PHYSICAL REVIEW D VOLUME 15, NUMBER 2 15 JANUARY 1977

Patterns of symmetry breaking in a gauge-theory model*

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(Received 24 May 1976)

We examine the $SU(2)_L \otimes U(1) \otimes U(1)$ gauge model of Yang by a group-theoretical approach and also by explicit computation to determine the number of physically inequivalent solutions of spontaneous symmetry breakdown and the physical particle content of these solutions. We find only two pseudo-Goldstone bosons in this particular model. An error is pointed out in Yang's paper in the determination of the number of pseudo-Goldstone bosons present and in the identification of the physical particle fields. Although our work deals with a specific model, it does illustrate the general type of detailed analysis necessary to determine the possible physical particle content of a gauge theory.

I. INTRODUCTION

The proposal that the pion may be a pseudo-Goldstone boson[1,2] has been widely investigated.[3,4] The desirable feature of this idea is that the mass of the pion is calculable and finite in higher orders of perturbation theory and, as a result, the mass of the pion can be related to other parameters of the theory. The finiteness of the calculated mass is a consequence of renormalizability and the absence of a counterterm which is not allowed by the symmetries imposed on the Lagrangian. In this note we will examine the structure of a model of T. C. Yang[4] which was motivated by this idea. We find that there are two different families of potentials in this model which are characterized by whether a certain coefficient in the scalar-field potential is positive or negative. By group-theoretical arguments and by explicit computation, only one physical solution of spontaneous symmetry breakdown of the model will be found to exist for each family of potentials. The solution in each case is determined by using the definition of the vacuum state as the lowest-energy state of the theory facilitated by use of the requirement that the solutions satisfy the "tadpole condition" of Weinberg.[2] The physical particle content of the model for one family of potentials will include one photon and four massive vector bosons, and for the other family of potentials it will include five massive vector bosons and no photon. There will exist two pseudo-Goldstone bosons in the theory for either family of potentials instead of the three pseudo-Goldstone bosons stated by Yang. The field taken to represent the neutral pion by Yang will be shown to be an unphysical Higgs-Kibble scalar field,[5] and a second field listed by him as a Goldstone boson will be shown to have a mass. The one-loop mass correction to the two pseudo-Goldstone bosons in the theory is obtained in an integral form, extending the mass correction computed by Yang to more general values of the parameters of the theory. We find, contrary to his claim, however, that this one-loop mass correction cannot be made to vanish if there is a photon present in the theory and all other vector bosons are massive. The mass correction, in this case, is too large to represent a physical pion mass if the heavy vector bosons in the theory have masses on the order of 30 GeV or larger. For the solution of spontaneous breakdown in which all of the vector bosons are massive, we find the interesting result that the mass correction to the pseudo-Goldstone bosons can be made arbitrarily small to all orders of perturbation theory by an appropriate choice of the parameters of the model.

Let us first consider a general gauge theory. We will define G to be the group of coordinate-dependent gauge symmetries and G_g to be the group of coordinate-independent continuous global symmetries satisfied by the Lagrangian. Each of these are symmetries before spontaneous symmetry breakdown. The group $\overline{G}$ will be defined as the continuous symmetry group of the scalar-field potential term of the Lagrangian before spontaneous symmetry breakdown. The groups S, S_g, and $\overline{S}$ will be defined as the gauge symmetry of the Lagrangian, continuous global symmetry of the Lagrangian, and continuous symmetry of the scalar-field potential, respectively, after spontaneous symmetry breakdown. The number of generators of a symmetry group $\mathcal{G}$ will be denoted $d[\mathcal{G}]$.

In a particular subclass of gauge theories, each continuous symmetry of the scalar-field potential that is broken after spontaneous symmetry breakdown will give rise to a scalar boson which is massless to zeroth order. Hence, there will be $d[\overline{G}] - d[\overline{S}]$ scalar bosons which are massless to zeroth order after spontaneous breakdown.[6] These zeroth-order massless scalar bosons can be divided into three categories: unphysical Higgs-

Kibble bosons, physical Goldstone bosons, and pseudo-Goldstone bosons.

If the symmetry of the potential that is broken is also a gauge symmetry of the Lagrangian, then the massless vector boson associated with the gauge symmetry will acquire a mass after spontaneous breakdown, and the massless scalar boson associated with the broken symmetry is an unphysical Higgs-Kibble scalar boson. This scalar boson is not a physical particle and may be gauged away by the Higgs-Kibble mechanism[5] to become the longitudinal component of the vector boson. The number of massive vector bosons and related unphysical Higgs-Kibble scalar fields is equivalent to the number of broken gauge symmetries,

$$n_V = d[G] - d[S]\,. \tag{1}$$

If the symmetry of the potential that is broken is a continuous global symmetry of the Lagrangian which is distinct from the gauge symmetries contained in G, then the associated massless scalar boson is a physical Goldstone boson. This scalar boson will remain massless to all orders of perturbation theory. Since this Goldstone boson is not associated with any massive vector boson, it is an excitation which cannot be gauged away. Hence, there will be

$$n_G = d[G_g] - d[S_g] - \{d[G] - d[S]\} \tag{2}$$

physical Goldstone bosons in the theory.

Now it may turn out, because of the symmetries imposed on the Lagrangian and the requirement of renormalizability, that the symmetry $\overline{G}$ of the scalar-field potential is *forced* to be larger than the global symmetry G_g of the Lagrangian. Those scalar bosons associated with symmetries contained in $\overline{G}$ but not in G_g that are broken after spontaneous breakdown are pseudo-Goldstone bosons. These scalar bosons are massless to zeroth order but then pick up a mass in higher orders of perturbation theory. This mass comes from terms in the Lagrangian that did not satisfy the symmetry in $\overline{G}$ that is associated with the pseudo-Goldstone boson. The number of such pseudo-Goldstone (PG) bosons is given by[1]

$$n_{PG} = d[\overline{G}] - d[\overline{S}] - \{d[G_g] - d[S_g]\}\ . \tag{3}$$

In a theory with pseudo-Goldstone bosons present there will in general be an arbitrariness in the vacuum state at the tree-graph level. This arbitrariness is reflected in the existence of a continuous infinity of physically inequivalent sets of vacuum expectation values which are solutions of the model at this level of calculation. Since the vacuum state is not completely specified at the tree-graph level, higher-order perturbative corrections must be carried out in order to determine the correct vacuum state of the theory.[7] As Weinberg has shown,[2] this can be achieved by imposing the condition that the vacuum expectation values have a well-behaved perturbative expansion. To implement this condition, Weinberg considers the one-loop correction to the vacuum expectation values of the scalar fields that is given by tadpole diagrams with an external scalar-boson line. A typical tadpole diagram is shown in Fig. 1. When the external line is that of a pseudo-Goldstone boson, which is massless to zeroth order, the tadpole diagram is singular unless the sum of the tadpole loops for that particular line is identically zero. In order for the vacuum expectation values to have a well-behaved perturbative expansion, the choice of vacuum expectation values is required to be such that the sum of the tadpole loops is identically zero for any external line which is a pseudo-Goldstone boson. Additionally, the vacuum expectation values must be chosen such that the calculated masses of the pseudo-Goldstone bosons in higher orders of perturbation theory are positive real quantities. We shall henceforth refer to the combination of these two conditions as the tadpole condition.

This raises the interesting question concerning how many physically inequivalent solutions of spontaneous breakdown are allowed for a gauge model by the tadpole condition. Further complicating this question is the fact that these solutions may have a discontinuous dependence upon the parameters of the scalar-field potential and thus, must be determined for each of different families of potentials. We shall illustrate this question concerning the number of allowed, inequivalent solutions by examining a particular gauge model of T. C. Yang.[4] The general symmetry properties of the model will be discussed in Sec. II. The

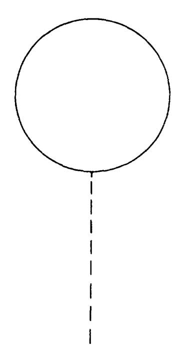

FIG. 1. Feynman diagram for the tadpole with an external pseudo-Goldstone boson line. The pseudo-Goldstone field is represented by the dashed line. The solid line represents a vector field or a scalar field.

solution of spontaneous breakdown of the model will be obtained for each of two families of scalar-field potentials in Secs. III and IV.

II. THE MODEL

The model of T. C. Yang[4] to be examined is invariant under an $SU(2)_L \otimes U(1) \otimes U(1)$ gauge group. The interesting feature which Yang investigated in this model was whether or not the pseudo-Goldstone bosons which he identified as pions could have small masses arising from the electromagnetic weak interaction even though the masses of the massive vector bosons were large. The model consists of three complex doublets of scalar bosons,

$$\phi = \begin{pmatrix} i\phi_1 + \phi_2 \\ \phi_4 - i\phi_3 \end{pmatrix}, \quad \psi = \begin{pmatrix} i\pi_1 + \pi_2 \\ \sigma - i\pi_3 \end{pmatrix}, \quad \xi = \begin{pmatrix} i\Pi_1 + \Pi_2 \\ \Sigma - i\Pi_3 \end{pmatrix}, \tag{4}$$

three $SU(2)_L$ vector bosons A_a^μ, and two $U(1)_B \otimes U(1)_C$ vector bosons B^μ and C^μ. The $SU(2)_L \otimes U(1)_B \otimes U(1)_C$ gauge-invariant Lagrangian formed from these fields is

$$\begin{aligned} \mathcal{L} = {} & \tfrac{1}{2}|(\partial^\mu - ig\tau_a A_a^\mu - ig'B^\mu)\phi|^2 \\ & + \tfrac{1}{2}|(\partial^\mu - ig\tau_a A_a^\mu - ig'C^\mu)\psi|^2 \\ & + \tfrac{1}{2}|(\partial^\mu - ig\tau_a A_a^\mu - ig'C^\mu)\xi|^2 \\ & - \tfrac{1}{4}(F_{(A)a}^{\mu\nu})^2 - \tfrac{1}{4}(F_{(B)}^{\mu\nu})^2 - \tfrac{1}{4}(F_{(C)}^{\mu\nu})^2 - P(\phi, \psi, \xi), \end{aligned} \tag{5}$$

where $F_{(A)}^{\mu\nu}$, $F_{(B)}^{\mu\nu}$, and $F_{(C)}^{\mu\nu}$ are the field strengths of the vector bosons, and P is the potential term of the scalar bosons.

Following Yang, we will impose a charge-conjugation symmetry $\phi \underset{C}{\rightleftarrows} \tau_2\phi^*$, $B^\mu \underset{C}{\rightleftarrows} -B^\mu$ on the Lagrangian in addition to the

$$G = SU(2)_L \otimes U(1)_B \otimes U(1)_C \tag{6}$$

gauge symmetry. The $SU(2)_L$ gauge group rotates the component fields of each of the three scalar doublets ϕ, ψ, and ξ, and makes a rotation and gauge translation on the vector fields A_a^μ. The $U(1)_B$ gauge symmetry rotates the ϕ scalar-doublet component fields and makes a gauge translation on the vector field B^μ. The $U(1)_C$ gauge symmetry rotates the component fields of each of the ψ and ξ doublets while making a gauge translation on the vector field C^μ. We can further require that the Lagrangian be invariant under two separate global coordinate-independent symmetry transformations $U(1)_\psi \otimes U(1)_\xi$. This allows the component fields of the ψ doublet and ξ doublet to rotate independently of each other under two global U(1) groups. From the gauge group above and the arbitrarily imposed $U(1)_\psi \otimes U(1)_\xi$ global symmetry, we conclude that the total global symmetry of the Lagrangian before spontaneous breakdown will be

$$G_g = SU(2)_L \otimes U(1)_\phi \otimes U(1)_\psi \otimes U(1)_\xi, \tag{7}$$

where the subscripts denote the particular scalar doublets that transform under each group.

The most general scalar-field potential which is consistent with this global symmetry, charge-conjugation invariance, and renormalizability is

$$\begin{aligned} P(\phi, \psi, \xi) = {} & -h_1\phi^2 - h_2\psi^2 - h_3\xi^2 \\ & + h_4(\phi^2)^2 + h_5(\psi^2)^2 + h_6(\xi^2)^2 \\ & + h_7\phi^2\psi^2 + h_8\phi^2\xi^2 + h_9\psi^2\xi^2 \\ & + h_{10}\psi^\dagger\xi\xi^\dagger\psi. \end{aligned} \tag{8}$$

The coefficients h_i are to be chosen such that each of the doublets ϕ, ψ, and ξ develops a vacuum expectation value. To accomplish this, the coefficients h_1, h_2, and h_3 are chosen to be positive. To guarantee a stable theory, the potential must be positive for large classical values of its fields. Therefore, to have a stable theory, the coefficients h_4, h_5, and h_6 must be chosen to be greater than zero, and each coefficient h_7, h_8, h_9, and h_{10} must be greater than some negative constant. The negative lower bound for each coefficient h_7, h_8, h_9, and h_{10} is determined by the values of the other coefficients in the potential. If h_7, h_8, and h_9 are too large, then not all of the doublets will develop a vacuum expectation value. For the type of spontaneous breakdown which we desire, each of these three coefficients must be less than some upper bound which is positive and is determined by the values of the other coefficients in the potential. The term with coefficient h_{10} will merit special attention since the sign of h_{10} has an important bearing on the solution of spontaneous breakdown and, as a consequence, the physical particle content of the theory.

From the definition of the complex doublet ϕ, Eq. (4), ϕ^2 can be written in terms of its real fields ϕ_i as

$$\phi^2 = \sum_{i=1}^{4} (\phi_i)^2. \tag{9}$$

We see that ϕ^2 is invariant under an $O(4)_\phi$ symmetry group of transformations of its real fields ϕ_i. This simple remark leads to a larger symmetry group. Using the definition of ψ and ξ, Eq. (4), the term $\psi^\dagger\xi\xi^\dagger\psi$ of the scalar-field potential can be written as

$$\begin{aligned} h_{10}\psi^\dagger\xi\xi^\dagger\psi = h_{10}[& (\pi_1\Pi_1 + \pi_2\Pi_2 + \pi_3\Pi_3 + \sigma\Sigma)^2 \\ & + (\pi_2\Pi_1 - \pi_1\Pi_2 + \pi_3\Sigma - \sigma\Pi_3)^2]. \end{aligned} \tag{10}$$

This term breaks down the $O(4)_\psi \otimes O(4)_\xi$ symmetry satisfied by the rest of the terms in the potential to an $SU(2)_{\psi\xi} \otimes U(1)_\psi \otimes U(1)_\xi$ symmetry. Consequently, the full symmetry $\overline{G}$ satisfied by the potential P before spontaneous breakdown is

$$\overline{G} = O(4)_\phi \otimes SU(2)_{\psi\xi} \otimes U(1)_\psi \otimes U(1)_\xi . \quad (11)$$

In the potential which Yang wrote down, the terms involving h_7, h_8, h_9, and h_{10} of Eq. (8) were omitted and a term of the form $(\psi^\dagger\xi + \xi^\dagger\psi)^2$ was included. His potential is invariant under an $O(4)_\phi \otimes O(4)_{\psi\xi}$ symmetry group before spontaneous breakdown, and if the electromagnetic and weak interactions are turned off, this becomes the symmetry of his entire Lagrangian. The $O(4)_\phi \otimes O(4)_{\psi\xi}$ symmetry group contains an $O(4)_{\phi\psi\xi}$ subgroup which is isomorphic to the $SU(2) \otimes SU(2)$ group. This subgroup was interpreted by Yang to be a chiral symmetry of the Lagrangian in the limit that the electromagnetic weak interaction is turned off. For Yang's purpose of identifying the physical particles and computing the one-loop mass correction to the pseudo-Goldstone bosons of the theory, his potential is sufficient. However, the terms omitted from Yang's potential satisfy the criteria of being of dimension less than or equal to four and consistent with the symmetries imposed on the complete Lagrangian, and thus are necessary to insure the renormalizability of the theory. The term of the form $(\psi^\dagger\xi + \xi^\dagger\psi)^2$ is not required for renormalizability since it is eliminated with the arbitrarily imposed $U(1)_\psi \otimes U(1)_\xi$ global symmetry. The potential of Eq. (8) contains the minimum number of scalar terms necessary to ensure the renormalizability of the model. But as we have seen [cf. Eqs. (10) and (11)], the global symmetry of the Lagrangian with this potential, in the limit of vanishing electromagnetic weak interaction, will contain an $O(4)_{\phi\psi\xi}$ symmetry subgroup to be identified as a chiral symmetry only if h_{10} is set equal to zero.

After spontaneous symmetry breakdown, the fields of the three doublets, Eq. (4), will acquire vacuum expectation values. Let us now determine the number of physically different theories that are allowed by Yang's Lagrangian. The number of physically different theories will correspond to the number of sets of vacuum expectation values that can be found which are compatible with the Lagrangian, and which are not related to each other by the symmetries of the Lagrangian. To find the number of physically different theories, we first suppose that each of the fields of the doublets, Eq. (4), acquire a vacuum expectation value. However, not all of these vacuum expectation values are independent of each other. The number of vacuum expectation values which are independent of each other and which are determined by the parameters of the scalar-field potential will be less than or equal to the canonical number of Bludman and Klein.[8] This canonical number is defined as the number of independent invariants contained in the scalar-field potential. For the potential of Eq. (8), the invariants are ϕ^2, ψ^2, ξ^2, and $\psi^\dagger\xi\xi^\dagger\psi$. Hence, the number of independent vacuum expectation values for this potential must be less than or equal to four. Our first step in determining the number of physically inequivalent solutions allowed by the Lagrangian will be to reduce the arbitrary set of vacuum expectation values developed by the scalar fields to a physically equivalent set of four or less. We proceed to do this by beginning with the ψ doublet and using the $SU(2)_L$ symmetry of the Lagrangian to rotate the fields of this doublet until only the new σ field has a nonzero vacuum expectation value. At this point, we make use of the definition of the vacuum state. The vacuum state is defined as the lowest-energy state of the theory; hence a classical minimum of the scalar-field potential must be reached when the fields therein are replaced by their zeroth-order vacuum expectation values. A classical minimum point of the potential in the scalar-field space is characterized by the first derivative of the potential with respect to any of its fields being zero and the second-derivative matrix being positive-definite when evaluated at this point.

III. FIRST SOLUTION

We will first consider the case when the coefficient h_{10} in the potential, Eq. (8), is negative. Turning to the ξ doublet, we apply the criteria resulting from the definition of the vacuum state. An examination of the scalar-field potential reveals that the minimum of the potential is reached only if the fields Σ and Π_3 contain the nonzero vacuum expectation values of the ξ doublet, with Π_1 and Π_2 having zero vacuum expectation values. There is only one term of the potential that is responsible for this restriction on the vacuum expectation values, and that is the term with coefficient h_{10}. As a result of the fields Π_1 and Π_2 having zero vacuum expectation values, the $U(1)_\xi$ symmetry can be used to rotate the fields of the ξ doublet until only the new Σ field develops a vacuum expectation value. Finally we consider the ϕ doublet. There is a little group of the $SU(2)_L \otimes U(1)_\phi \otimes U(1)_\psi \otimes U(1)_\xi$ global symmetry group of the Lagrangian that can be used to rotate the fields of the ϕ doublet until only the new ϕ_2 and ϕ_4 fields develop vacuum expectation values, while still retaining the vacuum expectation values in the new σ and Σ fields of the ψ and ξ doublets. We shall designate the vacuum expectation values of the ϕ_2, ϕ_4, σ, and Σ fields as $\lambda\sin\theta$, $\lambda\cos\theta$, $\langle\sigma\rangle$, and $\langle\Sigma\rangle$, respectively; and define new primed fields ϕ_2', ϕ_4', σ', and Σ', which have zero vacuum expectation values, by

$$\phi_2=\lambda\sin\theta+\phi_2', \quad \phi_4=\lambda\cos\theta+\phi_4' ,$$
$$\sigma=\langle\sigma\rangle+\sigma', \quad \Sigma=\langle\Sigma\rangle+\Sigma' . \tag{12}$$

A calculation of the classical minimum of the scalar-field potential using the above mentioned conditions on the derivatives of the potential shows that the magnitude of the zeroth-order vacuum expectation value of each doublet ϕ, ψ, and ξ can be determined as a function of the coefficients contained in the potential. Hence, the absolute values of λ, $\langle\sigma\rangle$, and $\langle\Sigma\rangle$ are determined by the coefficients in the potential, and these are the independent vacuum expectation values of the theory. As a further consequence of the global symmetry G_g which the Lagrangian satisfies, we may choose the vacuum expectation values λ, $\langle\sigma\rangle$, and $\langle\Sigma\rangle$ to be positive without any loss of generality.

The one parameter yet to be determined is the angle θ. The scalar-field potential does not put a constraint on this angle because it corresponds to a rotation contained in the symmetry group $\overline{G}$ under which the potential is invariant. This rotation does not belong to the global symmetry group G_g of the entire Lagrangian, however, which leaves open the possibility that there may be different allowed values of θ corresponding to different physical theories which are solutions of the model. To find the allowed values of the angle θ, we must look to the one-loop correction to the theory and apply the previously discussed tadpole condition. This consists of identifying the pseudo-Goldstone bosons and then finding the angles θ for which these bosons do not develop tadpoles and also for which their calculated masses are positive real quantities. We will show by explicit computation that there are two angles θ corresponding to two different sets of vacuum expectation values which satisfy the tadpole condition. These two angles will be shown to be $\theta=0$ and $\theta=\frac{1}{2}\pi$. For the moment, however, we will concern ourselves with the physical content of these solutions. The $\theta=\frac{1}{2}\pi$ solution can be transformed into the $\theta=0$ solution by the charge-conjugation symmetry operation on the ϕ and B^μ fields under which the Lagrangian is invariant. Thus, we are left with the $\theta=0$ solution as the physical solution of spontaneous breakdown. The original set of vacuum expectation values has been reduced down to a unique set by using the symmetries of the Lagrangian, and this set is completely determined by the parameters of the model. What we have shown, then, is that there is only one physical solution for this type of spontaneous symmetry breakdown of the model when the coefficient h_{10} in the scalar-field potential is negative.

Now, let us examine the symmetry structure of Yang's model after spontaneous breakdown. By consideration of ϕ^2, Eq. (9), and $\psi^\dagger\xi\xi^\dagger\psi$, Eq. (10), the presence of the vacuum expectation values can be seen to reduce the symmetry $\overline{G}$ of the scalar-field potential P, Eq. (8), to

$$\overline{S}^{(-)}=\mathrm{O}(3)_\phi\otimes\mathrm{U}(1)_{\psi\xi} . \tag{13}$$

The superscript (–) indicates that this result is obtained in the case when the coefficient h_{10} in the scalar-field potential is negative. Similarly, the global symmetry G_g and the gauge symmetry G of the Lagrangian, Eq. (5), are reduced to

$$S_g^{(-)}=\mathrm{U}(1), \quad S^{(-)}=\mathrm{U}(1) , \tag{14}$$

respectively, after spontaneous breakdown. This remaining U(1) gauge symmetry is an electromagnetic gauge invariance with an accompanying massless photon. There is an electromagnetic gauge symmetry of the theory only because the angle θ is equal to an integer multiple of $\frac{1}{2}\pi$. If the angle θ dictated by the tadpole condition had not been an integer multiple of $\frac{1}{2}\pi$, then there would not have been a remaining U(1) electromagnetic symmetry, and the theory would not have contained a massless photon.

The physical particle content of the Lagrangian can now be derived from the symmetry structure. From the preceding results, we find that

$$n_V^{(-)}=d[G]-d[S^{(-)}]$$
$$=5-1=4 \tag{15}$$

vector bosons will acquire a mass, leaving a fifth to remain massless. This massless vector boson is the photon associated with the remaining U(1) gauge invariance. We also find that there will be

$$n_G^{(-)}=d[G_g]-d[S_g^{(-)}]-\{d[G]-d[S^{(-)}]\}$$
$$=6-1-(5-1)=1 \tag{16}$$

physical Goldstone boson which remains massless to all orders of perturbation theory. In addition, there will be

$$n_{PG}^{(-)}=d[\overline{G}]-d[\overline{S}^{(-)}]-\{d[G_g]-d[S_g^{(-)}]\}$$
$$=11-4-(6-1)=2 \tag{17}$$

pseudo-Goldstone bosons that are massless to zeroth order but which acquire a mass in higher order of perturbation theory.

We now turn to verify our group-theoretical analysis of the Lagrangian by explicit computation. The tadpole condition on the angle θ, Eq. (12), will be derived, and as a secondary result, the one-loop mass correction to the pseudo-Goldstone bosons will be obtained.

The classical minimum of the potential P,

$$\left.\frac{\partial P(\phi,\psi,\xi)}{\partial\xi_i}\right|_{\phi=\langle\phi\rangle,\,\psi=\langle\psi\rangle,\,\xi=\langle\xi\rangle}=0\,, \tag{18}$$

with ξ_i any field occurring in the potential, defines the three vacuum expectation values λ, $\langle\sigma\rangle$, and $\langle\Sigma\rangle$, Eq. (12), to zeroth order in terms of the coefficients of the potential. Let us temporarily include the additional term $h_{11}(\psi^\dagger\xi+\xi^\dagger\psi)^2$ in the potential, Eq. (8), in order to identify the physical Goldstone boson. The coefficient h_{11} must be chosen negative so that the vacuum expectation values of Eq. (12) will be compatible with the definition of the vacuum state as being the lowest-energy state of the theory. This additional term breaks the global $U(1)_\psi\otimes U(1)_\xi$ symmetry contained in G_g down to $U(1)_{\psi\xi}$. As a result, the original Goldstone boson associated with the symmetry broken by this extra term will now, instead, be a massive scalar boson with a mass coefficient proportional to h_{11} allowing us to easily identify it. There will be three nontrivial relations of Eq. (18) defining λ, $\langle\sigma\rangle$, and $\langle\Sigma\rangle$. These three relations can be used to replace three parameters of the potential in terms of the vacuum expectation values $\lambda\sin\theta$, $\lambda\cos\theta$, $\langle\sigma\rangle$, and $\langle\Sigma\rangle$. We shall choose h_4, h_5, and h_6 as the parameters to be replaced. Having done this, the mass term of the potential P is computed to be

$$\begin{aligned}P^{(2)}(\phi,\psi,\xi)=&\,2h_1(\cos\theta\,\phi_4'+\sin\theta\,\phi_2')^2+2h_2\sigma'^2+2h_3\Sigma'^2-2h_7[\langle\sigma\rangle(\cos\theta\,\phi_4'+\sin\theta\,\phi_2')-\lambda\sigma']^2\\&-2h_8[\langle\Sigma\rangle(\cos\theta\,\phi_4'+\sin\theta\,\phi_2')-\lambda\,\Sigma']^2-(2h_9+2h_{10}+8h_{11})(\langle\Sigma\rangle\,\sigma'-\langle\sigma\rangle\,\Sigma')^2\\&-(h_{10}+4h_{11})[(\langle\Sigma\rangle\,\pi_1-\langle\sigma\rangle\,\Pi_1)^2+(\langle\Sigma\rangle\pi_2-\langle\sigma\rangle\,\Pi_2)^2]-4h_{11}(\langle\Sigma\rangle\,\pi_3-\langle\sigma\rangle\,\Pi_3)^2\,.\end{aligned}\tag{19}$$

The previously discussed constraints on the coefficeints are sufficient to ensure that the mass eigenvalues of this term are positive.

In general, there are three independent combinations of $\cos\theta\,\phi_4'+\sin\theta\,\phi_2'$, σ', and Σ' that are massive as well as $\langle\Sigma\rangle\,\pi_1-\langle\sigma\rangle\,\Pi_1$ and $\langle\Sigma\rangle\,\pi_2-\langle\sigma\rangle\,\Pi_2$. (Yang's paper had listed one combination of ϕ_4', σ', and Σ' as an unphysical Goldstone boson for the $\theta=0$ solution, when actually it is a massive scalar boson.) The field with mass proportional to h_{11} identifies the physical Goldstone boson ($h_{11}=0$) as

$$\tilde{\phi}_G=\frac{1}{(\langle\sigma\rangle^2+\langle\Sigma\rangle^2)^{1/2}}\,(\langle\Sigma\rangle\,\pi_3-\langle\sigma\rangle\,\Pi_3)\,. \tag{20}$$

Thus, we have accounted for six of the twelve scalar fields.

From the Lagrangian, Eq. (5), we find that the bilinear derivative coupling of the massive vector bosons to the unphysical Higgs-Kibble scalar fields is

$$\begin{aligned}\mathcal{L}^{(\mu)}=&-gA_1^\mu\partial_\mu[\lambda(\cos\theta\,\phi_1-\sin\theta\phi_3)+\langle\sigma\rangle\pi_1+\langle\Sigma\rangle\,\Pi_1]\;-gA_2^\mu\partial_\mu[\lambda(\cos\theta\,\phi_2'-\sin\theta\phi_4')+\langle\sigma\rangle\,\pi_2+\langle\Sigma\rangle\,\Pi_2]\\&-gA_3^\mu\partial_\mu[\lambda(\sin\theta\,\phi_1+\cos\theta\,\phi_3)+\langle\sigma\rangle\pi_3+\langle\Sigma\rangle\,\Pi_3]\;+g'\lambda B^\mu\partial_\mu(-\sin\theta\,\phi_1+\cos\theta\,\phi_3)\;+g'C^\mu\partial_\mu(\langle\sigma\rangle\pi_3+\langle\Sigma\rangle\Pi_3)\,.\end{aligned}\tag{21}$$

Four Higgs-Kibble fields can be identified from this bilinear coupling for an arbitrary angle θ. We will write these fields as the following orthogonal combination:

$$\begin{aligned}\tilde{\phi}_1&=\frac{1}{r_1}\,[\lambda(\cos\theta\,\phi_1-\sin\theta\,\phi_3)+\langle\sigma\rangle\pi_1+\langle\Sigma\rangle\,\Pi_1]\,,\\\tilde{\phi}_2&=\frac{1}{r_1}\,[\lambda(\cos\theta\,\phi_2'-\sin\theta\,\phi_4')+\langle\sigma\rangle\pi_2+\langle\Sigma\rangle\,\Pi_2]\,,\\\tilde{\phi}_3&=\frac{1}{r_1}\,[\lambda(\sin\theta\,\phi_1+\cos\theta\,\phi_3)+\langle\sigma\rangle\,\pi_3+\langle\Sigma\rangle\Pi_3]\,,\\\tilde{\phi}_4&=\frac{1}{r_1r_2}\,[r_2^{\,2}(\sin\theta\,\phi_1+\cos\theta\,\phi_3)-\lambda(\langle\sigma\rangle\,\pi_3+\langle\Sigma\rangle\,\Pi_3)]\,.\end{aligned}\tag{22}$$

The coefficients r_1 and r_2 are defined in terms of the vacuum expectation values λ, $\langle\sigma\rangle$, and $\langle\Sigma\rangle$ as $r_1^{\,2}=\lambda^2+\langle\sigma\rangle^2+\langle\Sigma\rangle^2$ and $r_2^{\,2}=\langle\sigma\rangle^2+\langle\Sigma\rangle^2$. The calculation by Yang of the one-loop electromagnetic weak mass correction to the "neutral pion" was based on identifying $\tilde{\phi}_4$ as the neutral pseudo-Goldstone pion field for the $\theta=0$ solution. But, it is actually an unphysical Higgs-Kibble scalar field which can be gauged away.

There is a fifth orthogonal field,

$$\begin{aligned}\tilde{\pi}_1=\frac{1}{r_1r_2}\,&[r_2^{\,2}(\cos\theta\,\phi_1-\sin\theta\,\phi_3)\\&-\lambda(\langle\sigma\rangle\,\pi_1+\langle\Sigma\rangle\,\Pi_1)]\,,\end{aligned}\tag{23}$$

which can be found from the bilinear coupling when θ is not an integer multiple of $\frac{1}{2}\pi$. This field $\tilde{\pi}_1$, however, does not have a bilinear derivative coupling to a vector boson field for θ equal to an integer multiple of $\frac{1}{2}\pi$. If θ were not equal to $\frac{1}{2}n\pi$, then $\bar{\pi}_1$ would be a Higgs-Kibble field. But, we will show that $\theta=\frac{1}{2}n\pi$ is the proper solution to the model, and therefore $\bar{\pi}_1$ which is massless to zeroth order is, in fact, a pseudo-Goldstone boson field.

The one remaining scalar field which we have not identified is

$$\bar{\pi}_2=\frac{1}{r_1 r_2}\left[r_2{}^2(\cos\theta\,\phi_2'-\sin\theta\,\phi_4')-\lambda(\langle\sigma\rangle\pi_2+\langle\Sigma\rangle\Pi_2)\right]. \tag{24}$$

It is massless to zeroth order, and it is the other pseudo-Goldstone boson of the theory. For $\theta=\frac{1}{2}n\pi$, there is a remaining U(1) electromagnetic gauge symmetry of the model under which $\bar{\pi}_1$ and $\bar{\pi}_2$ can be rotated into each other. For these special values of θ, the fields $\bar{\pi}_1$ and $\bar{\pi}_2$ can be combined to represent two equal but oppositely charged scalar bosons.

Now, let us determine the angles θ for which the pseudo-Goldstone bosons do not develop singular tadpoles. Using Weinberg's notation,[2] we will write the tadpole graph as

$$\delta\langle\bar{\pi}_i\rangle=i(2\pi)^{-4}\Delta^{\bar{\pi}}(0)T[\bar{\pi}_i], \tag{25}$$

where $\Delta^{\bar{\pi}}(k)$ is the propagator of the $\bar{\pi}_i$ field carrying four-momentum k, and $T[\bar{\pi}_i]$ is the total tadpole-loop contribution for the $\bar{\pi}_i$ external line. There are no scalar-boson loops that contribute to a tadpole graph with $\bar{\pi}_1$ or $\bar{\pi}_2$ as the external line. There are, however, vector-boson loops that can contribute to a tadpole graph with a $\bar{\pi}_1$ or $\bar{\pi}_2$ external line. The interaction terms for these vector-boson loop tadpoles, determined from the Lagrangian, Eq. (5), are

$$\mathcal{L}^{(T)}=2gg'\frac{\lambda r_2}{r_1}\left[\bar{\pi}_1(-\cos2\theta A_2^\mu B_\mu+A_2^\mu C_\mu)+\bar{\pi}_2(\cos2\theta A_1^\mu B_\mu+\sin2\theta A_3^\mu B_\mu-A_1^\mu C_\mu)\right]. \tag{26}$$

Weinberg[2] has shown that the part of the tadpole graph which becomes singular with an external, zero-mass pseudo-Goldstone-boson line is gauge invariant. It is this singular part of the tadpole graph with which we are concerned. Consequently, we may choose to calculate the tadpole loops in the Landau gauge without any loss of generality. There will be no contribution to the tadpole graphs by ghost loops in this gauge.

From Eq. (5), we find that the free-field Lagrangian for the vector bosons is

$$\begin{aligned}\mathcal{L}^{(V)}=&-\tfrac{1}{4}(F^{\mu\nu}_{(A)a})^2-\tfrac{1}{4}(F^{\mu\nu}_{(B)})^2-\tfrac{1}{4}(F^{\mu\nu}_{(C)})^2\\&+\tfrac{1}{2}g^2r_1{}^2A_a^{\mu 2}+\tfrac{1}{2}g'^2\lambda^2B^{\mu 2}+\tfrac{1}{2}g'^2r_2{}^2C^{\mu 2}\\&+gg'(\lambda^2\sin2\theta A_1^\mu B_\mu-\lambda^2\cos2\theta A_3^\mu B_\mu-r_2{}^2A_3^\mu C_\mu).\end{aligned} \tag{27}$$

Using the interaction terms of Eq. (26), the total tadpole-loop contribution to $\bar{\pi}_1$ and $\bar{\pi}_2$ can now be computed. The vector-boson loops are to be evaluated with the vector-boson propagator matrix of the above free-field Lagrangian in the Landau gauge. A general form defining the free-field vector-boson propagator in an arbitrary gauge is contained in Weinberg's paper.[2] The tadpole loops $T[\bar{\pi}_1]$ and $T[\bar{\pi}_2]$ are found to be

$$\begin{aligned}&T[\bar{\pi}_1]=0,\\&T[\bar{\pi}_2]=-3i(gg')^4\lambda^3r_1r_2{}^3\sin4\theta\int d_E{}^4k\frac{1}{d_1(k,\theta)},\end{aligned} \tag{28}$$

where $d_1(k,\theta)$ is defined as

$$\begin{aligned}d_1(k,\theta)&\\&=k^2(k^2+g^2r_1{}^2)\\&\quad\times\{[k^2+(2g^2+g'^2)\lambda^2](k^2+g'^2r_2{}^2)-k^2g^2(\lambda^2-r_2{}^2)\}\\&\quad+(gg')^4(\lambda^2r_2{}^2\sin2\theta)^2\end{aligned} \tag{29}$$

and the coefficients r_1 and r_2 were defined above. The integration contour of $T[\bar{\pi}_2]$ has been rotated such that it is a four-dimensional Euclidian integral. The vanishing of $T[\bar{\pi}_1]$ results from the particular form into which we have rotated the vacuum expectation values. The field $\bar{\pi}_1$ would develop a nonvanishing tadpole if the vacuum expectation values, Eq. (12), were rotated relative to the two fields $\bar{\pi}_1$ and $\bar{\pi}_2$ by an appropriate global symmetry rotation.

By inspection of the above form of $T[\bar{\pi}_2]$, we see that the tadpole loop vanishes for θ equal to an integer multiple of $\frac{1}{4}\pi$. By our tadpole condition then, only these values of θ are allowed as possible solutions to the model. To further reduce this number of possible solutions, we will compute the one-loop mass correction to the pseudo-Goldstone bosons. Only those values of θ for which the calculated pseudo-Goldstone boson masses are positive real will be allowed as solutions of the theory.

The pseudo-Goldstone boson masses can be easily obtained from the tadpole loop $T[\bar{\pi}_2]$ by making use of the general formalism for perturbative calculations of symmetry breaking derived by Weinberg.[2] The first Weinberg result that we shall use is that the one-loop calculation of the pseudo-Goldstone-boson mass matrix can be written as

$$m_{\tilde{\pi}}{}^2{}_{ij} = \frac{i}{(2\pi)^4} L[\tilde{\pi}_i] T[\tilde{\pi}_j] , \tag{30}$$

where $L[\tilde{\pi}_i]$ is an appropriately normalized Lie derivative which is associated with the pseudo-Goldstone boson $\tilde{\pi}_i$, and $T[\tilde{\pi}_j]$ is the tadpole loop for $\tilde{\pi}_j$ evaluated in the Landau gauge. For the mass matrix of our two pseudo-Goldstone bosons to be positive definite, its diagonal elements must be positive. Therefore, we will compute the diagonal element $m_{\tilde{\pi}}{}^2{}_{22}$ to determine the constraint placed on the angle θ by this positivity condition. The normalized Weinberg Lie derivative for $\tilde{\pi}_2$, Eq. (24), is

$$L[\tilde{\pi}_2] = \frac{1}{r_1 r_2}\left[r_2{}^2 \left(\cos\theta \frac{\partial}{\partial\langle\phi_2\rangle} - \sin\theta \frac{\partial}{\partial\langle\phi_4\rangle}\right) - \lambda\left(\langle\sigma\rangle \frac{\partial}{\partial\langle\pi_2\rangle} + \langle\Sigma\rangle \frac{\partial}{\partial\langle\Pi_2\rangle}\right)\right] . \tag{31}$$

To be able to use this derivative on the tadpole loop $T[\tilde{\pi}_2]$, we would need $T[\tilde{\pi}_2]$ evaluated with nonzero vacuum expectation values $\langle\pi_2\rangle$ and $\langle\Pi_2\rangle$. But, we have already rotated the vacuum expectation values of the ψ and ξ doublets in a direction in which $\langle\pi_2\rangle$ and $\langle\Pi_2\rangle$ vanish. To get around this, we construct the Lie derivative L_2,

$$L_2 = \langle\phi_4\rangle \frac{\partial}{\partial\langle\phi_2\rangle} - \langle\phi_2\rangle \frac{\partial}{\partial\langle\phi_4\rangle} + \langle\sigma\rangle \frac{\partial}{\partial\langle\pi_2\rangle} + \langle\Sigma\rangle \frac{\partial}{\partial\langle\Pi_2\rangle} . \tag{32}$$

It is a derivative with respect to an SU(2) rotation of the nonzero vacuum expectation values $\langle\phi_2\rangle$, $\langle\phi_4\rangle$, $\langle\sigma\rangle$, and $\langle\Sigma\rangle$ of the ϕ, ψ, and ξ doublets. This rotation is contained in the global symmetry group G_g of the Lagrangian. We now make use of a second Weinberg result, that

$$L_2 T[\tilde{\pi}_j] = 0 . \tag{33}$$

We are assuming that the vacuum expectation values $\langle\pi_2\rangle$ and $\langle\Pi_2\rangle$ contained in $T[\tilde{\pi}_j]$ have not been set to zero until after the derivative L_2 has been taken. This last equation is a consequence of the tadpole loop $T[\tilde{\pi}_j]$ calculated in the Landau gauge being invariant under a rotation of the vacuum expectation values contained in it if the rotation belongs to the global symmetry group G_g. The derivative $L[\tilde{\pi}_2]$, on the other hand, is a derivative with respect to a rotation of the ϕ doublet vacuum expectation values multiplied by one coefficient, plus a derivative with respect to a rotation of the ψ and ξ doublet vacuum expectation values multiplied by a different coefficient. Making use of the definition of $\langle\phi_4\rangle$ and $\langle\phi_2\rangle$, Eq. (12), we can combine the last four equations to obtain

$$m_{\tilde{\pi}}{}^2{}_{22} = \frac{i}{(2\pi)^4} \frac{r_1}{\lambda r_2} \frac{\partial}{\partial\theta} T[\tilde{\pi}_2] . \tag{34}$$

The one-loop mass correction matrix element $m_{\tilde{\pi}}{}^2{}_{22}$ is now formulated as a derivative of the tadpole loop $T[\tilde{\pi}_2]$ with respect to a relative rotation of the vacuum expectation values of the ϕ doublet to those of the ψ and ξ doublets. We can now use our expression for $T[\tilde{\pi}_2]$, Eq. (28), in this last equation to find the matrix element $m_{\tilde{\pi}}{}^2{}_{22}$. The values of θ for which the tadpole loop vanished were integer multiples of $\frac{1}{4}\pi$. Of these values of θ, our expressions for $m_{\tilde{\pi}}{}^2{}_{22}$ and $T[\tilde{\pi}_2]$ show that only the even-integer multiples of $\frac{1}{4}\pi$ will result in a positive $m_{\tilde{\pi}}{}^2{}_{22}$. Of these remaining values of θ, only two values correspond to different sets of vacuum expectation values. We will take these two values to be 0 and $\frac{1}{2}\pi$. The charge-conjugation symmetry operation involving the ϕ and B^μ fields transforms the $\theta = \frac{1}{2}\pi$ solution into the $\theta = 0$ solution. We are left then with $\theta = 0$ as our solution of spontaneous symmetry breakdown. For $\theta = 0$, the two pseudo-Goldstone bosons $\tilde{\pi}_1$ and $\tilde{\pi}_2$ will have the same mass since they are related by the remaining U(1) electromagnetic gauge symmetry. Hence, the mass matrix of these two bosons will necessarily be diagonal, and the matrix element $m_{\tilde{\pi}}{}^2{}_{22}$ represents the full mass correction for both $\tilde{\pi}_1$ and $\tilde{\pi}_2$. We shall henceforth denote this mass correction as $m_{\tilde{\pi}}{}^2$. Evaluating Eq. (34) with $\theta = 0$, we find that the one-loop mass correction to the pseudo-Goldstone bosons $\tilde{\pi}_1$ and $\tilde{\pi}_2$ is

$$m_{\tilde{\pi}}{}^2 = 12(gg')^4(\lambda r_1 r_2)^2 \int \frac{d_E{}^4 k}{(2\pi)^4} \frac{1}{d_1(k,0)} . \tag{35}$$

To obtain the pseudo-Goldstone-boson mass correction computed by Yang, we follow his procedure of making the special choice of vacuum expectation values $\lambda^2 = \langle\sigma\rangle^2 + \langle\Sigma\rangle^2$. With this choice of vacuum expectation values, $d_1(k,0)$ of Eq. (29) is in a factorized form, and the integral for $m_{\tilde{\pi}}{}^2$ can be straightforwardly evaluated. As an additional result of this special choice of vacuum expectation values, the mass-squared eigenvalues of the massive vector bosons are easily determined from the roots of $d_1(k,0) = 0$ to be $2g^2\lambda^2$, $(2g^2 + g'^2)\lambda^2$, and $g'^2\lambda^2$. The electric charge constant e is related to the two coupling constants g and g' in this theory by $e = 2gg'/(2g^2 + g'^2)^{1/2}$. Again following Yang, we define a mixing angle τ by the relation $g = e/(2\cos\tau)$, $g' = e/(\sqrt{2}\sin\tau)$. Carrying out the integration for $m_{\tilde{\pi}}{}^2$, and using the relation for the electric charge constant e and mixing angle τ, we obtain

$$m_{\bar{\pi}}^2(\tau) = \frac{3}{2}\left(\frac{e^2}{4\pi}\lambda\right)^2 \frac{1}{\cos^2\tau \sin^2\tau} \frac{1}{\cos^2\tau - \sin^2\tau} \times (\cos^2\tau \ln\cos^2\tau - \sin^2\tau \ln\sin^2\tau) . \tag{36}$$

This is the result found by Yang for the pseudo-Goldstone boson mass correction. However, we disagree with his conclusion that the pseudo-Goldstone boson mass becomes small as τ goes to 0 or $\frac{1}{2}\pi$. We find that the minimum mass occurs for $\tau = \frac{1}{4}\pi$. For this value of τ, $m_{\bar{\pi}}^2$ becomes

$$m_{\bar{\pi}}^2(\tfrac{1}{4}\pi) = \frac{3}{2\pi}\left(\frac{e^2}{4\pi}\right) m_x^2(1 - \ln 2) . \tag{37}$$

The mass $m_x = e\lambda$ is the mass of the lightest of the massive vector bosons in the theory, when $\tau = \frac{1}{4}\pi$.

IV. SECOND SOLUTION

We now return to the Lagrangian to determine what the particle content of the model would be if the coefficient h_{10} in the potential, Eq. (8), were chosen to be positive. Since the procedure for this investigation will be the same as in the case $h_{10} < 0$, we will only sketch the major differences found for the physical content of the theory when $h_{10} > 0$.

First, let us find the number of physically different theories that can occur when each of the doublets ϕ, ψ, and ξ develop a vacuum expectation value with $h_{10} > 0$. We again suppose that each of the fields of the doublets, Eq. (4), acquire a vacuum expectation value. Then, using the global $SU(2)_L \otimes U(1)_\phi \otimes U(1)_\psi \otimes U(1)_\xi$ symmetry group under which the Lagrangian is invariant, we rotate away as many of the vacuum expectation values as possible. The condition that a classical minimum of the scalar-field potential must be reached when the scalar fields are replaced by their zeroth-order vacuum expectation values puts an additional restriction on the number of fields that can acquire a nonzero vacuum expectation value. We find in this case that the vacuum expectation values can be rotated into the ϕ_2, ϕ_4, σ, and Π_2 fields. In the ξ doublet, the vacuum expectation value of the Π_2 field cannot be rotated into the Σ field, while still keeping the vacuum expectation value of the ψ doublet in the σ field. Therefore, this is a significantly different set of vacuum expectation values as contrasted to the case when h_{10} was negative. A set of new primed fields ϕ_2', ϕ_4', σ', and Π_2' having zero vacuum expectation values will be defined as

$$\phi_2 = \lambda \sin\theta + \phi_2' , \quad \phi_4 = \lambda\cos\theta + \phi_4' , \\ \sigma = \langle\sigma\rangle + \sigma' , \quad \Pi_2 = \langle \Pi_2 \rangle + \Pi_2' . \tag{38}$$

The zeroth-order values of λ, $\langle\sigma\rangle$, and $\langle\Pi_2\rangle$ can be determined as a function of the coefficients of the scalar-field potential from the defining relation of Eq. (18). To determine the allowed values of the angle θ, we will identify the pseudo-Goldstone bosons of the theory and find the angles θ for which these bosons do not develop tadpoles and for which their calculated masses are positive real quantities. We will show by explicit computation that the angles θ satisfying these conditions and corresponding to different sets of vacuum expectation values are $\theta = 0$ and $\theta = \frac{1}{2}\pi$. The $\theta = \frac{1}{2}\pi$ solution is physically equivalent to the $\theta = 0$ solution because of the charge conjugation symmetry involving the ϕ and B^μ fields. Therefore, the $\theta = 0$ solution can be chosen to represent the physical solution of spontaneous breakdown. The original set of vacuum expectation values has been reduced down to a unique set which is completely determined by the parameters of the Lagrangian. Thus, we show that there is only one physical solution for this type of spontaneous symmetry breakdown of the model when the coefficient h_{10} in the scalar-field potential is positive.

Next, let us examine the symmetry structure of the model after spontaneous breakdown. Consideration of the potential terms, especially ϕ^2, Eq. (9), and $\psi^\dagger\xi\xi^\dagger\psi$, Eq. (10), and also of the kinetic-energy terms of the Lagrangian reveals that the symmetry $\bar{G}$ of the scalar-field potential and the global symmetry G_g of the Lagrangian are reduced to

$$\bar{S}^{(+)} = O(3)_\phi \otimes U(1)_{\psi\xi} , \quad S_g^{(+)} = U(1)_{\phi\psi\xi} , \tag{39}$$

respectively, after spontaneous symmetry breakdown. There is no remaining gauge symmetry $S^{(+)}$ of the model after spontaneous breakdown. The global $U(1)_{\phi\psi\xi}$ symmetry is present only for θ equal to an integer multiple of $\frac{1}{2}\pi$, which is the solution dictated by the tadpole condition. For any other values of θ, there would have been no remaining global symmetry.

From these results, the physical particle content of the theory can be derived. We find that all of the vector bosons acquire a mass,

$$n_V^{(+)} = d[G] - d[S^{(+)}] \\ = 5 - 0 = 5 . \tag{40}$$

There is no longer a massless photon in the theory when the coefficient h_{10} becomes greater than zero. There is also no physical Goldstone boson in the theory

$$n_G^{(+)} = d[G_g] - d[S_g^{(+)}] - \{d[G] - d[S^{(+)}]\} \\ = 6 - 1 - (5 - 0) = 0 . \tag{41}$$

The number of pseudo-Goldstone bosons is given by

$$n_{\rm PG}^{(+)} = d[\overline{G}] - d[\overline{S}^{(+)}] - \{d[G_g] - d[S_g^{(+)}]\}$$
$$= 11 - 4 - (6 - 1) = 2\,. \qquad (42)$$

Two pseudo-Goldstone bosons are present only because the angle θ is an integer multiple of $\frac{1}{2}\pi$, which results in a remaining U(1) global symmetry. If the solution for θ had not been an integer multiple of $\frac{1}{2}\pi$, the group-theory analysis of the model would have indicated that there would be instead one pseudo-Goldstone boson and one physical Goldstone boson in the theory.

There are five scalar bosons which acquire a mass for this solution of spontaneous breakdown. These scalar bosons can be identified from the mass term of the scalar-field potential as three combinations of $\cos\theta\,\phi_4' + \sin\theta\,\phi_2'$, σ', and Π_2' along with $\langle\Pi_2\rangle\pi_2 + \langle\sigma\rangle\Sigma$ and $\langle\Pi_2\rangle\pi_1 + \langle\sigma\rangle\Pi_3$. There are also five unphysical Higgs-Kibble fields which can be identified from their bilinear derivative coupling to the massive vector bosons.

Our main interest is with the pseudo-Goldstone bosons. The two pseudo-Goldstone boson fields of the theory can be identified as those two remaining massless fields which are orthogonal to the Higgs-Kibble fields. Hence, following the same method as in Sec. III, we determine the pseudo-Goldstone bosons to be

$$\tilde{\Pi}_1 = \frac{1}{\overline{r}_3}\{\overline{r}_2{}^2[2\langle\sigma\rangle\langle\Pi_2\rangle(\cos\theta\,\phi_1 + \sin\theta\,\phi_3) - \lambda\sin2\theta(\langle\Pi_2\rangle\pi_3 + \langle\sigma\rangle\Pi_1)] - 2\lambda\cos2\theta\langle\sigma\rangle\langle\Pi_2\rangle(\langle\sigma\rangle\pi_1 - \langle\Pi_2\rangle\Pi_3)\}\,, \qquad (43)$$

$$\tilde{\Pi}_2 = \frac{1}{\overline{r}_1\overline{r}_2}[\overline{r}_2{}^2(\cos\theta\,\phi_2' - \sin\theta\,\phi_4') - \lambda(\langle\sigma\rangle\pi_2 - \langle\Pi_2\rangle\Sigma)]\,.$$

The coefficients $\overline{r}_1$ and $\overline{r}_2$ are again defined in terms of the squares of the vacuum expectation values of the doublets ϕ, ψ, and ξ as $\overline{r}_1{}^2 = \lambda^2 + \langle\sigma\rangle^2 + \langle\Pi_2\rangle^2$ and $\overline{r}_2{}^2 = \langle\sigma\rangle^2 + \langle\Pi_2\rangle^2$. The coefficient $\overline{r}_3$ is chosen to normalize the field $\tilde{\Pi}_1$ such that it has unit strength.

Now, let us find the angles θ for which the pseudo-Goldstone bosons do not develop singular tadpoles. Of the fields appearing in the Lagrangian, only the vector bosons will contribute tadpole loops to $\tilde{\Pi}_1$ and $\tilde{\Pi}_2$. The relevant interaction terms from the Lagrangian, Eq. (5), for these tadpole vertices are

$$\mathcal{L}^{(T)} = 2gg'\lambda\left\{2\frac{1}{\overline{r}_3}\langle\sigma\rangle\langle\Pi_2\rangle\tilde{\Pi}_1[\overline{r}_2{}^2A_2^\mu B_\mu + (\langle\sigma\rangle^2 - \langle\Pi_2\rangle^2)\cos2\theta A_2^\mu C_\mu] + \frac{1}{\overline{r}_1\overline{r}_2}\tilde{\Pi}_2[\overline{r}_2{}^2\cos2\theta A_1^\mu B_\mu + \overline{r}_2{}^2\sin2\theta A_3^\mu B_\mu - (\langle\sigma\rangle^2 - \langle\Pi_2\rangle^2)A_1^\mu C_\mu]\right\}. \qquad (44)$$

The tadpole loops will be calculated in the Landau gauge. The free-field vector-boson Lagrangian relevant to our calculation is found from Eq. (5) to be

$$\mathcal{L}^{(V)} = -\tfrac{1}{4}(F^{\mu\nu}_{(A)a}) - \tfrac{1}{4}(F^{\mu\nu}_{(B)})^2 - \tfrac{1}{4}(F^{\mu\nu}_{(C)})^2 + \tfrac{1}{2}g^2\overline{r}_1{}^2A_a^{\mu2} + \tfrac{1}{2}g'^2\lambda^2B^{\mu2} + \tfrac{1}{2}g'^2\overline{r}_2{}^2C^{\mu2} + gg'[\lambda^2\sin2\theta A_1^\mu B_\mu - \lambda^2\cos2\theta A_3^\mu B_\mu - (\langle\sigma\rangle^2 - \langle\Pi_2\rangle^2)A_3^\mu C_\mu]\,. \qquad (45)$$

The vector-boson propagator matrix of this free-field Lagrangian will be used to calculate the tadpoles. Evaluating $T[\tilde{\Pi}_i]$ of Eq. (25) from the interaction terms of Eq. (44), the total tadpole loop contribution for $\tilde{\Pi}_1$ and $\tilde{\Pi}_2$ is found to be

$$T[\tilde{\Pi}_1] = 0\,,$$
$$T[\tilde{\Pi}_2] = -3i(gg')^4\frac{1}{\overline{r}_2}\lambda^3\overline{r}_1(\langle\sigma\rangle^2 - \langle\Pi_2\rangle^2)^2\sin4\theta\int d_E{}^4k\frac{1}{d_2(k,\theta)}\,, \qquad (46)$$

where $d_2(k,\theta)$ is defined to be

$$d_2(k,\theta) = (k^2 + g^2\overline{r}_1{}^2)[(k^2 + g'^2\lambda^2)(k^2 + g^2\overline{r}_1{}^2)(k^2 + g'^2\overline{r}_2{}^2) - (gg')^2(\langle\sigma\rangle^2 - \langle\Pi_2\rangle^2)^2(k^2 + g'^2\lambda^2) - (gg'\lambda^2)^2(k^2 + g'^2\overline{r}_2{}^2)] + (gg'\lambda)^4(\langle\sigma\rangle^2 - \langle\Pi_2\rangle^2)^2(\sin2\theta)^2\,. \qquad (47)$$

Since $T[\tilde{\Pi}_2]$ vanishes for values of θ which are integer multiples of $\frac{1}{4}\pi$, these values of θ are the allowed possible solutions of spontaneous breakdown. For the situation when $\langle\sigma\rangle^2 = \langle\Pi_2\rangle^2$, the tadpole loops vanish for all angles θ, and the physical solution of spontaneous breakdown is not determined at the one-loop level for this special choice of vacuum expectation values. One must go to higher levels of perturbation theory to determine the correct solution of spontaneous symmetry breakdown when $\langle\sigma\rangle^2=\langle\Pi_2\rangle^2$.

We now turn to the one-loop pseudo-Goldstone-boson mass correction. The final constraint on the angle θ can be determined by computing the diagonal element $m_{\tilde{\Pi}}{}^2{}_{22}$ of the mass matrix, Eq. (30), for the pseudo-Goldstone bosons $\tilde{\Pi}_1$ and $\tilde{\Pi}_2$. Proceeding as before, we obtain

$$m_{\tilde{\Pi}}{}^2{}_{22}=\frac{i}{(2\pi)^4}\frac{\overline{r}_1}{\lambda\overline{r}_2}\frac{\partial}{\partial\theta}T[\tilde{\Pi}_2]\,, \tag{48}$$

which has the same form as our previous mass-correction matrix element, Eq. (34). Using our tadpole loop of Eq. (46) and evaluating this expression at those values of θ for which the tadpole loop vanished, we again find that positive real pseudo-Goldstone-boson masses will occur only for θ equal to an even-integer multiple of $\frac{1}{4}\pi$. These values of θ are physically equivalent by the global-rotational and charge-conjugation symmetries. Thus, our one physical solution of spontaneous breakdown may be represented by the $\theta=0$ solution.

For θ equal to zero, there is a remaining $U(1)_{\phi\psi\xi}$ global symmetry after spontaneous breakdown under which the pseudo-Goldstone bosons $\tilde{\Pi}_1$ and $\tilde{\Pi}_2$ can be rotated into each other. As a consequence of this global U(1) symmetry, these two bosons will have the same mass correction in higher orders of perturbation theory. Hence, their one-loop mass-correction matrix is diagonal, and the matrix element $m_{\tilde{\Pi}}{}^2{}_{22}$ is the complete one-loop mass correction for $\tilde{\Pi}_1$ and $\tilde{\Pi}_2$.

In this solution of spontaneous breakdown the one-loop mass correction for $\tilde{\Pi}_1$ and $\tilde{\Pi}_2$ vanishes when $\langle\sigma\rangle^2=\langle\Pi_2\rangle^2$. A general underlying reason for this vanishing of the one-loop pseudo-Goldstone-boson mass correction can be found from symmetry considerations. If an additional global symmetry which involves two separate rotations of the ψ doublet with the ξ doublet is imposed upon the Lagrangian,[9] we find as a consequence that $\langle\sigma\rangle^2$ must be equal to $\langle\Pi_2\rangle^2$. Furthermore, with this additional symmetry imposed on the Lagrangian, there will be two extra global-rotational symmetries which are broken after spontaneous breakdown. The two extra broken symmetries are associated with the bosons $\tilde{\Pi}_1$ and $\tilde{\Pi}_2$. Thus these two pseudo-Goldstone bosons have now become Goldstone bosons which will remain massless to all orders of perturbation theory. Returning in our original scalar-field potential, we note that its coefficients can be chosen such that the Lagrangian is arbitrarily close to the limit of satisfying this larger ψ,ξ doublet rotational symmetry. Assuming that the calculated mass of the pseudo-Goldstone bosons $\tilde{\Pi}_1$ and $\tilde{\Pi}_2$ has a continuous dependence on the parameters of the scalar-field potential, we infer that this mass can be made arbitrarily small by letting the Lagrangian approach the global ψ,ξ rotational symmetry.

V. SUMMARY

As an illustration of the tadpole condition, we investigated an $SU(2)_L\otimes U(1)\otimes U(1)$ gauge model to determine the allowed number of physically inequivalent solutions of spontaneous symmetry breakdown. First, the global symmetries of the model were used to reduce the number of solutions of a general type of spontaneous symmetry breakdown to a continuous infinity of physically inequivalent solutions. Then, the first part of the tadpole condition requiring that the perturbative expansion of the vacuum expectation values be well behaved was shown to limit the model to only two of these solutions. The second part of the tadpole condition requiring that the calculated pseudo-Goldstone-boson masses be positive real further restricted the model to a unique physical solution of spontaneous breakdown. This solution was found to have a disjoint dependence upon the parameters of the scalar-field potential. Therefore, the physical particle content of the solution was determined for each of two families of potentials. For each family of potentials the tadpole condition was found to choose the solution of spontaneous breakdown which left the maximum number of remaining symmetries of the theory. For one family of potentials the remaining symmetry was an electromagnetic gauge symmetry. For the other family of potentials, it was a U(1) global symmetry. Also, the one-loop mass correction to the pseudo-Goldstone bosons was found to vanish for certain values of the parameters of the scalar-field potential. A relationship was then shown between the vanishing of this mass correction and the invariance of the Lagrangian under a larger global symmetry group.

ACKNOWLEDGMENTS

The author would like to thank Professor Lowell S. Brown for valuable discussions and a critical reading of the manuscript and would also like to thank Professor D. G. Boulware and D. H. Fujimoto for helpful comments.

*Work supported in part by the U. S. Energy Research and Development Administration.

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[9]The two additional global symmetry rotations are

$$\psi \to \psi\cos\theta_1 + \xi\sin\theta_1,$$
$$\xi \to \xi\cos\theta_1 - \psi\sin\theta_1,$$
$$\psi \to \psi\cos\theta_2 + i\xi\sin\theta_2,$$
$$\xi \to \xi\cos\theta_2 + i\psi\sin\theta_2.$$

With this symmetry imposed upon the Lagrangian, the most general renormalizable scalar-field potential is

$$P = -h_1\phi^2 - h_2(\psi^2+\xi^2) + h_4(\phi^2)^2 + h_5(\psi^2+\xi^2)^2 + h_7\phi^2(\psi^2+\xi^2) + h_{10}(\psi^\dagger\xi\xi^\dagger\psi - \psi^2\xi^2).$$