III. MESON CURRENTS

In analogy with Sec. II, form the bilinear combinations

$$
\begin{aligned}
\mathcal{V}_{\alpha\beta} &= \phi_\alpha \dot{\phi}_\beta + \chi_\alpha \dot{\chi}_\beta,\\
\tilde{\mathcal{V}}_{\alpha\beta} &= \phi_\alpha \dot{\phi}_\beta - \chi_\alpha \dot{\chi}_\beta,\\
\mathcal{A}_{\alpha\beta} &= \chi_\alpha \dot{\phi}_\beta - \phi_\alpha \dot{\chi}_\beta,\\
\tilde{\mathcal{A}}_{\alpha\beta} &= \chi_\alpha \dot{\phi}_\beta + \phi_\alpha \dot{\chi}_\beta,
\end{aligned}
\tag{3.1}
$$

where the ϕ and χ are distinct Hermitian spin-0 fields. The scalar or pseudoscalar nature of these fields has no meaning except in terms of the strong interactions, nor does that parity have to be specified in order to develop the current algebras. However, in order to connect this analysis to past and future work we may as well consider the ϕ to be pseudoscalar and the χ to be scalar.

Next, referring to the prerequisite form (2.6), it is readily checked that any SU(3)-based algebra involving the expression (3.1) can contain only the sets

$$\mathcal{V} \text{ and } \mathcal{A} \tag{3.2}$$

or

$$\mathcal{V} \text{ and } \tilde{\mathcal{A}} \tag{3.3}$$

or

$$\mathcal{V} \text{ and } \tilde{\mathcal{V}}. \tag{3.4}$$

As an illustration,

$$[\mathcal{V}_{\alpha\beta}, \mathcal{A}_{\gamma\delta}] = -i\delta_{\beta\gamma}\mathcal{A}_{\alpha\delta} + i\delta_{\alpha\delta}\mathcal{A}_{\gamma\beta}, \tag{3.5}$$

which fits (2.6), while

$$[\tilde{\mathcal{V}}_{\alpha\beta}, \tilde{\mathcal{A}}_{\gamma\delta}] = +i\delta_{\beta\gamma}\mathcal{A}_{\alpha\delta} + i\delta_{\alpha\delta}\mathcal{A}_{\gamma\beta}, \tag{3.6}$$

which does not. [The algebras using (3.4) are of course more simply expressed in terms of the separate ϕ and χ algebras.]

In Tables II(a), II(b), and II(c) we list for completeness the algebras involving the Hermitian currents

$\mathcal{V}^{-f}, \mathcal{V}^{-d}, \mathcal{A}^{f}, \mathcal{A}^{d}$ [Table II(a)],

$\mathcal{V}^{-f}, \mathcal{V}^{-d}, \tilde{\mathcal{A}}^{f}, \tilde{\mathcal{A}}^{d}$ [Table II(b)],

$\mathcal{V}^{-f}, \mathcal{V}^{-d}, \tilde{\mathcal{V}}^{-f}, \tilde{\mathcal{V}}^{-d}$ [Table II(c)].

IV. "LEFT-HANDED" MESON CURRENT ALGEBRA

The mesonic part J_{mes} of the dominant weak currents must satisfy the same algebra as the baryonic part $J_{\text{bar}} = V^{if} - A^{if}$ in order that they might be combined as $J_{\text{bar}} + J_{\text{mes}}$, a set of currents satisfying their own algebra. [This argument is not affected by the fact that the W bosons, occurring in a completely written-out Lagrangian, will contribute a further term to J. There is no need to consider the W bosons for the purpose of this article.]

First compare Tables I(a) and II(a). The relevant information actually consists of the signs along the tables' diagonals. The signs multiplying the tables' entries are obtained by comparing those entries with the row and column headings. We see that changing the sign of an algebra's element can never change the relative sign of two diagonal entries. Thus, the "signature" of Table I(a) is $++++$. If, for example, the headings V^{if} were replaced by $-V^{if}$, the signature would become $----$, i.e., the diagonal entries would now be $-(-V^{if}), -(-V^{if}), -(-V^{if}), -(-V^{if})$. For the time being, we are interested in the algebra of V^{if} and A^{if}. From the signature of Table II(a) we see that $\mathcal{V}^{-f}$ and $\mathcal{A}^{d}$ are the only

TABLE II. In (a) a meson current algebra using the $\mathcal{V}$ and the $\mathcal{A}$ is given. In (b) the meson current algebra using the $\mathcal{V}$ and the $\tilde{\mathcal{A}}$ is given. In (c) the meson current algebra using the $\mathcal{V}$ and the $\tilde{\mathcal{V}}$ is given. In (d) the "right-handed" meson algebra based on (a) is given.

(a)

J' \ J	$\mathcal{V}^{-f}$	$\mathcal{A}^{f}$	$\mathcal{V}^{-d}$	$\mathcal{A}^{d}$
$\mathcal{V}^{-f}$	$\mathcal{V}^{-f}$	$\mathcal{A}^{f}$	$\mathcal{V}^{-d}$	$\mathcal{A}^{d}$
$\mathcal{A}^{f}$	$\mathcal{A}^{f}$	$-\mathcal{V}^{-f}$	$\mathcal{A}^{d}$	$-\mathcal{V}^{-d}$
$\mathcal{V}^{-d}$	$\mathcal{V}^{-d}$	$\mathcal{A}^{d}$	$-\mathcal{V}^{-f}$	$-\mathcal{A}^{f}$
$\mathcal{A}^{d}$	$\mathcal{A}^{d}$	$-\mathcal{V}^{-d}$	$-\mathcal{A}^{f}$	$\mathcal{V}^{-f}$

(b)

J' \ J	$\mathcal{V}^{-f}$	$\tilde{\mathcal{A}}^{f}$	$\mathcal{V}^{-d}$	$\tilde{\mathcal{A}}^{d}$
$\mathcal{V}^{-f}$	$\mathcal{V}^{-f}$	$\tilde{\mathcal{A}}^{f}$	$\mathcal{V}^{-d}$	$\tilde{\mathcal{A}}^{d}$
$\tilde{\mathcal{A}}^{f}$	$\tilde{\mathcal{A}}^{f}$	$\mathcal{V}^{-f}$	$\tilde{\mathcal{A}}^{d}$	$\mathcal{V}^{-d}$
$\mathcal{V}^{-d}$	$\mathcal{V}^{-d}$	$\tilde{\mathcal{A}}^{d}$	$-\mathcal{V}^{-f}$	$-\tilde{\mathcal{A}}^{f}$
$\tilde{\mathcal{A}}^{d}$	$\tilde{\mathcal{A}}^{d}$	$\mathcal{V}^{-d}$	$-\tilde{\mathcal{A}}^{f}$	$-\mathcal{V}^{-f}$

(c)

J' \ J	$\mathcal{V}^{-f}$	$\tilde{\mathcal{V}}^{-f}$	$\mathcal{V}^{-d}$	$\tilde{\mathcal{V}}^{-d}$
$\mathcal{V}^{-f}$	$\mathcal{V}^{-f}$	$\tilde{\mathcal{V}}^{-f}$	$\mathcal{V}^{-d}$	$\tilde{\mathcal{V}}^{-d}$
$\tilde{\mathcal{V}}^{-f}$	$\tilde{\mathcal{V}}^{-f}$	$\mathcal{V}^{-f}$	$\tilde{\mathcal{V}}^{-d}$	$\mathcal{V}^{-d}$
$\mathcal{V}^{-d}$	$\mathcal{V}^{-d}$	$\tilde{\mathcal{V}}^{-d}$	$-\mathcal{V}^{-f}$	$-\tilde{\mathcal{V}}^{-f}$
$\tilde{\mathcal{V}}^{-d}$	$\tilde{\mathcal{V}}^{-d}$	$\mathcal{V}^{-d}$	$-\tilde{\mathcal{V}}^{-f}$	$-\mathcal{V}^{-f}$

(d)

J' \ J	$\mathcal{V}^{-f} + \mathcal{A}^{d}$	$\mathcal{V}^{-d} - \mathcal{A}^{f}$
$\mathcal{V}^{-f} + \mathcal{A}^{d}$	$2(\mathcal{V}^{-f} + \mathcal{A}^{d})$	$2(\mathcal{V}^{-d} - \mathcal{A}^{f})$
$\mathcal{V}^{-d} - \mathcal{A}^{f}$	$2(\mathcal{V}^{-d} - \mathcal{A}^{f})$	$-2(\mathcal{V}^{-f} + \mathcal{A}^{d})$

analogous candidates. Thus, we must take

$$J_{\text{mes}(a)} = \mathcal{V}^{-f} \pm \mathcal{A}^{d}. \tag{4.1}$$

(The relative sign is arbitrary and can be changed by setting e.g. $\phi \to \phi$, $\chi \to -\chi$.) Similarly, from Table II(b),

$$J_{\text{mes}(b)} = \mathcal{V}^{-f} \pm \mathcal{A}^{f} = (\mathcal{V} \pm \mathcal{A})^{-f}. \tag{4.2}$$

From Table II(c),

$$J_{\text{mes}(c)} = \mathcal{V}^{-f} \pm \tilde{\mathcal{V}}^{-f} = \begin{cases} -2f_{\alpha\beta\gamma}\phi_\alpha\dot{\phi}_\beta \\ \text{or} \\ -2f_{\alpha\beta\gamma}\chi_\alpha\dot{\chi}_\beta. \end{cases} \tag{4.3}$$

The three possibilities (4.1), (4.2), and (4.3) are essentially different from each other. Some physical information from the weak interactions can be used to narrow the choice to (4.1). We rule out (4.2) by noting that the currents $\mathcal{A}^{-f}$ and $\mathcal{V}^{-f}$ have the same transformation properties under charge conjugation (determined by the charge interpretation of the f coefficients), but opposite transformation properties under space inversion. Hence, the $J_{\text{mes}(b)}$ have mixed CP parity, the amount of mixing being large and hence incompatible with its observed superweak magnitude. Such is not the case with $J_{\text{mes}(a)}$, owing to the presence of d coefficients. As an illustration, for $\alpha = 3$ in octet space, and using subscripts P for pseudoscalar, S for scalar,

$$(\mathcal{V}^{-f})_3 \propto i(\pi_P^+ \dot{\pi}_P^- - \pi_P^- \dot{\pi}_P^+) + \cdots, \tag{4.4}$$

while

$$(\mathcal{A}^d)_3 \propto \pi_P^0 \dot{\eta}_S + \eta_P \dot{\pi}_S^0 + \cdots, \tag{4.5}$$

showing opposite C transformations *and* opposite P transformations.

The choice (4.3) can be ruled out by appealing to conservation of the weak vector current, i.e., of the vector part of $J_{\text{bar}} + J_{\text{mes}}$. According to (4.3) this vector part amounts to a component of

$$f_{\alpha\beta\gamma}(i\psi_\alpha^\dagger \psi_\beta - 2\phi_\alpha \dot{\phi}_\beta), \tag{4.6}$$

or alternatively of

$$f_{\alpha\beta\gamma}(i\psi_\alpha^\dagger \psi_\beta - 2\chi_\alpha \dot{\chi}_\beta). \tag{4.7}$$

The strong interactions, on the other hand, will have to conserve the isospin components of

$$f_{\alpha\beta\gamma}(i\psi_\alpha^\dagger \psi_\beta - \phi_\alpha \dot{\phi}_\beta - \chi_\alpha \dot{\chi}_\beta). \tag{4.8}$$

Hence (4.3) is unacceptable. Thus, the "left-handed" choice is narrowed down to (4.1), in which we arbitrarily take the minus sign,

$$J_{\text{mes}} = \mathcal{V}^{-f} - \mathcal{A}^d. \tag{4.9}$$

V. "RIGHT-HANDED" 16-DIMENSIONAL MESON CURRENT ALGEBRA

Can we find a meson current algebra which commutes with (4.9) and which has 16 elements, similarly to Table I(c)? We now know it must be formed from elements of Table II(a). By using this table it is readily verified that the combinations $\mathcal{V}^{-f} + \mathcal{A}^d$ and $\mathcal{V}^{-d} - \mathcal{A}^f$ commute with (4.9). The remaining independent combination $\mathcal{V}^{-d} + \mathcal{A}^f$ does not. Thus, the required algebra consists of $\mathcal{V}^{-f} + \mathcal{A}^d$ and $\mathcal{V}^{-d} - \mathcal{A}^f$. It is recorded in Table II(d). We conclude from that table that an algebra of the desired dimensionality exists, but a comparison with the baryonic Table I(b) shows that the signatures of these two tables are inequivalent. In fact, the factorization of Table I(c) cannot be achieved, and the 16-dimensional meson current algebra is fundamentally different from the baryonic one. A look at a current×current meson interaction commuting with the algebra of Table II(d),

$$\mathcal{L} \propto (\mathcal{V}^{-f} + \mathcal{A}^d)^2 - (\mathcal{V}^{-d} - \mathcal{A}^f)^2 \tag{5.1}$$

[sums over SU(3) and space-time indices are understood], shows that, if $\mathcal{L}$ is mediated by W bosons, half of these have negative metric. The group corresponding to the Lie algebra of Table II(d) is noncompact.

As pointed out in connection with Eq. (3.8), the currents $\mathcal{V}^{-d} - \mathcal{A}^f$ are of second class. Any breaking of the SU(3) symmetry such that a baryon ×meson interaction of the form

$$J_{\text{bar}} \cdot (\mathcal{V}^{-d} - \mathcal{A}^f) \tag{5.2}$$

is allowed to occur also necessarily breaks CP invariance—acceptably so since the right-handed currents are presumably associated with very weak (as distinct from weak) couplings. However, the main conclusion of this section is that *there is no 16-dimensional right-handed algebra of the form $J_{\text{bar}} + J_{\text{mes}}$, although individual such algebras J_{bar} and J_{mes} exist.*

VI. CONSTRUCTING A 16-DIMENSIONAL RIGHT-HANDED CURRENT ALGEBRA $J_{\text{bar}} + J_{\text{mes}}$

The signature (+ −) of Table II(d) can be changed into (+ +), while keeping the currents Hermitian, by the introduction of two new nonets of spin-0 mesons. Let the indices μ, ν range over the values 1, 2. Consider two sets of 18 mesons each: $\phi_{\alpha\mu}, \chi_{\alpha\mu}$ ($\alpha = 0, \ldots, 8$, $\mu = 1, 2$). We may think of $\phi_{\alpha 1}, \chi_{\alpha 1}$ as the ϕ_α, χ_α of the preceding sections; the new mesons $\phi_{\alpha 2}, \chi_{\alpha 2}$ are not necessarily committed to a definite space parity. Using a matrix notation, set

$$\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}, \quad \chi = \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}, \quad \epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \tag{6.1}$$

and let a superscript T denote the transpose. Then define the following generalization of (3.1):

$$\begin{aligned} \mathcal{V}_{\alpha\beta} &= \phi_\alpha^T \dot{\phi}_\beta + \chi_\alpha^T \dot{\chi}_\beta, \\ \hat{\mathcal{V}}_{\alpha\beta} &= \phi_\alpha^T \epsilon \dot{\phi}_\beta + \chi_\alpha^T \epsilon \dot{\chi}_\beta, \\ \mathcal{A}_{\alpha\beta} &= \chi_\alpha^T \dot{\phi}_\beta - \phi_\alpha^T \dot{\chi}_\beta, \\ \hat{\mathcal{A}}_{\alpha\beta} &= \chi_\alpha^T \epsilon \dot{\phi}_\beta - \phi_\alpha^T \epsilon \dot{\chi}_\beta, \end{aligned} \tag{6.2}$$

where the relative sign of the two terms in each expression is essential for the properties to be derived.

By the method of Sec. II, the algebra recorded in Table III and involving $\mathcal{V}^{-f}, \hat{\mathcal{A}}^f, \hat{\mathcal{V}}^d, \mathcal{A}^d$ may be readily checked. That algebra is seen to be identical to the baryonic algebra of Table I(a), with the mapping

TABLE III. Expanded meson current algebra using four meson nonets. This turns out to be the same algebra as in Table I(a).

J' \ J	$\mathcal{V}^{-f}$	$\mathcal{A}^{d}$	$\hat{\mathcal{V}}^{d}$	$\hat{\mathcal{A}}^{f}$
$\mathcal{V}^{-f}$	$\mathcal{V}^{-f}$	$\mathcal{A}^{d}$	$\hat{\mathcal{V}}^{d}$	$\hat{\mathcal{A}}^{f}$
$\mathcal{A}^{d}$	$\mathcal{A}^{d}$	$\mathcal{V}^{-f}$	$\hat{\mathcal{A}}^{f}$	$\hat{\mathcal{V}}^{d}$
$\hat{\mathcal{V}}^{d}$	$\hat{\mathcal{V}}^{d}$	$\hat{\mathcal{A}}^{f}$	$\mathcal{V}^{-f}$	$\mathcal{A}^{d}$
$\hat{\mathcal{A}}^{f}$	$\hat{\mathcal{A}}^{f}$	$\hat{\mathcal{V}}^{d}$	$\mathcal{A}^{d}$	$\mathcal{V}^{-f}$

$$
\begin{aligned}
&V^{if} \leftrightarrow \mathcal{V}^{-f},\\
&A^{if} \leftrightarrow \mathcal{A}^{d},\\
&V^{d} \leftrightarrow \hat{\mathcal{V}}^{d},\\
&A^{d} \leftrightarrow \hat{\mathcal{A}}^{f}.
\end{aligned} \tag{6.3}
$$

Thus, we may construct a left-handed 8 dimensional weak current algebra

$$(V-A)^{if} + (\mathcal{V}^{-f} - \mathcal{A}^{d}), \tag{6.4}$$

which commutes with a right-handed 16-dimensional very weak current algebra

$$(V+A)^{if} + (\mathcal{V}^{-f} + \mathcal{A}^{d}), \quad (V+A)^{d} + (\hat{\mathcal{V}}^{d} + \hat{\mathcal{A}}^{f}). \tag{6.5}$$

If the ϕ_2 and χ_2 interact strongly, and if the ϕ_2 are pseudoscalar and the χ_2 are scalar, then $\hat{\mathcal{V}}^{d}$ and $\hat{\mathcal{A}}^{f}$ are second-class currents and the baryon + meson combination violates CP conservation. However, with the opposite P assignment for ϕ_2 and χ_2, CP is conserved. Hence, on this basis, there would be no objection to enlarging the algebra (6.4) by the inclusion of another current octet

$$(V-A)^{d} + (\hat{\mathcal{V}}^{d} - \hat{\mathcal{A}}^{f}). \tag{6.6}$$

At present, no motivation other than esthetic seems to exist for this, in contrast to the right-handed counterpart of this octet occurring in (6.5). There the baryonic part seems required by compensation theory.

In view of the present mounting evidence for a charmed degree of freedom, it is worth pointing out that, as far as mesons are concerned, all the considerations of this paper are essentially unchanged (apart from unitary multiplet sizes) if SU(3) is replaced by SU(n) ($n > 3$). Indeed, the availability of the f and d coefficients is all that is required.

We finally note as a curiosity the invariance of each current (6.2) under two separate O(2) groups:

$$
\begin{aligned}
&\phi \rightarrow \phi\cos\zeta + \chi\sin\zeta,\\
&\chi \rightarrow \chi\cos\zeta - \phi\sin\zeta,
\end{aligned} \tag{6.7}
$$

$$
\begin{aligned}
&\phi_1 \rightarrow \phi_1\cos\zeta' + \phi_2\sin\zeta',\\
&\chi_1 \rightarrow \chi_1\cos\zeta' + \chi_2\sin\zeta',\\
&\phi_2 \rightarrow \phi_2\cos\zeta' - \phi_1\sin\zeta',\\
&\chi_2 \rightarrow \chi_2\cos\zeta' - \chi_1\sin\zeta'.
\end{aligned} \tag{6.8}
$$

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[1] See, for example, M. Wellner, Ann. Phys. (N.Y.) 73, 180 (1972); 76, 549 (1973); 86, 331 (1974); R. Pappas and M. Wellner, Phys. Rev. D 11, 318 (1975); 12, 3341 (1975).

[2] M. Lévy, Nuovo Cimento 52A, 23 (1967); M. Gell-Mann, Physics 1, 63 (1964); S. L. Adler and R. F. Dashen, *Current Algebras* (Benjamin, New York, 1968).

[3] In the case of spinless mesons, chirality is of course divorced from helicity.

[4] Correspondingly, we set $Q_\alpha^{if} = if_{\alpha\beta\gamma}Q_{\beta\gamma}$, $Q_\alpha^{-f} = -f_{\alpha\beta\gamma}Q_{\beta\gamma}$, etc.

[5] Of course one does not get the Cabibbo currents in this way. For a discussion of this point, see the first citation in Ref. 1.