Can the quark-parton model give the correct neutron charge radius?*

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The question posed cannot be answered straightforwardly because no unique quark-parton distribution functions are available. By using only the data on deep-inelastic electron-nucleon structure functions, we have obtained an inequality which must be satisfied for an affirmative answer.

I. INTRODUCTION

So far, the quark-parton model has not succeeded in giving a satisfactory account for the mean-square charge radius $\langle r_n^2\rangle$ of the neutron.[1] In I it has been shown that in order to yield the correct (negative) sign for $\langle r_n^2\rangle$, the quark-parton distribution functions would most likely have to behave somewhat unexpectedly in the region $x \lesssim 0.3$. In this paper we consider in some detail this type of distribution functions which could give the correct sign, and we arrive at an inequality that must be satisfied for the magnitude of $\langle r_n^2\rangle$ to be correct as well. This inequality does not need the explicit form of the distribution functions.

In the next section the basic assumptions employed are clearly stated, and the expression for $\langle r_n^2\rangle$ is given within the framework of the quark-parton model. In Sec. III the above-mentioned inequality is obtained. An numerical estimation of the inequality is given in the final section.

II. NEUTRON CHARGE RADIUS IN THE QUARK-PARTON MODEL

In the quark-parton model the mean-square charge radius of the neutron is given by[1,2]

$$\langle r_n^2\rangle = \int_0^1 dx\,\{d(x)g_d(x) - \bar{d}(x)g_{\bar{d}}(x) - \tfrac{1}{2}[u(x)g_u(x) - \bar{u}(x)g_{\bar{u}}(x) + s(x)g_s(x) - \bar{s}(x)g_{\bar{s}}(x)]\}\,,$$

where the u, $\bar{u}$, d, $\bar{d}$, s, and $\bar{s}$ denote the quark-parton distribution functions inside the *proton*, and

$$g_i(x) \equiv \langle b^2\rangle_{i,x} = \frac{\int d^2b\, b^2 h_i(x,b)}{\int d^2b\, h_i(x,b)} = \int d^2b\, b^2 h_i(x,b)/i(x)\,. \tag{2.1}$$

In Eq. (2.1) $h_i(x,b)$ is the distribution function of quark-parton of type i in the transverse plane, with a fraction x of the total longitudinal momentum. In the valence-sea version[3] of the quark-parton model, we have

$$u(x)g_u(x) = u_v(x)g_{u_v}(x) + c(x)g_c(x)\,,$$
$$d(x)g_d(x) = d_v(x)g_{d_v}(x) + c(x)g_c(x)\,,$$
$$\bar{u}(x)g_{\bar{u}}(x) = \bar{d}(x)g_{\bar{d}}(x) \equiv c(x)g_c(x)\,,$$
$$s(x)g_s(x) = \bar{s}(x)g_{\bar{s}}(x) \equiv c'(x)g_{c'}(x)\,,$$

where suffix v refers to the valence part and c refers to the core part. We then have the simpler expression

$$\langle r_n^2\rangle = \int_0^1 dx\,[d_v(x)g_{d_v}(x) - \tfrac{1}{2}u_v(x)g_{u_v}(x)]\,. \tag{2.2}$$

Since we restrict ourselves throughout to the valence-sea version only, hereafter we shall drop the suffix v.

As in I, we assume that

(i) $g_u(x)$ and $g_d(x)$ are montonically decreasing functions of x,

and

$$\text{(ii)}\quad g_u(x) \leqslant g_d(x)\,. \tag{2.3}$$

These assumptions have some experimental support[4] and (i) is also suggested by certain theoretical models.[5]

In principle, the quark-parton distribution functions needed in Eq. (2.2) for the evaluation of $\langle r_n^2\rangle$ can be extracted from the directly measurable deep-inelastic lepton-nucleon structure functions. In practice, however, the data are rather crude and are not sufficient for an unique or clear-cut extraction. Therefore, instead of performing a somewhat arbitrary parametrization for the quark-parton distribution functions and calculating $\langle r_n^2\rangle$, we prefer to adopt a more general, less specific approach to get an inequality instead. This is carried out in the next section.

III. AN INEQUALITY INVOLVING $\langle r_n^2\rangle$

Using Eq. (2.2) (we have dropped the suffix v) and Eq. (2.3), we get

$$\langle r_n^2\rangle \geqslant \int_0^1 dx\, q(x)g_u(x)\,. \tag{3.1}$$

Here

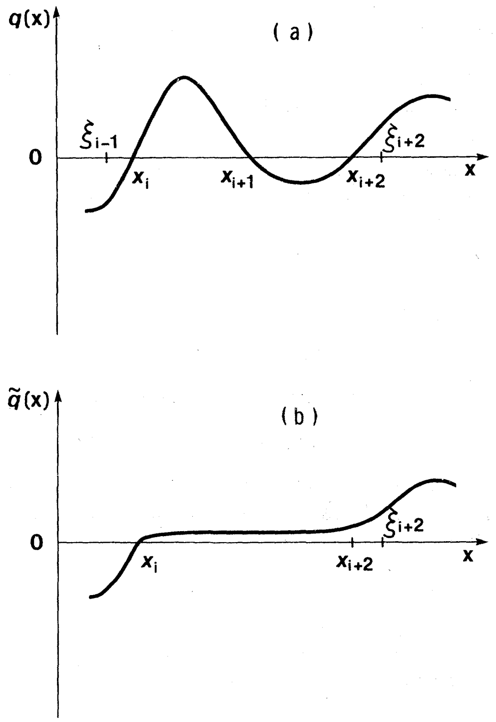

FIG. 1. (a) An example of a portion of the quark-parton distribution function $q(x)$. (b) The same portion after deformation.

$$q(x) \equiv d(x) - \tfrac{1}{2}u(x) \ ,$$

and the charge neutrality of the neutron requires

$$\int_0^1 dx\, q(x) = 0 \ .$$

Let us suppose, quite generally, that $q(x)$ have zeroes at $x = x_j$ $(j = 1, 2, 3, \ldots, N)$. Experimentally we know that the largest zero $x_N \lesssim 0.3$ and the function $q(x)$ is negative for $x_N < x < 1$. [See Eq. (3.10) and Fig. 3 below.] Consider the segment between x_i and x_{i+2} [see Fig. 1(a)]. Let us define

$$S(x, x') \equiv \int_x^{x'} q(x)dx \ .$$

Two cases can be distinguished.

(*a*) $S(x_i, x_{i+2}) \geq 0$. While keeping the value of the integral $S(x_i, \xi_{i+2})$ unchanged, we deform $q(x)$ in this interval (x_i, ξ_{i+2}) such that it is now nonnegative. Here ξ_{i+2} is any point in (x_{i+2}, x_{i+3}). [See illustration in Fig. 1(b).] We denote this deformed function by $\tilde{q}(x)$. Since $g_u(x)$ is a monotonically decreasing (positive) function [assumption (i)], the above deformation can be made in such a way that

$$\int_{x_i}^{\xi_{i+2}} q(x)g_u(x)dx \geq \int_{x_i}^{\xi_{i+2}} \tilde{q}(x)g_u(x)dx \ . \qquad (3.2)$$

(*b*) $S(x_i, x_{i+2}) < 0$. We deform $q(x)$ such that the new $\tilde{q}(x)$ is negative in the interval (ξ_{i-1}, x_{i+2}), but again keeping the value of $S(\xi_{i-1}, x_{i+2})$ unchanged. Again Eq. (3.2) can be made to hold. The above procedure can be repeated until finally we get a reference function $\tilde{q}(x)$ which is either of type A or type B, as illustrated in Fig. 2. We have now labeled the smallest zeroes by α, and the other one (in type B) by β. By our construction, it is clear that

$$\int_{x_k}^{x_l} \tilde{q}(x)dx = \int_{x_k}^{x_l} q(x)dx$$

and

$$\int_0^1 \tilde{q}(x)g_u(x)dx \leq \int_0^1 q(x)g_u(x)dx \ , \qquad (3.3)$$

where x_k and x_l are any of the zeroes (including $x=0$ and $x=1$) of $\tilde{q}(x)$. [These are of course also the remaining zeroes of the original $q(x)$.]

Naturally there are still an infinite number of $\tilde{q}(x)$ having these required properties. We can further require

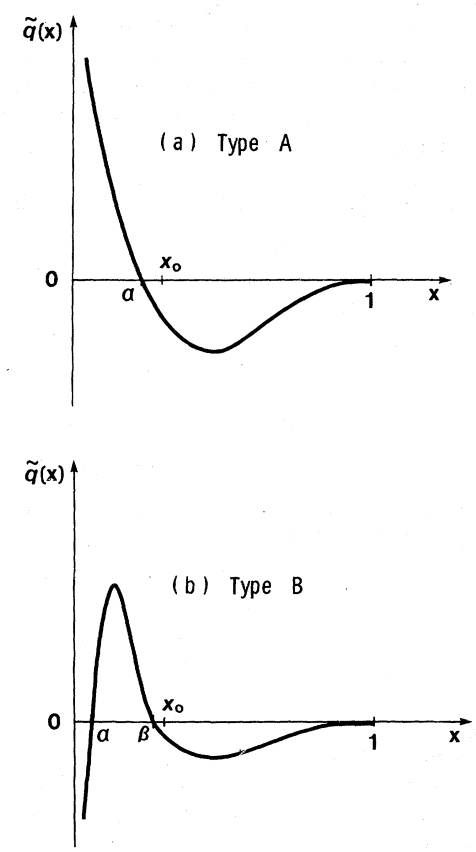

FIG. 2. Two types of the reference function $\tilde{q}(x)$. $x_0 \approx 0.3$ is a zero point of the function $q^{\max}(x)$ (see Fig. 3).

$$\tilde{q}(x) \leqslant q^{\max}(x)\ , \tag{3.4}$$

where $q^{\max}(x)$ is defined in Eq. (3.9) below. A little reflection will convince one that the constraint (3.4) can be accommodated without compromising any of our earlier requirements. [Of course, even with Eq. (3.4), there are still an infinite number of acceptable $\tilde{q}(x)$.]

As mentioned in I, type A is favored by Regge-pole theory, and it is the actual quark-parton distribution function employed by many authors.[1] It was the purpose of I to show that a positive value for the neutron mean-sqaure charge radius would result from this type of $q(x)$. We now investigate type B in detail. (Unlike many others, Van Hove's model[6] is in fact of this type.)

In view of the mentioned properties of $g_u(x)$ $\tilde{q}(x)$, and $q(x)$, we have

$$\int_0^1 \tilde{q}(x) g_u(x) dx$$

$$\geqslant g_u(0)\int_0^\alpha \tilde{q}(x)dx + g_u(\beta)\int_\alpha^1 \tilde{q}(x)dx$$

$$= -[g_u(0) - g_u(\beta)]\int_\alpha^1 q(x)dx\ . \tag{3.5}$$

Combining Eqs. (3.1), (3.3), and (3.5) we get

$$\int_\alpha^1 q(x)dx \geqslant \frac{|\langle r_n{}^2\rangle|}{g_u(0) - g_u(\beta)} \equiv Q\ . \tag{3.6}$$

Since in the valence-sea version of the quark-parton model we have[3]

$$F_1^{ep}(x) = \tfrac{1}{9}[4u(x) + d(x)] + \tfrac{10}{9}[c(x) + \tfrac{1}{5}c'(x)]\ ,$$
$$F_1^{en}(x) = \tfrac{1}{9}[u(x) + 4d(x)] + \tfrac{10}{9}[c(x) + \tfrac{1}{5}c'(x)]\ , \tag{3.7}$$

where $F_1^{ep}(x)$ and $F_1^{en}(x)$ are the usual deep-inelastic structure functions[7] and are measurable quantities, we can write

$$q(x) \equiv d(x) - \tfrac{1}{2}u(x)$$
$$= \tfrac{9}{10}[3F_1^{en}(x) - 2F_1^{ep}(x)] - [c(x) + \tfrac{1}{5}c'(x)]\ . \tag{3.8}$$

If we introduce

$$q^{\max}(x) \equiv \tfrac{9}{10}[3F_1^{en}(x) - 2F_1^{ep}(x)]\ , \tag{3.9}$$

we have

$$q(x) \leqslant q^{\max}(x)\ . \tag{3.10}$$

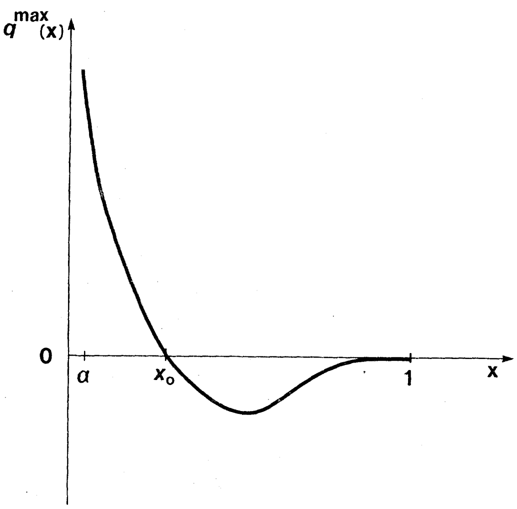

FIG. 3. Schematic sketch of the function $q^{\max}(x)$.

It is seen that $q^{\max}(x)$ is directly measurable in deep-inelastic electron-nucleon scattering, whereas $q(x)$ is not. Figure 3 sketches $q^{\max}(x)$ schematically.

We now combine Eqs. (3.6) and (3.10) to get

$$\int_\alpha^1 q^{\max}(x)dx \geqslant Q\ . \tag{3.11}$$

For a given Q, there is a maximum value of α, say $\alpha^{\max}(Q)$, beyond which Eq. (3.11) no longer holds. It is seen that $\alpha^{\max}(q)$ is a monotonically decreasing function of Q (see Fig. 3).

Equation (3.6) with Eqs. (3.8) and (3.7) yield

$$\int_\alpha^1 d(x)dx \geqslant 3\int_\alpha^1 [F_1^{ep}(x) - F_1^{en}(x)]dx + 2Q\ . \tag{3.12}$$

In view of the quark-parton-model sum rule, the left-hand side of Eq. (3.12) is smaller than unity. Thus Eq. (3.12) becomes

$$3\int_\alpha^1 [F_1^{ep}(x) - F_1^{en}(x)]dx \leqslant 1 - 2Q\ . \tag{3.13}$$

For a given Q, there is a minimum of α, say $\alpha^{\min}(Q)$, below which Eq. (3.13) no longer holds. $\alpha^{\min}(Q)$ is a montonically increasing function of Q.

Thus Eqs. (3.11) and (3.13) together require $\alpha^{\min}(Q) \leqslant \alpha \leqslant \alpha^{\max}(Q)$ for a given Q. There is a critical value of Q, say Q_c, for which we have $\alpha^{\min}(Q_c) = \alpha^{\max}(Q_c) \equiv \alpha_c$. Clearly, we have $Q \leqslant Q_c$. Hence from Eq. (3.6) we have[8]

$$g_u(0) - g_u(\beta) = \frac{|\langle r_n{}^2\rangle|}{Q} \geqslant \frac{|\langle r_n{}^2\rangle|}{Q_c} = \frac{0.12}{Q_c}\ \text{fm}^2 \equiv [g_u(0) - g_u(\beta)]_{\min}\ . \tag{3.14}$$

TABLE I. Calculated values of Q_c, α_c, and $[g_u(0) - g_u(\beta)]_{\min}$.

Data source	Q_c	α_c	$[g_u(0)-g_u(\beta)]_{\min}$ (fm^2)
Ref. 12	0.12	0.032	0.98
Ref. 13	0.14	0.030	0.84
Ref. 3 [a]	0.15, 0.14	0.032, 0.029	0.78, 0.84
Ref. 14	0.14	0.045	0.84
Ref. 15 [b]	0.16–0.18	0.046–0.048	0.65–0.73

[a]The first and second values correspond to their x and x' fits, respectively.

[b]The range corresponds to the choice $0.69 < \epsilon < 1$ where ϵ is a parameter in the model.

IV. ESTIMATION OF $[g_u(0) - g_u(\beta)]_{\min}$

Recent deep-inelastic lepton-nucleon scattering data suggest a violation of Bjorken scaling.[9] If we follow the spirit of the scale-invariant parton model proposed by Kogut and Susskind,[10] we need the data for the structure functions $F_1^{ep}(x, Q^2)$ and $F_1^{en}(x, Q^2)$ in the Q^2 range where we can "see the quark partons." The SLAC-MIT data in the small-x region, however, correspond to small Q^2, say $Q^2 \leq 2$ $(\mathrm{GeV}/c)^2$, and these may not be in the range we need. Besides, any straightforward fit of the SLAC-MIT data does not seem to satisfy the quark-parton-model sum rules.[11] Thus, in this paper, as our "data" for the structure functions, we adopt the fits performed by various authors[3,12-15] *based on the quark-parton models*. It should be emphasized that we do not use their quark-parton distribution functions, which are further extracted from the structure functions F_1^{ep} and F_1^{en}.

Table I summarizes the results for $[g_u(0) - g_u(\beta)]_{\min}$. It is seen that the results are quite consistent, and it is safe to say $[g_u(0) - g_u(\beta)]_{\min} \gtrsim 0.65$ fm^2. We may mention that, with the explicit quark-parton distribution functions of Van Hove,[6] we get, from Eq. (3.6), $g_u(0) - g_u(\beta) \geq 2.1$ fm^2.

In view of the fact that many inequalities have been used in arriving at the final result, Eq. (3.14), it seems likely that in fact

$$g_u(0) \geq g_u(0) - g_u(\beta) \gg 0.65 \text{ fm}^2 ,$$

or

$$\langle p_\perp{}^2 \rangle_{u,x=0} \approx 1/g_u(0) \ll (240 \text{ MeV}/c)^2 , \qquad (4.1)$$

where $\langle p_\perp{}^2 \rangle_{u,x=0}$ is the mean-square transverse momentum of the u quark with $x=0$ in a proton. Here it might be added that the data[8] on the proton mean-square charge radius demands only $g_u(0) > 0.47$ fm^2 in our framework. While it is not possible to determine $\langle p_\perp{}^2 \rangle_{u,x=0}$ precisely from other experimental evidence, it appears doubtful that Eq. (4.1) can be realized.[16] This, together with our findings in I, leads us to a pessimistic view about the successful evaluation of the neutron mean-square charge radius in the framework of the valence-sea version of the quark-parton model.

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[1]A. Niégawa and D. Kiang, Phys. Rev. D 14, 3235 (1976). This is referred to as I hereafter. See also the references cited therein. The left-hand side of Eq. (8) in I should read $f_2^{ep}(x)/x$ and $f_2^{en}(x)/x$.

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[3]See, e.g., V. Barger and R. J. N. Phillips, Nucl. Phys. B73, 269 (1974).

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[5]J. Kogut and L. Susskind, Phys. Rep. 8C, 75 (1973); P. V. Landshoff, Phys. Lett. 66B, 452 (1977).

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[7]For convenience we simply write $F_1(x)$ in place of $F_1(x,Q^2)$, though this does not necessarily mean that we assume Bjorken's scaling. See Sec. IV.

[8]We have used the values $\langle r_n{}^2 \rangle = -0.12$ fm^2 and $\langle r_p{}^2 \rangle = 0.70$ fm^2 which are quoted in G. Höhler *et al.*, Nucl. Phys. B114, 505 (1976), with relevant references.

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[11]See, e.g., E. D. Bloom, in *Proceedings of the Sixth International Symposium on Electron and Photon Interactions at High Energy, Bonn, Germany, 1973,* edited by H. Rollnik and W. Pfeil (North-Holland, Amsterdam, 1974), p. 227.

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