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Unified model of the Nambu–Jona-Lasinio type for all elementary-particle forces

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The unified model for all elementary-particle forces recently proposed by us is discussed in detail. Starting with a nonlinear fermion Lagrangian of the Nambu–Jona-Lasinio type and imposing the massless conditions of Bjorken on vector fields, we construct an effective Lagrangian which combines the unified gauge theory of Weinberg and Salam for the weak and electromagnetic interactions of leptons and quarks and the asymptotically free gauge theory of Gross, Wilczek, and Politzer for the strong interaction of quarks. The photon, weak vector bosons, and Higgs scalars appear as composites of lepton-antilepton or quark-antiquark pairs, while the color-octet gluons appear as composites of quark-antiquark pairs. As a result, the Weinberg angle is determined to be $\sin^2\theta_W = 3/8$ for fractionally charged quarks, which coincides with the prediction of Georgi and Glashow in their unified SU(5) gauge model. The gluon coupling constant is also determined to be 8/3 times the fine-structure constant. The masses of the weak vector bosons and physical Higgs scalars are related to those of leptons and quarks. We also propose a unified spinor-subquark model in which the gauge bosons and Higgs scalars as well as leptons and quarks are all composites of subquarks of spin 1/2. In such a model, we predict, among other things, the mass of the charged weak vector bosons to be approximately $\sqrt{3}$ times the subquark mass. From these results, we strongly suggest that there exist much heavier leptons and/or quarks whose masses reach or go beyond the weak-vector-boson masses or that there exist heavy subquarks whose pair-production threshold lies very close to the weak-vector-boson masses.

I. INTRODUCTION

In 1961, by adopting the nonlinear fermion interaction of the Heisenberg type, Nambu and Jona-Lasinio[1] proposed a dynamical model of elementary particles based on an analogy with superconductivity. In the original model, it was made clear, among other things, that an idealized pion, the massless pseudoscalar bound state of a nucleon-antinucleon pair, appears as a Nambu-Goldstone boson when the nucleon mass is generated spontaneously, breaking the chiral symmetry possessed by the Lagrangian. Subsequently, in the model with a nonlinear vector interaction, Bjorken[2] demonstrated that the photon can be considered as a "collective excitation" of a fermion-antifermion pair. He showed that the model is equivalent to quantum electrodynamics in spite of its different appearances. In 1974, in the modified Nambu–Jona-Lasinio model, Eguchi and Sugawara[3] found a set of equations which describes the collective motion of fermions and which is equivalent to the Higgs Lagrangian. They thereby clarified the nature of interactions among bound states of fermion-antifermion pairs. Recently, Kikkawa[4] made this approach to the collective motion very transparent by using the functional integration method, which gives the easiest way to find an effective Lagrangian for bosonic bound states.

On the other hand, as a result of the extensive theoretical works performed for the last several years, an attractive picture for interactions of elementary particles has been given by the unified gauge theory of Weinberg and Salam[5] for the weak and electromagnetic interactions of leptons and quarks and by the asymptotically free gauge theory of Gross, Wilczek, and Politzer[6] for the strong interaction of quarks. In addition to the familiar photon, the charged and neutral weak vector bosons and the color-octet massless vector gluons form a set of elementary gauge fields inherent to these theories. Recently, two of the present authors (K.A. and H.T.)[7] have suggested the possibility that all of these gauge bosons are composite states of fermion subquarks which are the building blocks of the ordinary quarks.

Recently, Saito and Shigemoto,[8] starting with a Lagrangian of self-interacting leptons, have constructed an effective Lagrangian of the Weinberg-Salam type and have proposed that the photon, the weak vector bosons, and the Higgs scalars are all composites of lepton-antilepton pairs. They have fixed, among other things, the Weinberg angle to be $\sin^2\theta_W=\frac{1}{4}$. However, their model is certainly incomplete since it does not include quarks. In our previous paper,[9] we have extended their model to a more realistic one including quarks. In our picture, the photon and the weak vector bosons are considered as composite states of lepton-antilepton or quark-antiquark pairs, while the color-octet gluons are considered as those of quark-antiquark pairs. As a result, the Weinberg angle is determined to be $\sin^2\theta_W = \frac{3}{8}$ for fractionally charged quarks, which coincides with the prediction of Georgi and Glashow[10] in their unified gauge model of all elementary-particle forces. The gluon coupling constant is also determined to be $\frac{8}{3}$ times the fine-structure constant. In this paper, we shall present all the details of our model which we omitted in the previous paper. They include a new important result that the masses of the weak vector bosons and the Higgs scalar are unambiguously related to those of leptons and quarks. We shall also propose a unified spinor-subquark model for all interactions of leptons and quarks in which the photon, the weak vector bosons, and the color-octet gluons are bound states of subquark-antisubquark pairs.

In Sec. II, our unified lepton-quark model is presented, and the Weinberg angle and various coupling constants are determined. Spontaneous generation of the lepton, quark, and weak-vector-boson masses is discussed in some detail in Sec. III. The unified spinor-subquark model is presented in Sec. IV. Finally, Sec. V is devoted to a summary and concluding remarks.

II. UNIFIED LEPTON-QUARK MODEL

Our unified lepton-quark model for all elementary-particle forces consists of the leptons l_L and l_R and the quarks q_L, u_R, and d_R. They belong to representations of $(\underline{1},\underline{2},\underline{1})$, $(\underline{1},\underline{1},\underline{1})$, $(\underline{3},\underline{2},\underline{1})$, $(\underline{3},\underline{1},\underline{1})$, and $(\underline{3},\underline{1},\underline{1})$, respectively, of the SU(3) $\times$ SU(2) $\times$ U(1) group, where SU(3) is the color symmetry of quarks and SU(2) $\times$ U(1) is the symmetry of Weinberg and Salam. The left- and right-handed fields are defined by $\psi_L = \frac{1}{2}(1-\gamma_5)\psi$ and $\psi_R = \frac{1}{2}(1+\gamma_5)\psi$. The leptons l_L and l_R can be identified either with

$$l_L \equiv \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \text{ and } l_R \equiv e^-_R$$

or with

$$l_L \equiv \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \text{ and } l_R \equiv \mu^-_R ,$$

while the quarks q_L, u_R, and d_R can be identified either with

$$q_L \equiv \begin{pmatrix} \mathcal{P} \\ \mathcal{N} \end{pmatrix}_L , \quad u_R \equiv \mathcal{P}_R , \text{ and } d_R \equiv \mathcal{N}_R$$

or with

$$q_L \equiv \begin{pmatrix} c \\ \lambda \end{pmatrix}_L , \quad u_R \equiv c_R , \text{ and } d_R \equiv \lambda_R .$$

An extension of the model to the one including the above four leptons and four flavors of quarks at the same time is trivial. A further extension to an arbitrary number of leptons and quarks, which is an extremely interesting possibility, will be discussed at the end of Sec. III. The Cabibbo rotation is ignored in this paper.

Let us start with the nonlinear Lagrangian for massless leptons and quarks

$$\begin{aligned} L = {} & \bar{l}_L i\partial\!\!\!/ l_L + \bar{l}_R i\partial\!\!\!/ l_R + \bar{q}_L i\partial\!\!\!/ q_L + \bar{u}_R i\partial\!\!\!/ u_R + \bar{d}_R i\partial\!\!\!/ d_R \\ & + f_1(Y_{l_L}\bar{l}_L\gamma_\mu l_L + Y_{l_R}\bar{l}_R\gamma_\mu l_R + Y_{q_L}\bar{q}_L\gamma_\mu q_L + Y_{u_R}\bar{u}_R\gamma_\mu u_R + Y_{d_R}\bar{d}_R\gamma_\mu d_R)^2 \\ & + f_2(\bar{l}_L\gamma_\mu\vec{\tau} l_L + \bar{q}_L\gamma_\mu\vec{\tau} q_L)^2 + f_3(\bar{q}_L\gamma_\mu\lambda^a q_L + \bar{u}_R\gamma_\mu\lambda^a u_R + \bar{d}_R\gamma_\mu\lambda^a d_R)^2 \\ & + f_4(b_l\bar{l}_L l_R - b_u\overline{q^G_R} u^c_L + b_d\bar{q}_L d_R)(b_l\bar{l}_R l_L - b_u\overline{u^c_L} q^G_R + b_d\bar{d}_R q_L) , \end{aligned} \tag{2.1}$$

where f's and b's are real constants, Y's are the weak hypercharges related to the charges and the weak isospins of leptons and quarks by $Q = I_3 + \frac{1}{2}Y$, $\tau_i (i = 1, 2, 3)$ and λ^a $(a = 1, 2, \ldots, 8)$ are the 2×2 Pauli matrices for SU(2) and the 3×3 Gell-Mann matrices for SU(3), and u^c and q^G are the charge-conjugate field of u and the G-parity-conjugate field of q $(\equiv i\tau_2 q^c)$, respectively. Obviously, this Lagrangian is invariant under the *global* SU(3) $\times$ SU(2) $\times$ U(1) symmetry.

Introducing the auxiliary vector fields V_μ, $\vec{V}_\mu$, and v^a_μ and the scalar field K,[11] which transform as $(\underline{1},\underline{1},\underline{1})$, $(\underline{1},\underline{3},\underline{1})$, $(\underline{8},\underline{1},\underline{1})$, and $(\underline{1},\underline{2},\underline{1})$ of SU(3) $\times$ SU(2) $\times$ U(1), respectively, we construct the Lagrangian

$$\begin{aligned} L' = {} & \bar{l}_L i\gamma^\mu(\partial_\mu + iY_{l_L}V_\mu + i\vec{\tau}\cdot\vec{V}_\mu)l_L + \bar{l}_R i\gamma^\mu(\partial_\mu + iY_{l_R}V_\mu)l_R \\ & + \bar{q}_L i\gamma^\mu(\partial_\mu + iY_{q_L}V_\mu + i\vec{\tau}\cdot\vec{V}_\mu + i\lambda^a v^a_\mu)q_L \\ & + \bar{u}_R i\gamma^\mu(\partial_\mu + iY_{u_R}V_\mu + i\lambda^a v^a_\mu)u_R + \bar{d}_R i\gamma^\mu(\partial_\mu + iY_{d_R}V_\mu + i\lambda^a v^a_\mu)d_R \\ & + b_l(\bar{l}_L K l_R + \bar{l}_R K^\dagger l_L) + b_u(\bar{q}_L K^G u_R + \bar{u}_R K^{G\dagger} q_L) + b_d(\bar{q}_L K d_R + \bar{d}_R K^\dagger q_L) \\ & + c_1(V_\mu)^2 + c_2(\vec{V}_\mu)^2 + c_3(v^a_\mu)^2 + c_4 K^\dagger K , \end{aligned} \tag{2.2}$$

where K^G is the G-parity-conjugate field of K $(=i\tau_2 K^*)$. Variations with respect to these auxiliary fields give the "equations of motion"

$$
\begin{aligned}
V_\mu &= \frac{1}{2c_1}(Y_{l_L}\bar{l}_L\gamma_\mu l_L + Y_{l_R}\bar{l}_R\gamma_\mu l_R + Y_{q_L}\bar{q}_L\gamma_\mu q_L + Y_{u_R}\bar{u}_R\gamma_\mu u_R + Y_{d_R}\bar{d}_R\gamma_\mu d_R)\,,\\
\vec{V}_\mu &= \frac{1}{2c_2}(\bar{l}_L\gamma_\mu\vec{\tau} l_L + \bar{q}_L\gamma_\mu\vec{\tau} q_L)\,,\\
v_\mu^a &= \frac{1}{2c_3}(\bar{q}_L\gamma_\mu\lambda^a q_L + \bar{u}_R\gamma_\mu\lambda^a u_R + \bar{d}_R\gamma_\mu\lambda^a d_R)\,,\\
K^\dagger &= -\frac{1}{c_4}(b_l\bar{l}_L l_R - b_u\overline{q_R^G}u_L^c + b_d\bar{q}_L d_R)\,,\\
K &= -\frac{1}{c_4}(b_l\bar{l}_R l_L - b_u\overline{u_L^c}q_R^G + b_d\bar{d}_R q_L)\,.
\end{aligned}
\tag{2.3}
$$

Substitution of these relations shows that the Lagrangian (2.2) is effectively equivalent to the original Lagrangian (2.1) if

$$
c_i = -\frac{1}{4f_i}\ \text{for}\ i=1,2,3\ \text{and}\ c_4 = -\frac{1}{f_4}\,. \tag{2.4}
$$

Define the effective Lagrangian $L_{\rm eff}$ for the auxiliary fields by the path integrals over the lepton and quark fields[4]:

$$
\exp\left(i\int d^4x\, L_{\rm eff}\right) = \int (d\bar{l}_L)(dl_L)(d\bar{l}_R)(dl_R)(d\bar{q}_L)(dq_L)(d\bar{u}_R)(du_R)(d\bar{d}_R)(dd_R)\exp\left(i\int d^4x\, L'\right)\,. \tag{2.5}
$$

Performing the path integration formally, we then obtain

$$
\begin{aligned}
\int d^4x\, L_{\rm eff} = \int d^4x[c_1(V_\mu)^2 + c_2(\vec{V}_\mu)^2 + c_3(v_\mu^a)^2 + c_4K^\dagger K] - i\,\mathrm{Tr}\ln
\begin{bmatrix}
1-\frac{1}{i\partial\!\!\!/}\gamma^\mu(Y_{l_L}V_\mu + \vec{\tau}\cdot\vec{V}_\mu) & \frac{1}{i\partial\!\!\!/}b_l K\\
\frac{1}{i\partial\!\!\!/}b_l K^\dagger & 1-\frac{1}{i\partial\!\!\!/}\gamma^\mu Y_{l_R}V_\mu
\end{bmatrix}\\
-i\,\mathrm{Tr}\ln
\begin{bmatrix}
1-\frac{1}{i\partial\!\!\!/}\gamma^\mu(Y_{q_L}V_\mu + \vec{\tau}\cdot\vec{V}_\mu + \lambda^a v_\mu^a) & \frac{1}{i\partial\!\!\!/}b_u K^G & \frac{1}{i\partial\!\!\!/}b_d K\\
\frac{1}{i\partial\!\!\!/}b_u K^{G\dagger} & 1-\frac{1}{i\partial\!\!\!/}\gamma^\mu(Y_{u_R}V_\mu + \lambda^a v_\mu^a) & 0\\
\frac{1}{i\partial\!\!\!/}b_d K^\dagger & 0 & 1-\frac{1}{i\partial\!\!\!/}\gamma^\mu(Y_{d_R}V_\mu + \lambda^a v_\mu^a)
\end{bmatrix}\,,
\end{aligned}
\tag{2.6}
$$

where Tr denotes the trace operation with respect to the space-time points, the γ matrices, and the matrices for the internal symmetry SU(3) × SU(2) ×U(1). The second and third terms in (2.6) correspond to a series of fermion-loop diagrams if they are expanded into a Taylor series in the auxiliary fields. Among them, one-fermion-loop diagrams, to which two, three, and four auxiliary fields are attached as external fields, involve divergent integrals. We must, therefore, introduce the cutoff momentum Λ. Then, the divergent part of $L_{\rm eff}$, which we call $L_{\rm div}$, can be calculated to be

$$
\begin{aligned}
L_{\rm div} = &-\tfrac{1}{6}I_2[(N_1+2)n_2(v_{\mu\nu}^a)^2 + N_2(1+n_1)(\vec{V}_{\mu\nu})^2 + \bar{Y}^2(V_{\mu\nu})^2]\\
&-\tfrac{1}{2}I_1[(N_1+2)n_2(v_\mu^a)^2 + N_2(1+n_1)(\vec{V}_\mu)^2 + \bar{Y}^2(V_\mu)^2] + I_2\bar{b}^2|D_\mu K|^2 + 2I_1\bar{b}^2|K|^2 - I_2B^4(|K|^2)^2\,,
\end{aligned}
\tag{2.7}
$$

with

$$
\begin{aligned}
v_{\mu\nu}^a &= \partial_\mu v_\nu^a - \partial_\nu v_\mu^a - 2f^{abc}v_\mu^b v_\nu^c\,,\\
\vec{V}_{\mu\nu} &= \partial_\mu\vec{V}_\nu - \partial_\nu\vec{V}_\mu - 2\vec{V}_\mu\times\vec{V}_\nu\,,\\
V_{\mu\nu} &= \partial_\mu V_\nu - \partial_\nu V_\mu\,,
\end{aligned}
\tag{2.8}
$$

and

$$
D_\mu K = (\partial_\mu + iV_\mu + i\vec{\tau}\cdot\vec{V}_\mu)K\,,
$$

where

$$
\begin{aligned}
&N_1 = \mathrm{tr}\,\underline{I}^2 = 2 \text{ and } N_2\delta_{ij} = \mathrm{tr}\,\tau_i\tau_j = 2\delta_{ij} \text{ for SU(2)},\\
&n_1 = \mathrm{tr}\,\underline{I}^2 = 3, \quad n_2\delta_{ab} = \mathrm{tr}\,\lambda^a\lambda^b = 2\delta_{ab}, \text{ and } [\lambda^a, \lambda^b] = 2if^{abc}\lambda^c \text{ for SU(3)},\\
&\tilde{Y}^2 = N_1 {Y_{l_L}}^2 + {Y_{l_R}}^2 + N_1 n_1 Y_{q_L} + n_1 {Y_{u_R}}^2 + n_1 {Y_{d_R}}^2 \; (=\mathrm{tr}\,Y^2),\\
&\tilde{b}^2 = {b_l}^2 + n_1 {b_u}^2 + n_1 {b_d}^2,\\
&B^4 = {b_l}^4 + n_1 {b_u}^4 + n_1 {b_d}^4,
\end{aligned}
\tag{2.9}
$$

and the constants $I_{1,2}$ are the quadratically and logarithmically divergent integrals defined by

$$
I_1 = \frac{i}{(2\pi)^4}\int^{\Lambda} \frac{d^4p}{p^2 + i\epsilon}
$$

and (2.10)

$$
I_2 = \frac{-i}{(2\pi)^4}\int^{\Lambda} \frac{d^4p}{(p^2 + i\epsilon)^2}.
$$

Let us now construct the new Lagrangian

$$
L'' = L' + L_{\mathrm{div}}, \tag{2.11}
$$

where the auxiliary fields, V_μ, $\vec{\mathrm{V}}_\mu$, v^a_μ, and K, have been promoted to become "genuine" Bose fields. Then the relation

$$
L' = L'' - L_{\mathrm{div}}, \tag{2.12}
$$

which looks trivial, indicates that the original Lagrangian L is effectively equivalent to the new one L'' if the divergent parts of one-fermion-loop diagrams are subtracted. Furthermore, the Lagrangian L'' becomes invariant under the *local* SU(3) × SU(2) × U(1) gauge symmetry if we require the massless conditions for the vector fields, V_μ, $\vec{\mathrm{V}}_\mu$, and v^a_μ:

$$
\begin{aligned}
&\tfrac{1}{2}\tilde{Y}^2 I_1 + \frac{1}{4f_1} = 0,\\
&\tfrac{1}{2}N_2(1+n_1)I_1 + \frac{1}{4f_2} = 0,
\end{aligned}
\tag{2.13}
$$

and

$$
\tfrac{1}{2}(N_1+2)n_2 I_1 + \frac{1}{4f_3} = 0.
$$

It is only by imposing these massless conditions of Bjorken[2] on vector fields that our results will follow from the original Lagrangian of the Nambu-Jona-Lasinio type (2.1). The conditions (2.13) fix the original coupling constants f_i for $i = 1, 2, 3$ in terms of the cutoff momentum Λ, hence also fixing the Weinberg angle, etc., as we shall see in what follows. It should also be emphasized here that we must impose the massless conditions of this type in order by order when more than one fermion loop is considered.

Assuming these conditions and rescaling the Bose fields by

$$
\begin{aligned}
V_\mu &= \frac{1}{2}\left(\frac{6}{\tilde{Y}^2 I_2}\right)^{1/2} B_\mu,\\
\vec{\mathrm{V}}_\mu &= -\frac{1}{2}\left[\frac{6}{N_2(1+n_1)I_2}\right]^{1/2} \vec{\mathrm{A}}_\mu,\\
v^a_\mu &= -\frac{1}{2}\left[\frac{6}{(N_1+2)n_2 I_2}\right]^{1/2} G^a_\mu,
\end{aligned}
\tag{2.14}
$$

and

$$
K = -(\tilde{b}^2 I_2)^{1/2}\phi,
$$

we finally obtain the desired form of the Lagrangian

$$
\begin{aligned}
L'' = {}& \bar{l}_L i\gamma^\mu(\partial_\mu + ig'\tfrac{1}{2}Y_{l_L}B_\mu - ig\tfrac{1}{2}\vec{\tau}\cdot\vec{\mathrm{A}}_\mu)l_L + \bar{l}_R i\gamma^\mu(\partial_\mu + ig'\tfrac{1}{2}Y_{l_R}B_\mu)l_R\\
&+ \bar{q}_L i\gamma^\mu(\partial_\mu + ig'\tfrac{1}{2}Y_{q_L}B_\mu - ig\tfrac{1}{2}\vec{\tau}\cdot\vec{\mathrm{A}}_\mu - if\tfrac{1}{2}\lambda^a G^a_\mu)q_L\\
&+ \bar{u}_R i\gamma^\mu(\partial_\mu + ig'\tfrac{1}{2}Y_{u_R}B_\mu - if\tfrac{1}{2}\lambda^a G^a_\mu)u_R + \bar{d}_R i\gamma^\mu(\partial_\mu + ig'\tfrac{1}{2}Y_{d_R}B_\mu - if\tfrac{1}{2}\lambda^a G^a_\mu)d_R\\
&- G_l(\bar{l}_L\phi l_R + \bar{l}_R\phi^\dagger l_L) - G_u(\bar{q}_L\phi^G u_R + \bar{u}_R\phi^{G^\dagger} q_L) - G_d(\bar{q}_L\phi d_R + \bar{d}_R\phi^\dagger q_L)\\
&- \tfrac{1}{4}(B_{\mu\nu})^2 - \tfrac{1}{4}(\vec{\mathrm{A}}_{\mu\nu})^2 - \tfrac{1}{4}(G^a_{\mu\nu})^2 + |D_\mu\phi|^2 - \mu^2|\phi|^2 - \lambda(|\phi|^2)^2,
\end{aligned}
\tag{2.15}
$$

where

$$
\begin{aligned}
B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu,\\
\vec{\mathrm{A}}_{\mu\nu} &= \partial_\mu\vec{\mathrm{A}}_\nu - \partial_\nu\vec{\mathrm{A}}_\mu + g\vec{\mathrm{A}}_\mu\times\vec{\mathrm{A}}_\nu,\\
G^a_{\mu\nu} &= \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + ff^{abc}G^b_\mu G^c_\nu,\\
D_\mu\phi &= (\partial_\mu + ig'\tfrac{1}{2}B_\mu - ig\tfrac{1}{2}\vec{\tau}\cdot\vec{\mathrm{A}}_\mu)\phi,
\end{aligned}
\tag{2.16}
$$

and

$$
\phi^G = i\tau_2\phi^*.
$$

This Lagrangian is exactly what is expected from the combination of the unified gauge theory of Weinberg and Salam[5] for the weak and electromagnetic interactions of leptons and quarks and the asymptotically free gauge theory of Gross, Wilczek, and Politzer[6] for the strong interaction of quarks. The difference here lies in the fact that the coupling constants f, g, g', λ, G_l, G_u, and G_d and the mass parameter μ^2 are arbitrary in the combined theory, whereas in our model they are completely fixed by the quantum numbers of leptons and quarks, the cutoff momentum, and the coupling constants in the original Lagrangian L. In fact, they are given by

$$\begin{aligned} f &= [6/(N_1+2)n_2I_2]^{1/2}\,, \\ g &= [6/N_2(1+n_1)I_2]^{1/2}\,, \\ g' &= (6/\tilde{Y}^2 I_2)^{1/2}\,, \\ -\mu^2 &= \left(2I_1 - \frac{1}{\tilde{b}^2 f_4}\right)\Big/ I_2\,, \\ \lambda &= B^4/(\tilde{b}^2)^2 I_2\,, \end{aligned} \tag{2.17}$$

and

$$G_i = b_i/(\tilde{b}^2 I_2)^{1/2} \text{ for } i=l,\ u, \text{ and } d\,.$$

From these equations, we reach the important results in our model. The Winberg angle defined by $\tan\theta_W = g'/g$ and the ratio of the coupling constants f and g are determined as follows:

$$\sin^2\theta_W = \frac{g'^2}{g^2+g'^2} = \frac{N_2(1+n_1)}{N_2(1+n_1)+\tilde{Y}^2}\quad\left(= \frac{\mathrm{tr}I_3^2}{\mathrm{tr}Q^2}\right) \tag{2.18}$$

and

$$\frac{f^2}{g^2} = \frac{N_2(1+n_1)}{n_2(N_1+2)}\quad\left(= \frac{\text{the number of isodoublets}}{\text{the number of color triplets}}\right) = 1\,. \tag{2.19}$$

Since $Y_{l_L} = -1$, $Y_{l_R} = -2$, $Y_{q_L} = \frac{1}{3}$, $Y_{u_R} = \frac{4}{3}$, and $Y_{d_R} = -\frac{2}{3}$ for fractionally charged quarks ($\tilde{Y} = \frac{40}{3}$), the relation (2.18) gives $\sin^2\theta_W = \frac{3}{8}$, which coincides with the prediction of Georgi and Glashow[10] in their unified SU(5) gauge theory. Also, the relation (2.19) together with the familiar one[5] $g^2 = e^2/\sin^2\theta_W$ fixes the gluon coupling constant to be $f^2/4\pi = \alpha/\sin^2\theta_W = \frac{8}{3}\alpha$, where α is the fine-structure constant. Therefore, all the gauge coupling constants, f, g, and g', are related to a single coupling constant, the fine-structure constant. In the light of the general argument by Georgi, Quinn, and Weinberg,[12] this situation suggests that our model may be extended to the one with a symmetry larger than SU(3) × SU(2) × U(1). In fact, such an extension to U(8) × U(7), where the eight left-handed and seven right-handed fermions form fundamental multiplets, or even to U(15), seems to be possible, though it may not be relevant. Here, however, it cannot be stressed too strongly that the relations (2.18) and (2.19) are due not to such an assumed higher symmetry but to the dynamics in our model.

III. SPONTANEOUS GENERATION OF MASSES

Although we have started with the massless leptons and quarks and have required the massless conditions (2.13) for the vector fields, the Higgs potential in the Lagrangian L'' (2.15) produces the familiar breakdown of SU(2) × U(1) symmetry of the vacuum[5] if

$$-\mu^2 > 0\,, \text{ i.e., } 2I_1 - \frac{1}{\tilde{b}^2 f_4} > 0\,. \tag{3.1}$$

The Higgs scalar ϕ will thus acquire the nonvanishing vacuum expectation value

$$\langle\phi\rangle_0 = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix} \text{ with } v = \left(\frac{-\mu^2}{\lambda}\right)^{1/2}. \tag{3.2}$$

This vacuum expectation value generates, through the Yukawa interactions in L'', the masses of leptons and quarks given by

$$m_i = \frac{1}{\sqrt{2}} G_i v = b_i\left(\frac{-\mu^2\tilde{b}^2}{2B^4}\right)^{1/2} \text{ for } i=l,\ u, \text{ and } d\,. \tag{3.3}$$

Equation (3.3) is essentially the same as the Nambu–Jona-Lasinio self-consistency condition for the fermion mass. In fact, it can be transformed into

$$\frac{1}{2f_4\tilde{b}^2 I_1} = 1 - \left(\frac{m_i}{b_i}\right)^2 \frac{B^4 I_2}{\tilde{b}^2 I_1}\,, \tag{3.4}$$

which corresponds to Eq. (3.9) in Ref. 1.

Define as usual the four real components ξ_i and η of the Higgs scalar ϕ by[13]

$$\phi = \exp(i\vec{\tau}\cdot\vec{\xi}/2v)\frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v+\eta\end{pmatrix}\,. \tag{3.5}$$

The three massless components $\vec{\xi}$ are "fictitious" Nambu-Goldstone bosons which will be absorbed by the Higgs mechanism, whereas the physical Higgs scalar has the mass given by

$$m_\eta = (-2\mu^2)^{1/2} = 2\left(\frac{m_l^4 + n_1 m_u^4 + n_1 m_d^4}{m_l^2 + n_1 m_u^2 + n_1 m_d^2}\right)^{1/2}. \tag{3.6}$$

This result—that the mass of the physical scalar boson is roughly twice the fermion mass—also agrees with the result of Nambu and Jona-Lasinio.[1]

The nonvanishing vacuum expectation value of the Higgs scalar also generates the masses of the charged weak vector bosons $W_\mu^\pm$ $[=(1/\sqrt{2})(A_\mu^1 \mp iA_\mu^2)]$ and the neutral one Z_μ $(=\cos\theta_W A_\mu^3 + \sin\theta_W B_\mu)$, leaving the photon A_μ $(=-\sin\theta_W A_\mu^3 + \cos\theta_W B_\mu)$ massless in a familiar way. The acquired masses of the weak vector bosons are given by

$$m_{W^\pm}=\tfrac{1}{2}gv=\left[\frac{-3\mu^2(\tilde{b}^2)^2}{2N_2(1+n_1)B^4}\right]^{1/2}$$
$$=\left[\frac{3(m_l^2+n_1m_u^2+n_1m_d^2)}{N_2(1+n_1)}\right]^{1/2} \quad (3.7)$$

and

$$m_Z=m_{W^\pm}/\cos\theta_W$$
$$=\left\{\frac{3[N_2(1+n_1)+\tilde{Y}^2](m_l^2+n_1m_u^2+n_1m_d^2)}{N_2(1+n_1)\tilde{Y}^2}\right\}^{1/2}. \quad (3.8)$$

These relations between the masses of the weak vector bosons and those of the leptons and quarks are entirely new and proper to our model. More practically, however, the weak-vector-boson masses can be predicted by the well-known relations[5] $g^2/8m_{W^\pm}{}^2=G_F/\sqrt{2}$ (where G_F is the Fermi coupling constant), $g^2=e^2/\sin^2\theta_W$, and the present result $\sin^2\theta_W=\frac{3}{8}$ to be

$$M_{W^\pm}=\left(\frac{\sqrt{2}\,\pi\alpha}{2G_F\sin^2\theta_W}\right)^{1/2}=61\text{ GeV}$$

and (3.9)

$$m_Z=m_{W^\pm}/\cos\theta_W=77\text{ GeV}.$$

Notice that the SU(3) color-gauge symmetry is unbroken so that the color-octet gluons G_μ^a remain massless.

In concluding this section, let us briefly discuss a possible extension of the present model to an arbitrary number of leptons and quarks which form a series of the Weinberg-Salam multiplets

$$(l_L,l_R,q_L,u_R,d_R)_i\,,\quad i=1,2,\ldots,N\,. \quad (3.10)$$

A necessary modification of all the results so far presented is easy and almost trivial. The predictions for the Weinberg angle and the ratio of the gauge-coupling constants are unchanged. The new relations (3.6)–(3.8) should be modified as

$$m_\eta=2\left[\frac{\sum_i(m_{l_i}{}^4+n_1m_{u_i}+n_1m_{d_i}{}^4)}{\sum_i(m_{l_i}{}^2+n_1m_{u_i}{}^2+n_1m_{d_i}{}^2)}\right]^{1/2}, \quad (3.11)$$

$$m_{W^\pm}=\left[\frac{3\sum_i(m_{l_i}{}^2+n_1m_{u_i}{}^2+n_1m_{d_i}{}^2)}{N_2(1+n_1)N}\right]^{1/2}, \quad (3.12)$$

and

$$m_Z=\left\{\frac{3[N_2(1+n_1)+\tilde{Y}^2]\sum_i(m_{l_i}{}^2+n_1m_{u_i}{}^2+n_1m_{d_i}{}^2)}{N_2(1+n_1)\tilde{Y}^2N}\right\}^{1/2}. \quad (3.13)$$

These extended relations indicate that the Higgs-scalar and weak-vector-boson masses are related to certain averages of the lepton and quark masses. This situation strongly suggests that there exist much heavier leptons and/or quarks whose masses reach or go beyond the weak-vector-boson masses given by (3.9).

IV. UNIFIED SPINOR-SUBQUARK MODEL

In the previous paper,[7] we have proposed the spinor-subquark model for quarks in which quarks are made of three subquarks of spin $\frac{1}{2}$:

$$w_i\ (i=1,2)\,,\quad h_i\ (i=1,2,\ldots,N)\,,\ \text{and}\ C_i\ (i=1,2,3)\,. \quad (4.1)$$

The w subquarks w_L, w_{1R}, and w_{2R} are a doublet, a singlet, and a singlet of the Weinberg-Salam SU(2) group. The h subquarks form an N-plet of the unknown H symmetry of SU(N). Also, the C subquarks form a triplet of the SU(3) color symmetry. For example, the familiar quarks are given by

$$\mathcal{P}_i=(w_1h_1C_i)\,,\quad c_i=(w_1h_2C_i)\,,$$
$$\mathfrak{N}_i=(w_2h_1C_i)\,,\quad \lambda_i=(w_2h_2C_i)\,. \quad (4.2)$$

Leptons can also be considered as composites of a w subquark, an h subquark, and an additional C subquark, C_0, which is singlet under the SU(3) color symmetry.

Let us assume the Lagrangian for the w subquarks,

$$L_w=\bar{w}_Li\partial\!\!\!/ w_L+\bar{w}_{1R}i\partial\!\!\!/ w_{1R}+\bar{w}_{2R}i\partial\!\!\!/ w_{2R}$$
$$+F_1(Y_{w_L}\bar{w}_L\gamma_\mu w_L+Y_{w_{1R}}\bar{w}_{1R}\gamma_\mu w_{1R}+Y_{w_{2R}}\bar{w}_{2R}\gamma_\mu w_{2R})^2$$
$$+F_2(\bar{w}_L\gamma_\mu\vec{\tau}w_L)^2$$
$$+F_4(-a_1\overline{w_R^G}\,w_{1L}^c+a_2\bar{w}_Lw_{2R})(-a_1\overline{w_{1L}^c}\,w_R^G+a_2\bar{w}_{2R}w_L)\,, \quad (4.3)$$

where F's and a's are real constants, Y's are the weak hypercharges of subquarks, and the superscripts c and G denote the charge-conjugate and G-parity-conjugate states, respectively. The Lagrangian (4.3) is very much similar to the original one of Saito and Shigemoto.[8] Without repeating the same procedure as in Secs. II and III, we simply present the following results. This model can be effectively equivalent to the unified gauge theory of Weinberg and Salam for the w subquarks. The gauge bosons $\vec{A}_\mu$ and B_μ appear as the bound states of w subquark-antisubquark pairs which behave as $\bar{w}_L\gamma_\mu\vec{\tau}w_L$ and $Y_{w_L}\bar{w}_L\gamma_\mu w_L+Y_{w_{1R}}\bar{w}_{1R}\gamma_\mu w_{1R}+Y_{w_{2R}}\bar{w}_{2R}\gamma_\mu w_{2R}$, respectively. This is exactly what we have suggested in Ref. 7.

The Weinberg angle is determined to be

$$\sin^2\theta_W = \frac{N_2}{N_2 + N_1 Y_{w_L}{}^2 + Y_{w_{1R}}{}^2 + Y_{w_{2R}}{}^2} = \frac{\frac{1}{4}}{Q^2 + (Q-1)^2}, \tag{4.4}$$

where Q is the charge of w_1. For $Q=1$ and $\frac{2}{3}$, for example, $\sin^2\theta_W = \frac{1}{4}$ and $\frac{9}{20}$, respectively. Although Q is rather arbitrary, the Weinberg angle is bounded by $\sin^2\theta_W \le \frac{1}{2}$. The relations between the masses of the Higgs scalar and weak vector bosons and those of the w subquarks are

$$m_\eta = 2\left(\frac{m_{w_1}{}^4 + m_{w_2}{}^4}{m_{w_1}{}^2 + m_{w_2}{}^2}\right)^{1/2} \cong 2m_w, \tag{4.5}$$

$$m_{W^\pm} = \left[\frac{3(m_{w_1}{}^2 + m_{w_2}{}^2)}{N_2}\right]^{1/2} \cong \sqrt{3}m_w, \tag{4.6}$$

and

$$m_Z = m_{W^\pm}/\cos\theta_W \cong \sqrt{3}m_w/\cos\theta_W. \tag{4.7}$$

Also, the model Lagrangian for the C subquarks is assumed to be

$$L_C = \bar{C}i\partial\!\!\!/ C + F_3(\bar{C}\gamma_\mu\lambda^a C)^2, \tag{4.8}$$

where F_3 is the real coupling constant. This model can be effectively equivalent to the SU(3) color-gauge theory for the C subquarks. The color-octet gluons appear as the bound states of C subquark-antisubquark pairs which behave as $\bar{C}\gamma_\mu\lambda^a C$. This also coincides with our conjecture in Ref. 7. However, the C subquarks are massless since the SU(3) gauge symmetry is unbroken.

We shall not discuss interactions of the h subquarks, which are ambiguous, though some speculations are made in Ref. 7, nor shall we discuss the mechanism of binding subquarks into a quark, which is more ambiguous.

Although some contents in this section seem to be still premature, an interesting picture of the gauge fields as bound states of subquarks has emerged in a model of the Nambu–Jona-Lasinio type. The relations (4.5)–(4.7) especially suggest that the masses of the Higgs scalar and weak vector bosons may be very close to the threshold of subquark-pair production, if any.

V. SUMMARY AND CONCLUDING REMARKS

Starting with a nonlinear fermion Lagrangian of the Nambu–Jona-Lasinio type, we have constructed an effective Lagrangian which combines the unified gauge theory of Weinberg and Salam for the weak and electromagnetic interactions of leptons and quarks and the asymptotically free gauge theory of Gross, Wilczek, and Politzer for the strong interaction of quarks. All the gauge bosons and the Higgs scalars are created as bound states of fermion-antifermion pairs. Arbitrary parameters involved in the combined theory are all determined by the quantum numbers of leptons and quarks, the cutoff momentum, and the coupling constants in the original Lagrangian. As a result, they are constrained by many mutual relations derived. The gauge coupling constants, for example, are all related, leaving only a single free parameter, the fine-structure constant. The model is, therefore, potentially a unified theory for all elementary-particle forces. The results, which are especially interesting are the relations between the masses of the Higgs scalar and the weak vector bosons and those of leptons and quarks or of subquarks. These relations strongly suggest either the existence of much heavier leptons and/or quarks whose masses reach or go beyond the weak-vector-boson masses or the existence of heavy subquarks whose pair-production threshold lies very close to the weak-vector-boson masses. We hope that this fascinating possibility of superheavy leptons, quarks, or subquarks will be checked in future experiments by the presently proposed big colliding-beam machines such as Isabelle, Popae, and Tristan.

After this work had been completed, we received a report by Saito and Shigemoto.[14] They have further extended the model discussed in Sec. II, including the axial-vector color gluon. Notice the main difference: They have introduced three kinds (for leptons, up-quarks, and down-quarks) of Higgs scalars while we have introduced only one kind.

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