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Magnetic and quadrupole moments of the W boson*

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The validity of the external-field treatment for the electromagnetic scattering of a W boson in the Weinberg model is examined, and calculation of the magnetic and quadrupole moments of the W boson is carried out in the renormalizable gauge instead of the unitary gauge used by earlier authors. Our results agree with those of Bardeen, Gastmans, and Lautrup in the static approximation, and thus gauge independence of these results is demonstrated.

I. INTRODUCTION

Magnetic and quadrupole moments of the W boson in the Weinberg model[1] were calculated by Bardeen, Gastmans, and Lautrup[2] by using the unitary gauge and applying dimensional regularization. However, recently DeRaad, Milton and Tsai[3] have discussed an anomaly in the results obtained in Ref. 2, and questioned the use of dimensional regularization in the unitary gauge. It would, therefore, be desirable to recalculate the moments in the renormalizable gauge to test the consistency of the dimensional regularization procedure.

It is customary to derive the magnetic and quadrupole moments of a particle by considering its scattering by an external electromagnetic field. We shall first examine the validity of the external-field treatment in the Weinberg model, and then carry out an independent calculation of the moments with the use of the renormalizable gauge instead of the unitary gauge. By showing that our static results agree with those given in Ref. 2 and confirmed in Ref. 3, we shall demonstrate gauge independence of the results obtained with the application of dimensional regularization. The vector anomaly, however, will not be discussed here.

In order to explain our notation, we shall state the Lagrangian density for the Weinberg model without leptons in the form used for our calculations. The Lagrangian density is

$$L = L_0 + L_{\rm int} \ , \tag{1.1}$$

with

$$\begin{aligned} L_0 = &-(\partial_\nu W_\mu^* \partial_\nu W_\mu + M_W{}^2 W_\mu^* W_\mu) - (\partial_\mu b^* \partial_\mu b + M_W{}^2 b^* b) - \tfrac{1}{2}[(\partial_\nu Z_\mu)^2 + M_Z{}^2 Z_\mu{}^2] - \tfrac{1}{2}[(\partial_\mu b_0)^2 + M_Z{}^2 b_0{}^2] \\ &- \tfrac{1}{2}(\partial_\nu A_\mu)^2 - \tfrac{1}{2}[(\partial_\mu s)^2 + \mu^2 s^2] - (\partial_\mu C_W^* \partial_\mu C_W + M_W{}^2 C_W^* C_W) - (\partial_\mu C_{W^*}^* \partial_\mu C_{W^*} + M_W{}^2 C_{W^*}^* C_{W^*}) \\ &- (\partial_\mu C_Z^* \partial_\mu C_Z + M_Z{}^2 C_Z^* C_Z) - (\partial_\mu C_A^* \partial_\mu C_A) \end{aligned} \tag{1.2}$$

and

$$\begin{aligned} L_{\rm int} = &\, i\left(g\frac{M_W}{M_Z} Z_\nu - eA_\nu\right)[(\partial_\mu W_\nu^* - \partial_\nu W_\mu^*)W_\mu - W_\mu^*(\partial_\mu W_\nu - \partial_\nu W_\mu)] \\ &+ i\left(g\frac{M_W}{M_Z}(\partial_\mu Z_\nu - \partial_\nu Z_\mu) - e(\partial_\mu A_\nu - \partial_\nu A_\mu)\right) W_\mu^* W_\nu \\ &- i\left(g\frac{M_W{}^2 - M_Z{}^2}{M_Z} Z_\mu - eM_W A_\mu\right)(W_\mu^* b - b^* W_\mu) - gM_W s W_\mu^* W_\mu - g\frac{M_Z{}^2}{2M_W} sZ_\mu{}^2 \\ &- \tfrac{1}{2}g(s + ib_0)W_\mu^*\partial_\mu b - \tfrac{1}{2}g(s - ib_0)\partial_\mu b^* W_\mu + \tfrac{1}{2}g(\partial_\mu s + i\partial_\mu b_0)W_\mu^* b \\ &+ \tfrac{1}{2}g(\partial_\mu s - i\partial_\mu b_0)b^* W_\mu - \tfrac{1}{2}g\frac{M_Z}{M_W}Z_\mu(s\partial_\mu b_0 - b_0\partial_\mu s) \\ &- i\left(g\frac{2M_W{}^2 - M_Z{}^2}{2M_W M_Z} Z_\mu - eA_\mu\right)(\partial_\mu b^* b - b^*\partial_\mu b) - g\frac{\mu^2}{4M_W}s(s^2 + 2b^* b + b_0{}^2) \\ &- \left(g\frac{M_W}{M_Z}Z_\nu - eA_\nu\right)^2 W_\mu^* W_\mu + \left(g\frac{M_W}{M_Z}Z_\mu - eA_\mu\right)\left(g\frac{M_W}{M_Z}Z_\nu - eA_\nu\right)W_\mu^* W_\nu \end{aligned}$$

$$+\tfrac{1}{4}g^2(W_\mu^* W_\nu - W_\nu^* W_\mu)^2 - \tfrac{1}{4}g^2\left(W_\mu^* W_\mu + \frac{M_Z^2}{2M_W^2} Z_\mu^2\right)(s^2 + 2b^*b + b_0^2)$$
$$-\tfrac{1}{2}ig\left(g\frac{M_W^2 - M_Z^2}{M_W M_Z} Z_\mu - eA_\mu\right)[(s+ib_0)W_\mu^* b - (s-ib_0)b^* W_\mu]$$
$$-\left(e^2 A_\mu^2 - eg\frac{2M_W^2 - M_Z^2}{M_W M_Z} A_\mu Z_\mu + g^2\frac{M_W^2 - M_Z^2}{M_Z^2} Z_\mu^2\right)b^*b - g^2\frac{\mu^2}{32M_W^2}(s^2 + 2b^*b + b_0^2)^2$$
$$+i\left(g\frac{M_W}{M_Z}Z_\mu - eA_\mu\right)(C_W^*\partial_\mu C_W - C_{W^*}^*\partial_\mu C_{W^*}) - \tfrac{1}{2}gM_W(s+ib_0)C_W^* C_W$$
$$-\tfrac{1}{2}gM_W(s-ib_0)C_{W^*}^* C_{W^*} + i\left(g\frac{M_W}{M_Z}\partial_\mu C_Z^* - e\partial_\mu C_A^*\right)(W_\mu^* C_W - W_\mu C_{W^*})$$
$$-i(C_W^* W_\mu - C_{W^*}^* W_\mu^*)\left(g\frac{M_W}{M_Z}\partial_\mu C_Z - e\partial_\mu C_A\right) - \tfrac{1}{2}igM_Z C_Z^*(b^* C_W - bC_{W^*})$$
$$+\tfrac{1}{2}igM_Z(C_W^* b - C_{W^*}^* b^*)C_Z - g\frac{M_Z^2}{2M_W} sC_Z^* C_Z\,, \tag{1.3}$$

where b and b_0 are nonphysical fields related to W_μ and Z_μ, respectively, C_W, C_{W^*}, C_Z, and C_A are nonphysical gauge-compensating fields obeying the Fermi statistics, and the coupling constants e and g are related as

$$e^2 = g^2(1 - M_W^2/M_Z^2)\,. \tag{1.4}$$

II. VALIDITY OF EXTERNAL-FIELD TREATMENT

The external-field treatment, which is often used in quantum electrodynamics, must be carefully examined before it can be applied to the Weinberg model. In order to understand the situation, consider the lowest-order scattering of two W bosons. Such a scattering can proceed not only through the exchange of a photon, but also through the exchange of a Z particle as well as through a direct interaction. It is easy to verify that the one-photon-exchange contribution predominates when[4]

$$q^2/M_W^2 \ll 1\,, \tag{2.1}$$

where q is the momentum transfer during the scattering. Since the external-field treatment corresponds only to the one-photon exchange, such a treatment is valid only under the condition (2.1).

The necessity of the condition (2.1) for the validity of the external-field treatment becomes especially transparent in the renormalizable gauge. Let us consider the first-order contribution of the scattering operator for the scattering of a W boson by an external electromagnetic field, which is given by

$$S_1 = ie(2\pi)^4\delta(p' - p - q)W_\mu^*(p')W_\nu(p)A_\lambda(q)$$
$$\times[(p_\lambda + p'_\lambda)\delta_{\mu\nu} + (q_\mu - p_\mu)\delta_{\nu\lambda} - (q_\nu + p'_\nu)\delta_{\mu\lambda}]\,, \tag{2.2}$$

where p and p' are the initial- and final-momentum four-vectors of the W boson. It is well known that $W_\mu(p)$ contains a nonphysical component $W(p)$, which must be treated by means of an indefinite metric.[5] The nonphysical component can be eliminated from $W_\mu(p)$ by subjecting it to the constraint

$$p_\mu W_\mu(p) = 0\,, \tag{2.3}$$

while the nonphysical component can be extracted from $W_\mu(p)$ by the replacement

$$W_\mu(p) \to ip_\mu M_W^{-1}W(p)\,. \tag{2.4}$$

When the initial W boson is physical while the final one is nonphysical, the application of

$$W_\mu^*(p') \to -ip'_\mu M_W^{-1}W^*(p'),\quad p_\nu W_\nu(p) = 0,\quad p^2 = -M_W^2 \tag{2.5}$$

to (2.2) yields

$$S'_1 = e(2\pi)^4\delta(p' - p - q)W^*(p')W_\nu(p)A_\lambda(q)$$
$$\times M_W^{-1}(M_W^2\delta_{\nu\lambda} + q^2\delta_{\nu\lambda} - q_\nu q_\lambda)\,. \tag{2.6}$$

On the other hand, the corresponding contribution for the process in which a physical W boson is converted into a b particle is

$$S''_1 = e(2\pi)^4\delta(p' - p - q)b^*(p')W_\nu(p)A_\lambda(q)M_W\delta_{\nu\lambda}\,. \tag{2.7}$$

A comparison of (2.6) and (2.7) shows that the negative probability for the creation of the nonphysical component of W boson will be canceled by the positive probability for the creation of the b particle only if, in addition to the supplementary condition $q_\lambda A_\lambda(q) = 0$ for the external field, we also impose the condition (2.1). Thus, the external-field treatment requires the condition (2.1)

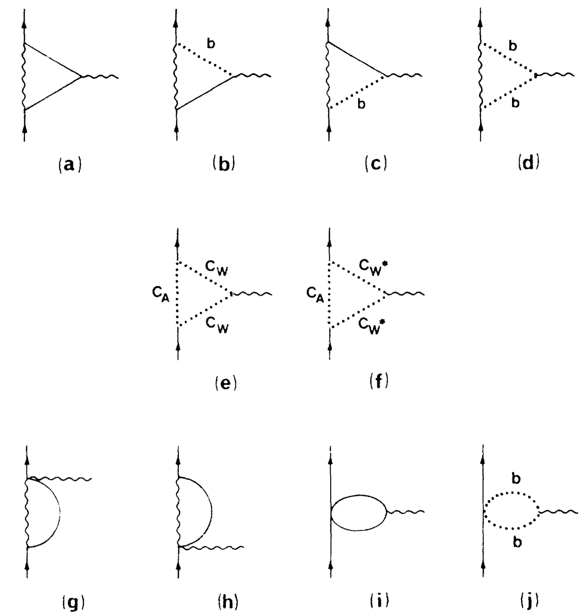

FIG. 1. Scattering diagrams without Z and s lines. In Figs. 1, 2, and 3, solid, broken, wavy, and crossed lines represent the W, Z, A, and s particles, respectively, while labeled dotted lines represent various nonphysical particles.

to maintain consistency even in the lowest order, where divergent integrals are not involved.

III. EVALUATION OF MAGNETIC AND QUADRUPOLE MOMENTS IN RENORMALIZABLE GAUGE

Let us again consider the scattering of a W boson by an external electromagnetic field, and de-

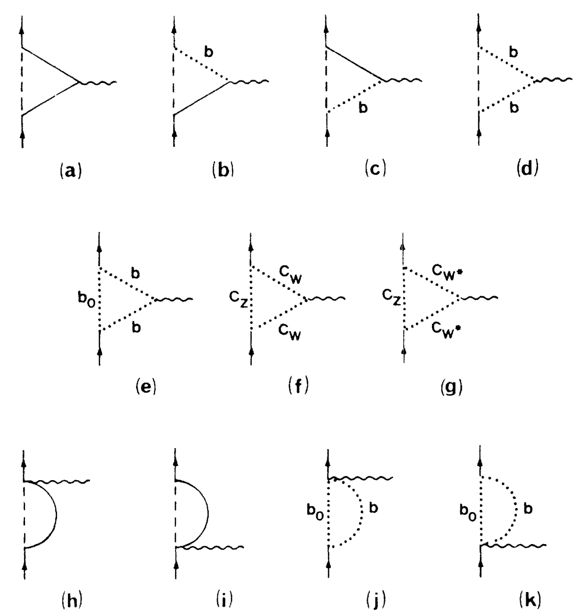

FIG. 2. Scattering diagrams with Z lines.

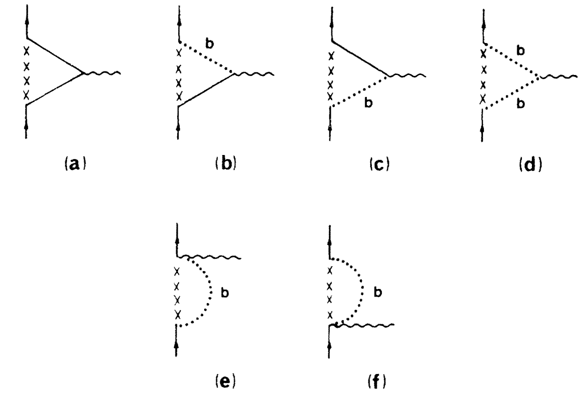

FIG. 3. Scattering diagrams with s lines.

note its initial- and final-momentum four-vectors as p and p'. We shall now take the initial as well as the final W boson as physical, so that

$$p'_\mu W^*_\mu(p') = p_\nu W_\nu(p) = 0, \quad p^2 = p'^2 = -M_W^2 . \qquad (3.1)$$

Moreover, in order to ensure the validity of the external-field treatment as explained in Sec. II, we shall impose the condition (2.1).

With the use of (3.1), the first-order contribution of the scattering operator, given by (2.2), can be expressed as

$$S_1 = ie(2\pi)^4\delta(p' - p - q)W^*_\mu(p')W_\nu(p)A_\lambda(q) \times[(p_\lambda + p'_\lambda)\delta_{\mu\nu} + 2(q_\mu\delta_{\nu\lambda} - q_\nu\delta_{\mu\lambda})] . \qquad (3.2)$$

In order to obtain the third-order contribution in the renormalizable gauge, it is necessary to consider a large number of diagrams. Since the calculations for diagrams with lepton loops remain unchanged when the renormalizable gauge is used instead of the unitary gauge, we shall not evaluate such diagrams.[6] We shall also ignore self-energy diagrams which do not give rise to any contribution to the moments. The remaining diagrams can be divided into three categories, which are shown in Figs. 1, 2, and 3. These diagrams can be treated by means of dimensional regularization,[7] and the total contribution can be put, with the use of (3.1) and (2.1), in the form

$$S_3 = ie(2\pi)^4\delta(p' - p - q)W^*_\mu(p')W_\nu(p)A_\lambda(q) \times\{A[(p_\lambda + p'_\lambda)\delta_{\mu\nu} + 2(q_\mu\delta_{\nu\lambda} - q_\nu\delta_{\mu\lambda})] + \kappa(q_\mu\delta_{\nu\lambda} - q_\nu\delta_{\mu\lambda}) + (Q/2M_W^2)q_\mu q_\nu(p_\lambda + p'_\lambda)\} , \qquad (3.3)$$

where the constant A is absorbed by charge renormalization, while κ and Q represent the anomalous magnetic and quadrupole moments.

The contributions to the magnetic moment κ arising from the diagrams in Figs. 1, 2, and 3 are

$$\kappa_{1a} = -\frac{e^2}{16\pi^2}\int_0^1 dx\left[3x^2 - 7x + \frac{15}{2} - \frac{2}{x} + 15(3x^2-2x)\ln x\right],$$

$$\kappa_{1b} + \kappa_{1c} = \frac{3e^2}{32\pi^2},$$

$$\kappa_{1d} = \frac{e^2}{16\pi^2}\int_0^1 dx\left(2 - \frac{2}{x}\right),$$

$$\kappa_{1e} + \kappa_{1f} = \frac{e^2}{16\pi^2}\int_0^1 dx(3x^2-2x)\ln x,$$

$$\kappa_{1g} + \kappa_{1h} = \frac{e^2}{16\pi^2}\left[\tfrac{9}{2}\eta + 6\int_0^1 dx(x-2)\ln x\right],$$

$$\kappa_{1i} = -\frac{g^2}{16\pi^2}(\tfrac{9}{2}\eta),$$

$$\kappa_{1j} = 0,$$

$$\kappa_{2a} = -\frac{g^2 M_W^2}{32\pi^2 M_Z^2}\int_0^1 dx\left(\frac{6x^4 - 14x^3 + 15x^2 - 4x}{x^2 + (1-x)(M_Z^2/M_W^2)} + 15(3x^2-2x)\times\ln[x^2 + (1-x)(M_Z^2/M_W^2)]\right),$$

$$\kappa_{2b} + \kappa_{2c} = \frac{g^2}{32\pi^2}\frac{M_W^2 - M_Z^2}{M_Z^2}\times\int_0^1 dx\,\frac{3x^2}{x^2 + (1-x)(M_Z^2/M_W^2)},$$

$$\kappa_{2d} = \frac{g^2}{16\pi^2}\frac{(M_W^2 - M_Z^2)^2}{M_W^2 M_Z^2}\int_0^1 dx\,\frac{2(x^2-x)}{x^2 + (1-x)(M_Z^2/M_W^2)},$$

$$\kappa_{2e} = -\frac{g^2}{32\pi^2}\int_0^1 dx(3x^2-2x)\ln[x^2 + (1-x)(M_Z^2/M_W^2)],$$

$$\kappa_{2f} + \kappa_{2g} = \frac{g^2 M_W^2}{32\pi^2 M_Z^2}\times\int_0^1 dx(3x^2-2x)\ln[x^2 + (1-x)(M_Z^2/M_W^2)],$$

$$\kappa_{2h} + \kappa_{2i} = \frac{g^2 M_W^2}{16\pi^2 M_Z^2}\times\left(\tfrac{9}{2}\eta + 3\int_0^1 dx(x-2)\times\ln[x^2 + (1-x)(M_Z^2/M_W^2)]\right),$$

$$\kappa_{2j} = \kappa_{2k} = 0,$$

$$\kappa_{3a} = \frac{g^2}{32\pi^2}\int_0^1 dx\,\frac{3x^2}{x^2 + (1-x)(\mu^2/M_W^2)},$$

$$\kappa_{3b} + \kappa_{3c} = \frac{g^2}{32\pi^2}\int_0^1 dx\,\frac{x^2}{x^2 + (1-x)(\mu^2/M_W^2)},$$

$$\kappa_{3d} = -\frac{g^2}{32\pi^2}\int_0^1 dx(3x^2-2x)\ln[x^2 + (1-x)(\mu^2/M_W^2)],$$

$$\kappa_{3e} + \kappa_{3f} = 0,$$

where the divergent constant η is given by[8]

$$\eta = \lim_{n\to 4}\left(\frac{1}{2-n/2}\right) - \gamma - \ln\pi + \ln\left(\frac{M^2}{M_W^2}\right) \qquad (3.4)$$

(γ is Euler's constant).

Further, the nonvanishing contributions to the quadrupole moment Q, which arise from only eight diagrams, are

$$Q_{1a} = -\frac{e^2}{32\pi^2},$$

$$Q_{1e} + Q_{1f} = \frac{1}{9}\left(\frac{e^2}{32\pi^2}\right),$$

$$Q_{2a} = \left(\frac{g^2 M_W^2}{16\pi^2 M_Z^2}\right)3\int_0^1 dx\,\frac{x^4 - x^3}{x^2 + (1-x)(M_Z^2/M_W^2)},$$

$$Q_{2e} = \left(\frac{g^2}{16\pi^2}\right)\frac{1}{3}\int_0^1 dx\,\frac{x^4 - x^3}{x^2 + (1-x)(M_Z^2/M_W^2)},$$

$$Q_{2f} + Q_{2g} = -\left(\frac{g^2 M_W^2}{16\pi^2 M_Z^2}\right)\times\frac{1}{3}\int_0^1 dx\,\frac{x^4 - x^3}{x^2 + (1-x)(M_Z^2/M_W^2)},$$

$$Q_{3d} = \left(\frac{g^2}{16\pi^2}\right)\frac{1}{3}\int_0^1 dx\,\frac{x^4 - x^3}{x^2 + (1-x)(\mu^2/M_W^2)}.$$

Thus, the total contributions to the magnetic and quadrupole moments are found to be

$$\kappa = \frac{5\alpha}{3\pi} + \left(\frac{GM_W^2}{2\pi^2\sqrt{2}}\right)\frac{1}{R}\int_0^1 dx\,x\,\frac{8x^3 - 8x^2 + 8x + R(x^3 - 5x^2 - 2x) + \frac{1}{2}R^2(-x^2 + 5x - 4)}{x^2 + R(1-x)} + \left(\frac{GM_W^2}{2\pi^2\sqrt{2}}\right)\int_0^1 dx\,x^2\,\frac{x^2 - x + 2 - \frac{1}{2}R'(x-1)}{x^2 + R'(1-x)}, \qquad (3.5)$$

and

$$Q=-\frac{\alpha}{9\pi}-\left(\frac{GM_W^{\,2}}{2\pi^2\sqrt{2}}\right)\frac{1}{3R}\int_0^1 dx\,\frac{x^3(1-x)(8+R)}{x^2+R(1-x)}-\left(\frac{GM_W^{\,2}}{2\pi^2\sqrt{2}}\right)\frac{1}{3}\int_0^1 dx\,\frac{x^3(1-x)}{x^2+R'(1-x)}\,, \tag{3.6}$$

where

$$\alpha=\frac{e^2}{4\pi}\,,\quad \frac{G}{\sqrt{2}}=\frac{g^2}{8M_W^{\,2}}\,,\quad R=\frac{M_Z^{\,2}}{M_W^{\,2}}\,,\quad R'=\frac{\mu^2}{M_W^{\,2}}\,, \tag{3.7}$$

and it is to be noted that terms involving the divergent constant η in $\kappa_{1g}+\kappa_{1h}$, κ_{1l}, and $\kappa_{2h}+\kappa_{2l}$ are mutually canceled by virtue of (1.4). The above results agree with those obtained by Bardeen *et al.* in the unitary gauge,[9] which can be regarded as a confirmation of the gauge invariance of these results.

The arguments presented in this paper clearly establish that the anomalous magnetic and quadrupole moments of the W boson are given by (3.5) and (3.6) within the limits of validity of the external-field treatment.

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[4]Since $M_Z > M_W$, the condition (2.1) also implies that $q^2/M_Z^{\,2} \ll 1$.

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[6]For the treatment of diagrams with lepton loops, see Ref. 2 and L. L. DeRaad, K. A. Milton, and W.-Y. Tsai, Phys. Rev. D 9, 2847 (1974).

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[8]The mass parameter M appearing in (3.4) is introduced to preserve the dimension of a dimensionally continued integral. See P. Salomonson and Y. Ueda, Phys. Rev. D 11, 2606 (1975).

[9]Note that in Ref. 2 the sign of the result for the quadrupole moment of the W boson should be changed, and the result for the magnetic moment due to Z exchange should be divided by 2.