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Bremsstrahlung production of high-energy lepton pairs in quantum chromodynamics*

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We study high-energy quark-quark scattering with bremsstrahlung emission of a photon of mass q^2, where q^2 is a finite fraction of the energy, using a leading-logarithm approximation in quantum chromodynamics. The calculation exhibits the importance of current conservation in a cancellation among gauge-invariant sets of graphs. To sixth order in the strong-coupling constant the amplitude is a power series in $g^2\ln^2(q^2/\lambda^2)$ dominated by infrared singularities (λ is the infrared cutoff). This series does not exponentiate. Leading graphs have color octet structure in a crossed channel.

I. INTRODUCTION

There is great interest in the calculation of the asymptotic behavior of physical processes in non-Abelian gauge theories. The motivation for this interest is the possibility that the strong interactions are described by quantum chromodynamics (QCD), a gauge theory of massless colored quarks and gluons with SU(3) of color as an exact gauge symmetry.[1-3] Such a model is asymptotically free so that it would be consistent with the results of deep-inelastic lepton-nucleon scattering.[4,5] It may also result in confinement of color thereby accounting for the nonobservation of isolated quarks or gluons.

The processes for which the asymptotic behavior has been previously calculated are the vertex functions and quark-quark scattering amplitudes.[6-11] The technique which is used to study the asymptotic behavior is the leading-logarithm approximation. In this approximation, one sums the leading asymptotic terms of each order in perturbation theory. One hopes the qualitative features of the exact amplitude are revealed by this procedure. This approximation has resulted in simple mathematical expressions, primarily exponentiation, in calculations performed to date for non-Abelian gauge theories. Typically in these calculations, the leading behavior came from the infrared contributions of virtual gluons. It has been shown that these results can be summarized as the solutions of a differential equation in the vector-gluon mass.[6,12-14] This equation, which has not been proved, has a form which is very similar to that of a renormalization-group equation. On the basis of this equation (or of the form of the answer) Cornwall and Tiktopoulos (CT) have argued that colored particles are confined by the infrared singularities of the theory. Others have calculated the infrared singularities for certain inclusive processes, and have found a pattern of cancellation familiar from QED; they therefore do not see confinement.[15-17] These two points of view involve a difference in orders of limits, and we do not take sides. However, the spirit of our calculation is in line with the CT approach.

We have applied these ideas to another process in quantum chromodynamics. We report here a study of the asymptotic behavior of hard-photon emission in high-energy ($\sqrt{s}$) quark-quark scattering. We consider a photon of mass q^2, q^2/s fixed as $s\to\infty$, emitted as a bremsstrahlung. We shall perform this calculation only to lowest order in the electromagnetic coupling. Production of massive lepton pairs in hadron-hadron collisions is an interesting process because, among other things, it has been proposed as a test of the quark-parton model.[18] In this model, the massive lepton pairs arise solely from parton-antipartion annihilation and the contribution from the emission of the massive pair as a bremsstrahlung is negligible. This is based upon a postulate that the amplitudes become small as the virtual partons go far off their mass shell.[19] Despite this, the bremsstrahlung mechanism has been studied by several people.[20,21] Bremsstrahlung may be in fact the dominant mechanism in kinematic regions in which the annihilation process is not prominent, for example, when the fractional change of the longitudinal momenta for nucleon-nucleon scattering is large.[22]

The main result which we wish to report here is an apparent nonexponentiation of the amplitude for production of massive lepton pairs. We have calculated the leading contributions to this process through sixth order and have found a series in $g^2\ln^2(q^2/\lambda^2)$ which is not the expansion of an exponential series. Here g is the coupling constant and λ is the gluon mass (or infrared cutoff). The leading diagrams in each order have the structure of a color octet in some of the crossed channels and always involve the trigluon couplings. The requirements of gauge invariance (or current conservation) lead to an important cancellation among gauge-invariant sets of graphs. The methods of calculation used are Feynman parametrization and momentum-space techniques. We have restricted our attention to the process in which the momentum transfer between the quark lines is held fixed.

This paper is organized as follows. In Sec. II we present the relevant kinematics and notation. We shall also discuss the Born term and a cancellation which occurs in this order betewen the gauge-invariant sum of graphs. This is a cancellation of forward divergences which has the effect of reducing but not eliminating the forward peak of the amplitude. This may be related to cancellations found by Thacker[21] in his calculation of the cross section in a particular model. In Sec. III we present arguments which, when combined with our calculations in perturbation theory, show that this feature of the Born term is maintained for the higher-order amplitudes. In Sec. IV we present the results of our fourth- and sixth-order calculation. We identify the diagrams which are asymptotically dominant and illustrate the calculational method which is used. In Sec. V we present our conclusions. In the Appendix we study in detail an example of a leading Feynman graph in sixth order.

II. KINEMATICS AND THE BORN TERM

We consider the exclusive emission of a massive timelike photon in fermion-fermion scattering, labeled as in Fig. 1.[23] The interesting limits are q^2, $s\to\infty$, $z\equiv q^2/s$ finite (generally speaking, $0<z<1$). We take t_1, t_2 fixed. The subenergies s_1 and s_2 then satisfy the constraint $s_1 s_2 \approx q^2 s$. Finally we choose for simplicity $q_{\text{space component}}=0$ so that $t=t_1=t_2=$ fixed.

Because of some cancellations which occur in the calculation, it is convenient to be explicit about a useful frame, the p_1-p_2 center-of-mass frame. Choose the x_1-x_3 plane to be the fermion scattering plane, labeled as in Fig. 2. In this frame, to $O(\theta)$

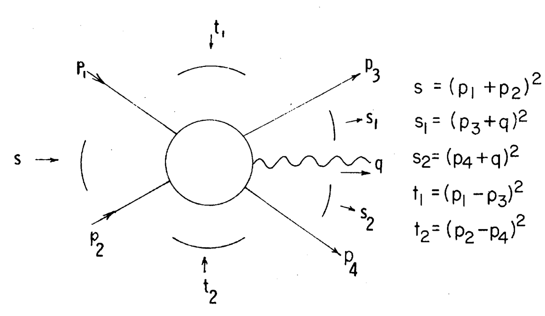

FIG. 1. A symmetric choice of the five independent kinematic invariants for the process which we study.

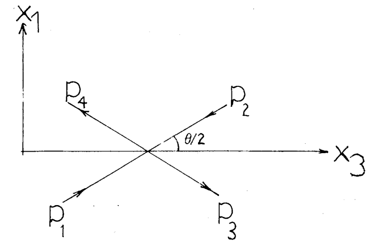

FIG. 2. Specification of the center-of-mass scattering angle θ for the case $\vec{q}=0$. Scattering is in the x_1-x_3 plane.

$$\begin{aligned}
&p_1 \approx (\sqrt{s}, \tfrac{1}{4}\sqrt{s}\theta, 0, 0),\\
&p_2 \approx (0, -\tfrac{1}{4}\sqrt{s}\theta, 0, \sqrt{s}),\\
&p_3 \approx (\sqrt{w_{34}}, -\tfrac{1}{4}\sqrt{w_{34}}\theta, 0, 0),\\
&p_4 \approx (0, \tfrac{1}{4}\sqrt{w_{34}}\theta, 0, \sqrt{w_{34}}),\\
&q = (q_0, 0, 0, q_0),
\end{aligned} \tag{2.1}$$

where we write an arbitrary four-vector

$$p^\mu = (p_+, \vec{p}, p_-) = (p^0+p^3, p^1, p^2, p^0-p^3),$$

$w_{34}=(p_3+p_4)^2$, and zero means of order θ^2. We are retaining $O(\theta)$ terms because of a cancellation of factors which occurs in this process and which requires keeping this order.

As discussed in Sec. I, we study bremsstrahlung contributions in powers of the bare coupling g and leading logarithms of large quantities in QCD—indeed the lowest-order graph is such a contribution. In order to handle the infrared divergences which dominate the calculation, we give the gluons a mass λ, a procedure known to be gauge invariant for the leading-logarithmic terms. Finally we study emission of the photon from only one of the fermion lines (the 1–3 lines); this procedure does not destroy gauge invariance and does not affect the result in a significant way.

The amplitude for this process, T^μ, is dominated by the component $\mu=\perp$ in our coordinate system. In Sec. III we shall show this result to be a general one; however, it is useful to set the stage by examination of the lowest-order contribution, the Born term, shown in Fig. 3. In the process we

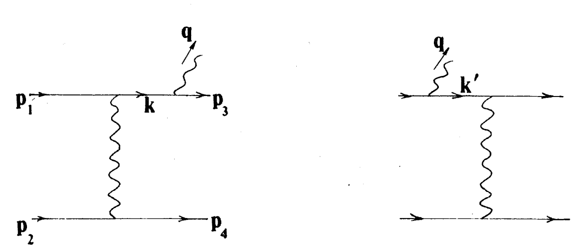

FIG. 3. The Feynman diagrams for the Born term.

demonstrate a (not unconnected) cancellation between the graphs of Figs. 3(a) and 3(b). In particular, the forward peaking in t (in the Feynman gauge) from either graph is partially canceled in the gauge-invariant sum.

The structure of the Born term is that of a color octet in the t_1 or t_2 channel. This structure persists in higher orders and may mean we are calculating a (somehow) dynamically suppressed amplitude for which perturbation theory is irrelevant.

The amplitude corresponding to Fig. 3 is[24]

$$T^{\mu}_{\text{Born}} = +ieg^2 \frac{1}{t-\lambda^2} (\vec{t})_{i_1 i_3} \cdot (\vec{t})_{i_2 i_4} \bar{u}(p_4,\lambda_4)\gamma_\lambda u(p_2,\lambda_2) \times \left[\frac{1}{k^2}\bar{u}(p_3,\lambda_3)\gamma^{\mu}\not{k}\gamma^{\lambda}u(p_1,\lambda_1) + \frac{1}{k'^2}\bar{u}(p_3,\lambda_3)\gamma^{\lambda}\not{k}'\gamma^{\mu}u(p_1,\lambda_1)\right] \tag{2.2}$$

$$= B^{\mu}_{(a)} + B^{\mu}_{(b)} , \tag{2.3}$$

where $\vec{t}$ are the color matrices and we define

$$k = (p_3 + q) , \quad k' = (p_1 - q) . \tag{2.4}$$

We reduce the spinor factors in $B^{\mu}_{(a)}$ as follows[25]:

$$\bar{u}(p_3,\lambda_3)\gamma^{\mu}\not{k}\gamma^{\lambda}u(p_1,\lambda_1)\bar{u}(p_4,\lambda_4)\gamma_\lambda u(p_2,\lambda_2) = k_\nu \bar{u}(p_4,\lambda_4)\gamma_\lambda u(p_2,\lambda_2)\bar{u}(p_3,\lambda_3)(g^{\nu\lambda}\gamma^{\mu} - g^{\mu\lambda}\gamma^{\nu} + g^{\mu\nu}\lambda^{\gamma} + i\epsilon^{\mu\nu\lambda\rho}\gamma_\rho\gamma_5)u(p_1,\lambda_1) . \tag{2.5}$$

For our coordinate system we have the following explicit asymptotic forms to $O(\theta)$:

$$\begin{aligned}
&\bar{u}(p_3,\lambda_3)\gamma_+ u(p_1,\lambda_1) \approx 2(sw_{34})^{1/4}\delta_{\lambda_3\lambda_1} ,\\
&\bar{u}(p_3,\lambda_3)\gamma_- u(p_1,\lambda_1) \approx 0 ,\\
&\bar{u}(p_3,\lambda_3)\gamma^1 u(p_1,\lambda_1) = 0 ,\\
&\bar{u}(p_3,\lambda_3)\gamma^2 u(p_1,\lambda_1) \approx (-1)^{\lambda_3+1/2}(i/2)\theta(sw_{34})^{1/4}\delta_{\lambda_3\lambda_1} ,\\
&\bar{u}(p_4,\lambda_4)\gamma_+ u(p_2,\lambda_2) \approx 0 ,\\
&\bar{u}(p_4,\lambda_4)\gamma_- u(p_2,\lambda_2) \approx 2(sw_{34})^{1/4}\delta_{\lambda_4\lambda_2} ,\\
&\bar{u}(p_4,\lambda_4)\gamma^1 u(p_2,\lambda_2) = 0 ,\\
&\bar{u}(p_4,\lambda_4)\gamma^2 u(p_2,\lambda_2) \approx (-1)^{\lambda_4+1/2}(i/2)\theta(sw_{34})^{1/4}\delta_{\lambda_4\lambda_2} ,\\
&\bar{u}(p_3,\lambda_3)\gamma^{\mu}\gamma^5 u(p_1,\lambda_1) \approx (-1)^{\lambda_3-1/2}\bar{u}(p_3,\lambda_3)\gamma^{\mu}u(p_1,\lambda_1) .
\end{aligned} \tag{2.6}$$

Substituting these forms into Eq. (2.5), one finds to $O(\theta)$

$$B^{\mu}_{(a)} = ieg^2 \frac{1}{t-\lambda^2} (\vec{t})_{i_1 i_3} \cdot (\vec{t})_{i_2 i_4} \left(\frac{1}{k^2}\right) 4(sw_{34})^{1/2}\delta_{\lambda_3\lambda_1}\delta_{\lambda_4\lambda_2} \times \begin{cases} \sqrt{s} \quad (\mu=+) ,\\ 0 \quad (\mu=-) ,\\ -(\theta/8)[\sqrt{s} + \sqrt{w_{34}} + q_0(-1)^{\lambda_3-1/2}(-1)^{\lambda_4-1/2}] \quad (\mu=1) ,\\ -i\,q_0(\theta/8)[(-1)^{\lambda_3-1/2} - (-1)^{\lambda_4-1/2}] \quad (\mu=2) , \end{cases} \tag{2.7}$$

and similarly

$$B^{\mu}_{(b)}=ieg^2\frac{1}{t-\lambda^2}(\vec{t})_{i_1i_3}\cdot(\vec{t})_{i_2i_4}\left(\frac{1}{k'^2}\right)4(sw_{34})^{1/2}\delta_{\lambda_3\lambda_1}\delta_{\lambda_4\lambda_2}$$
$$\times\begin{cases}\sqrt{w_{34}}\quad(\mu=+)\,,\\ 0\quad(\mu=-)\,,\\ (\theta/8)[\sqrt{s}+\sqrt{w_{34}}-q_0(-1)^{\lambda_3-1/2}(-1)^{\lambda_4-1/2}]\quad(\mu=1)\,,\\ iq_0(\theta/8)[(-1)^{\lambda_3-1/2}-(-1)^{\lambda_4-1/2}]\quad(\mu=2)\,.\end{cases}\tag{2.8}$$

Finally, we use

$$k^2=\sqrt{s}\;q_0=-\sqrt{s/w_{34}}\;k'^2=s_1\tag{2.9}$$

to sum Eqs. (2.7) and (2.8). We find

$$T^{\mu}_{\text{Born}}=ieg^2\frac{1}{t-\lambda^2}(\vec{t})_{i_1i_3}\cdot(\vec{t})_{i_2i_4}4(sw_{34})^{1/2}\delta_{\lambda_3\lambda_1}\delta_{\lambda_4\lambda_2}$$
$$\times\begin{cases}0\quad(\mu=+)\,,\\ 0\quad(\mu=-)\,,\\ -\theta/(2q_0)+(\theta/8)\left[\frac{1}{\sqrt{s}}-\frac{1}{\sqrt{w_{34}}}\right][1-(-1)^{\lambda_3-1/2}(-1)^{\lambda_4-1/2}]\quad(\mu=1)\,,\\ -i(\theta/8)\left[\frac{1}{\sqrt{s}}+\frac{1}{\sqrt{w_{34}}}\right][(-1)^{\lambda_3-1/2}-(-1)^{\lambda_4-1/2}]\quad(\mu=2)\,.\end{cases}\tag{2.10}$$

The term with $\mu=+$, which was independent of the scattering angle θ, has been canceled out in the sum.[26]

III. GAUGE-INVARIANT CANCELLATION

In the preceding section, we demonstrated a cancellation among a gauge-invariant set of $O(g^2)$ graphs which produces a reduction in the forward peaking, or equivalently (see Ref. 21) reduces an integrated cross section from one which grows linearly in s for fixed q^2 to one which is constant. We shall now show that this feature is maintained for diagrams of higher order in g^2.

This argument is crucial for our work since if in a given order of g^2 there were a piece of the exact amplitude which did not exhibit this cancellation it would be the leading asymptotic term by a power, namely, of order $\sqrt{s/t}\to\infty$ for t fixed relative to the Born term. We would therefore not obtain a series which was a multiplicative factor of the Born term.

An outline of our argument is as follows: The general t structure of the Feynman integrals for the leading graphs is, neglecting numerator algebra and logarithmic factors, $(t-\lambda^2)^{-1}$. (Special care would have to be given to ϕ^3 graphs corresponding to infinite-momentum "shortcuts"[27]; we neglect these because they will fall as a power when the numerator structure of QCD is restored.) We shall show that the $\mu=+,\,-$ terms in the numerator factor of the full amplitude T^{μ} [with the $(t-\lambda^2)^{-1}$ factor extracted] vanish as t [more precisely vanish to $O(\theta)$] while the $\mu=\perp$ term behaves for small t as $\sqrt{t}$ (as θ). Thus the $\mu=\perp$ terms will be dominant as $s\to\infty$, and, moreover, will behave like the Born term up to logarithms.

To show the vanishing of the $\mu=+,\,-$ terms, we first show the explicit vanishing of the $\mu=-$ terms [an example of this is given in Eqs. (2.7) and (2.8)]. The $\mu=+$ terms then vanish by current conservation, or in other words as a result of the gauge-invariance requirement. In the example of Sec. II this was shown explicitly. However, we can show that it must follow more generally as follows: Current conservation for the electromagnetic field requires for us

$$q_{\mu}T^{\mu}=\tfrac{1}{2}(q_+T_-+q_-T_+)$$
$$=\tfrac{1}{2}q_0(T_++T_-)=0\,,\tag{3.1}$$

or

$$T_+=-T_-\,.\tag{3.2}$$

Thus if the $\mu=-$ piece vanishes, then the $\mu=+$ piece must likewise vanish when a gauge-invariant set is added, as explicitly shown in Eq. (2.10). We now turn, with some preliminaries, to the $\mu=-$ terms.

The product of the bilinear forms for the two fermion lines will have the following general structure:

$$F^\mu = \bar{u}_3\Gamma^1\gamma^\mu\Gamma^2 u_1 \bar{u}_4\Gamma^3 u_2 . \qquad (3.3)$$

The quantities Γ^i, $i=1-3$, are matrices which contain all of the numerator algebra. All possible Lorentz indices of the Γ's have been suppressed; note, however, that these must be contracted over in (3.3). We shall consider the integrations over internal momenta to have been performed so that the Γ's are functions only of the external momenta p_i, $i=1-4$ and q. All of the Feynman graphs can be accommodated in the above structure. For example, the case in which the photon emission is external to the strong interactions is obtained by setting Γ^1 or Γ^2 equal to the unit matrix.

Since we study the case of massless fermions, both Γ^1 and Γ^2 are necessarily the product of an even number of γ matrices and Γ^3 the product of an odd number. Thus we can write Γ^1 and Γ^2 as a unique linear combination of the following set of matrices:

$$(1, \gamma_+\gamma_-, \gamma_+\gamma_\perp, \gamma_-\gamma_\perp, \gamma_1\gamma_2, \gamma_5)$$

(Ref. 28). All other products of an even number of γ matrices can be expressed as a sum of the above set. Similarly, the matrix Γ^3 will be expressed as a linear combination of the set $(\gamma_+, \gamma_-, \gamma_\perp, \gamma_+\gamma_-\gamma_\perp, \gamma_1\gamma_2\gamma_+, \gamma_1\gamma_2\gamma_-)$.

We shall now argue that the product of the bilinear forms for $\mu = -$ will be of second order in θ, for small scattering angles θ. We show by enumerating all of the possibilities that the zeroth- and first-order terms in the expansion for F^- vanish.

Since the spinors $\bar{u}_3$ and u_1 are for particles which have a large + component only of four-momenta, we see immediately that the spinors which result from the operations $\bar{u}_3\Gamma^1\gamma_-$ and $\gamma_-\Gamma^2 u_1$ are no larger than first order in θ for all terms in the expansions of Γ^1 and Γ^2 except for the $\gamma_+\gamma_\perp$ part. This is a consequence of the fact that $\gamma_- u_1$ and $\bar{u}_3\gamma_-$ are no larger than $O(\theta)$. But the $\gamma_+\gamma_\perp$ terms will result in the bilinear form

$$\begin{aligned}\bar{u}_3\gamma_+\gamma_{\perp_1}\gamma_-\gamma_{\perp_2}u_1 &= 4\bar{u}_3\gamma_-\gamma_{\perp_1}\gamma_{\perp_2}u_1\\ &= \pm 4i\bar{u}_3\gamma_-\gamma^5 u_1\\ &= O(\theta^2).\end{aligned}$$

Thus the bilinear form corresponding to the "1–3" fermion lines must be no larger than $O(\theta)$. If the product of the two bilinear forms is to be of first order in θ, then the bilinear form for the "2–4" lines must be of zeroth order. Thus we see from Eq. (2.6) that we can omit all matrices in the expansion for Γ^3 except γ_- and $\gamma_-\gamma^5 = -i\gamma^1\gamma^2\gamma_-$. Additionally, we see that any of the matrices in our expansion sets which contain a $\gamma_\perp$ must be contracted with another matrix containing a $\gamma_\perp$ and not a transverse component of an external momentum $(p_i)_\perp$, since these are all of first order in the scattering angle θ.

We first examine the contribution from the γ_- term in the expansion of Γ^3. For this case, we need only consider the matrices 1 and $\gamma_+\gamma_-$ (the only combinations without any $\gamma_\perp$ terms) in the expansions of Γ^1 and Γ^2. These matrices will always result in a bilinear form for the "1–3" lines which either vanishes identically or is proportional to $\bar{u}_3\gamma_- u_1$ which is of second order in θ.

The remaining matrix in the expansion set for Γ^3 is $\gamma_-\gamma^5$, which contains two transverse γ matrices. These, as indicated above, must be contracted with $\gamma_\perp$'s which occur in Γ^1 and Γ^2. The possibilities are that either both Γ^1 and Γ^2 contain one $\gamma_\perp$

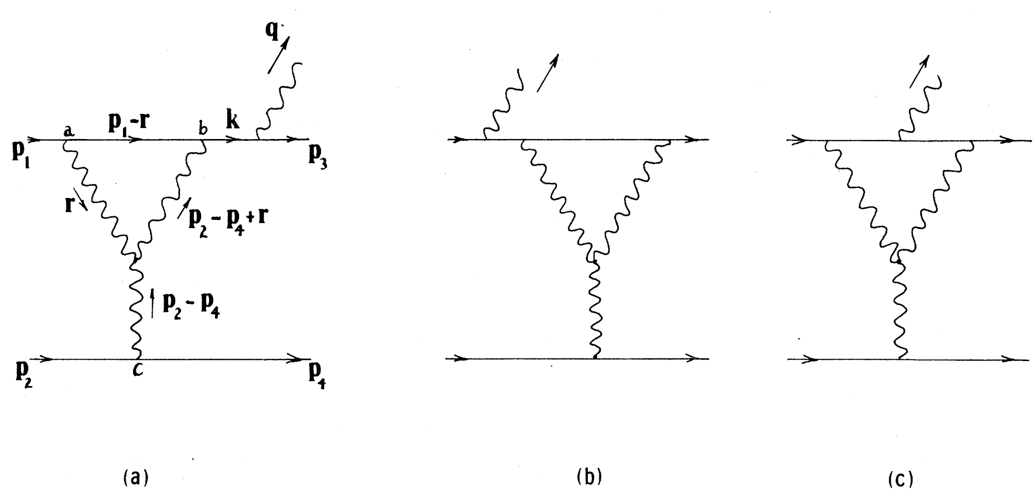

FIG. 4. The leading diagrams in $O(eg^4)$. Internal momenta and color indices are labeled for (a).

each, or that one of them contains two $\gamma_\perp$'s and the other none. For the former, we have already seen that the $\gamma_+\gamma_\perp$ terms result in a bilinear form of $O(\theta^2)$ and the $\gamma_-\gamma_\perp$ term vanishes identically. For the latter, one of the Γ's must be expressible as a combination of γ_5 and $\gamma_1\gamma_2$ and the other as a combination of 1 and $\gamma_+\gamma_-$. All possible combinations of these result in a bilinear form which is proportional to $\bar{u}_3\gamma_-\gamma^5 u_1 = O(\theta^2)$. Hence, $F^- = O(\theta^2)$ for small θ.

The same type of argument shows that $T^{\mu=\perp}$ is, in general, of $O(\theta)$. In particular, for $\bar{u}_3\Gamma^1\gamma_\perp\Gamma^2 \times u_1\bar{u}_4\Gamma^3 u_2$ to be independent of θ, Γ^3 must be $\propto\gamma_-$ or $\gamma_-\gamma_5$. By tracing through the cases as above, we find that it is not possible to construct an amplitude independent of θ with $\mu=\perp$.

IV. PERTURBATIVE CALCULATIONS

A. Order eg^4

The diagrams which were found to be leading in this order are shown and labeled in Fig. 4. Let us first consider graph 4(a),

$$T^\mu_{4(a)} = ieg^4 \frac{1}{t-\lambda^2}\frac{1}{k^2}\bar{u}(p_4)\gamma^\lambda u(p_2) f_{abc}(t_b t_a)_{i_1 i_3}(t_c)_{i_2 i_4}$$
$$\times\int\frac{d^4 r}{(2\pi)^4}\frac{\bar{u}(p_3)\gamma^\mu \not{k}\gamma^\alpha(\not{p}_1-\not{r})\gamma^\nu u(p_1)}{(r^2-\lambda^2)[(p_2-p_4+r)^2-\lambda^2](p_1-r)^2}$$
$$\times\{(2r+p_2-p_4)_\lambda g_{\alpha\nu}+[2(p_4-p_2)-r]_\nu g_{\alpha\lambda}+(p_2-p_4-r)_\alpha g_{\nu\lambda}\}\,, \tag{4.1}$$

where f_{abc} is the structure constant of the gauge group. The color weight of the above diagram is[29]

$$f_{abc}(t_b t_a)_{i_1 i_3}(t_c)_{i_2 i_4} = -\tfrac{1}{2} i C_A (\vec{t})_{i_1 i_3}\cdot(\vec{t})_{i_2 i_4}\,, \tag{4.2}$$

where C_A is the Casimir operator defined by

$$f_{abc}f_{abe} = C_A\delta_{ce}\,.$$

We perform the integrations in Eq. (4.1) by standard Feynman-parameter techniques. These integrations are of the same form as those which arise in the second-order form-factor calculations; here the time-like k plays the role of the far-off-shell gluon while the lines labeled by p_1 and p_2-p_4 play the role of the on-shell fermions. [This analogy will carry through the $O(eg^6)$ calculations of subsection B.] The important range of integration will therefore be the infrared region $r\to 0$. We simplify the numerator by neglecting r everywhere. The numerator factor then becomes

$$\bar{u}_3\gamma^\mu\not{k}\gamma^\alpha\not{p}_1\gamma^\nu u_1\bar{u}_4\gamma^\lambda u_2[(p_2-p_4)_\lambda g_{\alpha\nu}+2(p_4-p_2)_\nu g_{\alpha\lambda}+(p_2-p_4)_\alpha g_{\nu\lambda}] = 2k^2\bar{u}_3\gamma^\mu u_1\bar{u}_4\not{p}_1 u_2 - 2k^2\bar{u}_3\gamma^\mu\not{k}\gamma^\lambda u_1\bar{u}_4\gamma_\lambda u_2\,, \tag{4.3}$$

while for the remaining r integral we have

$$(2\pi)^{-4}\int d^4 r[(r^2-\lambda^2)((p_2-p_4+r)^2-\lambda^2)(p_1-r)^2]^{-1} = \frac{i}{16\pi^2}\int_0^1\frac{d\alpha_1 d\alpha_2 d\alpha_3\,\delta(1-\Sigma\alpha_i)}{k^2\alpha_2\alpha_3+t\alpha_1\alpha_2-\lambda^2(\alpha_1+\alpha_2)}\,. \tag{4.4}$$

We extract the leading behavior of (4.4) by evaluating only the contribution to the integral from the region where[30] $\alpha_2,\alpha_3\to 0$:

$$\int_0^\epsilon\frac{d\alpha_2 d\alpha_3}{(k^2\alpha_2\alpha_3-\lambda^2)} = \frac{\ln^2(k^2/\lambda^2)}{2k^2}\,. \tag{4.5}$$

Thus

$$T^\mu_{4(a)} = \frac{+ieg^4 C_A}{32\pi^2}\frac{1}{t-\lambda^2}\frac{\ln^2(k^2/\lambda^2)}{k^2}(\vec{t})_{i_1 i_3}\cdot(\vec{t})_{i_2 i_4}(\bar{u}_3\gamma^\mu u_1\bar{u}_4\not{p}_1 u_2-\bar{u}_3\gamma^\mu\not{k}\gamma^\lambda u_1\bar{u}_4\lambda_\lambda u_2)\,. \tag{4.6}$$

Similarly we find for diagram 4(b)

$$T^\mu_{4(b)} = \frac{+ieg^4 C_A}{32\pi^2}\frac{1}{t-\lambda^2}\frac{\ln^2(-k'^2/\lambda^2)}{k'^2}(\vec{t})_{i_1 i_3}\cdot(\vec{t})_{i_2 i_4}(\bar{u}_3\gamma^\mu u_1\bar{u}_4\not{p}_3 u_2-\bar{u}_3\gamma^\lambda\not{k}'\gamma^\mu u_1\bar{u}_4\gamma_\lambda u_2)\,. \tag{4.7}$$

(We avoid the case where $|k'^2| = |q^2-k^2| \ll |s|$, which is on the boundary of the physical region.) Note that for both diagrams 4(a) and 4(b) the large minus-component transfer $(p_2-p_4)_-$, which must be given to q, is routed through the gluon which is attached most closely to the emitted photon.

The sum of the above two diagrams can be written as (note that for the argument of the logarithms all of the large variables are equivalent in a leading-logarithm approximation)

$$T^{\mu}_{4(a)+4(b)} = T^{\mu}_{\rm Born}\left[-\frac{g^2C_A}{32\pi^2}\ln^2(q^2/\lambda^2)\right] + \frac{ieg^4C_A}{32\pi^2}\frac{1}{t-\lambda^2}\ln^2(q^2/\lambda^2)(\vec{t})_{i_1i_3}\cdot(\vec{t})_{i_2i_4}\bar{u}_3\gamma^{\mu}u_1\bar{u}_4\left(\frac{\not{p}_1}{k^2}+\frac{\not{p}_3}{k'^2}\right)u_2 . \quad (4.8)$$

Likewise diagram 4(c) can be calculated. The leading piece comes from two different regions of parameter space. These regions correspond to routing the $(p_2-p_4)_-$ transfer through either the gluon line which is attached before or after the photon emission. The result is

$$T^{\mu}_{4(c)} = T^{\mu}_{\rm Born}\left[-\frac{g^2C_A}{32\pi^2}\ln^2(q^2/\lambda^2)\right] - \frac{ieg^4C_A}{32\pi^2}\frac{1}{t-\lambda^2}\ln^2(q^2/\lambda^2)(\vec{t})_{i_1i_3}\cdot(\vec{t})_{i_2i_4}\bar{u}_3\gamma^{\mu}u_1\bar{u}_4\left(\frac{\not{p}_1}{k^2}+\frac{\not{p}_3}{k'^2}\right)u_2 . \quad (4.9)$$

The non-Born terms, which would not satisfy current conservation on their own, cancel in the gauge-invariant sum 4(a) + 4(b) + 4(c). The result of our leading-log calculation through fourth order is

$$T^{\mu} = T^{\mu}_{\rm Born}\left[1-\frac{g^2C_A}{16\pi^2}\ln^2(q^2/\lambda^2)\right] . \quad (4.10)$$

All remaining diagrams in fourth order were found to contribute only to the nonleading terms. Note in particular that we have not attempted to separate the color-singlet (in the t_1 and t_2 channels) amplitude which first appears in this order.

B. Order eg^6

In Fig. 5 we show one-half of the leading diagrams and their weights h. We have drawn only those pieces which are multiples of one of the two Born terms, namely Fig. 3(a). There is another set which is the mirror image of those drawn and which is a multiple of the other Born term. The result in sixth order is of the form

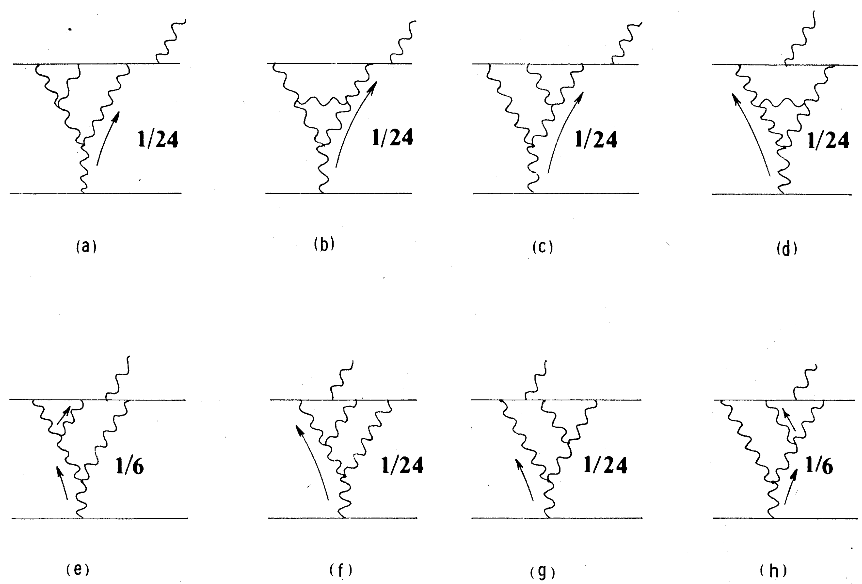

FIG. 5. Half of the leading diagrams in $O(eg^6)$ with weights indicated. There is symmetry with regard to the remaining graphs found by taking the mirror image. Routing for large $(p_2-p_4)_-$ is indicated by the direction of the arrow.

$$T_{O(g^6)} = T^{\mu}_{\text{Born}}\left[-\frac{g^2}{16\pi^2} C_A \ln^2(q^2/\lambda^2)\right]^2 (h_{5(a)}+h_{5(b)}+\cdots+h_{5(h)})$$
$$= T^{\mu}_{\text{Born}}\frac{7}{12}\left[-\frac{g^2}{16\pi^2} C_A \ln^2(q^2/\lambda^2)\right]^2 . \quad (4.11)$$

As for the fourth-order calculation, there are leading pieces in any given graph which do not have the structure of the Born term but which cancel when the full gauge-invariant set of graphs is summed. We do not show these contributions.

Also shown in Fig. 5 are arrows indicating the dominant region of large-minus-momentum-component flow. (These arrows should be included when taking the mirror image of these graphs.)

The technique which is used in the calculation of Figs. 5(a)–5(c) is familiar from the form-factor calculation. If one chops off the pieces of the diagram which do not contain the loop integrations, it is seen that these graphs are the quark-quark-vector form factor with one quark far-off-shell. This form factor has been previously calculated so that we shall omit the details of the calculation,[6] except to state that we have verified the results. The nonplanar graph shown in Fig. 6 is identically zero owing to the vanishing of the color weight.[31] Note that, as for the fourth-order graphs, the momentum flow is along that path lying closest to the emitted photon.

Attaching the photon to interior lines results in five more leading diagrams. As before, these graphs have two types of dominant momentum flow, contributing to multiples of the two separate pieces of the Born term. The results for graphs 5(d)–5(h) have been checked in part by two methods of finding the leading behavior of Feynman integrals, Feynman parameters and infinite-momenta techniques. We illustrate the method by a detailed calculation of graph 5(*e*) in the Appendix.

The sum of the leading contributions through sixth order is

$$T^{\mu} = T^{\mu}_{B}\left\{1+\left[-\frac{g^2}{16\pi^2} C_A \ln^2(q^2/\lambda^2)\right] + \frac{7}{12}\left[-\frac{g^2}{16\pi^2} C_A \ln^2(q^2/\lambda^2)\right]^2\right\}. \quad (4.12)$$

This expression is of course not the expansion of an exponential series.

As in previous calculations of asymptotic behavior in non-Abelian theories, we have found the contributions of graphs involving four-gluon couplings, ghosts, scalars, etc. to be nonleading.

V. DISCUSSION

A main result of our paper is the apparent nonexponentiation of the leading-logarithm series for far-off-shell bremsstrahlung. We recognize that occasionally such claims have proved faulty in the past. However, this type of process was found to exhibit features which were different from previous calculations of asymptotic behavior of vertex functions and fermion-fermion scattering amplitudes. In particular, we mention the cancellation arising from inclusion of gauge invariance of terms normally present in, say, fermion-fermion scattering. It is also interesting that for this process, the non-Abelian gauge theories give an answer which is qualitatively different from the result of massive quantum electrodynamics, in contrast to results found for calculations of the vertex functions.[6,10] This may be because the leading contributions were found to arise solely from the contributions of trigluon couplings.

The leading graphs are color octet in, say, the t_2 channel. Should our result hold up, it would appear to spell difficulty for the program in which confinement is postulated on the basis of the rapid vanishing of the sum of leading-logarithmic terms as $\lambda \to 0$. The analogous result for the color-singlet piece would be useful to know.

Nonexponentiation of leading terms is known to be correct for color-singlet channels in fermion-fermion scattering.[12,13] These amplitudes satisfy a differential equation superficially similar to the renormalization-group equation, as do the exponentiating amplitudes. It would be interesting to know whether our result can be described by a differential equation of this type.

From a phenomenological point of view, experimental production of lepton pairs is an important process. The lore of the parton model is that bremsstrahlung-type processes (as opposed to annihilation of quarks and antiquarks) are strongly suppressed. The parton model does not

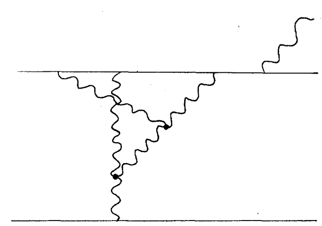

FIG. 6. Nonplanar graph which has vanishing color weight. It corresponds to crossed ladder exchange in a form-factor calculation.

reckon with the infrared effects we encounter but rather with short-distance behavior; we therefore cannot say what relevance our calculations have for the parton model. Regardless, bremsstrahlung processes may be important in kinematical regions (and for processes) for which annihilation is not allowed. These questions deserve further attention.

ACKNOWLEDGMENT

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APPENDIX

We illustrate the method used in extracting the leading behavior of the sixth-order graphs by sketching the calculation for the graph shown and labeled in Fig. 7. The amplitude for this graph is given by

$$T^{\mu}=ieg^{6}(t-\lambda^{2})^{-1}f_{abc}f_{cde}(t_{d}t_{b}t_{a})_{i_1i_3}(t_{e})_{i_2i_4}\int\frac{d^4r_1d^4r_2}{(2\pi)^8}\,\frac{N^{\mu}}{D}\;, \tag{A1}$$

where

$$\begin{aligned}N^{\mu}&=\bar{u}_3\gamma^{\rho}(\not{k}'+\not{r}_1)\gamma^{\mu}(\not{p}_1+\not{r}_1)\gamma^{\alpha}(\not{p}_1+\not{r}_2)\gamma^{\nu}u_1\bar{u}_4\gamma^{\lambda}u_2\\&\quad\times[(r_1-2r_2)_{\beta}\,g_{\nu\alpha}+(r_2-2r_1)\,g_{\alpha\beta}+(r_1+r_2)_{\alpha}\,g_{\nu\beta}]\\&\quad\times\{(p_2-p_4-2r_1)_{\lambda}\,g^{\beta}_{\rho}+[r_1-2(p_2-p_4)]^{\beta}g_{\rho\lambda}+(p_2-p_4+r_1)_{\rho}\,g^{\beta}_{\lambda}\}\end{aligned} \tag{A2}$$

and

$$D=(r_1^{\,2}-\lambda^2)(r_2^{\,2}-\lambda^2)[(r_1-r_2)^2-\lambda^2][(p_2-p_4-r_1)^2-\lambda^2](p_1+r_2)^2(p_1+r_1)^2(k'+r_1)^2\,. \tag{A3}$$

The group weight can be simplified to

$$f_{abc}f_{cde}(t_{d}t_{b}t_{a})_{i_1i_3}(t_{e})_{i_2i_4}=-\tfrac{1}{4}\,C_A^{\,2}(\vec{t})_{i_1i_3}\cdot(\vec{t})_{i_2i_4}\,. \tag{A4}$$

In the usual manner,[30] one combines the propagators of Eq. (A1) by introducing Feynman parameters α_i as labeled in Fig. 7. Then by performing the integrations over the internal momenta one arrives at a parametric representation for the amplitude,

$$T^{\mu}=\frac{ieg^{6}}{2(16\pi^{2})^{2}}\,(t-\lambda^{2})^{-1}C_A^{\,2}(\vec{t})_{i_1i_3}\cdot(\vec{t})_{i_2i_4}\int_0^1 d\alpha_1\cdots d\alpha_7\,\delta\Big(\sum_{i=1}^{7}\alpha_i-1\Big)\,U^{-2}\Big(\frac{N_0^{\mu}}{V^3}+\frac{N_1^{\mu}}{2V^2}+\frac{N_2^{\mu}}{2V}\Big)\;, \tag{A5}$$

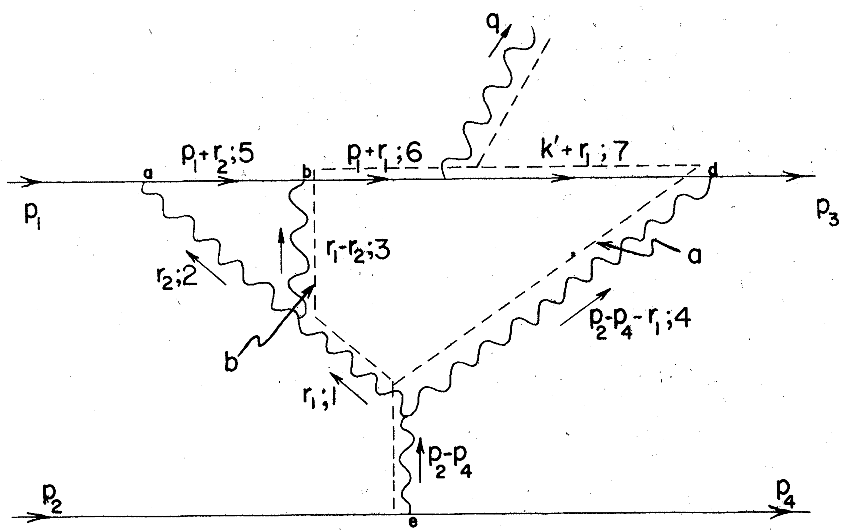

FIG. 7. Labeling of internal momenta, color indices, and Feynman parameters for sixth-order graph that we present in detail. Paths (a) and (b) for momentum transfer are indicated by broken lines.

where U and V are the parametric determinants:

$$U=(\alpha_2+\alpha_3+\alpha_5)(\alpha_1+\alpha_4+\alpha_6+\alpha_7)+\alpha_3(\alpha_2+\alpha_5), \tag{A6}$$

$$V=-\lambda^2(\alpha_1+\alpha_2+\alpha_3+\alpha_4)+U^{-1}\{t\alpha_4[\alpha_1(\alpha_2+\alpha_3+\alpha_5)+\alpha_2\alpha_3]+k'^2\alpha_7[\boldsymbol{\alpha_1}(\alpha_2+\alpha_3+\alpha_5)+\alpha_2\alpha_3] + q^2\alpha_7[\alpha_6(\alpha_2+\alpha_3+\alpha_5)+\alpha_3\alpha_5]+k^2[\alpha_3\alpha_4\alpha_5+\alpha_4\alpha_6(\alpha_2+\alpha_3+\alpha_5)]\}. \tag{A7}$$

The numerator function N_0^μ is given by the following substitution for the internal momenta in Eq. (A2):

$$\begin{aligned} r_1 &\to U^{-1}\{\alpha_2+\alpha_3+\alpha_5)[(p_2-p_4)\alpha_4-p_1\alpha_6-k'\alpha_7]-\alpha_3\alpha_5 p_1\},\\ r_2 &\to U^{-1}\{\alpha_3[(p_2-p_4)\alpha_4-p_1\alpha_6-k'\alpha_7]-\alpha_5(\alpha_1+\alpha_3+\alpha_4+\alpha_6+\alpha_7)p_1\}. \end{aligned} \tag{A8}$$

The functions N_1^μ and N_2^μ involve the two-index Lorentz tensor X_{ij}, in the notation of Nakanishi.[32] The terms in Eq. (A5) which involve these functions do not contribute leading logarithms and they will be omitted from the remainder of the discussion.

Using well-known techniques, we now extract the leading behavior from Eq. (A5). The regions of parameter space which give the dominant contributions are the neighborhoods of the point where the coefficients of the large variables k^2 and k'^2 (note $q^2=k^2+k'^2$) in Eq. (A7) vanish. These may include regions where all of the parameters of a closed loop vanish. To facilitate the inclusion of these regions, we first perform a scale transformation on the parameters of each loop. Specifically, we scale the variables

$$\alpha_2=\sigma_1\alpha_2', \quad \alpha_3=\sigma_1\alpha_3',$$
$$\alpha_5=\sigma_1\alpha_5', \quad \alpha_2'+\alpha_3'+\alpha_5'=1$$

and

$$\alpha_1=\sigma_2\alpha_1', \quad \alpha_3'=\sigma_2\alpha_3'', \quad \alpha_4=\sigma_2\alpha_4',$$
$$\alpha_6=\sigma_2\alpha_6', \quad \alpha_7=\sigma_2\alpha_7',$$
$$\alpha_1'+\alpha_3''+\alpha_4'+\alpha_6'+\alpha_7'=1.$$

We shall consider the parametric functions appearing in Eq. (A5) expressed in terms of these scaled variables. Next we perform scale transformations, with a scale factor ρ, on the sets of parameters of minimum effective length which cause the coefficients of the large variables to vanish when they themselves vanish. (The leading terms will come from end-point contributions.) The dominant asymptotic behavior is then extracted by performing the integrations in the limit where $\rho\to 0$.

For our calculation, we construct the following independent sets of scaling sequences:

$$\begin{aligned} &\text{(i)}\ (\sigma_2)(\alpha_3\alpha_6\alpha_7)(\alpha_5\alpha_6\alpha_7)(\alpha_4\alpha_7),\\ &\text{(ii)}\ (\alpha_3\alpha_6\alpha_7)(\alpha_5\alpha_6\alpha_7)(\sigma_1\alpha_6\alpha_7)(\alpha_4\alpha_7) \end{aligned} \tag{A9}$$

and

$$\begin{aligned} &\text{(i)}\ (\sigma_2)(\alpha_1\alpha_3\alpha_6)(\alpha_5\alpha_6\alpha_7)(\alpha_3\alpha_6\alpha_7),\\ &\text{(ii)}\ (\alpha_1\alpha_3\alpha_6)(\alpha_5\alpha_6\alpha_7)(\alpha_3\alpha_6\alpha_7)(\sigma_1\alpha_6\alpha_7),\\ &\text{(iii)}\ (\sigma_1\alpha_1\alpha_6)(\alpha_5\alpha_6\alpha_7)(\alpha_3\alpha_6\alpha_7)(\alpha_1\alpha_3\alpha_6),\\ &\text{(iv)}\ (\alpha_5\alpha_6\alpha_7)(\alpha_3\alpha_6\alpha_7)(\sigma_1\alpha_6\alpha_7)(\sigma_1\alpha_1\alpha_6). \end{aligned} \tag{A10}$$

Typically, we shall find that the leading contributions come from regions where a continuous path of gluon lines carries the full large minus component of the momentum transfer (p_2-p_4) and all remaining gluons carry no large momentum components. The possible paths are indicated by the dashed lines in Fig. 7. For the region of parameter space corresponding to the scalings of Eq. (A9) we find that the substitutions for the internal momenta in Eq. (A8) can be approximated by

$$\begin{aligned} r_1 &\to \alpha_4'(p_2-p_4),\\ r_2 &\to -\alpha_5' p_1. \end{aligned} \tag{A11}$$

Both of these substitutions are linear in a vanishing scale factor ρ. We have not retained the higher-order terms in ρ. Because the effective r_i is small, these two scaling sets will correspond to the momentum routing of path (a).

Using the above substitution, we find an approximation for the numerator function to $O(\rho)$:

$$N_0^\mu \to 2k^2\alpha_4'(k^2\bar{u}_3\gamma^\lambda \not{k}'\gamma^\mu u_1\bar{u}_4\gamma_\lambda u_2+k'^2\bar{u}_3\gamma^\mu u_1\bar{u}_4\not{p}_1 u_2). \tag{A12}$$

It is necessary to retain the terms of $O(\rho)$ because for this momentum routing there is a trigluon vertex involving three soft gluons. The neglected terms of higher order in ρ will not produce leading logarithms.

Evaluation of the parametric integrals for these two scaling sets gives the result

$$T^{\mu}_{(a)} = ieg^2(t-\lambda^2)^{-1}\left(\frac{1}{24}\right)\left[\frac{-g^2}{16\pi^2}C_A \ln^2(q^2/\lambda^2)\right]^2 (\vec{t})_{i_1i_3}\cdot(\vec{t})_{i_2i_4}\left\{\frac{1}{k'^2}\bar{u}_3\gamma^\lambda \not{k}'\gamma^\mu u_1\bar{u}_4\gamma_\lambda u_2 + \frac{1}{k^2}\bar{u}_3\gamma^\mu u_1\bar{u}_4\not{p}_1 u_2\right\}. \tag{A13}$$

The first term in curly brackets corresponds to a piece of the Born term with a weight $\frac{1}{24}$ as shown for the mirror image in Fig. 5(g). The second term in the curly brackets, which does not have the form of any part of the Born term, will be canceled when the contribution of a gauge-invariant (electromagnetic) subset of diagrams is included.

Similarly, the scaling of Eq. (A10) corresponds to the momentum routing

$$r_1 \to (p_2 - p_4)\,, \quad r_2 \to 0\,, \tag{A14}$$

as indicated by path (b) in Fig. 7. There are no vanishing trigluon couplings for this routing and the numerator can be approximated by

$$N_0^\mu \to 2k^2(2k'^2\bar{u}_3\gamma^\mu \not{k}\gamma^\lambda u_1\bar{u}_4\gamma_\lambda u_2 + k^2\bar{u}_3\gamma^\mu u_1\bar{u}_4\not{p}_3 u_2 - 2k'^2\bar{u}_3\gamma^\mu u_1\bar{u}_4\not{p}_1 u_2)\,. \tag{A15}$$

Extraction of leading-logarithmic factors for these four sets of scalings gives

$$T^{\mu}_{(b)} = ieg^2(t-\lambda^2)^{-1}\left(\frac{1}{6}\right)\left[\frac{-g^2}{16\pi^2}C_A \ln^2(q^2/\lambda^2)\right]^2$$
$$\times(\vec{t})_{i_1i_3}\cdot(\vec{t})_{i_2i_4}\left\{\frac{1}{k^2}\bar{u}_3\gamma^\mu \not{k}\gamma^\lambda u_1\bar{u}_4\gamma_\lambda u_2 + \frac{1}{2k'^2}\bar{u}_3\gamma^\mu u_1\bar{u}_4\not{p}_3 u_2 - \frac{1}{k^2}\bar{u}_3\gamma^\mu u_1\bar{u}_4\not{p}_1 u_2\right\}. \tag{A16}$$

The first term in the curly brackets is a contribution to the Born term structure with a weight factor of $\frac{1}{6}$ as shown in Fig. 5(g). The other two terms in the curly brackets are non-Born pieces which again cancel in the gauge-invariant sum.

We shall also illustrate the use of the infinite-momentum technique by calculating the contribution for momentum routing of path (b). For this routing, where there is no trigluon vertex involving the soft gluons, the numerator factors play no role in the integrations, but instead they can be approximated by a leading power, as in Eq. (A15). This power then constitutes the only difference between the full theory and the equivalent ϕ^3 graph (except for "shortcuts" allowed in ϕ^3—these will not concern us here). For such cases infinite-momentum methods are quite straightforward.

Kinematics is as in Eq. (2.1):

$$T^\mu_{(b)} = -\tfrac{1}{4}ieg^6 C_A{}^2(t-\lambda^2)^{-1}(\vec{t})_{i_1i_3}\cdot(\vec{t})_{i_2i_4}(2\pi)^{-8}N^\mu_{(b)}I_{(b)}\,, \tag{A17}$$

where $N^\mu_{(b)}$ is given by Eq. (A15) and $I_{(b)}$ is the contribution of

$$I = \int d^4r_1 d^4r_2 D^{-1}$$

from the momentum routing of path (b). We write D in terms of $p_+, p_-, p_\perp$, and use the volume element $d^4r_i = \frac{1}{2}dr_{i+}dr_{i-}d^2\vec{r}_\perp$. The $\int_{-\infty}^{\infty} dr_{i+}$ is performed by closing the contour and using Cauchy's theorem; these integrals are in general nonzero only for finite values of the r_{i-} integrations. It is convenient to write r_{i-} as scaled variables:

$$r_{i-} = x_i(p_2 - p_4)_-\,.$$

This is significant because in our frame $(p_2-p_4)_-$ is the large minus component which must be carried over to q. We find

$$I_{(b)} = \frac{-1}{4k'^2}(2\pi)^2\int d^2r_2 d^2r_2 \int_0^1 dx_2 \int_{x_2}^1 dx_1 \frac{1}{x_1{}^2x_2{}^2(x_1-x_2)(1-x_1)^2}$$
$$\times\left[-\left(\frac{\Delta_1}{1-x_1}+\frac{\Delta_1}{x_1}\right)^{-1}\left(k^2-\frac{\Delta_1}{1-x_1}-\frac{\Delta_1}{x_1}\right)^{-1}\left(\frac{\Delta_1}{1-x_1}+\frac{\Delta_2}{x_2}+\frac{\Delta_{12}}{x_1-x_2}\right)^{-1}\right.$$
$$\times\left(k^2-\frac{\Delta_1}{1-x_1}-\frac{\Delta_2}{x_2}-\frac{\Delta_{12}}{x_1-x_2}\right)^{-1}\left(k'^2-\frac{\Delta_1}{1-x_1}+\frac{\Delta_1}{x_1}\right)^{-1}\left(k^2+k'^2-\frac{\Delta_1}{1-x_1}-\frac{\Delta_1}{x_1}\right)^{-1}$$
$$\left.\times\left|\left(k'^2-\frac{\Delta_1}{1-x_1}-\frac{\Delta_2}{x_2}-\frac{\Delta_{12}}{x_1-x_2}\right)^{-1}\right/\left(k'^2+\frac{\Delta_1}{1-x_1}+\frac{\Delta_2}{x_2}+\frac{\Delta_{12}}{x_1-x_2}\right)^{-1}\right]\,, \tag{A18}$$

where we have defined

$$\Delta_i = \vec{r}_i{}^2+\lambda^2\,, \quad \Delta_{12} = (\vec{r}_1-\vec{r}_2)^2+\lambda^2\,.$$

(We ignore the distinction between Δ_i and the factor $\vec{r}_i^{\,2}$ which appears from fermion denominators. This will have no effect on our final answer.)

In this integral, there are potential logarithmic contributions from end points in the x_i integrations. For path (b), we pick up contributions only from $x_1 \to 1$, $x_2 \to 0$ (any $x_i \to 1$ corresponds to the associated line carrying the full minus component).

The contribution from this path is correctly given by retaining only the singular x behavior and dropping nonsingular terms, i.e., $1-x_1 = y$ and x_2 compared to 1. We find after some algebra that the x_1 and x_2 integrals of Eq. (A18) are given by

$$\frac{1}{\Delta_2 k^2}\int_0^\epsilon dy\left\{\frac{-1}{\Delta_1 k^2}\left(y-\frac{\Delta_1}{k^2}\right)^{-1}\ln\left[\frac{-y(k^2-\Delta_2)+\Delta_1}{\Delta_1+y\Delta_2}\right] + \frac{s}{2k'^2(yk'^2-\Delta_1)[y(k^2+k'^2)-\Delta_1]}\ln\left[\frac{y(k'^2+\Delta_2)+\Delta_1}{-y(k'^2-\Delta_2)+\Delta_1}\right]\right\} \approx \frac{-1}{\Delta_1\Delta_2 k^4}\,\tfrac{1}{2}\ln^2(\Delta_1/k^2)\,,$$

where we have used more algebra and recognized that the leading behavior in the $(\vec{r}_1, \vec{r}_2)$ integration comes from the region where Δ_i can be dropped compared to s. Thus

$$\begin{aligned} I_{(b)} &= \frac{-(2\pi)^2}{8k'^2k^4}\int d^2\vec{r}_1 d^2\vec{r}_2 \frac{1}{\Delta_1\Delta_2}\ln^2(\Delta_1/k^2) \\ &= -\frac{\pi^4}{6k'^2k^4}\ln^3(q^2/\lambda^2)\int_{\lambda^2}^{\epsilon q^2}\frac{d\Delta_2}{\Delta_2} \\ &= -\frac{\pi^4}{6k'^2k^4}\ln^4(q^2/\lambda^2)\,. \end{aligned}$$

Substituting this result in Eq. (A17) gives Eq. (A16).

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