Electromagnetic mass differences of charmed baryons in the charmed-quark model

Seiji Ono*†
III. Physikalisches Institut, Technische Hochschule Aachen, Aachen, West-Germany
(Received 30 August 1976)

Electromagnetic mass differences of charmed baryons are calculated using the harmonic-osciallator charmed-quark model.

After the discovery of charmed mesons D and D^*,[1,2] charmed baryons are expected to be found. In this note we study electromagnetic mass differences of charmed baryons in the charmed-quark model. In our previous papers[3,4] it was shown that the harmonic-oscillator quark model with R^2 (wave-function radius) $=2.75$ GeV^{-2} can be used consistently and with remarkable success to explain electromagnetic mass differences of baryons, amplitudes of the processes $\gamma N \rightarrow N^* \rightarrow N\pi$, and other properties of baryons. In an analogous way we studied properties of uncharmed mesons[4] and charmed mesons.[5]

Following Refs. 3–5 we make the following assumptions in studying charmed baryons:

(a) The electromagnetic mass differences of charmed baryons are caused (i) by the mass difference (Δm_e) between the p quark (up quark) and the n quark (down quark), (ii) by the Coulomb force among quarks, and (iii) by the magnetic hyperfine interaction.

(b) The value of the gyromagnetic ratio of all quarks is unity, i.e., the magnetic moment $=$charge/2$\times$ mass. From magnetic moments of uncharmed baryons one gets $\mu_q = \mu_p = 2.793\ e/2m_p$ and m_q (mass of uncharmed quarks) ~ 336 MeV.

(c) From the mass spectrum of mesons (ψ, D^0, ρ, π, etc.) we guess $m_c = 4m_q - 5m_q$.

We use harmonic-oscillator wave functions for s-state cqq systems and for s-state ccq systems (q is the uncharmed quark):

$$\psi_{cqq} = N \exp\left(-\frac{|\vec{r}_1|^2}{2R_q^2} - \frac{|\vec{r}_2|^2}{2R_q^2} - \frac{|\vec{r}_3|^2}{2R_c^2}\right), \tag{1}$$

$$\psi_{qcc} = N \exp\left(-\frac{|\vec{r}_1|^2}{2r_c^2} - \frac{|\vec{r}_2|^2}{2r_c^2} - \frac{|\vec{r}_3|^2}{2r_q^2}\right). \tag{2}$$

In Eq. (1) [Eq. (2)] $\vec{r}_1$ and $\vec{r}_2$ are coordinates of two uncharmed (charmed) quarks and $\vec{r}_3$ is a coordinate of a charmed (uncharmed) quark. Since we know that SU(3)-symmetric wave functions work very well,[3-5] we assume the same R_q (or r_q) for all uncharmed quarks.

Taking into account the large mass difference between m_q and m_c one gets for the cqq system.

$$\langle |\vec{r}_1 - \vec{r}_2|^2 \rangle = \frac{\int |\psi|^2 |\vec{r}_1 - \vec{r}_2|^2 \delta(\vec{R} - [m_q(\vec{r}_1 + \vec{r}_2) + m_c \vec{r}_3]/(2m_q + m_c))\, d\vec{r}_1 d\vec{r}_2 d\vec{r}_3}{\int |\psi|^2 \delta(\vec{R} - [m_q(\vec{r}_1 + \vec{r}_2) + m_c \vec{r}_3]/(2m_q + m_c))\, d\vec{r}_1 d\vec{r}_2 d\vec{r}_3} = 3R_q^2, \tag{3}$$

$$\langle |\vec{r}_2 - \vec{r}_3|^2 \rangle = 3\left\{\frac{2}{(m_q + m_c)^2}\left(\frac{m_q^2}{R_c^2} + \frac{m_c^2}{R_q^2}\right) - \frac{2m_q^2(m_q/R_c^2 - m_c/R_q^2)^2}{(m_q + m_c)^2\{m_q^2/R_c^2 + [m_q^2 + (m_q + m_c)^2]/R_q^2\}}\right\} \equiv 3R_{cq}^2, \tag{4}$$

$$\left\langle \frac{1}{|\vec{r}_1 - \vec{r}_2|} \right\rangle = \left(\frac{2}{\pi}\right)^{1/2} \frac{1}{R_q}, \quad \left\langle \frac{1}{|\vec{r}_2 - \vec{r}_3|} \right\rangle = \left(\frac{2}{\pi}\right)^{1/2} \frac{1}{R_{cq}}, \tag{5}$$

$$\langle \delta(\vec{r}_1 - \vec{r}_2) \rangle = \frac{1}{(2\pi)^{3/2} R_q^3}, \qquad \langle \delta(\vec{r}_2 - \vec{r}_3) \rangle = \frac{1}{(2\pi)^{3/2} R_{cq}^3}. \tag{6}$$

In the same way we get for the qcc system

$$\langle |\vec{r}_1 - \vec{r}_2|^2 \rangle = 3r_c^2, \quad \langle |\vec{r}_2 - \vec{r}_3|^2 \rangle \equiv 3r_{cq}^2, \tag{7}$$

$$\left\langle \frac{1}{|\vec{r}_1 - \vec{r}_2|} \right\rangle = \left(\frac{2}{\pi}\right)^{1/2} \frac{1}{r_c}, \qquad \left\langle \frac{1}{|\vec{r}_2 - \vec{r}_3|} \right\rangle = \left(\frac{2}{\pi}\right) \frac{1}{r_{cq}}, \tag{8}$$

$$\langle\delta(\vec{r}_1-\vec{r}_2)\rangle=\frac{1}{(2\pi)^{3/2}r_{c3}},$$
$$\langle\delta(\vec{r}_2-\vec{r}_3)\rangle=\frac{1}{(2\pi)^{3/2}r_{cq}{}^3}. \quad (9)$$

We take

$$R_q{}^2=2.75\ \text{GeV}^{-2},\quad r_c=\tfrac{1}{2}R_q=\tfrac{2}{3}R_{cq}=\tfrac{2}{3}r_{cq},$$
$$\Delta m_e=-1.83\ \text{MeV}. \quad (10)$$

The value $\Delta m_e=-1.83$ MeV for baryons is different from that for bosons.[4,6] However, we have shown in Ref. 3 that this choice of parameters $\Delta m_e=-1.83$ MeV and $R^2=2.75\ \text{GeV}^{-2}$ is a very successful one in that it can explain the properties of baryons (electromagnetic mass difference, photoproduction and electroproduction processes).

Now we must construct an SU(8) wave function for charmed baryons. We assume that each wave function must be totally symmetric under an exchange of any two quarks. However, the spatial s-state wave function of the charmed baryon is not symmetric because of the large mass difference between m_q and m_c and because of the difference between R_q and R_c (or r_q and r_c). Therefore, in order to construct the symmetric total wave function we must use the broken SU(8) wave function. As an example we show the symmetric s-state wave function for C_1^{++} (we use the notation introduced by Gaillard, Lee, and Rosner[7] for charmed baryons):

$$C_1^{++}=\frac{1}{\sqrt{18}}[2\mathcal{P}_\uparrow c_\downarrow\mathcal{P}_\uparrow\psi(1,3,2)+2\mathcal{P}_\uparrow\mathcal{P}_\uparrow c_\downarrow\psi(1,2,3)+2c_\downarrow\mathcal{P}_\uparrow\mathcal{P}_\uparrow\psi(3,2,1)-\mathcal{P}_\uparrow\mathcal{P}_\downarrow c_\uparrow\psi(1,2,3)-\mathcal{P}_\uparrow c_\uparrow\mathcal{P}_\downarrow\psi(1,3,2)$$
$$-\mathcal{P}_\downarrow c_\uparrow\mathcal{P}_\uparrow\psi(1,3,2)-c_\uparrow\mathcal{P}_\downarrow\mathcal{P}_\uparrow\psi(3,2,1)-c_\uparrow\mathcal{P}_\uparrow\mathcal{P}_\downarrow\psi(3,2,1)-\mathcal{P}_\downarrow\mathcal{P}_\uparrow c_\uparrow\psi(1,2,3)], \quad (11)$$

where $\psi(1,3,2)$ can be obtained by the permutation of coordinates r_1, r_2, r_3 in Eq. (1). Wave functions of other states are constructed in a similar way.

The symmetric s-state wave function belongs to the representation $120=(20,4)+(20',2)$ in SU(8), which corresponds to $56=(10,4)+(8,2)$ in SU(6). We list our results of electromagnetic mass differences of charmed baryons in Table I, where we define

$$a=R_{cq}/R_q,\quad b=r_{cq}/r_c,$$
$$d=(R_{cq}/R_q)^3,\quad f=(r_{cq}/r_c)^3, \quad (12)$$
$$r=m_q/m_c\sim\tfrac{1}{4}.$$

The electromagnetic mass difference can be calculated according to the following formula:

$$\Delta m=\Delta m_e\alpha+\left(\frac{2}{\pi}\right)^{1/2}\frac{e^2}{R_0}C$$
$$-\tfrac{8}{3}\pi\mu_p{}^2\frac{1}{(2\pi)^{3/2}}\frac{1}{R_0{}^3}H, \quad (13)$$

where

$R_0=R_{cq}$ for the cqq system,

$R_0=r_{cq}$ for the ccq system,

$\mu_p=2.793\ e/2m_p$.

From Table I we can derive the following sum rules:

$$C_1^{*+}-C_1^{*0}=S^{*+}-S^{*0},$$
$$C_1^{+}-C_1^{0}=S^{+}-S^{0},$$

TABLE I. Electromagnetic mass differences of charmed baryons. We follow the notation introduced by Gaillard *et al.*, Ref. 7, for charmed baryons. Parameters a, d, and r are defined in Eq. (12).

(SU(4), 2S+1)		α	C	H	Mass difference (MeV)
(20,4)	$C_1^{*++}-C_1^{*+}$	1	$\frac{2}{3}a+\frac{2}{3}$	$\frac{2}{3}d+\frac{2}{3}r$	1.63
	$C_1^{*+}-C_1^{*0}=S^{*+}-S^{*0}$	1	$-\frac{1}{3}a+\frac{2}{3}$	$-\frac{1}{3}d+\frac{2}{3}r$	~0.00
	$X_u^{*++}-X_d^{*+}$	1	$\frac{4}{3}$	$\frac{4}{3}r$	2.92
(20′,2)	$C_1^{++}-C_1^{+}$	1	$\frac{2}{3}a+\frac{2}{3}$	$\frac{2}{3}d-\frac{4}{3}r$	3.86
	$C_1^{+}-C_1^{0}=S^{+}-S^{0}$	1	$-\frac{1}{3}a+\frac{2}{3}$	$-\frac{1}{3}d-\frac{4}{3}r$	2.24
	$X_u^{++}-X_d^{+}$	1	$\frac{4}{3}$	$-\frac{8}{3}r$	7.39
	$A^{+}-A^{0}$	1	$-\frac{1}{3}a+\frac{2}{3}$	d	−1.77

$$(C_1^{*++} - C_1^{*+}) - (C_1^{++} - C_1^{+}) = (C_1^{*+} - C_1^{*0}) - (C_1^{+} - C_1^{0})$$
$$= \tfrac{1}{2}[(X_u^{*++} - X_d^{*+}) - (X_u^{++} - X_d^{+})]$$
$$= 2\frac{m_q}{m_c}\left(-\frac{8}{3}\pi\right)\mu_p^2 \frac{1}{(2\pi)^{3/2}}\frac{1}{R_0^3}$$
$$\equiv -2H_s < 0 . \quad (14)$$

In Eq. (13) the last term which comes from the magnetic hyperfine interaction is not negligibly small but gives a (10–20)% contribution. H_s in Eq. (14) comes only from the magnetic hyperfine interaction and we can determine the strength of the magnetic hyperfine interaction by measuring the mass differences in Eq. (14).

From Table I we notice that as a rule in the same isospin multiplet the larger the charmed baryon's charge, the heavier it becomes. These results are opposite to that of the uncharmed baryons, in which the mass difference between the n quark (down quark) and the p quark (up quark) plays an important role. This opposite result in the charmed baryon is caused by the large Coulomb interaction between the charmed quark with the rather large positive charge and uncharmed quarks. This result is also dependent on the small value of $|\Delta m_e|$. If we use $\Delta m_e = -7.06$ MeV as is used for mesons, the conclusion is reversed.

The author is grateful to Dr. D. G. Sutherland for discussions. He would like to thank the Alexander von Humboldt Foundation for financial support.

---

*Work supported in part by the Alexander von Humbolt Foundation.

†Present address: Department of Physics, University of California, Davis, California 95616.

[1]V. Lüth, work presented at Neutrino Conference, Aachen, W. Germany, 1976 (unpublished).

[2]A. De Rûjula, work presented at Neutrino Conference, Aachen, W. Germany, 1976 (unpublished).

[3]S. Ono, Nucl. Phys. B107, 522 (1976).

[4]S. Ono (unpublished).

[5]S. Ono, Phys. Rev. Lett. 37, 655 (1976).

[6]T. Minamikawa *et al.*, Prog. Theor. Phys. Suppl. 37–38, 56 (1976).

[7]M. K. Gaillard, B. W. Lee, and J. L. Rosner, Rev. Mod. Phys. 47, 227 (1975).