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Neutron-star mass limit in the bimetric theory of gravitation

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The "neutron"-star upper mass limit is examined in Rosen's bimetric theory of gravitation. An exact solution, approximate scaling law, and numerical integration of the hydrostatic equilibrium equation show the dependence of the mass limit on the assumed equation of state. As in general relativity, that limit varies roughly as $1/\sqrt{\rho_0}$, where ρ_0 is the density above which the equation of state becomes "stiff." Unlike general relativity, the stiffer the equation of state, the higher the mass limit. For $\rho_0 = 2\times10^{14}$ g/cm^3 and $P=(\rho-\rho_0)c^2$, we found $M_{\max}=81M_\odot$. This mass is consistent with causality and experimental tests of gravitation and nuclear physics. For $dp/d\rho > c^2$ it appears that the upper mass limit can become arbitrarily large.

I. INTRODUCTION

The mass of a neutron star arising from a particular choice of equation of state and gravitational theory is of interest for two reasons. First, since there are now several neutron stars with approximately known masses, one can begin to compare observational data and theoretical models to set constraints on the equation of state $P=P(\rho)$, the theory of gravity, or both. Second, since the best case for the existence of black holes depends crucially on the value of the upper mass limit, it is important to know what this limiting mass $M_{\max}$ is and on what assumptions it depends. Many papers have appeared in the literature on this subject (for a recent review see Brecher and Caporaso[1]). In a previous paper[2] we have examined the effects of varying $P(\rho)$ in general relativity. In this paper we wish to examine the effect on $M_{\max}$ of varying $P(\rho)$ in an alternative theory of gravity and compare these results with the general-relativistic case.

The bimetric theory proposed by Rosen[3] appears to be the only currently viable alternative theory of gravity to general relativity[4] [other than the parametrized post-Newtonian (PPN) theories constructed explicitly to satisfy the classic tests, but which are otherwise without physical foundation]. It conforms to all of the experimental tests. Furthermore, it does not permit the existence of black holes. It does appear to allow binary systems to emit "negative-energy" gravitational waves causing the stars to move apart with time.[5] As a matter of principle, this may seem more disturbing than violations of parity or of time-reversal invariance. However, we take the point of view that only logical inconsistencies or observationally incorrect predictions should be used as arguments against the bimetric theory, neither of which exist at present. In this paper, we will be concerned only with particular consequences of the bimetric theory. At the very least, they will help to clarify the very singular predictions of general relativity. For further details and some unique features of the theory itself, we refer the reader to the original papers of Rosen.[3,6]

The masses of neutron stars in the bimetric theory were previously investigated numerically by Rosen and Rosen[7] for a particular equation of state. They found neutron-star masses about five times larger than those calculated in general relativity with the same equation of state. In a subsequent numerical calculation[8] these results were extended to "stiffer" equations of state. In the present paper we present an exact solution for a specific equation of state, an approximate mass-limit scaling law for any equation of state, and the results of numerical solutions for three different equations of state in the bimetric theory. In addition, we compare these results to some corresponding general-relativistic results.

II. THE GENERAL PROBLEM

If we choose $\gamma_{\mu\nu}=\eta_{\mu\nu}=\mathrm{diag}(1,-1,-1,-1)$, then the line element appropriate to the study of a nonrotating, spherically symmetric neutron star may be taken as[3]

$$ds^2=e^{2\phi}dt^2-e^{2\psi}(dx^2+dy^2+dz^2)\,, \tag{1}$$

where $\psi=\psi(r)$, $\phi=\phi(r)$, $r^2=x^2+y^2+z^2$. Taking $T_\mu{}^\nu$ as the stress-energy tensor of a perfect fluid, namely

$$T_0{}^0=\rho\,,\quad T_1{}^1=T_2{}^2=T_3{}^3=-P\,,\quad \text{all others } 0\,,$$

with density $\rho=\rho(r)$ and pressure $P=P(r)$, Rosen's field equations become[7] (in units where $G=c^2=1$)

$$\nabla^2\phi=\phi''+\frac{2}{r}\phi'=4\pi e^{\phi+3\psi}(\rho+3P)\,, \tag{2}$$

$$\nabla^2\psi=\psi''+\frac{2}{r}\psi'=-4\pi e^{\phi+3\psi}(\rho-P)\,. \tag{3}$$

The energy-momentum relations[6] $T_\mu{}^\nu{}_{;\nu}=0$ yield the hydrostatic equilibrium equation

$$P'+(\rho+P)\phi'=0\,. \tag{4}$$

Outside the star[6,7] (for $r\geq R$, R is the neutron-star radius)

$$\phi=-M/r\,,\quad \psi=M'/r\,, \tag{5}$$

where

$$M=\int_0^R 4\pi r^2(\rho+3P)e^{\phi+3\psi}dr$$

and

$$M'=\int_0^R 4\pi r^2(\rho-P)e^{\phi+3\psi}dr\,.$$

(Note that M' is not the derivative of M but is another mass distinct from M which contributes to the deflection of light near the star.) The boundary conditions for the field equations are

$$\phi'=\psi'=0 \quad \text{at } r=0\,, \tag{6}$$

$$r\phi'+\phi=0\,,\quad r\psi'+\psi=0 \quad \text{at } r=R\,. \tag{7}$$

Now, for $r<R$,

$$M(r)=\int_0^r 4\pi r^2(\rho+3P)e^{\phi+3\psi}dr\,, \tag{8}$$

$$M'(r)=\int_0^r 4\pi r^2(\rho-P)e^{\phi+3\psi}dr\,. \tag{9}$$

With these forms for M and M' we note that (2) and (8) and (3) and (9) yield, respectively,

$$\frac{d}{dr}(r^2\phi')=\frac{dM}{dr} \tag{10}$$

and

$$\frac{d}{dr}(r^2\psi')=\frac{-dM'}{dr}\,. \tag{11}$$

Solving for ϕ' and ψ' in these expressions and integrating gives

$$\phi(r)=\phi(0)+\int_0^r \frac{M(r')dr'}{r'^2}\,, \tag{12}$$

$$\psi(r)=\psi(0)-\int_0^r \frac{M'(r')dr'}{r'^2}\,. \tag{13}$$

Now, defining $\phi_0\equiv\phi(0)$ and $\psi_0\equiv\psi(0)$ and integrating the hydrostatic equilibrium equation (4) from $r=0$ to $r=R$, we have

$$\phi(R)-\phi_0=-\int_{\rho_c}^{\rho_s}\frac{dP}{P+\rho}\,, \tag{14}$$

where ρ_c is the central density (at $r=0$) and ρ_s is the surface density (at $r=R$) and where P is related to ρ by an equation of state

$$P=P(\rho)\,. \tag{15}$$

It should be noted at this point that numerical solution of the field equations for a neutron star would involve correctly choosing the two initial parameters ϕ_0 and ψ_0 until a point is found such that both boundary conditions (7) are satisfied. However, by making appropriate use of information contained in (14), this two-dimensional space of trial initial conditions can be reduced to a line.

To accomplish this we define a new variable $\theta(r)$, following Rosen,[7]

$$\theta(r)\equiv\phi(r)-\phi(R)\,. \tag{16}$$

With this definition the field equations (2) and (3) become

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\theta}{dr}\right)=\frac{1}{r^2}\frac{dM}{dr}=4\pi e^{\theta+3\psi+\phi(R)}(\rho+3P)\,, \tag{17}$$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\psi}{dr}\right)=-\frac{1}{r^2}\frac{dM'}{dr}=4\pi e^{\theta+3\psi+\phi(R)}(\rho-P)\,. \tag{18}$$

If we now make the transformation

$$r=\alpha\bar{r}\,, \tag{19}$$

where α is a dimensionless constant, we can reduce Eqs. (17) and (18) to

$$\frac{d\bar{M}(\bar{r})}{d\bar{r}}=4\pi\bar{r}^2e^{\bar{\theta}(\bar{r})+3\bar{\psi}(\bar{r})}[\bar{\rho}(\bar{r})+3\bar{P}(\bar{r})]\,, \tag{20}$$

$$\frac{d\bar{M}'(\bar{r})}{d\bar{r}}=4\pi\bar{r}^2e^{\bar{\theta}(\bar{r})+3\bar{\psi}(\bar{r})}[\bar{\rho}(\bar{r})-\bar{P}(\bar{r})]\,, \tag{21}$$

where $\bar{\theta}(\bar{r})=\theta(r)$; $\bar{\psi}(\bar{r})=\psi(r)$, $\bar{P}(\bar{r})=P(r)$; $\bar{\rho}(\bar{r})=\rho(r)$, $\alpha\bar{M}(\bar{r})=M(r)$, and $\alpha\bar{M}'(\bar{r})=M'(r)$, and these functions are given by

$$\bar{\theta}(\bar{r})=\theta_0+\int_0^{\bar{r}}\frac{\bar{M}(\bar{r})d\bar{r}}{\bar{r}^2}\,, \tag{22}$$

$$\bar{\psi}(\bar{r})=\psi_0-\int_0^{\bar{r}}\frac{\bar{M}'(\bar{r})d\bar{r}}{\bar{r}^2}\,, \tag{23}$$

$$\bar{M}(\bar{r})=\int_0^{\bar{r}}4\pi\bar{r}^2e^{\bar{\theta}(\bar{r})+3\bar{\psi}(\bar{r})}[\bar{\rho}(\bar{r})+3\bar{P}(\bar{r})]d\bar{r}\,, \tag{24}$$

$$\bar{M}'(\bar{r})=\int_0^{\bar{r}}4\pi\bar{r}^2e^{\bar{\theta}(\bar{r})+3\bar{\psi}(\bar{r})}[\bar{\rho}(\bar{r})-\bar{P}(\bar{r})]d\bar{r}\,, \tag{25}$$

if we choose

$$\alpha=e^{-\phi(R)/2}\,. \tag{26}$$

With (16) and (19) the hydrostatic equilibrium equation (4) can be written as

$$\frac{d\bar{P}(\bar{r})}{d\bar{r}}=-[\bar{\rho}(\bar{r})+\bar{P}(\bar{r})]\frac{d\bar{\theta}(\bar{r})}{d\bar{r}}\,, \tag{27}$$

while (14) and (16) imply that

$$\theta_0=-\int_{\rho_s}^{\rho_c}\frac{dP}{P+\rho}\,. \tag{28}$$

The boundary condition (7) on ψ now becomes

$$\bar{R}\,\frac{d\bar{\psi}(\bar{R})}{d\bar{r}}+\bar{\psi}(\bar{R})=0 \tag{29}$$

so that the boundary conditions are now $\bar{\theta}(\bar{R})=0$ and Eqs. (27) and (28).

To solve a neutron-star structure problem an equation of state along with a central and surface density is chosen. This gives θ_0 through (27). Next a ψ_0 is chosen and the equations for $\bar{\rho}$, $\bar{P}$, $\bar{M}$, $\bar{M}'$, $\bar{\theta}$, and $\bar{\psi}$ are iterated radially outward from $\bar{r}=0$ until $\bar{\theta}(\bar{r})=0$. At this point the boundary condition on $\bar{\psi}$, Eq. (38), is tested. If the boundary condition is not satisfied to within the desired precision ψ_0 is changed and the procedure repeated as often as necessary until the boundary condition is met. At this point $\bar{R}$, $\bar{M}(\bar{R})$, and $\bar{M}'(\bar{R})$ are known. Now (26) becomes

$$\alpha=e^{-\phi(R)/2}=e^{\bar{M}(\bar{R})/2\bar{R}}. \tag{30}$$

Thus we have

$$M(R)=\bar{M}(\bar{R})e^{\bar{M}(\bar{R})/2\bar{R}}, \tag{31}$$

$$M'(R)=\bar{M}'(\bar{R})e^{\bar{M}(\bar{R})/2\bar{R}}, \tag{32}$$

$$R=\bar{R}e^{\bar{M}(\bar{R})/2\bar{R}} \tag{33}$$

completing the solution of the problem.

III. AN EXACT SOLUTION

Consider the field equations for the equation of state

$$P=\tfrac{1}{3}\rho. \tag{34}$$

Adding Eq. (2) to three times (3) yields

$$\nabla^2\phi+3\nabla^2\psi=\nabla^2(\phi+3\psi)=0. \tag{35}$$

The problem is spherically symmetric so only r dependence is possible. Thus

$$\phi+3\psi=C_0+C_1/r. \tag{36}$$

Now from (10) we have that

$$\frac{d}{dr}(r^2\phi')=\frac{dM}{dr}. \tag{37}$$

If we integrate and assume that

$$\lim_{r\to 0} r^2\frac{d\phi}{dr}=0, \tag{38}$$

then

$$r^2\phi'(r)=M(r). \tag{39}$$

Similarly

$$r^2\psi'(r)=-M'(r). \tag{40}$$

Now adding (39) to three times (40) and recalling (8) and (9) we have

$$r^2(\phi+3\psi)'=0. \tag{41}$$

From (36) it is seen that C_1 must vanish, that is,

$$\phi(r)+3\psi(r)=C_0. \tag{42}$$

The hydrostatic equilibrium equation (4) and the equation of state (34) yields

$$\frac{d\rho}{dr}=-\frac{4\rho M(r)}{r^2}. \tag{43}$$

Noting the similarity of this equation to the general-relativistic and Newtonian cases for $P=\beta\rho$ and recalling[2] that $\rho\propto 1/r^2$ is a solution in both instances, we seek a solution of the form

$$\rho(r)=k/r^2. \tag{44}$$

Substituting this into (43) we find that

$$\rho(r)=\frac{e^{-C_0}}{16\pi r^2} \tag{45}$$

and

$$M(r)=\tfrac{1}{2}r. \tag{46}$$

Now from (10)

$$\frac{d\phi}{dr}=\frac{M(r)}{r^2}=\frac{1}{2r}. \tag{47}$$

Similarly,

$$\frac{d\psi}{dr}=-\frac{1}{6r} \tag{48}$$

so that

$$\phi(r)=A+\tfrac{1}{2}\ln r, \tag{49}$$

$$\psi(r)=B-\tfrac{1}{6}\ln r. \tag{50}$$

Now at some $r=R$, ϕ and ψ will both satisfy their respective boundary conditions (7) if a ρ_s greater than zero is chosen. Then (7) becomes

$$A+\tfrac{1}{2}(\ln R+1)=0, \tag{51}$$

$$B-\tfrac{1}{6}(\ln R+1)=0. \tag{52}$$

Adding (51) and three times (52) we obtain $A+3B=0$. But, by (42), $A+3B=C_0$, thus

$$C_0=0 \tag{53}$$

and

$$\rho(r)=\frac{1}{16\pi r^2}. \tag{54}$$

Note from (49) that

$$\phi'(r)=\frac{1}{2r}. \tag{55}$$

This violates the boundary condition (6), but condition (38) which was assumed in the derivation is satisfied. It is not too surprising that condition (6) is violated in this special case as ρ_c is infinite.

Now, selecting a density ρ_N below which nuclear physics is assumed known [i.e., where (34) no longer applies] we may find the "core" mass. Inserting the appropriate physical constants, (45) and (46) become

$$\rho(r)=\frac{c^2}{16\pi G r^2}\,, \tag{56}$$

$$M(r)=\frac{c^2 r}{2G}\,. \tag{57}$$

Now if r_N is the core radius

$$r_N=\left(\frac{c^2}{16\pi G\rho_N}\right)^{1/2} \tag{58}$$

and the core mass $M(\rho_N)$ is

$$M(\rho_N)=\frac{1}{8(\pi\rho_0)^{1/2}}\left(\frac{c^2}{G}\right)^{3/2}\left(\frac{\rho_0}{\rho_N}\right)^{1/2}$$

$$=3.91\left(\frac{\rho_0}{\rho_N}\right)^{1/2}M_\odot\,, \tag{59}$$

where $\rho_0=2\times10^{14}$ g/cm^3. For $P=\alpha\rho$ in general relativity, the core mass (for $\rho_c=\infty$) is[2]

$$M(\rho_n)=\left(\frac{2}{\pi\rho_0}\right)^{1/2}\left(\frac{c^2}{G}\right)^{3/2}\left(\frac{\alpha}{\alpha^2+6\alpha+1}\right)^{3/2}\left(\frac{\rho_0}{\rho_N}\right)^{1/2}. \tag{60}$$

For $P=\frac{1}{3}\rho$ and $P=\rho$, respectively, this is

$$M(\rho_N)=1.55\left(\frac{\rho_0}{\rho_N}\right)^{1/2}M_\odot\,, \tag{61}$$

$$M(\rho_N)=1.96\left(\frac{\rho_0}{\rho_N}\right)^{1/2}M_\odot\,. \tag{62}$$

Equation (60) has a maximum when $\alpha=1$. Thus we immediately see that the core mass for $P=\frac{1}{3}\rho$ in the bimetric theory is twice as large as the largest core mass for $P=\alpha\rho$ in general relativity. This reinforces the notion[7] that neutron stars have larger masses in the bimetric theory than in general relativity. As we shall show, we have found that this is the case for many numerical solutions.

IV. APPROXIMATE SCALING LAW

Qualitatively, we may obtain some idea of how the maximum mass of a neutron star depends on the equation of state in the bimetric theory. By Eqs. (10), (12), and (16) we see that

$$\theta(R)=\theta_0+\int_0^{\bar R}\frac{\bar M(\bar r)d\bar r}{\bar r^2}=0\,.$$

We expect θ, $\bar M$, and $\bar R$ to be related approximately by

$$\theta_0=-\int_0^{\bar R}\frac{\bar M(\bar r)d\bar r}{\bar r^2}\cong-\frac{\bar M(\bar R)}{\bar R}\,. \tag{63}$$

Thus (31) becomes

$$M(R)\cong\bar M(\bar R)e^{-\theta_0/2}\,. \tag{64}$$

If we now define an average density $\hat\rho$ by

$$\bar M(\bar R)\equiv\frac{4\pi\hat\rho\bar R^3}{3}\,, \tag{65}$$

then

$$\frac{\bar M(\bar R)}{\bar R}=\frac{4\pi\bar R^2\hat\rho}{3}\cong-\theta_0 \tag{66}$$

and

$$\bar R\cong\left(-\frac{3\theta_0}{4\pi\hat\rho}\right)^{1/2}, \tag{67}$$

thus

$$\bar M(\bar R)\cong\left(\frac{3}{4\pi\hat\rho}\right)^{1/2}(-\theta_0)^{3/2}\,.$$

Using (64) and inserting appropriate physical constants the mass becomes

$$M(R)\cong\left(\frac{3}{4\pi\hat\rho}\right)^{1/2}\left(\frac{-c^2\theta_0}{G}\right)^{3/2}e^{-\theta_0/2} \tag{68}$$

and the radius is

$$R\cong\left(\frac{3}{4\pi\hat\rho}\right)^{1/2}\left(\frac{-c^2\theta_0}{G}\right)^{1/2}e^{-\theta_0/2}\,. \tag{69}$$

As an illustration consider the equation of state

$$P=\alpha\rho c^2\,. \tag{70}$$

Then θ_0 becomes

$$\theta_0=-\frac{\alpha}{1+\alpha}\ln\left(\frac{\rho_c}{\rho_s}\right). \tag{71}$$

If we put $\hat\rho=\rho_c$ then M and R can be written as

$$M\cong\left(\frac{3}{4\pi\rho_s}\right)^{1/2}\left[\frac{c^2\alpha}{G(1+\alpha)}\right]^{3/2}f(x)\,, \tag{72}$$

$$R\cong\left(\frac{3}{4\pi\rho_s}\right)^{1/2}\left[\frac{c^2\alpha}{G(1+\alpha)}\right]^{1/2}g(x)\,, \tag{73}$$

where

$$x=\ln\left(\frac{\rho_c}{\rho_s}\right) \tag{74}$$

and

$$f(x)=x^{3/2}e^{-x/2(1+\alpha)}\,, \tag{75}$$

$$g(x)=x^{1/2}e^{-x/2(1+\alpha)}\,. \tag{76}$$

Maximizing these functions yields

$$x_f=3(1+\alpha)=\ln\left(\frac{\rho_c}{\rho_s}\right), \tag{77}$$

$$x_g=1+\alpha=\ln\left(\frac{\rho_c}{\rho_s}\right). \tag{78}$$

So the maxima for M and R are

$$M_{\max} \cong 31.4\left(\frac{\rho_0}{\rho_s}\right)^{1/2} \alpha^{3/2} M_\odot \,, \tag{79}$$

$$R_{\max} \cong 24.3\left(\frac{\rho_0}{\rho_s}\right)^{1/2} \alpha^{1/2} \text{ km} \,, \tag{80}$$

where $\rho_0 = 2\times 10^{14}$ g/cm³. With $\rho_s = 2\times 10^{14}$ g/cm³ we find for $\alpha = \frac{1}{3}$ that $M \cong 6M_\odot$ and $R \cong 14$ km. For $\alpha = 1$, one has $M \cong 31M_\odot$, $R \cong 24$ km.

Note that the disagreement between M here for $\alpha = \frac{1}{3}$ and the exact solution found earlier is due to the fact that the exact solution corresponds to the case of infinite central density, while here the maximum mass occurs when $\rho_c = \rho_s \exp[3(1+\alpha)]$.

Note also, however, that unlike the general-relativistic case,[2] where $M(\alpha)$ peaks for $\alpha = 1$, in the bimetric theory $M \sim \alpha^{3/2}$ so that there does not appear to be an upper mass limit (independent of the equation of state) to a neutron star in the bimetric theory. This $\alpha^{3/2}$ dependence also arises for Newtonian stars with $P = \alpha\rho$ and infinite central densities. Rosen and Rosen[7] found that the masses calculated for an equation of state with one parameter in the bimetric theory were about 5.6 times the corresponding maxima in general relativity. Their equation of state was "soft" at low density but $dP/d\rho$ approached 1 as ρ increased to infinity. That is, the effective α for their equation of state was less than 1. Thus they obtained lower masses than the present results.

In Rosen's theory there are two distinct masses M and M'. As has been noted,[7] M corresponds to the Newtonian mass and influences orbits of bodies at large distances from the star, while M' contributes to the deflection of light rays passing close to the star. In fact, to first order in GM/r_0c^2 and GM'/r_0c^2 the angular deflection of a ray incident from infinite distance with "impact" parameter r_0 is, following Weinberg,[9]

$$\Delta\varphi = \frac{2G}{r_0 c^2}(M + M') \,. \tag{81}$$

Now recalling Eq. (9),

$$M' = \int_0^R 4\pi r^2 e^{\phi+3\psi}(\rho - P)\,dr \,, \tag{9}$$

for the case $P = \alpha\rho$ we see that

$$M' = (1-\alpha)\int_0^R 4\pi r^2 e^{\phi+3\psi}\rho\,dr \,. \tag{82}$$

As $\alpha \to 1$ (P approaches ρ), M' shrinks relative to M. When $\alpha = 1$ (the so-called "causality limit"), M' vanishes and for "superluminal" equations of state,[10] where $P > \rho$ ($\alpha > 1$), M' can become negative. In any case, $M > M'$ so that (81) will always be smaller than its general-relativisitic counterpart

$$\Delta\varphi = \frac{4GM}{r_0 c^2} \tag{83}$$

and, in the extreme case $P \gg \rho$, $M \to -3M'$ and

$$\Delta\varphi \cong \frac{1}{3}\left(\frac{4GM}{r_0 c^2}\right) \tag{84}$$

or $\frac{1}{3}$ the general-relativistic value. In this case, however, GM/r_0c^2 is likely to be of order 1, and thus more terms in the expansion for $\Delta\varphi$ must be examined.

V. NUMERICAL TECHNIQUES AND RESULTS

We now turn to the numerical solution of the neutron-star structure problem for any equation of state. We first find θ_0 from Eq. (28):

$$\theta_0 = \bar\theta_0 = -\int_{\rho_s}^{\rho_c} \frac{dP}{P + \rho c^2} \,. \tag{85}$$

Next we choose a radial increment $\Delta\bar r$ and iterate the equations for $\bar\theta$, $\bar\psi$, $\bar M$, $\bar M'$, and $\bar\rho$. A $\Delta\bar r$ of 10 meters was used for the calculations. Equations (22) and (23) in iterated form are

$$\bar\theta = \bar\theta + \frac{G}{c^2}\frac{\bar M \Delta\bar r}{\bar r^2} \,, \tag{86}$$

$$\bar\psi = \bar\psi - \frac{G}{c^2}\frac{\bar M' \Delta\bar r}{\bar r^2} \,. \tag{87}$$

Similarly, Eqs. (24) and (25) for $\bar M$ and $\bar M'$ become

$$\bar M = \bar M + 4\pi\bar r^2 \Delta\bar r\, e^{\bar\theta+3\bar\psi}(\bar\rho + 3\bar P/c^2) \,, \tag{88}$$

$$\bar M' = \bar M' + 4\pi\bar r^2 \Delta\bar r\, e^{\bar\theta+3\bar\psi}(\bar\rho - \bar P/c^2) \,. \tag{89}$$

The hydrostatic equilibrium equation (27) becomes

$$\bar\rho = \bar\rho - \frac{\Delta\bar r(\bar\rho + \bar P/c^2)\bar M G}{\bar r^2 (d\bar P/d\bar\rho)} \,, \tag{90}$$

where we have used the fact that $dP/dr = (dP/d\rho) \times (d\rho/dr)$. The last relation needed is the equation of state

$$\bar P = \bar P(\bar\rho) \,. \tag{91}$$

First the equation of state was selected, then ρ_s and ρ_c were chosen, and θ_0 was found from (85). A ψ_0 was chosen and the equations were iterated from $\bar r = 0$ outward until $\bar\theta(\bar r)$ vanished. At that point, boundary condition (29) was tested

$$\bar R\bar\psi' + \bar\psi = 0 \,. \tag{29}$$

The prime denotes differentiation with respect to $\bar r$. By (11) and (23) this is

$$\bar\psi(\bar R) - \frac{G\bar M'(\bar R)}{\bar R c^2} = 0 \,. \tag{92}$$

If this condition were not satisfied to the desired precision ψ_0 would be indexed to a new value and the equations would be iterated again until

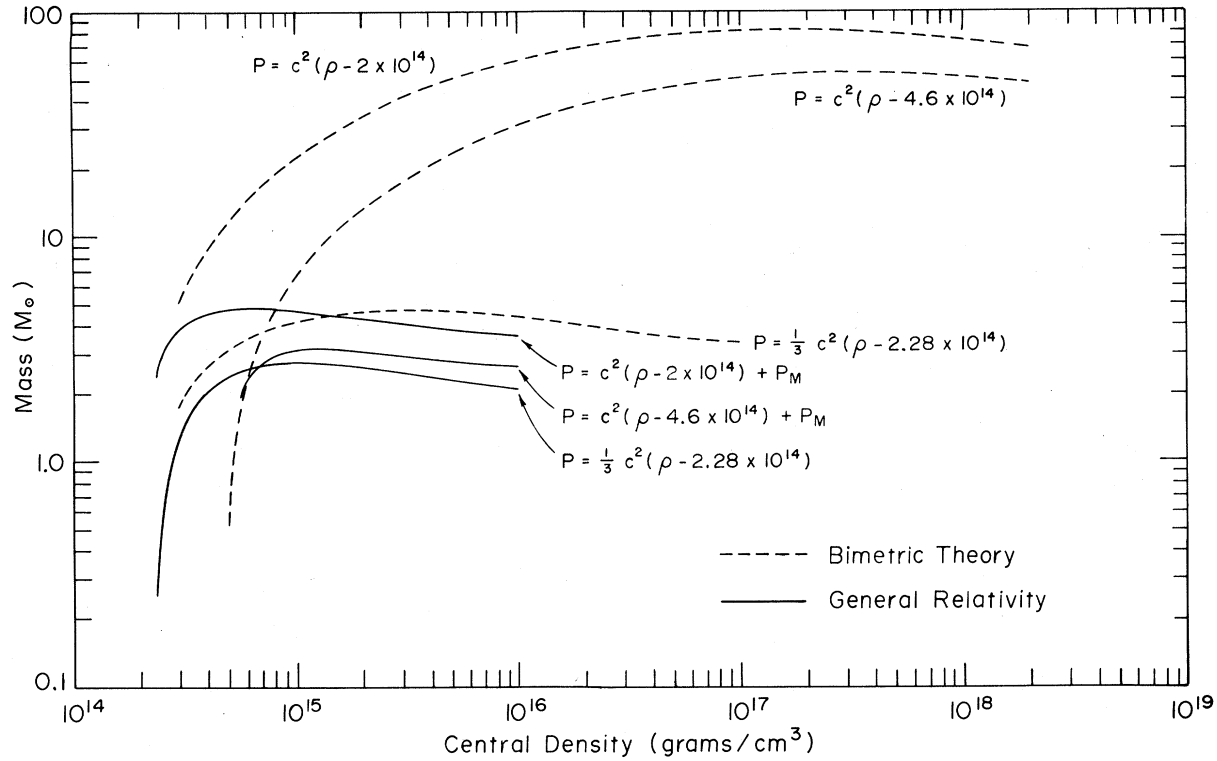

FIG. 1. Results of numerical solutions to neutron-star structure for the three equations of state mentioned in the text in Rosen's bimetric theory (dashed) and in general relativity (solid). Note that the masses in the bimetric theory are much larger than in general relativity and that M vs ρ_c peaks at much higher central densities than in general relativity.

(92) was satisfied. The masses M, M' and the radius R would then be found from Eqs. (31), (32), and (33).

Curves of M vs ρ_c were found for 3 equations of state which have been studied previously in general relativity.[2] The equations[11] used were

$$P=c^2(\rho-2\times10^{14}),$$

$$P=c^2(\rho-4.6\times10^{14}),$$

$$P=\tfrac{1}{3}c^2(\rho-2.28\times10^{14}).$$

The resulting curves are shown in Fig. 1. For comparison the corresponding general-relativistic curves are also shown.

Several points were also computed using 1-meter iterations in $\bar{r}$ to test the sensitivity of the results to step size. The points generally agreed to within 1%. The equation of state used by Rosen[7] with $\lambda=40$ was also used. As a check of the technique the point corresponding to $\rho_c=1.8\times10^5\rho_s$ (the M vs ρ_c maximum) was computed. The resulting M, M', and R all agreed with Rosen's results to within a few percent.

The maximum mass obtained was 81 $M_\odot$. When computations for $P=2\rho c^2$ were attempted, the resulting masses were orders of magnitude greater than that obtained for $P=\rho c^2$ and the quantitiy $\exp(\frac{1}{2}G\bar{M}/\bar{R}c^2)$ became on the order of 10^2–10^3. The step size was probably too large at that point to give accurate results.

VI. DISCUSSION

The importance of these results lies in the fact that they clearly demonstrate that the upper mass limit of a neutron star is critically dependent on the theory of gravity used to calculate it.

It is conventionally assumed that if a collapsed object with a mass greater than a few $M_\odot$ is detected it must be a black hole. Since this is the only way currently to find a black hole, it is now clear that one must simultaneously know which theory of gravity is correct.

Current gravitational theories have only been tested in the weak-field domain. However, all viable theories of gravity are constructed so that they will agree in this domain. But the upper mass limit depends critically on the behavior of gravity in the strong-field limit where there have been no tests of gravitational theories. The mass determined by doing orbital mechanics on a neutron star in a binary system is M. Therefore in the case of CYG X-1, whose mass is estimated[12] at between 3 and 10 $M_{\odot}$, we see that one cannot draw the immediate conclusion that it must be a black hole.

If one could study the deflection of light, say by a neutron star in a binary system, one should observe a smaller effect than in general relativity since $M'<M$; indeed M' may even become negative for sufficiently massive neutron stars. Will[4] has noted that the binary pulsar PSR1913+16 could emit gravitational dipole radiation at a sufficient rate to produce a detectable change in the orbital period. In the absence of either of these observational tests of the theory the mass limits derived here must be taken as allowable within the present experimentally tested laws of physics. On the other hand, if a collapsed object is ever discovered whose mass is greater than the mass limit set by general relativity, and the object is found to be a neutron star (because it is a pulsar, for example), one may reverse the above arguments to support the bimetric theory as more viable than general relativity.

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[1]K. Brecher and G. Caporaso, in Proceedings of the Eighth Texas Symposium on Relativistic Astrophysics, 1977 (unpublished).

[2]K. Brecher and G. Caporaso, Nature (London) 259, 377 (1976).

[3]N. Rosen, Gen. Relativ. Gravit. 4, 935 (1973).

[4]C. Will, 1977 (unpublished).

[5]C. Will and D. Eardley, Astrophys. J. Lett. 212, L1 (1977).

[6]N. Rosen, Ann. Phys. (N.Y.) 94, 455 (1974).

[7]J. Rosen and N. Rosen, Astrophys. J. 202, 782 (1975).

[8]J. Rosen and N. Rosen, Astrophys. J. 212, 605 (1977).

[9]S. Weinberg, *Gravitation and Cosmology* (Wiley, New York, 1972), p. 188. We use Weinberg's equations (8.5.5), (8.3.8), and (8.3.9) and note that (8.4.1) is equal to (1) to first order in GM/rc^2 and GM'/rc^2. Then we identify $B(r)$ and $A(r)$ with $e^{-2M/r}$ and $e^{2M'/r}$, respectively. We then replace γM by M' in (8.5.7) and put $r=r_0$.

[10]S. A. Bludman and M. A. Ruderman, Phys. Rev. D 1, 3243 (1970). Although it is clear that no signal velocity can be greater than c it has still not been satisfactorily determined whether or not P can exceed ρc^2.

[11]For matching to a realistic equation of state at nuclear densities, neglecting the matching pressure P_m makes a negligible change in the resulting mass as we have previously found in general relativity.

[12]Y. Avni, 1977 (unpublished).