Hadron and quark mass differences*

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An estimate $D^+ - D^0 \approx 5$ MeV is made on the basis of a phenomenological quark model in which baryon mass differences are fitted exactly and simple corrections made for the looser structure of bosons. The other boson fits are satisfactory except for predicting $\rho^+ - \rho^0 \approx 0.3$ MeV. An incidental quark mass difference is obtained: $m_d - m_u \approx 10$ MeV from sources other than ordinary QED. It is suggested that weak interactions with the heavy charmed quark are responsible, so that $[(m_d - m_u)/(m_s - m_u)]^{1/2} \approx \theta_C$, the Cabibbo angle.

Electromagnetic mass differences ($\Delta m_{\rm em}$) of hadrons is a topic that has long attracted attention.[1] In particular, it has proved exceedingly difficult to obtain the correct sign for the n-p mass difference.[1,2] Treatments involving symmetry principles like SU(3) have led to a number of specific relations among the mass differences, which have been successful as a phenomenology but do not predict absolute differences.[3] Recently, the surprising success of explicit quark models for hadron structure[4,5] has led to their use in predicting absolute electromagnetic mass differences. This is especially critical for the D mesons because of threshold effects on their decay sequences.[6-8] Several sets of estimates for $D^+ - D^0$ have been published: the more phenomenological ones[6,8] leading to values on the order of 12–15 MeV, the more detailed ones[9,10] to values around 7 MeV. In the face of this discrepancy it has been suggested[11] that all the models are inadequate; quadratic mass dependences have also been proposed.[12]

In the following note we attempt to counter this suggestion by providing an example of a phenomenological treatment, motivated by the Breit equation, that leads directly to $D^+ - D^0 \approx 5$ MeV. It depends on (a) detailed rehearsal of the situation for baryon $\Delta m_{\rm em}$ plus suitable correction of these parameters upon transference to bosons, using simple kinetic effects related to binding, and (b) use of the ratio of $|\psi(0)|^2$ to $\langle 1/r \rangle$ as a relative constant in this transference, as it seems to be reasonably constant in more than one phenomenological quark potential. As a consequence an intrinsic quark mass difference $m_d - m_u \approx 10$ MeV is estimated, which is rather large and seems to exacerbate the classical n-p dilemma at the level of quarks. We suggest that this difference reflects the influence of the weak interaction, with $(m_d - m_u)/(m_c - m_u) \approx \tan^2\theta_C$, where θ_C is the Cabibbo angle and m_c is the charmed-quark mass.

I. BARYONS

The phenomenological formula for baryon mass differences arising from electromagnetic interactions of quarks is taken from the Breit equation[5] to be

$$E_{\rm em} = \alpha \sum_{i>j} q_i q_j \{a_1 + a_2 \vec{\sigma}_1 \cdot \vec{\sigma}_2 [1 + \lambda(y_i + y_j)] + a_3 \lambda (y_i + y_j)\}, \quad (1)$$

The summation is over all quark pairs with q_i the quark charge in units of e, $\vec{\sigma}_i = 2\vec{s}_i$ its Pauli operator, and y_i its hypercharge. The mass differences of the quarks themselves are taken phenomenologically at this level and are represented by

$$m_s - \tfrac{1}{2}(m_u + m_d) = \lambda m_0 , \quad (1a)$$

$$m_0 = \tfrac{1}{3}(m_u + m_d + m_s) , \quad (1b)$$

$$m_d - m_u = \delta m. \quad (1c)$$

In Eq. (1) the expansion in λ is taken only to first order and effects of order $\delta m / m_0$ are neglected; observed baryon mass differences will of course contain direct terms δm in addition to $E_{\rm em}$.

In Eq. (1) the phenomenological coefficients correspond to the following two-body Breit terms:

$$a_1 = \langle \alpha / r \rangle + a_3 ,$$

$$a_2 = -\tfrac{2}{3}\pi\alpha\, |\psi_{ij}(0)|^2 / m_0^2 , \quad (2)$$

$$a_3 = \tfrac{3}{2} a_2 - \alpha \left\langle \frac{\vec{p}_i \cdot \vec{p}_j}{r} + \frac{\vec{r} \cdot (\vec{r} \cdot \vec{p}_i) \cdot \vec{p}_j}{r^3} \right\rangle .$$

To evaluate Eq. (1) it is necessary to decompose the three-body SU(6) baryon wave functions into two-body components with explicit SU(3)×SU(2) content. The $\underline{56}$ supermultiplet in SU(6) contains two multiplets,

$$(8|\tfrac{1}{2})^2 \to \tfrac{3}{2}[(6|1)^2 + (\bar{3}|0)^2] ,$$
$$(10|\tfrac{3}{2})^2 \to 3(6|1)^2 . \quad (3)$$

TABLE I. Baryon mass differences.

$n-p=\delta m-\frac{1}{3}a'_1$	$\Delta^+-\Delta^{++}=\delta m-\frac{4}{3}a'_1$
$\Sigma^0-\Sigma^+=\delta m-\frac{1}{3}a'_1-a'_2$	$\Delta^0-\Delta^+=\delta m-\frac{1}{3}a'_1$
$\Sigma^--\Sigma^0=\delta m+\frac{2}{3}a'_1-a'_2$	$\Delta^--\Delta^0=\delta m+\frac{2}{3}a'_1$
$\Xi^--\Xi^0=\delta m+\frac{2}{3}a'_1-2a'_2$	$\Sigma^{*0}-\Sigma^{*+}=\delta m-\frac{1}{3}a'_1-\frac{1}{3}a'_3\lambda$
	$\Sigma^{*-}-\Sigma^{*0}=\delta m+\frac{2}{3}a'_1-\frac{1}{3}a'_3\lambda$
	$\Xi^{*-}-\Xi^{*0}=\delta m+\frac{2}{3}a'_1-\frac{2}{3}a'_3\lambda$
$a'_1=\langle 1/r\rangle+a'_3(1+\frac{2}{3}\lambda)$,	
$a'_2=a_2+\frac{1}{3}a_3\lambda$,	
$a'_3=a_2+a_3$	

The notation in Eq. (3) defines the wave function as $\psi=(\mathrm{SU(3)}|\mathrm{SU(2)})$, and the reduction is $|\psi(3\text{ body})|^2\to|\psi(2\text{ body})|^2$, the left-hand sides being normalized to three particles. Of course the wave functions have more detailed spin-isospin structure, but it will not be needed for the simple considerations below. The only explicit feature required is to keep track of the exchange properties in the octet. For example, $p=uud$ decomposes as follows:

$$(uu)=(6|1)^2\ ,\qquad (ud)=\tfrac{1}{2}(6|1)^2+\tfrac{3}{2}(\bar{3}|0)^2\ , \tag{4}$$

used similarly for all other octet baryons. The decuplet is totally symmetric throughout.

In this way one arrives at Table I for Δm_{em} , displaying a number of familiar relations. The empirical values[13] are accurate for the first three entries only, but this is enough to determine the relevant coefficients:

$$\begin{aligned} &n-p=1.3\text{ MeV},\quad \delta m=1.9\text{ MeV}\ ,\\ &\Sigma^0-\Sigma^+=3.1\text{ MeV},\quad a'_1=1.8\text{ MeV}\ ,\\ &\Sigma^--\Sigma^0=4.9\text{ MeV},\quad a'_2=-1.8\text{ MeV}\ . \end{aligned} \tag{5}$$

One then predicts

$$\begin{aligned} &\Xi^--\Xi^0=6.7\text{ MeV},\\ &\Sigma^{*-}+\Sigma^{*+}-2\Sigma^{*0}=1.8\text{ MeV}\ ,\\ &\Delta^+-\Delta^{++}=-0.5\text{ MeV},\\ &(\Sigma^{*-}-\Sigma^{*+})-(\Xi^{*-}-\Xi^{*0})=1.3\text{ MeV}\ ,\\ &\Delta^0-\Delta^+=1.3\text{ MeV}\ ,\\ &\Delta^--\Delta^0=3.1\text{ MeV}\ , \end{aligned} \tag{6}$$

and

$$a'_3\lambda=3[(\Sigma^{*-}-\Sigma^{*0})-(\Xi^{*-}-\Xi^{*0})]\ . \tag{7}$$

The empirical agreement with Eq. (6) is satisfactory, the respective values[13] being 6.4 ± 0.6 MeV and 0.8 ± 1.5 MeV for the first and fourth entries; and $\Delta^0-\Delta^{++}\approx1\pm2$ MeV. There are no disagreements outside the quoted errors although, in some cases, these are so large as to render the comparison meaningless. This is unfortunately true of Eq. (7), so we must take for λ the estimate from strong-force correlations[5] that

$$\lambda\approx0.3\pm0.1\ . \tag{8a}$$

There still remain three quantities, $\langle 1/r\rangle$, a_2, and a_3, to determine from a'_1 and a'_2. For this purpose we rely on the estimate of $a_2\langle 1/r\rangle^{-1}=f$, obtainable from three popular shapes of nonrelativistic quark potential: Coulomb, linear, and Gaussian as in Table II where μ is the reduced mass. The linear and Gaussian forms are equivalent for this purpose. Then Eq. (5) implies that $a'_1+a'_2=0$ or hence

$$a_3/a_2=-(f^{-1}+2+\tfrac{2}{3}\lambda)/(1+\lambda)\approx\begin{cases}-1\text{ to }-2\\ 0\end{cases}, \tag{9}$$

respectively, for the Coulomb and linear Gaussian cases. But Eq. (2) suggests that a_3/a_2 is displaced from a zero-order value of 3/2; this displacement is less for the second case, which we therefore adopt. Then

TABLE II. Parameters for various potentials.

Potential	$\lvert\psi(0)\rvert^2$	$\langle 1/r\rangle$	f	Parameter (Refs. 4, 6, 8, 14, 15)	$-f$
Coulomb: $-\frac{\alpha_s}{r}$	$\frac{1}{\pi}(\mu\alpha_s)^3$	$\mu\alpha_s$	$-\frac{1}{6}\alpha_s^2$	$\alpha_s=0.2$–0.5	0.01–0.04
linear: $ar+b$	$\left(\frac{\mu a}{2\pi}\right)$	$\sim\left(\frac{32}{3\pi^2}a\mu\right)^{1/3}$	$-\frac{\pi}{12}\left(\frac{3}{2\pi}\frac{a^2}{m_0^4}\right)^{1/3}$	$a=0.2$–$0.3\ \mathrm{GeV}^2$	0.3–0.8
Gaussian: $\frac{1}{2}\mu\omega^2r^2=\frac{1}{2}kr^2$	$\left(\frac{\mu k}{\pi^2}\right)^{3/4}$	$2\left(\frac{\mu k}{\pi^2}\right)^{1/4}$	$-\frac{1}{3}\left(\frac{k}{2m_0^3}\right)^{1/2}$	$k=0.1$–$0.3\ \mathrm{GeV}^3$	0.3–0.8

$$a_2 \approx a_2' = -1.8 \text{ MeV} , \tag{8b}$$

$$\langle \alpha/r \rangle \approx 4.0 \text{ MeV} . \tag{8c}$$

II. BOSONS

The quark-antiquark states are already separated according to singlet and triplet spins. Equation (1) can again be applied, with the caveat *not* to reverse the signs of y_i and δm for antiquarks. The Δm formulas are

$$\begin{aligned} &\pi^+ - \pi^0 = \tfrac{1}{2}\langle 1/r \rangle + \tfrac{1}{2}a_4'(1+\tfrac{2}{3}\lambda) , \\ &\rho^+ - \rho^0 = \pi^+ - \pi^0 + \tfrac{1}{2}(a_3' - a_4')(1+\tfrac{2}{3}\lambda) , \\ &K^+ - K^0 = \delta m + \tfrac{1}{3}\langle 1/r \rangle + \tfrac{1}{3}a_4'(1-\tfrac{1}{3}\lambda) , \\ &K^{*+} - K^{*0} = K^+ - K^0 + \tfrac{1}{3}(a_3' - a_4')(1-\tfrac{1}{3}\lambda) , \\ &D^+ - D^0 = \delta m + \tfrac{2}{3}\langle 1/r \rangle + \tfrac{2}{3}a_4'\kappa + \tfrac{3}{4}a_2(1-2\kappa) , \\ &D^{*+} - D^{*0} = D^+ - D_0 + \tfrac{2}{3}(a_3' - a_4')\kappa , \end{aligned} \tag{10}$$

where $a_4' = a_3 - 3a_2$ and $\kappa = m_0/m_c \approx 0.2$, the ratio of ordinary- and charmed-quark masses. The empirical values[13] are $\pi^+ - \pi^0 = 4.6$ MeV, $K^+ - K^0 = 4.0$ MeV, $K^{*+} - K^{*0} = -4.1 \pm 0.6$ MeV; the difference $\rho^+ - \rho^0$ is not well defined but appears to be 0 to 6 MeV negative. We take the numerical values to be in decreasing order of reliability on the grounds that the broader the resonance, the more difficult it is to position exactly, as is true of the Δ baryons.

Nevertheless, the qualitative relation seems established that

$$(\pi^+ - \pi^0) - (\rho^+ - \rho^0) \gg (K^+ - K^0) - (K^{*+} - K^{*0}) ,$$

implying that λ has become relatively large for the bosons. This is a difference in effective quark masses, as is δm, so that both may be expected to change with the situation, i.e., baryons vs bosons. In fact, it appears below that δm increases by a factor 3 in going from baryon to light boson. Although we have no proof that λ behaves in an identical fashion, this is the most obvious phenomenological modification that suggests itself; accordingly, $\lambda \sim 1$ in the following. As before we take $a_3/a_2 \approx 0$ and $\langle 1/r \rangle = a_2/f$ with $f \approx -0.5$; then the $\pi^+ - \pi^0$ and $K^+ - K^0$ mass values yield

$$\begin{aligned} &\delta m \approx 5.8 \text{ MeV}, \quad \langle \alpha/r \rangle \approx 2.6 \text{ MeV}, \\ &a_2 \approx -1.3 \text{ MeV} . \end{aligned} \tag{11}$$

This value of $\langle 1/r \rangle$ and that of Ref. 9 are the same. From these we expect[16]

$$\begin{aligned} &K^{*+} - K^{*0} \approx -5.1 \text{ MeV} , \\ &\rho^+ - \rho^0 \approx 0.3 \text{ MeV}, \\ &D^+ - D^0 \approx 7.4 \text{ MeV} , \\ &D^{*+} - D^{+0} \approx 6.7 \text{ MeV} . \end{aligned} \tag{12}$$

The biggest change in Eq. (11) from the baryon values is in δm; it therefore seems plausible to account for the tighter structure of the D mesons relative to the light mesons by using a value of δm averaged between Eqs. (11) and (5). Then

$$\begin{aligned} &D^+ - D^0 \approx 5.4 \text{ MeV} , \\ &D^{*+} - D^{*0} \approx 4.7 \text{ MeV} . \end{aligned} \tag{13}$$

The uncertainty in these estimates appears to be of order 2 MeV.

III. INTRINSIC MASS DIFFERENCE m_d-m_u

The difference of effective masses $m_d - m_u$ has appeared to be strongly momentum-dependent in the above. This is in agreement with the first nonrelativistic correction,[5] which suggests that

$$(\delta m/\delta m_0) = 1 - \langle p^2 \rangle / 2m_0^2 . \tag{14}$$

If we now approximate $\langle p^2 \rangle \sim \langle 1/r \rangle^2$ for the two cases considered, use in Eq. (14) suggests that

$$\delta m_0 \approx 10 \text{ MeV} . \tag{15}$$

This value and the one in Eq. (5) are much like those obtained in Ref. 11 cited above.

Now the n-p mass-difference problem has been made to regress one stage to become the d-u mass-difference problem. This is in principle unsatisfactory; and in practice it is even worse, as the sequence seems to be diverging. The magnitude of δm_0 is five times as large as for n-p, the electric-charge-squared difference is only $\frac{1}{3}$ as great for the quarks, and the algebraic sign is still wrong. Thus it is tempting to look for another, not explicitly electromagnetic source of δm_0 that would be effective first at the quark level.

The weak interactions seem to offer this possibility, and we sketch an heuristic argument. The self-energy of a neutral quark arising from emission and absorption of a W^- will be $\Delta m \sim (2m - \frac{1}{2}m_0)\ln(\Lambda/m_0)$, where m is the mass of the intermediate quark, and Λ is an explicit cutoff. When $m = m_c$ this term is presumably much larger than when $m = m_0$. In terms of the Cabibbo angle θ_C, regarded as fixed by other constraints,[17]

$$\frac{\Delta m_d}{\Delta m_s} \approx \frac{\sin^2\theta_C}{\cos^2\theta_C} = \tan^2\theta_C , \tag{16}$$

these being the relative contributions of the dominant term with $m = m_c$.

To check with our numbers above,

$$\Delta m_s = \lambda_0 m_0 \approx m_0 \approx \frac{\Delta m_d}{\theta_C^2} \approx \frac{10 \text{ MeV}}{0.03} \approx 300 \text{ MeV} , \tag{17}$$

which is a currently popular value for m_0, provided that the relatively small θ_C is acceptable.

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[1]See review article by A. Zee, Phys. Rep. 3C, 127 (1972).

[2]A. Duncan and P. Schattner, Phys. Rev. D 7, 1861 (1973).

[3]L. A. Radicati, L. E. Picasso, D. P. Zanello, and J. J. Sakurai, Phys. Rev. Lett. 14, 160 (1965), and earlier references cited there.

[4]A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn, and V. F. Weisskopf, Phys. Rev. D 9, 3471 (1974); T. de Grand, R. L. Jaffe, K. Johnson, and J. Kiskis, *ibid*. 12, 2060 (1975).

[5]A. De Rújula, H. Georgi, and S. L. Glashow, Phys. Rev. D 12, 147 (1975).

[6]E. Eichten, M. Gottfried, T. Kinoshita, J. Kogut, K. D. Lane, and T.-M. Yan, Phys. Rev. Lett. 34, 369 (1975); A. de Rújula, H. Georgi, and S. L. Glashow, *ibid*. 37, 398 (1976).

[7]K. Lane and E. Eichten, Phys. Rev. Lett. 37, 477 (1976).

[8]S. Ono, Phys. Rev. Lett. 37, 655 (1976).

[9]K. Lane and S. Weinberg, Phys. Rev. Lett. 37, 717 (1976).

[10]N. G. Deshpande, D. A. Dicus, K. Johnson, and K. V. Teplitz, Phys. Rev. Lett. 37, 1305 (1976).

[11]W. Celmaster, Phys. Rev. Lett. 37, 1042 (1976).

[12]Simple quadratic mass dependence of the effect determined here would reduce our estimate by a factor of order (m_K/m_D) to $D^+ - D^0 \approx 2$ MeV, in agreement with J. Daboul and M. Krammer, Phys. Lett. 64B, 341 (1976). By a combination of quadratic and linear mass dependences, H. Fritzsch, Phys. Lett. 63B, 419 (1976), obtains a result midway between the maximum (linear mass dependence) of ~13 MeV and the minimum (quadratic mass dependence) of ~2 MeV.

[13]Particle Data Group, Rev. Mod. Phys. 48, S1 (1976).

[14]T. Applequist and H. D. Politzer, Phys. Rev. Lett. 34, 41 (1975).

[15]J. S. Kang and H. J. Schnitzer, Phys. Rev. D 12, 2791 (1975); H. J. Schnitzer, *ibid*. 13, 74 (1976).

[16]D. B. Lichtenberg, Phys. Rev. D 12, 3760 (1975).

[17]E.g., R. J. Oakes, Phys. Lett. 29B, 683 (1969).