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Origin of the particles in black-hole evaporation

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Arguments are advanced to indicate that the radiation responsible for black-hole evaporation originates in the vacuum region outside the black hole which is formed by a collapsing body.

There are two prevalent schools of thought on the question of where the "particles" that produce the black-hole evaporation process originate. One view, espoused originally by Hawking,[1] is that the evaporation process results from an instability of the metric itself. The "particles" are produced at a constant rate just outside the horizon of the black hole. Those traveling out to infinity carry energy away with them, while those traveling into the black hole carry a compensating negative energy with them. (These "particles" are therefore not particles in the true sense, but are a metaphor for the actual physical events taking place.)

The alternative view[2-4] suggests that the particles are produced by the collapsing matter just before the formation of a horizon. These are trapped near the horizon for varying lengths of time by the strong gravitational field and gradually leak out to infinity. As a corollary to this viewpoint, the particles must be produced with larger and larger proper energy to compensate for the red-shift suffered by them on their way out to infinity.

This paper will present three arguments to support the former rather than the latter interpretation of black-hole evaporation.

It must be remembered that talk about particles is a very crude and metaphorical way of talking about the physics occurring near the horizon of the black hole. The only justification of any such picture is if it does present a coherent view of the physical processes occurring near the horizon.

The first question one can ask is about the energy-momentum tensor of a quantum field near the horizon of a collapsing body. The second viewpoint would lead to the conclusion that the horizon itself would never form. As formation became imminent, the matter would create a sufficient number of particles and would lose sufficient mass to prevent its formation. Instead of a black hole, the collapse would result in a sort of geon[5] (a gravitationally bound aggregate of principally massless particles) which would leak away. (The baryons, etc. of the collapsing body would presumably lose all their mass to this cloud of photons, neutrinos, etc., while their baryon number would become condensed to a point.)

The first viewpoint, on the other hand, would suggest that the energy would constantly appear just outside the horizon at a constant rate. Energy would flow out to infinity while a corresponding negative amount would flow down the black hole.

The actual calculation of the energy-momentum tensor outside a black hole is extremely difficult. The expectation value of the energy-momentum tensor itself is formally infinite, and some regularization technique must be used to extract a finite part. The result is in general rather ambiguous. However, Davies, Fulling, and Unruh[6] have recently developed a so-called "point-separation" approach to the regularization of the energy-momentum tensor for massless fields in two-dimensional spacetimes. (The technique had been suggested by DeWitt,[7] made covariant in flat two-dimensional spacetimes by Davies and Fulling,[8] and extended to general two-dimensional spacetimes by Davies, Fulling, and Unruh.) The result obtained by them is that the regularized energy-momentum tensor is given by

$$T_{\mu\nu}=\frac{-\theta_{\mu\nu}}{12\pi}+\frac{\tilde{R}}{48\pi}g_{\mu\nu}\ ,$$
$$\theta_{uu}=\Omega^{-1}\Omega_{,u,u},\ \ \theta_{vv}=\Omega^{-1}\Omega_{,v,v},\ \ \theta_{uv}=0\ , \tag{1}$$
$$\tilde{R}=\text{Ricci scalar}\ ,$$

where Ω is the conformal factor appearing in the metric

$$ds^2=\Omega^{-2}du\,dv\ , \tag{2}$$

and where u and v are a specially chosen set of null coordinates in which the positive-frequency normal modes of the field take a simple form.

The above expression was used by them to investigate the regularized energy-momentum tensor in a two-dimensional metric which mimics the four-dimensional collapse metric. The metric is the t,r portion of the four-dimensional metric of a shell of matter which is static with radius R until some time when it collapses to form a black hole.

Before the collapse the metric has the form

$$ds^2 = (1 - 2M/r)dt^2 - dr^2/(1 - 2M/r)$$
$$= (1 - 2M/r)du\, dv \qquad (3)$$

outside the shell and

$$ds^2 = (1 - 2M/R)dt^2 - dr^2$$
$$= (1 - 2M/R)dudv \qquad (4)$$

inside the shell. The regularized energy-momentum tensor has components

$$T_{uu} = T_{vv} = (24\pi)^{-1}\left(\frac{3M^2}{2r^4} - \frac{M}{r^3}\right),$$
$$T_{uv} = (24\pi)^{-1}\left(\frac{2M^2}{r^4} - \frac{M}{r^3}\right),$$
$$T_{tt} = (24\pi)^{-1}\left(\frac{7M^2}{r^4} - \frac{4M}{r^3}\right), \qquad (5)$$
$$T_{rr} = -(24\pi)^{-1}\frac{M^2}{r^4}(1 - 2M/r)^{-2},$$
$$T_{tr} = 0$$

outside the shell, is zero inside the shell, and has a δ-function singularity on the shell.

After the collapse, if terms which die out a long time after the collapse has occurred are neglected, one obtains

$$T_{tt} = (24\pi)^{-1}\left(\frac{7M^2}{r^4} - \frac{4M}{r^3} + \frac{1}{32M^2}\right),$$
$$T_{tr} = -(24\pi)^{-1}(1 - 2M/r)^{-1}\frac{1}{32M}, \qquad (6)$$
$$T_{rr} = -(24\pi)^{-1}(1 - 2M/r)^{-2}\left(\frac{M^2}{r^4} - \frac{1}{32M}\right)$$

outside the shell. The difference between these components and those outside the shell before collapse is of the form

$$(24\pi)^{-1}(32M^2)^{-1}u_\mu u_\nu ,$$

where u^μ is an outward-directed null vector. As this is true up to the surface of the collapsing shell, it has prompted Davies[3] to claim that this therefore supports the view that the particles are created by the collapsing matter.[9] However, this conclusion neglects the effect of the so-called static term [given by Eq. (5)]. These terms are valid only outside the surface of the matter, and are not valid near $r = 2M$. As R approaches $2M$, a steadily increasing force is needed to keep the shell static. In addition, the quantum field itself exerts a pressure on the matter. If one compares the total energy in the quantum field, i.e.,

$$E = \int_0^\infty T_t{}^t\, dr , \qquad (7)$$

for two shells with slightly different radii R and $R + dR$, one finds that E decreases as R decreases. In particular, using Eqs. (1), (3), and (4) one obtains

$$E = \frac{-1}{24\pi}\int_R^\infty \frac{1}{(1 - 2M/r)}\frac{M^2}{r^4}\, dr . \qquad (8)$$

(The integral over $\tilde{R}$ drops out as a complete divergence and the integral over $\theta_t{}^t$ can be put in the above form by an integration by parts.) Therefore one obtains

$$\frac{dE}{dR} = \frac{M^2}{(24\pi)(1 - 2M/R)R^4} . \qquad (9)$$

As the radius of the shell decreases, the energy in the quantum field decreases. This may be interpreted as the field's doing work on the body as it contracts. This implies that the field itself exerts a pressure on the matter in the body, trying to force it to collapse. For radii near the horizon, the so-called static (and it is static only because of large forces on the matter to keep it static) part of the energy-momentum tensor cannot be neglected, or subtracted off. It is a dominant feature of the physics there. If one examines the components of the energy-momentum tensor of Eq. (6) in a coordinate system adapted to the horizon (namely, Kruskal coordinates) in which the external metric has the form

$$ds^2 = \frac{2Me^{-r/2M}}{r}\, dU\, dV , \qquad (10)$$

with r an implicit function of U, V given by

$$UV = -(4M)^2(r/2M - 1)e^{r/2M} , \qquad (11)$$

one finds

$$T_{UU} = \frac{1}{(768\pi M^2)}\frac{V^2 e^{-r/M}}{4r^2}\left(1 + \frac{4M}{r} + \frac{12M^2}{r^2}\right),$$
$$T_{VV} = \frac{1}{6\pi}\frac{M^2}{V^2}\left(\frac{3M^2}{2r^4} - \frac{M}{r^3}\right), \qquad (12)$$
$$T_{UV} = -\frac{M^2}{12\pi r^4}e^{-r/2M} .$$

Note that the energy-momentum tensor is finite near the future horizon $U = 0$. Although T_{VV} appears to blow up on the past horizon ($V = 0$), the solution there is invalid as that is inside the collapsing matter.[10] The energy-momentum tensor is therefore finite near the horizon, whereas either the "static" part [Eq. (5)] or the difference between the collapse tensor and the "static" tensor would have blown up there. This suggests that the horizon does form, contrary to the expectation of the second viewpoint and in harmony with the first. In addition, the energy flow across the future horizon, given by T_{VV}, is seen to be negative as expected from the first viewpoint.

As the above expressions for the energy-momentum tensor are somewhat open to question, other arguments to support the first viewpoint can also be given. One such argument concerns the behavior of bodies falling into the black hole a long time after the collapse. If a massive stream of particles is flowing out just along the horizon of the black hole, one would expect a body dropped into the black hole to interact with those particles. For example, if they were photons, one would expect any body dropped into the black hole which could interact with the photons to be incinerated if the second viewpoint is correct. When they get to infinity the particles have an energy of the order of $1/M$, where M is the mass of the black hole. Near the horizon, at radius r, they must have an energy of the order of $1/M(1-2M/r)$ because of the large red-shifts the particles experience as they travel out to infinity.

However, under the first viewpoint there are no particles flowing out near the horizon. The particles are created constantly well outside the horizon, and a body dropped into the black hole would notice very little as it passed the horizon.[11]

This latter expectation is what actually happens. Let us consider the quantum field of interest to be a scalar field Φ. Expand this field in terms of the normal modes which go as $e^{-i\omega(t+r)}$ near infinity, where such modes are designated by ϕ_ω:

$$\Phi = \sum_{\omega>0} (a_\omega \phi_\omega + a_\omega^\dagger \bar{\phi}_\omega) . \tag{13}$$

The state corresponding to no particles coming in from infinity toward the black hole will be given by the state $|0\rangle$ satisfying

$$a_\omega |0\rangle = 0 . \tag{14}$$

The body falling into the black hole will interact with the field Φ with an interaction proportional to $j\Phi$, where j is some current operator within the body. The probability of transition of the body from some state S_1 to S_2 in a proper time T, as measured by the body itself, will be proportional to an expression of the form

$$\sum_{|p\rangle} \int_0^T |\langle S_2|j|S_1\rangle \langle p|\Phi|0\rangle|^2 d\tau , \tag{15}$$

where the sum is over all possible final states of the field Φ, τ is the time as measured by the body, and the integral is taken along the path of the body. If the state S_2 has an energy E higher than the state S_1, then the term $\langle S_2|j|S_1\rangle$ will be proportional to $e^{+iE\tau}$ and the integral over τ will pick out those components of Φ which go as $e^{-iE\tau}$ along the path of the body; i.e., the body will interact only with those components of Φ which have positive frequency as seen by the body itself. However, the only terms of $\langle p|\Phi|0\rangle$ which survive are those which went as $e^{i\omega t}$ near infinity; i.e., as $a_\omega|0\rangle$ is zero whatever the state $|p\rangle$ is, only the terms of Φ equal to $a_\omega^\dagger \bar{\phi}_\omega$ will survive. We must therefore ask what the frequency of a state $\bar{\phi}_\omega$ is as seen by the body falling into the black hole. A state $\bar{\phi}_\omega$ in traveling toward the black hole may, if its energy is too low, never get to the vicinity of the collapsing body. It will be reflected from the angular momentum barrier back out to infinity (or, in more conventional terms, it will miss the collapsing body because it was aimed too far out). On the other hand, if it does get into the collapsing body, the body will have collapsed somewhat by the time it tries to get out. Our chief concern will be with those modes which exit from the body very near the time when the horizon forms.

Construct a null coordinate system, u, v, such that near infinity the v coordinate is just given by $t+r$. Extend these null rays of constant v inward toward the collapsing body until they reach the center ($r=0$) of the body. Then define the null coordinate u such that $u+v=0$ along the center of the body, and extend these null rays of constant u out through the collapsing body. This construction has been done explicitly for the case of rapid collapse of a shell by Unruh,[12] and near the horizon the null coordinate u becomes essentially equivalent to the Kruskal null coordinate U defined earlier [Eqs. (10), (11)].

Now the lines of constant phase of those modes ϕ_ω which went as $e^{i\omega(t+r)}=e^{i\omega v}$ near infinity will follow the lines of constant v until they are very close to the center of the collapsing body. At this point they will pass the center and, on leaving the collapsing body, the lines of constant phase will follow the lines of constant u. Furthermore, the time Δv for a phase shift of 2π will go as $2\pi/\omega$ (i.e., $\bar{\phi}_\omega$ will go as $e^{i\omega v}$ near $r=0$) and thus the outgoing wave will have the form $e^{i\omega u}$.

The path which the body follows while falling into the black hole will be a geodesic. In Kruskal coordinates U, V the geodesic is some smooth straight line, and near the horizon we will have an equation for the geodesic of the form

$$\begin{aligned} U &= a\tau + O(\tau^2) , \\ V &= V_0 + b\tau + O(\tau^2) , \end{aligned} \tag{16}$$

where $O(\tau^2)$ will be small everywhere near the horizon, and a, b are positive constants. V_0 is the value of the V coordinate when the observer drops through the horizon.

The outgoing waves which go as $e^{i\omega u}$ therefore will have the form $e^{i\omega a\tau}$ along the path of the particle. In other words, the modes $\bar{\phi}_\omega$ will have negative frequency as seen by the falling body. The probability of transition to state S_2 will therefore be essentially zero. A more careful investigation would show that if $E \sim 1/M$, the body would see some particles, but this is to be expected as the metric around the falling observer is changing on time scales of order M. (Furthermore the observer runs into the singularity on time scales of order M.) But these energies are totally insignificant when one is looking for the highly energetic particles near the horizon predicted by the first viewpoint.

A third argument has to do with stimulated emission. Wald[13] has shown that one can produce stimulated emission from a black hole. If one is attempting to do so from near infinity, however, one must send in very highly energetic particles to do so, and must send them in at just the right time so that they exit from the collapsing body just at the point when the horizon is forming. Instead of trying to produce stimulated emission from infinity, however, let us do so by dropping an emitter into the black hole a long time after the collapse has taken place, and have it emit particles of a suitable energy just as it is crossing the horizon. Under the second viewpoint as to the origin of the Hawking particles, one would expect no stimulated emission in this case. The particles are produced by the collapsing matter itself and that has occurred long before the emitter was dropped in. The emitter is located between the source of the particles and the observer at infinity. Unless the emitter could stimulate emission in the past, one would expect no stimulated emission. One could argue that the particles already produced will stimulate the emitter, and thus produce stimulated emission. However, here the argument as to what a falling body sees near the horizon will also apply. An observer falling through the horizon will see no particles pouring out of the collapsing body, will see nothing to stimulate it to emit. The first viewpoint, that the particles originate in the empty spacetime around the black hole, will predict stimulated emission. If the emitter produces a wave packet with unit norm and of the correct frequency distribution, the observer at infinity will expect to see the usual background number of particles due to the quantum evaporation process, and will expect to see more than one extra particle coming out of the black hole (i.e., the one produced by the falling emitter plus the stimulated radiation from the region of spacetime around the black hole). Again it is this expectation which is upheld.

Wald's prescription for producing a wave packet which will produce stimulated emission is as follows: Take a wave packet representing a single particle traveling away from the black hole toward infinity long after the collapse, with energy such that there exist spontaneously emitted particles like it. Use the wave equation to project this wave packet backwards in time. Part will scatter from the curvature of spacetime outside the black hole, and part will get into the collapsing star and finally emerge in the past with a much higher energy. Consider only the second part of the wave packet. It will be a combination of both positive- and negative-frequency parts as seen by an observer in the past near infinity. Take an appropriate linear combination of the positive-frequency part with the complex conjugate of the negative-frequency part near infinity, and let this new wave packet travel down toward the collapsing star. It will produce stimulated emission coming out of the collapse (i.e., the number of extra particles coming out at infinity will be greater than just the one extra particle which was sent in). Now Wald shows that this wave packet must have an energy of the order of $\kappa \exp[\kappa(t-t_0)]$, where κ is proportional to $1/M$, t_0 is the time of collapse, and t is the time at which the particle is seen as coming out of the collapsed object. An ingoing wave packet with so large an energy will go straight into the collapsing body, with no scattering off the curvature of spacetime. Consider an emitter dropped into the black hole a long time after the collapse. By our previous analysis, the above wave packet will be a positive-frequency wave packet as far as the emitter is concerned, and thus it could be emitted by the emitter. Furthermore, the energy of that packet as seen by the emitter is much lower than the energy it had when sent in toward the black hole.

To see this, we must look more closely at the velocity a, b in the Eq. (16). Choose the null coordinate v such that the collapsing body crosses the horizon at $v=0$. This implies $V=4Me^{v/4M}$ is equal to $4M$ when the star crosses the horizon. If the emitter is dropped into the black hole, so that it crosses at advanced time v after the collapse, the time the emitter crosses the horizon in Kruskal coordinates will be given by

$$V_0 = 4Me^{v/4M} .$$

If we assume the emitter drops along a geodesic with zero velocity when at infinity, its velocity in the U direction $[dU/d\tau = a$ as defined in Eq. (16)] can be calculated to be

$$a \approx 4M/(V_0) \approx e^{-v/4M} . \tag{17}$$

The wave which has frequency ω going into the collapsing body is seen to have frequency $\omega a \approx \omega e^{-v/4M}$ by the body falling into the black hole a time v after the collapse. As long as this remains larger than of order $1/M$ the packet will still be localized near the horizon. By choosing v appropriately, Wald's wave packet will be seen by the emitter as having positive frequency and being of a reasonable energy so that it could be emitted by a physically reasonable emitter. Now, as this wave packet could be the result of an emission by the emitter, program the emitter so that it does produce just such a wave packet as it falls through the horizon. This emission of such a wave packet will put the field into a one-particle state with respect to the vacuum at $\mathscr{I}^-$. But by Wald's analysis, such a one-particle in-state corresponds to a state at infinity with more than one extra particle when compared with the usual thermal spectrum of particles coming out of a collapsing star. This one particle emitted by the emitter has produced more than one particle at infinity, i.e., it has caused stimulated emission to occur. Again this supports the view that particle creation takes place in the empty spacetime near the horizon rather than inside the collapsing matter itself.

Conclusion. Three arguments have been presented to support the view that the particle creation, which occurs in the quantum evaporation of black holes, originates in a continuous manner in the empty region of spacetime near the horizon of a black hole rather than within the collapsing matter which formed the black hole in the first place. This suggests that it is the vacuum spacetime itself which is quantum-mechanically unstable.

The three arguments presented use the form of a regularized energy-momentum tensor near the horizon of a collapsing body, the behavior of bodies which drop through the horizon of the black hole, and, finally, the possibility of producing stimulated emission long after the collapse itself has taken place.

The last two arguments also indicate that one need not worry about the exact nature of the star or about interactions between the quantum field and the matter in the star. As long as the star behaves reasonably (i.e., does not emit an infinite flux of particles as seen by freely falling observers near the horizon) the Hawking result will obtain. Although most calculations till now have relied upon the behavior of waves propagating through the collapsing body itself, it is only the feature that freely falling detectors see little or no emission from the body that is needed to give black-hole evaporation.

The first argument on the implications of the regularized energy-momentum tensor to the origin of black-hole evaporation owes much to extensive conversations with S. Fulling[14] while both of us enjoyed the hospitality of L. Parker at the University of Wisconsin.

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[1]S. Hawking, Nature 248, 30 (1974); Commun. Math. Phys. 43, 199 (1975).

[2]D. Boulware, Phys. Rev. D 13, 2169 (1976).

[3]P. C. W. Davies, Proc. R. Soc. London A (to be published).

[4]U. Gerlach, Phys. Rev. D 14, 2479 (1976); see also proceedings of Marcel Grossman Conference, Trieste, 1975 (unpublished).

[5]See articles reprinted in J. A. Wheeler, *Geometrodynamics* (Academic, New York, 1962).

[6]P. C. W. Davies, S. Fulling, and W. Unruh, Phys. Rev. D 13, 2720 (1976).

[7]B. DeWitt, Phys. Rep. 19C, 295 (1975).

[8]P. C. W. Davies and S. Fulling, Proc. R. Soc. London A (to be published).

[9]Davies himself does not talk of particles, but rather of the "radiation" originating in the matter. His stress-energy tensor is equivalent to the one of this paper, but he offers a different interpretation of the significance of the finite part.

[10]Note that if one does regard this as the energy-momentum tensor in a full-vacuum Schwarzschild metric, the divergence of T_{VV} near $V=0$ corresponds to a quantum version of Eardley's "blue-sheet" instability for white holes. However, note that this energy-momentum tensor is negative near $V=0$ rather than positive as in Eardley's case [D. Eardley, Phys. Rev. Lett. 33, 442 (1974)].

[11]W. Unruh, report to Marcel Grossman Conference, Trieste, 1975 (unpublished).

[12]W. Unruh, Phys. Rev. D 14, 870 (1976).

[13]R. M. Wald, Phys. Rev. D 13, 3176 (1976).

[14]See also S. Fulling, Phys. Rev. D (to be published).