# Space-time structure in a generalization of gravitation theory

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A generalized theory of gravitation, formulated recently in terms of a nonsymmetric field structure, is derived from a variational principle, and its relation to the Einstein-Maxwell theory is studied in detail. The physical interpretation of a new universal constant $k$ that occurs in the theory is considered, and reasons are given to choose the constant to be $|k| = \hbar G/c^3 e = L^2/e$, where $L = (\hbar G/c^3)^{1/2}$ is the Planck length. The exact static spherically symmetric solution of the theory is shown to be world-line complete. The timelike and null (radial and nonradial) physical paths of test particles are deflected away from a sphere $S$ with a radius $r_s \sim L$. A discussion is given of the implications of the nonsingular solution of the theory for large- and small-scale physical phenomena.

## I. INTRODUCTION

It has long been thought that all of physics can be deduced from a purely geometrical theory. With the advent of Einstein's theory of general relativity,[1] in which the gravitational field is inferred from the curvature of space-time, the idea of describing physics as geometry received fresh impetus. For various reasons, this program has not met with any great success due mainly, perhaps, to a lack of a deeper understanding of the role of quantum theory in general relativity. The proliferation of new ideas and experimental data associated with elementary-particle physics has diverted attention away from efforts to geometrize all of physics.

In spite of the well-founded reasons to suppose that physics is not geometrical in origin, there is the persistent feeling that a physics which does not incorporate the general theory of relativity is ultimately prevented from reaching a logical, unified description of nature. Einstein devoted many years attempting to extend his gravitational theory to include the electromagnetic field in a consistent, natural field structure. This structure would constitute a generalization of the gravitational theory based on Riemannian geometry. The starting point of his theory of gravitation was the recognition of the unity of gravitation and inertia in the principle of equivalence. From this principle and the specific properties exhibited by the behavior of light in empty space, he deduced that the theory should be described by a symmetrical metric tensor; the mathematical description of the gravitational field was almost completely determined.[2]

Any attempt to extend the theory of general relativity to include, within the geometry, Maxwell's electromagnetic field, meets with difficulty because of the many possible approaches to the problem. The attempt based by Einstein on a (complex) nonsymmetric $g_{\mu\nu}$ is the mathematically most elegant of all the extensions proposed.[3-6] The tensor $g_{\mu\nu}$ is decomposed into its symmetric and skew-symmetric parts according to the equation

$$g_{\mu\nu}=g_{(\mu\nu)}+g_{[\mu\nu]}\,, \tag{1.1}$$

where

$$\begin{aligned} g_{(\mu\nu)} &\equiv \tfrac{1}{2}(g_{\mu\nu}+g_{\nu\mu})=s_{\mu\nu}\,,\\ g_{[\mu\nu]} &\equiv \tfrac{1}{2}(g_{\mu\nu}-g_{\nu\mu})=ia_{\mu\nu}\,, \end{aligned} \tag{1.2}$$

where the $s_{\mu\nu}$ and $a_{\mu\nu}$ are real quantities. The $g_{\mu\nu}$ satisfy the condition of Hermitian symmetry $g_{\mu\nu} = \bar{g}_{\nu\mu}$, which is the generalization of the condition of symmetry of the metric tensor of gravitation.

As emphasized by Einstein,[7] the principle of equivalence gives no clue as to the unequivocal choice of mathematical equations necessary to formulate the theory. This would appear to be a strong objection to any program to unify gravitation and electromagnetism. However, it is necessary, as in all extensions of physical theories, to formulate a close connection between the new theory and the old laws of physics that have been proved by experiment to be correct. Any generalization of gravitation theory should contain Maxwell's equations and Einstein's equations of gravitation in an unambiguous way through a principle of correspondence. Such a principle reduces considerably the possible choices of field equations based on a nonsymmetrical tensor $g_{\mu\nu}$.

A principle of this kind was recently proposed.[8,9] It represents a powerful physical argument which suggests that, in spite of the objection raised above, the nonsymmetrical theory is the natural and correct generalization of Einstein's theory of gravitation. The theory reduces to the Einstein-Maxwell field equations in the limit that a fundamental constant $k$ tends (formally) to zero.

Another objection riased against the nonsymmetrical theory[10,11] is that it is in disagreement with the principle that only *irreducible* quantities should be used in field theories. This principle seems to have been upheld in past field theories. However, if the theory is invariant under a *wider* group of transformations, then the property of the reducibility of the $\Gamma^\lambda_{\mu\nu}$ and $R_{\mu\nu}$ is, in part, removed. If the theory is developed in terms of a complex tetrad formalism,[12] then the scalar density $\mathcal{H}$ (Hamiltonian or Lagrangian density) can be shown to be invariant under the local gauge group of transformations of U(3,1) which contains $U(1)\otimes O(3,1)$.[13] It has also been shown that the theory is readily extended to a standard Yang-Mills scheme.[14] This feature of the theory could be of fundamental significance for our understanding of the interrelationship of gravitation and electromagnetism to other forces of nature and to quantum theory.

In the following, we shall investigate some of the implications of the new theory for our understanding of space-time structure, particularly at small distances. In practice, it is customary nowadays to study the field equations as a system of partial-differential equations with *global* properties.[15] We shall investigate the nature of the singularities that occur in the exact solutions of the differential equations. It is found that the exact spherically symmetric static solution of the field equations leads to the property of world-line completeness,[16] i.e., no physical timelike or null world lines can terminate at the point $r=0$, owing to an analytic boundary surface that occurs at small distances. *The rigorous solution is nonsingular.* The singular event horizons in the solution are caused by the choice of coordinates, and can be transformed away by analytically continuing to a maximally extended solution.

This interesting result could have far-reaching implications for our understanding of space-time structure for both large- and small-scale physical phenomena.

## II. THE FIELD STRUCTURE, THE VARIATIONAL PRINCIPLE, AND THE FIELD EQUATIONS

The concept of the infinitesimal parallel displacement of a vector $A^\mu$ can be extended to the nonsymmetric field by

$$\delta A^\mu = -\Gamma^\mu_{\alpha\beta} A^\alpha dx^\beta , \tag{2.1}$$

$$\delta A_\mu = \Gamma^\alpha_{\mu\beta} A_\alpha dx^\beta , \tag{2.2}$$

where $\Gamma^\lambda_{\mu\nu}$ is the affine connection decomposed according to

$$\Gamma^\lambda_{\mu\nu} = \Gamma^\lambda_{(\mu\nu)} + \Gamma^\lambda_{[\mu\nu]} . \tag{2.3}$$

The $\Gamma^\lambda_{\mu\nu}$ satisfy the inhomogeneous coordinate transformation law of an affine connection given by

$$\Gamma'^\lambda_{\mu\nu} = \frac{\partial x'_\lambda}{\partial x_\alpha}\frac{\partial x_\beta}{\partial x'_\mu}\frac{\partial x_\gamma}{\partial x'_\nu}\Gamma^\alpha_{\beta\gamma} + \frac{\partial x'_\lambda}{\partial x_\delta}\frac{\partial^2 x_\delta}{\partial x'_\mu \partial x'_\nu} . \tag{2.4}$$

It follows that the antisymmetric part of $\Gamma^\lambda_{\mu\nu}$ is a purely imaginary third-rank tensor.

Taking into account (2.1) and (2.2), it follows that

$$A^\mu{}_{|\nu} = \partial_\nu A^\mu + A^\alpha \Gamma^\mu_{\alpha\nu} \tag{2.5}$$

and

$$A_{\mu|\nu} = \partial_\nu A_\mu - A_\alpha \Gamma^\alpha_{\mu\nu} \tag{2.6}$$

have tensor character, just as in the case of the Christoffel symbols of general relativity.

The contravariant tensor $g^{\mu\nu}$ can be related to the covariant tensor $g_{\mu\nu}$ by the equation

$$g^{\mu\nu} g_{\sigma\nu} = g^{\nu\mu} g_{\nu\sigma} = \delta^\mu_\sigma , \tag{2.7}$$

where the order of the suffixes is important.

As in the symmetrical theory, a curvature tensor may be derived by parallel displacement of a vector along a boundary of an infinitesimal surface element:

$$R^\sigma_{\mu\nu\rho} = (\partial_\rho \Gamma^\sigma_{\mu\nu} - \Gamma^\sigma_{\alpha\nu}\Gamma^\alpha_{\mu\rho}) - (\partial_\nu \Gamma^\sigma_{\mu\rho} - \Gamma^\sigma_{\alpha\rho}\Gamma^\alpha_{\mu\nu}) . \tag{2.8}$$

The contracted curvature tensor is

$$R_{\mu\nu} = (\partial_\beta \Gamma^\beta_{\mu\nu} - \Gamma^\beta_{\alpha\nu}\Gamma^\alpha_{\mu\beta}) - (\partial_\nu \Gamma^\beta_{\mu\beta} - \Gamma^\beta_{\alpha\beta}\Gamma^\alpha_{\mu\nu}) . \tag{2.9}$$

The field equations will be derived using Schrödinger's[17] definition of the affine connection and the Palatini method.[18] The connection $\Gamma^\lambda_{\mu\nu}$ may be expressed in terms of another connection $W^\lambda_{\mu\nu}$ by the equation

$$\Gamma^\lambda_{\mu\nu} = W^\lambda_{\mu\nu} + \tfrac{2}{3}\delta^\lambda_\mu W_\nu , \tag{2.10}$$

where

$$W_\nu = \tfrac{1}{2}(W^\sigma_{\nu\sigma} - W^\sigma_{\sigma\nu}) . \tag{2.11}$$

It can be verified immediately that

$$\Gamma_\mu \equiv \Gamma^\sigma_{[\mu\sigma]} = 0 . \tag{2.12}$$

A "covariant derivative" will be defined in terms of the $W^\lambda_{\mu\nu}$ connection as

$$\mathfrak{g}^{\mu\nu}{}_{;\alpha} = \partial_\alpha \mathfrak{g}^{\mu\nu} + \mathfrak{g}^{\sigma\nu} W^\mu_{\sigma\alpha} + \mathfrak{g}^{\mu\sigma} W^\nu_{\sigma\alpha} - \mathfrak{g}^{\mu\nu} W^\sigma_{\sigma\alpha} , \tag{2.13}$$

in which $\mathfrak{g}^{\mu\nu} = \sqrt{-g}\, g^{\mu\nu}$ is the fundamental tensor density.

We shall choose as the integrand for our variational principle a scalar density $\mathcal{H}$, built out of the $\mathfrak{g}^{\mu\nu}$, $W^\lambda_{\mu\nu}$ (or out of $g^{\mu\nu}$, $\Gamma^\lambda_{\mu\nu}$, and $W_\lambda$) and their first and second derivatives. The variational principle requires that

$$\delta \int \mathcal{H}\, d^4x = 0 , \tag{2.14}$$

where the $\mathfrak{g}^{\mu\nu}$ and $\Gamma^\lambda_{\mu\nu}$ are to be varied independently of one another.

The scalar density $\mathcal{H}$ is chosen to be[8]

$$\mathcal{H}=\mathfrak{g}^{\mu\nu}R_{\mu\nu}(W)+\frac{4\pi G}{k^2c^4}\,\mathfrak{g}^{[\mu\nu]}g_{[\nu\mu]}\,, \tag{2.15}$$

where $R_{\mu\nu}(W)$ is the contracted curvature tensor of the connection $W^{\lambda}_{\mu\nu}$. Moreover, $k$ is a (purely imaginary) universal constant to be specified later. Equation (2.14) can now be written as

$$\int[\mathfrak{g}^{\mu\nu}\delta R_{\mu\nu}(W)+{}^*R_{\mu\nu}(W)\delta\mathfrak{g}^{\mu\nu}]d^4x=0\,. \tag{2.16}$$

The second term in (2.16) immediately gives the field equations

$$^*R_{\mu\nu}(W)=0\,. \tag{2.17}$$

It can be shown that[17]

$$\delta R_{\mu\nu}(W)=(\delta W^{\alpha}_{\mu\nu})_{;\alpha}-(\delta W^{\alpha}_{\mu\alpha})_{;\nu}+2W^{\alpha}_{[\nu\beta]}\delta W^{\beta}_{\mu\alpha}\,. \tag{2.18}$$

By partial integration, it follows that

$$\int\mathfrak{g}^{\mu\nu}\delta R_{\mu\nu}(W)d^4x=\int(\mathfrak{B}^{\mu\nu}_{\alpha}-\delta^{\nu}_{\alpha}\mathfrak{B}^{\mu\beta}_{\beta})\delta W^{\alpha}_{\mu\nu}d^4x\,, \tag{2.19}$$

where

$$\mathfrak{B}^{\mu\nu}_{\alpha}=-\mathfrak{g}^{\mu\nu}{}_{;\alpha}+2\mathfrak{g}^{\mu\nu}W_{\alpha}-\tfrac{2}{3}\delta^{\nu}_{\alpha}\mathfrak{g}^{\mu\beta}W_{\beta}-2\mathfrak{g}^{\mu\beta}W^{\nu}_{[\alpha\beta]}\,. \tag{2.20}$$

Equations (2.16) and (2.19) yield

$$\mathfrak{B}^{\mu\nu}_{\alpha}=0\,. \tag{2.21}$$

The variation of the second term in (2.15) gives

$$\begin{aligned}\delta(\mathfrak{g}^{[\mu\nu]}g_{[\nu\mu]})&=\delta(\mathfrak{g}^{\mu\nu}g_{[\nu\mu]})\\&=g_{[\nu\mu]}\delta\mathfrak{g}^{\mu\nu}+\mathfrak{g}^{[\mu\nu]}\delta g_{\nu\mu}\\&=(g_{[\nu\mu]}-g_{\mu\alpha}g^{[\alpha\sigma]}g_{\sigma\nu}\\&\quad+\tfrac{1}{2}g_{\mu\nu}g_{\sigma\rho}g^{[\rho\sigma]})\delta\mathfrak{g}^{\mu\nu}\,.\end{aligned} \tag{2.22}$$

If we now substitute $W^{\lambda}_{\mu\nu}$, defined by (2.10), into (2.20) then, using (2.17), (2.21), and (2.22), we are led to the field equations

$$\partial_{\alpha}\mathfrak{g}^{\mu\nu}+\mathfrak{g}^{\sigma\nu}\Gamma^{\mu}_{\sigma\alpha}+\mathfrak{g}^{\mu\sigma}\Gamma^{\nu}_{\alpha\sigma}-\mathfrak{g}^{\mu\nu}\Gamma^{\sigma}_{(\alpha\sigma)}=0\,, \tag{2.23}$$

$$\Gamma_{\mu}=0\,, \tag{2.24}$$

$$^*R_{\mu\nu}=\tfrac{2}{3}(\partial_{\nu}W_{\mu}-\partial_{\mu}W_{\nu})\,. \tag{2.25}$$

In the above

$$^*R_{\mu\nu}=R_{\mu\nu}(\Gamma)+\frac{4\pi G}{k^2c^4}I_{\mu\nu} \tag{2.26}$$

and

$$I_{\mu\nu}=-(g_{\mu\sigma}g^{[\sigma\rho]}g_{\rho\nu}+\tfrac{1}{2}g_{\mu\nu}g_{\sigma\rho}g^{[\sigma\rho]}+g_{[\mu\nu]})\,. \tag{2.27}$$

From (2.23) by contracting on $\nu$ and $\alpha$ we get, using (2.12),

$$\partial_{\alpha}\mathfrak{g}^{[\mu\alpha]}=0\,. \tag{2.28}$$

The vector $W_{\mu}$ is not determined by the variational principle.[17] The scalar density (2.15) is invariant under the *projective transformation* (2.10) and leaves the vector $W_{\mu}$ undetermined. Moreover, the 64 partial-differential equations (2.23) can be written in a simpler form in terms of $g_{\mu\nu}$. The field equations can finally be written in the form

$$\partial_{\alpha}g_{\mu\nu}-g_{\sigma\nu}\Gamma^{\sigma}_{\mu\alpha}-g_{\mu\sigma}\Gamma^{\sigma}_{\alpha\nu}=0\,, \tag{2.29}$$

$$\partial_{\nu}\mathfrak{g}^{[\mu\nu]}=0\,, \tag{2.30}$$

$$^*R_{(\mu\nu)}=0\,, \tag{2.31}$$

$$\partial_{\sigma}{}^*R_{[\mu\nu]}+\partial_{\nu}{}^*R_{[\sigma\mu]}+\partial_{\mu}{}^*R_{[\nu\sigma]}=0\,. \tag{2.32}$$

Equation (2.32) follows as a consequence of (2.25).

This is the set of field equations of the generalized theory.

If we define the skew tensor density

$$^*\mathfrak{R}^{[\mu\nu]}=\tfrac{1}{2}\epsilon^{\mu\nu\rho\sigma}{}^*R_{[\rho\sigma]}\,, \tag{2.33}$$

then Eq. (2.32) can be written in the alternative form

$$\partial_{\nu}{}^*\mathfrak{R}^{[\mu\nu]}=0\,. \tag{2.34}$$

Among the eight equations (2.30) and (2.34), two identities hold:

$$\partial_{\mu}(\partial_{\nu}\mathfrak{g}^{[\mu\nu]})=0\,, \tag{2.35}$$

$$\partial_{\mu}(\partial_{\nu}{}^*\mathfrak{R}^{[\mu\nu]})=0\,. \tag{2.36}$$

The scalar density (2.15) was considered by Einstein in 1953.[19,20] However, the scalar density chosen by him as the basis of his theory was of the form[21]

$$\mathcal{H}=\mathfrak{g}^{\mu\nu}R_{\mu\nu}\,. \tag{2.37}$$

It did not include terms of the form $g^{[\mu\nu]}g_{[\nu\mu]}$, etc., which depend on $g_{\mu\nu}$ alone. To quote Einstein[19]: "All such additional terms bring a heterogeneity into the system of equations, and can be disregarded, *provided that no strong physical argument is found to support them*" (our italics). We feel that strong physical arguments have been found to support a theory based on the scalar density (2.15), which includes a quadratic term $g^{[\mu\nu]}g_{[\nu\mu]}$. Indeed, in the next section, we shall show that such terms are *essential* to provide a firm physical interpretation of the nonsymmetrical unified field theory.

### III. THE CORRESPONDENCE PRINCIPLE, THE EINSTEIN-MAXWELL THEORY, AND THE PHYSICAL INTERPRETATION OF $k$

Now that we have surveyed the formal aspects of the theory, we must study in detail the physical content of the equations. The question which comes into the foreground in the theory is: How do Einstein's theory of gravitation and Maxwell's equations for the electromagnetic field fit into the scheme?

In earlier work, based on Einstein's nonsymmetric theory,[22] the field equations were studied in the weak-field approximation in which the system of equations decomposed into two sets of equations, one for the symmetric components of the field, the other for the antisymmetric components. The equations for the antisymmetric part were shown to correspond to a much weaker system of equations than Maxwell's equations. Moreover, the equations of motion for a charged particle did not follow, in a natural way, from the field equations.[23]

It would seem more plausible to expect that the covariant (nonlinear) Einstein-Maxwell theory should appear as the *underlying approximation* to the generalized theory.

We shall adopt the following identification:

$$g_{[\mu\nu]}=kF_{\mu\nu}\,, \tag{3.1}$$

where $k=i\kappa$ and $\kappa$ is a real constant. The metric of space-time in the generalized theory is determined, as in the gravitational theory, by

$$ds^2=g_{\mu\nu}dx^\mu dx^\nu\,. \tag{3.2}$$

The dimensions of $F_{\mu\nu}$ and the electromagnetic potentials $A_\mu$ are, in cgs units,

$$\begin{aligned}[F_{\mu\nu}]&=\mathrm{g}^{1/2}\mathrm{cm}^{-1/2}\mathrm{sec}^{-1}=\mathrm{statvolt\ cm}^{-1}\,,\\ [A_\mu]&=\mathrm{g}^{1/2}\mathrm{cm}^{1/2}\mathrm{sec}^{-1}=\mathrm{statvolt}\,.\end{aligned} \tag{3.3}$$

The $g_{\mu\nu}$ in the theory are physically dimensionless quantities. *Therefore, from purely dimensional arguments*, the dimensions of $\kappa$ are $[\kappa]=[1/F_{\mu\nu}]=L^2/e$, where $L$ is a length and $e$ is the charge of the electron. The characteristic length $L$ which can be formed from $\hbar=1.05\times10^{-27}$ g cm$^2$ sec$^{-1}$, $G=6.67\times10^{-8}$ g$^{-1}$ cm$^3$ sec$^{-2}$, and $c=3\times10^{10}$ cm sec$^{-1}$ is the Planck length[24]

$$L=(\hbar G/c^3)^{1/2}=1.62\times10^{-33}\ \mathrm{cm}. \tag{3.4}$$

In terms of $\hbar$, $G$, $c$, and $e=4.80\times10^{-10}$ g$^{1/2}$ cm$^{3/2}$ sec$^{-1}$, the constant $\kappa=L^2/e$ can be expressed as

$$\begin{aligned}\kappa=\frac{L^2}{e}&\sim\frac{\hbar G}{c^3 e}=5.44\times10^{-57}\mathrm{g}^{-1/2}\,\mathrm{cm}^{1/2}\,\mathrm{sec}\\ &=5.44\times10^{-57}\ \mathrm{cm\,(statvolt)}^{-1}.\end{aligned} \tag{3.5}$$

The unified theory *requires for its logical completeness* a fundamental length in physics, which we have chosen to be given by (3.4). A "classical" length might on dimensional grounds be $eG^{1/2}/c^2=1.38\times10^{-34}$ cm, which would yield $\kappa=eG/c^4=3.95\times10^{-59}$ g$^{-1/2}$ cm$^{1/2}$ sec. But this differs from (3.4) merely by the factor $\alpha^{-1}=\hbar c/e^2\sim137$.

The universal constant can be fixed by using a gauge-invariant formulation of Dirac's wave equation. In the notation of Ref. 14 [the normalization is different from the one we have adopted in Eq. (2.15)] and in general units, Borchsenius obtained the value

$$p=-\frac{2i\hbar G}{c^3 e}\,. \tag{3.6}$$

If we write

$$g_{\mu\nu}=g_{(\mu\nu)}-pF_{\mu\nu}, \tag{3.7}$$

then the result (3.6) forces us to use the (complex) Hermitian form for $g_{\mu\nu}$.

We shall make the identification

$$A_\mu=\frac{kc^4}{12\pi G}W_\mu, \tag{3.8}$$

where $W_\mu$ is defined by (2.11). The $A_\mu$ reduces in the limit $k\to0$ to the electromagnetic potentials in the Einstein-Maxwell theory. In general, the $A_\mu$ will be a function of $k$.

Decomposing $I_{\mu\nu}$, in (2.27), into its symmetric and antisymmetric parts we get

$$\begin{aligned}I_{(\mu\nu)}=-(&g_{(\mu\rho)}g^{[\rho\sigma]}g_{[\sigma\nu]}+g_{(\nu\rho)}g^{[\rho\sigma]}g_{[\sigma\mu]}\\ &+\tfrac{1}{2}g_{(\mu\nu)}g_{[\sigma\rho]}g^{[\sigma\rho]})\end{aligned} \tag{3.9}$$

and

$$\begin{aligned}I_{[\mu\nu]}=-(&g_{[\mu\rho]}g^{[\rho\sigma]}g_{[\sigma\nu]}+g_{(\mu\rho)}g^{[\rho\sigma]}g_{(\sigma\nu)}\\ &+\tfrac{1}{2}g_{[\mu\nu]}g_{[\rho\sigma]}g^{[\rho\sigma]}+g_{[\mu\nu]})\,.\end{aligned} \tag{3.10}$$

In the limit $k\to0$, we have

$$\begin{aligned}&g_{\mu\nu}=g_{(\mu\nu)}\,,\\ &{}^*R_{[\mu\nu]}=\frac{4\pi G}{k^2c^4}I_{[\mu\nu]}=-\frac{8\pi G}{kc^4}F_{\mu\nu}\,,\\ &R_{(\mu\nu)}=G_{\mu\nu}\,,\\ &\frac{4\pi G}{k^2c^4}I_{(\mu\nu)}=-\frac{8\pi G}{c^4}T_{\mu\nu}\\ &\qquad=\frac{8\pi G}{c^4}(F^\alpha{}_\mu F_{\alpha\nu}-\tfrac{1}{4}g_{(\mu\nu)}F_{\alpha\beta}F^{\alpha\beta})\,.\end{aligned} \tag{3.11}$$

In the above, $G_{\mu\nu}$ is the Ricci tensor, defined in terms of the Christoffel symbols $\{^{\lambda}_{\mu\nu}\}$ (Ref. 25),

$$G_{\mu\nu}=\partial_\alpha\{^{\alpha}_{\mu\nu}\}-\partial_\nu\{^{\alpha}_{\mu\alpha}\}-\{^{\alpha}_{\mu\sigma}\}\{^{\sigma}_{\alpha\nu}\}+\{^{\alpha}_{\mu\nu}\}\{^{\sigma}_{\alpha\sigma}\}\,, \tag{3.12}$$

and $T_{\mu\nu}$ is Maxwell's stress-energy tensor.

From (3.11) it follows that (2.29) just becomes the ordinary connection between the metric $g_{(\mu\nu)}$ of general relativity and $\{^{\lambda}_{\mu\nu}\}$, while (2.30), (2.31), and (2.32) give

$$G_{\mu\nu}=\frac{8\pi G}{c^4}T_{\mu\nu}\,, \tag{3.13}$$

$$\partial_\nu(\sqrt{-g}\,F^{\mu\nu})=0\,, \tag{3.14}$$

$$\partial_\sigma F_{\mu\nu}+\partial_\mu F_{\nu\sigma}+\partial_\nu F_{\sigma\mu}=0\,, \tag{3.15}$$

where now $g=\det(g_{(\mu\nu)})$.

This demonstrates that the field equations of the generalized theory reduce to those of the Einstein-Maxwell theory of empty space when $k\to 0$. For $k\to 0$, the skew part of (2.25) becomes, by (3.8),

$$F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu\,. \tag{3.16}$$

The scalar density (2.15) reduces in the limit $k\to 0$ to

$$\mathcal{H}=\sqrt{-g}\left(g^{(\mu\nu)}G_{\mu\nu}-\frac{4\pi G}{c^4}F^{\mu\nu}F_{\mu\nu}\right), \tag{3.17}$$

which is the scalar density of the Einstein-Maxwell theory. A schematic representation of the correspondence principle is given in Table I.

The theory is invariant under the ordinary Abelian gauge transformations of electromagnetism. This can be shown as follows. Consider the transformation

$$\tilde{W}^\alpha_{\mu\nu}=W^\alpha_{\mu\nu}-\tfrac{2}{3}\delta^\alpha_\mu\partial_\nu\lambda\,,\qquad \tilde{W}_\mu=W_\mu+\partial_\mu\lambda\,. \tag{3.18}$$

The vector $W_\nu$ is related to the electromagnetic 4-potential $A_\mu$ through Eq. (3.8) and $\lambda$ is an arbitrary scalar field. Equation (3.18) induces the transformation on the $\Gamma$ connection

$$\tilde{\Gamma}^\alpha_{\mu\nu}=\tilde{W}^\alpha_{\mu\nu}+\tfrac{2}{3}\delta^\alpha_\mu\tilde{W}_\nu=\Gamma^\alpha_{\mu\nu}\,, \tag{3.19}$$

where we have used the Schrödinger affinity (2.10). It can be easily shown that

$$R_{\mu\nu}(\tilde{W})=R_{\mu\nu}(W)\,, \tag{3.20}$$

so that the scalar density $\mathcal{H}$ and the field equations are invariant under the gauge transformations (3.18) and (3.19).

This is the correspondence principle of the theory. It shows that the system of field equations is founded on a (formally) well-established theory, namely, the Einstein-Maxwell system of equations. This secure basis for the nonsymmetric theory is eliminated when we exclude the quadratic term $g^{[\mu\nu]}g_{[\nu\mu]}$ from the scalar density (2.15).

A result that follows from the theory is that the complete laws of classical electrodynamics—including the Lorentz equations of motion for charged particles—*follow from the field equations alone in the first nontrivial approximation*, under the assumption that the field $g_{\mu\nu}$ becomes Minkowskian as $r\to\infty$.[8,26] This overcomes one of the grave objections raised against Einstein's nonsymmetric theory, formulated in terms of the scalar density (2.37).[23]

## IV. EXACT SOLUTION OF THE FIELD EQUATIONS

In a series of papers, exact static spherically symmetric solutions of the field equations have been derived.[8,9,27,28] From these solutions, it is clear that the physical content of the generalized theory differs significantly from that of the Einstein-Maxwell theory, although the solutions reduce to those of the latter theory when $k\to 0$. This fact will become clearer from the following.

The $g_{\mu\nu}$ corresponding to a static field with spherical symmetry is written in polar coordinates $x^1=r$, $x^2=\theta$, $x^3=\phi$, $x^4=ct$ as[29]

$$g_{\mu\nu}=\begin{pmatrix}-\alpha & 0 & 0 & w\\ 0 & -\beta & f\sin\theta & 0\\ 0 & -f\sin\theta & -\beta\sin^2\theta & 0\\ -w & 0 & 0 & \gamma\end{pmatrix}, \tag{4.1}$$

where $\alpha$, $\beta$, $\gamma$, $f$, and $w$ are functions of $r$ only ($f$ and $w$ are purely imaginary functions). We have from (4.1)

$$g=-(\alpha\gamma-w^2)(\beta^2+f^2)\sin^2\theta\,. \tag{4.2}$$

The $g^{\mu\nu}$ are well-defined provided (4.2) does not vanish.

A solution of the system of Eqs. (2.29)–(2.32) for the (complex) $g_{\mu\nu}$, corresponding to a charged massive particle at the origin of coordinates, is given by[8]

TABLE I. The one-to-one relation between the field equations of the generalized theory and those of the Einstein-Maxwell theory in the limit $k\to 0$.

| Generalized theory | Einstein-Maxwell theory |
|---|---|
| $\mathcal{H}=\sqrt{-g}\left(g^{\mu\nu}R_{\mu\nu}-\frac{4\pi G}{k^2c^4}g^{[\mu\nu]}g_{[\mu\nu]}\right)$ | $\mathcal{H}=\sqrt{-g}\left(g^{(\mu\nu)}G_{\mu\nu}-\frac{4\pi G}{c^4}F^{\mu\nu}F_{\mu\nu}\right)$ |
| $\partial_\alpha g_{\mu\nu}-g_{\sigma\nu}\Gamma^\sigma_{\mu\alpha}-g_{\mu\sigma}\Gamma^\sigma_{\alpha\nu}=0$ | $\partial_\alpha g_{(\mu\nu)}-g_{(\sigma\nu)}\left\{ {\sigma \atop \mu\alpha} \right\}-g_{(\mu\sigma)}\left\{ {\sigma \atop \nu\alpha} \right\}=0$ |
| $\partial_\nu(\sqrt{-g}\,g^{[\mu\nu]})=0$ | $\partial_\nu(\sqrt{-g}\,F^{\mu\nu})=0$ |
| $*R_{(\mu\nu)}=0$ | $G_{\mu\nu}=\frac{8\pi G}{c^4}T_{\mu\nu}$ |
| $\partial_\sigma *R_{[\mu\nu]}+\partial_\nu *R_{[\sigma\mu]}+\partial_\mu *R_{[\nu\sigma]}=0$ | $\partial_\sigma F_{\mu\nu}+\partial_\nu F_{\sigma\mu}+\partial_\mu F_{\nu\sigma}=0$ |

$$\alpha = \left(1 - \frac{2Gm}{c^2 r} + \frac{4\pi G Q^2}{c^4 r^2}\right)^{-1}, \quad \beta = r^2,$$

$$\gamma = \left(1 - \frac{2Gm}{c^2 r} + \frac{4\pi G Q^2}{c^4 r^2}\right)\left(1 - \frac{\kappa^2 Q^2}{r^4}\right), \tag{4.3}$$

$$w = \frac{kQ}{r^2}, \quad f = 0.$$

This solution obeys the boundary condition that $g_{\mu\nu} \to \eta_{\mu\nu}$ ($\eta_{\mu\nu}$ is the Minkowskian metric) for $r \to \infty$. The constants $m$ and $Q$ are the mass and charge of the particle, respectively. Equations (4.2) and (4.3) yield the result

$$(-g)^{1/2} = r^2 \sin\theta. \tag{4.4}$$

The field $E_r = w/k = Q/r^2$ is the static electric field due to the charge $Q$ on the particle.

In the limit $k = i\kappa \to 0$ (or large $r$), the solution (4.3) reduces to the Reissner-Nordström solution[30] of the Einstein-Maxwell equations (3.13)–(3.15).

## V. EQUATION OF PATHS

The parallel displacement of a complex vector $A^\mu$ is not a unique operation for a given complex $\Gamma^\lambda_{\mu\nu}$.[31] If we consider the complex conjugate of $\delta A^\mu$ in (2.1) we get

$$\overline{\delta A^\mu} = -\Gamma^\mu_{\beta\alpha} \bar{A}^\alpha dx^\beta, \tag{5.1}$$

where we have used the property of Hermitian symmetry $\Gamma^\lambda_{\mu\nu} = \bar{\Gamma}^\lambda_{\nu\mu}$. By using (2.1), (2.7), and (5.1) we obtain

$$\delta(A_\nu \bar{A}^\nu) = \delta(g_{\mu\nu} A^\mu \bar{A}^\nu)$$
$$= (\partial_\sigma g_{\mu\nu} - g_{\rho\nu}\Gamma^\rho_{\mu\sigma} - g_{\mu\rho}\Gamma^\rho_{\sigma\nu}) A^\mu \bar{A}^\nu dx^\sigma, \tag{5.2}$$

where we note the order of the suffixes in the third term in parentheses on the right-hand side of (5.2). The quantity $\delta(A_\nu \bar{A}^\nu)$ vanishes by virtue of (2.29) and the magnitude of the vector $A^\mu$ is conserved under parallel transfer.

Let us now consider the *paths* of (neutral) test particles in the non-Riemannian geometry determined by the theory. A necessary and sufficient condition for the vector $A^\mu(p)$ to be parallel to the curve $x^\mu = x^\mu(p)$ is[32]

$$A^\mu\left(\frac{dA^\nu}{dp} + \Gamma^\nu_{\alpha\beta} A^\alpha \frac{dx^\beta}{dp}\right) - A^\nu\left(\frac{dA^\mu}{dp} + \Gamma^\mu_{\alpha\beta} A^\alpha \frac{dx^\beta}{dp}\right) = 0. \tag{5.3}$$

A path is a curve whose tangents are parallel with respect to itself. For $A^\mu = dx^\mu/dp$, (5.3) demands that

$$\frac{d^2x^\mu}{dp^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{dp}\frac{dx^\beta}{dp} = K(p)\frac{dx^\mu}{dp}, \tag{5.4}$$

where $K(p)$ is an arbitrary function of $p$. A suitable change of parameter to $s = s(p)$ will give

$$\frac{d^2x^\mu}{ds^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds} = 0. \tag{5.5}$$

The affine parameter is only fixed to within a linear transformation with constant coefficients. Only the symmetric part of $\Gamma^\lambda_{\mu\nu}$ contributes to (5.5).

The equations of paths (5.5) are the generalization of geodesic paths of Riemannian geometry or, equivalently, the generalization of straight lines of Euclidean geometry.

It can be shown that the paths are the same for two symmetric connections, related by the projective transformation[33]

$$\tilde{\Gamma}^\lambda_{(\mu\nu)} = \Gamma^\lambda_{(\mu\nu)} + \delta^\lambda_\mu V_\nu + \delta^\lambda_\nu V_\mu, \tag{5.6}$$

where $V_\mu$ is an arbitrary covariant vector. If we set $V_\mu = \frac{1}{3}W_\mu$, then the $\Gamma^\lambda_{(\mu\nu)}$ and $W^\lambda_{(\mu\nu)}$ obtained from (2.10) have the same paths in the manifold.

There exists a class of coordinates called *normal coordinates*[34] such that the symmetric connection $\Gamma^\lambda_{(\mu\nu)}$ vanishes at a point. These coordinates ensure that the generalized theory satisfies the principle of equivalence.

Equations (5.5) reduce in the limit $k \to 0$ to the standard equations of geodesics in general relativity written in terms of the Christoffel symbols $\left\{{\lambda \atop \mu\nu}\right\}$.

## VI. WORLD-LINE COMPLETENESS OF THE EXACT SOLUTION

We shall study the properties of the exact solution by investigating the behavior of paths of test particles in the theory. These test particles have vanishing mass within a tube enclosing the world line. The rigorous solution (4.3) will be considered to be valid for all $r$. We shall treat the case where the orbits lie in the equatorial plane, whereby $\theta = \frac{1}{2}\pi$ and $\dot{\theta} \equiv d\theta/d\tau = 0$. Here $\tau$ is the proper time along the path of the particle and $ds^2 = E\,d\tau^2$, where $E$ is a constant.

Two of the four equations in (5.5) are, for $x^3 = \phi$ and $x^4 = ct$,

$$\ddot{\phi} + \frac{2}{r}\dot{r}\dot{\phi} = 0, \tag{6.1}$$

$$\ddot{t} - \frac{\alpha'}{\alpha}\dot{r}\dot{t} = 0, \tag{6.2}$$

where we have used (4.3) and the calculated values[8]

$$\Gamma^3_{(13)} = \frac{1}{r},$$
$$\Gamma^4_{(14)} = -\frac{\alpha'}{2\alpha} + \frac{2}{r} + \frac{w'}{w} = -\frac{\alpha'}{2\alpha}. \tag{6.3}$$

By means of the relations

$$\tfrac{1}{2} g_{\lambda\mu} \frac{d}{d\tau}(\dot{x}^\lambda \dot{x}^\mu) = g_{\lambda\mu}\ddot{x}^\lambda \dot{x}^\mu ,$$
$$g_{\alpha\rho}\dot{x}^\rho \Gamma^\alpha_{\lambda\sigma}\dot{x}^\lambda \dot{x}^\sigma = \frac{1}{2}\frac{d}{d\tau}(g_{\lambda\rho})\dot{x}^\lambda \dot{x}^\rho \tag{6.4}$$

obtained from (2.29) and (5.5), it can be shown that

$$\frac{d}{d\tau}(g_{\lambda\alpha}\dot{x}^\lambda \dot{x}^\alpha) = 0 \tag{6.5}$$

or

$$g_{\lambda\alpha}\dot{x}^\lambda \dot{x}^\alpha = E . \tag{6.6}$$

Then from (4.3), (6.1), (6.2), and (6.6) we obtain the first integrals

$$\gamma \dot{t}^2 - \alpha \dot{r}^2 - r^2 \dot{\phi}^2 = E , \tag{6.7}$$

$$r^2 \dot{\phi} = J , \tag{6.8}$$

$$\dot{t} = \alpha A , \tag{6.9}$$

where $A$ is a constant.

From (6.7)–(6.9) and (4.3) it follows that

$$\dot{r} = \pm\left[A^2\left(1 - \frac{\kappa^2 Q^2}{r^4}\right) - \left(1 - \frac{2Gm}{c^2 r} + \frac{4\pi G Q^2}{c^4 r^2}\right)\left(E + \frac{J^2}{r^2}\right)\right]^{1/2} \tag{6.10}$$

and

$$\tau - \tau_0 = \pm\int_{r_0}^{r} \frac{dr}{\left[A^2\left(1 - \frac{\kappa^2 Q^2}{r^4}\right) - \left(1 - \frac{2Gm}{c^2 r} + \frac{4\pi G Q^2}{c^4 r^2}\right)\left(E + \frac{J^2}{r^2}\right)\right]^{1/2}} . \tag{6.11}$$

Here $\tau_0$ and $r_0$ are some initial constant values of proper time and position, respectively.

As $r \to 0$, in (6.11), *the proper time $\tau$ eventually becomes either infinite or complex.* In the latter case the physical test particle is deflected away from $r \sim \sqrt{\kappa Q}$.

The velocity $\dot{r}$ given by (6.10) remains *finite* as $r$ approaches $\sqrt{\kappa Q}$ and eventually becomes complex for $r < \sqrt{\kappa Q}$.

The coordinate velocity $dr/dt$ is given by

$$\frac{1}{c}\frac{dr}{dt} \equiv \frac{1}{c}\frac{dr}{d\tau}\frac{d\tau}{dt}$$
$$= \left(1 - \frac{2Gm}{c^2 r} + \frac{4\pi G Q^2}{c^4 r^2}\right)\left[\left(1 - \frac{\kappa^2 Q^2}{r^4}\right) - \left(1 - \frac{2Gm}{c^2 r} + \frac{4\pi G Q^2}{c^4 r^2}\right)\left(E + \frac{J^2}{r^2}\right)\right]^{1/2} , \tag{6.12}$$

and the coordinate time $t$ for a radial null path is

$$t - t_0 = \frac{1}{c}\int_{r_0}^{r} \frac{dr}{\left(1 - \frac{2Gm}{c^2 r} + \frac{4\pi G Q^2}{c^4 r^2}\right)\left(1 - \frac{\kappa^2 Q^2}{r^4}\right)^{1/2}} . \tag{6.13}$$

If it is assumed that we are *inside* the "event horizons" located at $r_+$ and $r_-$, where

$$r_\pm = \frac{Gm}{c^2} \pm \left(\frac{G^2 m^2}{c^4} - \frac{4\pi G Q^2}{c^4}\right)^{1/2} , \tag{6.14}$$

then $1/\alpha$ is nonvanishing and by the mean-value theorem (6.13) yields

$$t = \frac{1}{c} H(\bar{r}) \int^{r} \frac{dr}{(r^2 - \kappa Q)^{1/2}}$$
$$= \frac{1}{c} H(\bar{r}) \ln[r + (r^2 - \kappa Q)^{1/2}], \tag{6.15}$$

where

$$H(\bar{r}) = \frac{\bar{r}^2}{(1 - 2Gm/c^2\bar{r} + 4\pi G Q^2/c^4\bar{r}^2)(\bar{r}^2 + \kappa Q)^{1/2}} . \tag{6.16}$$

This reveals that $t$ becomes purely imaginary for $r < \sqrt{\kappa Q}$, and physical null paths cannot reach $r = 0$.

We define a "physical" path to be one for which the proper time *$\tau$ remains real.*

These results for the behavior of paths in the neighborhood of $r \sim \sqrt{\kappa Q}$ show that the rigorous solution is world-line complete for timelike and null physical paths.

We see that for $k \to 0$ timelike and null world lines (geodesics) are complete.[15] Null radial geodesics are not prevented from reaching $r = 0$. Thus the Reissner-Nordström solution[30] has a geometrical singularity at $r = 0$.

In the exact solution of the generalized theory no physical test particles can hit $r = 0$ and, *in this sense, the solution is singularity-free.*

The solution (4.3) is of triple null surface form.

There are null surfaces at $r=r_+$, $r=r_-$, and $r=\sqrt{\kappa Q}$, where $r_\pm$ are given by (6.14). The singular *event horizons* at $r_\pm$ occur when $G^2m^2 > 4\pi G Q^2$, and they can be removed by an analytical completion of the manifold; i.e., they occur because of a special choice of coordinates.

The surface of the sphere $S$ defined by $r_s=\sqrt{\kappa Q}$ acts as a "surface of concealment" for the singularity at $r=0$. The surface of $S$ is nonsingular and analytic for the solution $g_{\mu\nu}$ and the field equations (2.29)–(2.32). We can see this by calculating $g^{\mu\nu}$ for $r=\sqrt{\kappa Q}=a$. We find that

$$g^{\mu\nu}=\begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & -\dfrac{1}{a^2} & 0 & 0 \\ 0 & 0 & -\dfrac{1}{a^2\sin^2\theta} & 0 \\ i & 0 & 0 & \left(1-\dfrac{2Gm}{c^2 a}+\dfrac{4\pi G Q^2}{c^4 a^2}\right)^{-1}\end{pmatrix}. \tag{6.17}$$

Moreover, from (4.4) we have

$$(-g)^{1/2}=a^2\sin\theta. \tag{6.18}$$

The curvature tensor $R^\lambda_{\mu\nu\sigma}$ and the contracted curvature tensor $R_{\mu\nu}$ are also regular at $r=\sqrt{\kappa Q}$. For example, a calculation yields

$$\begin{aligned} R_{44}&=-\frac{4\pi G}{c^4 k^2}I_{44}\\ &=\frac{4\pi G Q^2}{c^4 r^4}\left(1-\frac{2Gm}{c^2 r}+\frac{4\pi G Q^2}{c^4 r^2}\right)\left(1-\frac{\kappa^2 Q^2}{r^4}\right), \end{aligned} \tag{6.19}$$

which vanishes at $r=\sqrt{\kappa Q}$. The tensor $E_{\mu\nu}=-(1/2k^2)I_{(\mu\nu)}$ plays the role of a generalized electromagnetic stress-energy tensor in the theory.

We observe that

$$(-s)^{1/2}=[-\det(g_{(\mu\nu)})]^{1/2}=r^2\sin\theta\left(1-\frac{\kappa^2Q^2}{r^4}\right)^{1/2} \tag{6.20}$$

vanishes at $r=\sqrt{\kappa Q}$. Therefore the symmetric tensor $s^{\mu\nu}$, defined to be the inverse of $g_{(\mu\nu)}$, is singular at $r=\sqrt{\kappa Q}$. The Riemannian subspace of the theory is singular at $r=\sqrt{\kappa Q}$.

The space-time signature in the theory undergoes a *change of sign* as we pass through the surface of $S$. Within $S$ the space is locally an Euclidean $E^4$, for $ds^2$ is negative definite. We are unable to attach a Minkowskian null cone to a point in $E^4$. At the surface of $S$ the signature of space-time changes from the normal hyperbolic form $(---+)$ to the "elliptic" form $(----)$. Within $S$ the local gauge group of transformations is that of U(4), which contains the coordinate transformations of O(4).

## VII. IMPLICATIONS OF THE THEORY FOR THE LARGE- AND SMALL-SCALE PHENOMENA OF THE UNIVERSE

What are the probable consequences for physics that follow from a regular solution of a covariant set of field equations, describing the structure of space-time? If the basic constituents of matter are always electrically charged, as would be the case for fractionally charged quarks,[35] then the theory may not require infinite renormalization techniques. Present-day quantum field theory removes by renormalization theory infinite quantities associated with the mass, charge, and wave function of a particle, as well as the infinity associated with the energy density in the vacuum. The total density of the zero-point energy of, e.g., the electromagnetic field, is given by

$$(\hbar/2\pi^2)\int_0^\infty k^3 dk. \tag{7.1}$$

This integral formally diverges. Similar divergences occur with other fields and with vacuum fluctuations. They are removed by renormalization. In the unified field theory described here, space-time "ends" in an analytic fashion at the order of the Planck length $L=1.6\times10^{-33}$ cm. This implies that the effective upper limit or "cutoff" in the formal divergent integrals, such as (7.1), is to be taken to be of the order of the reciprocal Planck length[36]:

$$k_{\text{cutoff}}\sim\frac{1}{L}=\left(\frac{c^3}{\hbar G}\right)^{1/2}=6.2\times10^{32}\ \text{cm}^{-1}. \tag{7.2}$$

In effect, something radically new happens for wavelengths of the order of the Planck length, or for energies

$$\hbar c k_{\text{cutoff}}\sim10^{19}\ \text{GeV}. \tag{7.3}$$

The cutoff arises purely from the geometrical laws governing space-time.

In this picture, a neutral particle, such as a neutron, will be a composite system of fractionally charged quarks. Only the nonsingular *charged* constituents form the basic entities of field physics; these are to be described by the rigorous, regular solutions of the *nonlinear* field equations. This physical description of particles could lead to a logically complete theory of matter—free of infinities.

If the ultimate form of matter is indeed charged, then it follows unavoidably from the present theory that, at the location of a particle, there is an *ab-*

*solute* limit to how small a region of space-time we can measure.

It would seem from what we have learned that a deep understanding of space-time structure appears to be essential before we can hope to make any fundamental progress in particle physics.

In the universe, the large- and small-scale phenomena are inescapably linked together through "catastrophes" like gravitational collapse and the "initial" and "final" states of the universe.[37] If a star burns out all its nuclear fuel and undergoes gravitational collapse with a small but finite electrostatic charge $Q$, then after it implodes through the event horizon $r_+$ it cannot contract to infinite density according to the predictions of the generalized gravitation theory. The analytic surface $S$ will repel all the collapsing matter. A thorough analysis of these problems, within the context of the present theory, remains to be carried out.

The generalized theory also predicts that, in the "big-bang" cosmology,[37] the 3 °K microwave background is the relic of an initially highly dense state of the universe not smaller than a sphere with radius $r \sim \sqrt{\kappa Q}$. If the matter had even the smallest charge $Q$, i.e., *inhomogeneity*, the theory *prohibits the existence* of a singularity at $r=0$ at the origin of time.

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