Radiative meson decays in octet-broken SU(3)

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A vector-dominance model proposed by the present authors and Munczek to account for radiative and certain strong decays of mesons is completed by the inclusion of η-η' mixing, and updated in the light of recent new measurements. As in the earlier work, excellent supporting evidence is found for the current-mixing form of ω-ϕ mixing and for large octet-type symmetry breaking in the vertex involving two vector mesons and one pseudoscalar meson.

A great deal of interest in radiative decays of mesons has been stimulated by recent measurements,[1,2] particularly of $\phi\to\pi\gamma$, $\rho\to\pi\gamma$, $\phi\to\eta\gamma$, and $K^{*0}\to K^0\gamma$, which are not well accounted for by naive quark models or simple vector-dominance schemes.[3] About eight years ago the present authors and Munczek proposed a vector-dominance model[4] that included an octet-broken version of the Gell-Mann, Sharp, and Wagner interaction vertex[5] (two vector mesons and one pseudoscalar meson) and current mixing for the vector mesons.[6,7] In BMS we envisaged large symmetry-breaking effects in the PVV vertex other than those due to singlet-octet mixings, and particularly pointed to a $K^{0*}\to K^0+\gamma$ rate much smaller than the SU(6)-symmetric value.

Since the recent experiments appear to vindicate our general approach, and since sizable changes have occurred in certain input quantities (especially the rate[8] for $\eta\to 2\gamma$), we have decided to update our earlier work and to complete it by the inclusion of η-η' mixing.

The vector mesons V_μ^a in our model (only the nine of lowest mass) are described by a generalized Yang-Mills Lagrangian density

$$\mathcal{L}_V=-\tfrac{1}{4}K^{ab}V^a_{\mu\nu}V^b_{\mu\nu}+\tfrac{1}{2}m^2V^a_\mu V^a_\mu -\tfrac{1}{4}K^{00}V^0_{\mu\nu}V^0_{\mu\nu}+\tfrac{1}{2}m^2V^0_\mu V^0_\mu-\tfrac{1}{2}K^{80}V^8_{\mu\nu}V^0_{\mu\nu}\ , \tag{1}$$

with $a,\ b=1\cdots 8$, and

$$V^a_{\mu\nu}=\partial_\mu V^a_\nu-\partial_\nu V^a_\mu-gf^{abc}V^b_\mu V^c_\nu\ .$$

The K's are fitted to the mass splittings of the vector mesons, and provide for ω-ϕ mixing in the vector-mixing model; K^{ab} is the diagonal matrix

$$K^{ab}=\delta^{ab}+\sqrt{3}\,\epsilon_0 d^{ab8}. \tag{2}$$

Diagonalizing (1) in terms of the physical ω and ϕ, we obtain

$$\begin{aligned}V^{1,2,3}_\mu&=\frac{1}{(K_\rho)^{1/2}}\,\rho^{1,2,3}_\mu\\ V^{4,5,6,7}_\mu&=\frac{1}{(K_{K^*})^{1/2}}\,K^{*4,5,6,7}_\mu\ ,\\ V^8_\mu&=\frac{\sin\theta}{(K_\omega)^{1/2}}\,\omega_\mu-\frac{\cos\theta}{(K_\phi)^{1/2}}\,\phi_\mu\ ,\\ V^0_\mu&=\frac{\cos\theta}{(K_\omega)^{1/2}}\,\omega_\mu+\frac{\sin\theta}{(K_\phi)^{1/2}}\,\phi_\mu\ ,\end{aligned} \tag{3}$$

with

$$K_i=m^2/m_i^{\,2}\quad (i=\rho, K^*, \omega, \phi)\ .$$

Satisfactory agreement with observed masses is obtained for $\theta=30°$ and $m^2=0.718$ GeV.[2]

The pseudoscalar mesons P^a are described by the usual Lagrangian density with appropriate covariant derivative, octet-broken mass^2 matrix, and a singlet-octet mixing angle $\phi=-10.5°$. Thus,

$$P^8=\cos\phi\eta-\sin\phi\eta'\ ,\quad P^0=\sin\phi\eta+\cos\phi\eta'\ . \tag{4}$$

Using the vector-dominance approach of Gell-Mann, Sharp, and Wagner in general octet-broken SU(3), we have, as in BMS,

$$\mathcal{L}_{PVV}=\tfrac{1}{4}\epsilon_{\alpha\beta\mu\nu}(hD^{abc}V^a_{\alpha\beta}V^b_{\mu\nu}P^c+\lambda D^{ab}V^a_{\alpha\beta}P^bV^0_{\mu\nu}), \tag{5}$$

with $a, b, c=1\cdots 8$, and

$$\begin{aligned}D^{abc}&=d^{abc}+\sqrt{3}\,\epsilon_1 d^{abd}d^{d8c}\\&\quad+\tfrac{1}{2}\sqrt{3}\,\epsilon_2(d^{acd}d^{d8b}+d^{bcd}d^{d8a})\\&\quad+(\epsilon_3/\sqrt{3})\delta^{ab}\delta^{c8},\end{aligned} \tag{6}$$

$$D^{ab}=\delta^{ab}+\sqrt{3}\,\epsilon_4 d^{ab8}\ . \tag{7}$$

The interactions involving the ninth pseudoscalar P^0 have an analogous form which we will discuss below.

To complete the model we introduce the effective electromagnetic interaction[7]

$$\mathcal{L}_{\rm em} = (em^2/g)[K_\rho^{-1/2}\rho_\mu^3 + (3K_\omega)^{-1/2}\omega_\mu \sin\theta - (3K_\phi)^{-1/2}\phi_\mu \cos\theta]A_\mu . \quad (8)$$

In the formalism described, the quantity g is related to the $\rho\pi\pi$ form factor $g_{\rho\pi\pi}(p^2)$ by

$$g = g_{\rho\pi\pi} K_\rho^{1/2} . \quad (9)$$

We have used the value $g^2/4\pi = 3.40$, corresponding to a ρ-meson width of 146 MeV, when the momentum dependence of the form factor is neglected.

We shall consider four three-body processes: ω or $\phi \to 3\pi$, and η or $\eta' \to \pi^+\pi^-\gamma$. For these we calculate the invariant amplitude

$$F\epsilon_{\alpha\beta\mu\nu} e_\alpha^V k_\beta^V p_\mu^1 p_\nu^2 (p^2 - m_\rho^2)^{-1}, \quad (10)$$

where p_μ^1 and p_μ^2 are two pion four-momenta, and $p_\mu = p_\mu^1 + p_\mu^2$. The superscript V refers to the ω or ϕ in the three-pion decays and to the photon in the η and η' decays. Similarly, the two-body decays we consider, of types $P \to V + \gamma$, $V \to P + \gamma$, and $P \to 2\gamma$, have the invariant amplitude

$$F\epsilon_{\alpha\beta\mu\nu} e_\alpha^{V_1} k_\beta^{V_1} e_\mu^{V_2} k_\nu^{V_2} , \quad (11)$$

where the V_i refer either to vector mesons or photons.

In BMS, expressions for F are given for a dozen decays in the form of the SU(6)-symmetric limit for each decay mode, multiplied by a symmetry-breaking factor depending on θ and on the K's and ϵ's. It was assumed there that the small effect of η-η' mixing and the small $\phi \to 3\pi$ amplitude could be neglected. In the calculations presented here these two effects are included and this modifies the expressions for F, but we shall not quote the new results at length.

The six processes listed in Table I depend only upon the known coupling constant g and upon hD^{811} and λD^{11} of Eqs. (6) and (7). Thus, they depend only upon g, $h(1+\epsilon_1)$, and $\lambda(1+\epsilon_4)$. Using $\omega \to 3\pi$ and $\phi \to 3\pi$ experimental widths as input, two solutions are obtained, one of which has been used to obtain the predicted widths given in Table I. In dimensionless units, with $m_\pi = 138$ MeV, the solution adopted[9] is

$$H \equiv (m_\pi^2 h^2/4\pi)(1+\epsilon_1)^2 = 0.108 , \quad (12)$$

$$\Lambda \equiv (m_\pi^2 \lambda^2/4\pi)(1+\epsilon_4)^2 = 0.328 . \quad (13)$$

The calculated values in Table I are in excellent agreement with experiment, except for the case[10] of $\rho \to \pi\gamma$. If (for the time being, and subject to future confirmation) we can disregard this last result, Table I gives significant support to the assumptions underlying this model, in particular vector dominance and current mixing. However, it does not rely upon the octet-broken form of the PVV vertex, as the six decay processes in Table I involve only the $\omega\rho\pi$ and the $\phi\rho\pi$ vertices. This makes the $\rho \to \pi\gamma$ discrepancy even more striking; the same difficulty appears to be at least as much of a problem in all vector-dominance and naive quark models.[3]

Using the effective Lagrangian of Eq. (8) we can also calculate the leptonic decay rates of vector mesons; their widths are

$$\Gamma_{(\rho,\omega,\phi)}(V \to e^+e^-) = (4\pi\alpha^2 m_V K_V/3g^2) \times (1, \tfrac{1}{3}\sin^2\theta, \tfrac{1}{3}\cos^2\theta). \quad (14)$$

With the value $g^2/4\pi = 3.40$, which gives the remarkable fit of Table I as well as correct widths for $V \to P + P$ decays, we find values for $\Gamma(V \to e^+e^-)$ which are about 70% of the experimental ones. However, if we use $g^2/4\pi = 2.28$ for the leptonic decays, we obtain (in keV) $\Gamma_\rho = 7.17$ $\Gamma_\omega = 0.595$, $\Gamma_\phi = 1.37$. These are in excellent agreement with the experimental values: $\Gamma_\rho^{\rm exp} = 6.54 \pm 0.78$, $\Gamma_\omega^{\rm exp} = 0.76 \pm 0.17$, $\Gamma_\phi^{\rm exp} = 1.31 \pm 0.10$. We have no ready explanation to offer for the apparent change in value of $g(q^2)$ between $q^2 = 0$ and $q^2 \cong 1$ GeV^2, particularly in view of the fact that a single value of g fits the decays of Table I and of Table II, where transitions $V \to \gamma$ at $q^2 = 0$ are involved, and also fits the $V \to P + P$ decays. However, we note that the discrepancy between g_ρ determined from the leptonic decays and $g_{\rho\pi\pi}$ determined from the ρ width is a general problem, mentioned repeatedly in the literature (see, e.g., O'Donnell, Ref. 3).

In BMS it was pointed out that the experimental branching ratio of $\eta \to \pi^+\pi\gamma$ to $\eta \to 2\gamma$ is about four times smaller than the SU(6)-symmetric value, implying that symmetry-breaking effects are

TABLE I. Processes depending only upon g, H, and Λ.

Decay process	Calculated width (MeV)	Experiment (Ref. 2)
$\omega \to 3\pi$	8.99 (input)	8.99 ± 0.36
$\phi \to 3\pi$	0.677 (input)	0.677 ± 0.069
$\omega \to \pi\gamma$	0.900	0.880 ± 0.050
$\phi \to \pi\gamma$	4.8×10^{-3}	$(5.7 \pm 2.0) \times 10^{-3}$
$\pi^0 \to 2\gamma$	7.47×10^{-6}	$(7.95 \pm 0.55) \times 10^{-6}$
$\rho^- \to \pi^-\gamma$	77×10^{-3}	$(35 \pm 10) \times 10^{-3}$

TABLE II. Fitted values of widths for the parameters given in Eq. (16). The underlined processes are input, as is the branching ratio of $\eta' \to \pi^+\pi^-\gamma$ to $\eta' \to \gamma\gamma$, which is 15.

Decay process	Calculated width (keV)	Experiment (Ref. 2)
$\underline{\eta \to 2\gamma}$	0.323	0.323 ± 0.046
$\underline{\eta \to \pi^+\pi^-\gamma}$	0.0416	0.0416 ± 0.0059
$\underline{\phi \to \eta\gamma}$	82	82 ± 16
$\omega \to \eta\gamma$	0.17	<50
$\rho \to \eta\gamma$	16.7	<160
$\underline{K^{*0} \to K^0\gamma}$	75	75 ± 35
$\underline{K^{*+} \to K^+\gamma}$	$19(1+0.94\epsilon_2)^2$	<80
$\eta' \to \pi^+\pi^-\gamma$	21.4	<300
$\eta' \to \gamma\gamma$	1.44	<20
$\eta' \to \omega\gamma$	1.23	<50

large. These two decay rates were used as further input, determining two solutions for ϵ_1 and $\epsilon' \equiv \epsilon_2 + \epsilon_3$ that, in turn, gave predictions for ϕ, ω, and ρ decaying into $\eta + \gamma$ which were consistent with the quoted experimental upper limits. It also predicted a suppressed rate for $K^{*0} \to K^0 + \gamma$, in agreement with recent measurements.

In reexamining these decays in the scheme of octet-broken SU(3), with the inclusion of η-η' mixing, we face the introduction of new coupling parameters that are, however, constrained by the newer measurements available. A possible *theoretical* constraint is to consider that the ninth pseudoscalar P^0 and the pseudoscalar octet form a nonet, and that the *PVV* interaction is a combination of zeroth and eighth components of a $(3^*, 3) \oplus (3, 3^*)$ representation of chiral $SU(3) \otimes SU(3)$. In that case,[11] only one new ϵ parameter is introduced when P^0 is included. We shall consider this possibility below, as a modification of a proposal due to Singer.[12]

He adds to the interaction [Eq. (5)] a term

$$\mathcal{L}_{P^0VV} = \tfrac{1}{4}\epsilon_{\alpha\beta\mu\nu} h_0 \Delta^{ab} V^a_{\alpha\beta} V^b_{\mu\nu} P^0 , \quad (15)$$

with

$$\Delta^{ab} = \delta^{ab} + \sqrt{3}\beta d^{ab8} , \quad (16)$$

$a, b = 1 \cdots 8$. A possible term of the form $\beta_1 \epsilon_{\alpha\beta\mu\nu} V^8_{\alpha\beta} V^0_{\mu\nu} P^0$, which would induce, by symmetry breaking, singlet-to-singlet radiative transitions has been neglected. Such a term would affect only the $\omega \to \eta\gamma$, $\phi \to \eta\gamma$, and $\eta' \to \omega\gamma$, $\eta' \to \phi\gamma$ decays. The present state of experimental knowledge cannot determine whether such a term is needed or not. Hence, we have chosen to calculate the decay with $\beta_1 = 0$.

The solution which gives the fit displayed in Table II has

$$\epsilon_1 = 0.40, \quad \epsilon' = \epsilon_2 + \epsilon_3 = 0.51, \quad \epsilon_4 = 0.465, \quad h_0/h = 1.51, \quad \beta = -1.81, \quad (17)$$

and is the one that uniquely (aside from uncertainties due to experimental errors) satisfies the six upper-limit constraints shown in Table II.

We note, in connection with Table II, that $\rho \to \eta\gamma$ and $\eta \to \pi^+\pi^-\gamma$ have the same *PVV* vertex and that the inequality for $K^{*+} \to K^+\gamma$ is satisfied for $-3 \lesssim \epsilon_2 \lesssim 1$. The predicted total width for η' decay is about 70 keV, while its experimental upper limit is 1 MeV.

For processes which do not involve P^0 and V^0_μ simultaneously, the restriction that $\mathcal{L}_{PVV}$ transform as $(3^*, 3) \oplus (3, 3^*)$ in a chiral theory is equivalent to the constraint that

$$\beta h_0 = (\tfrac{2}{3})^{1/2} h \epsilon_1 \quad (18)$$

in Eqs. (15) and (16). Using the chiral constraint (18) allows us to eliminate β as a parameter, and to use one process less as input. A fit satisfying the upper limits in Table II is obtained with

$$\epsilon_1 = 1.3, \quad \epsilon' = 1.8, \quad \epsilon_4 = 0.24, \quad h_0/h = -0.66. \quad (19)$$

Results for $\omega \to \eta\gamma$ and $\rho \to \eta\gamma$ are unchanged but $\Gamma(K^{*0} \to K^0\gamma) = 5.7$ keV, compared with the experimental result (75 ± 35) keV. The η' decay widths are predicted to be considerably smaller than those in Table II; they are (in keV) $\Gamma(\eta' \to \pi^+\pi^-\gamma) = 2.80$, $\Gamma(\eta' \to \gamma\gamma) = 0.185$, $\Gamma(\eta' \to \omega\gamma) = 4.0$. The ratio of the last two widths violates, by at least a factor of 2, the upper limit given in Ref. 2. Thus, the chiral constraint is apparently ruled out.

In conclusion, we find that radiative and certain strong decays of mesons with masses of about 1 GeV or less are well accounted for by an octet-broken SU(3) model incorporating current mixing for the vector mesons with octet mass² breaking and singlet-octet mixing for the pseudoscalars. We stress that the BMS model alone,[4] although it predicted the large suppression of the $K^{*0} \to K^0\gamma$ transition, cannot accommodate all the measured rates, and the inclusion of η-η' mixing is necessary in order to achieve the fit of Table II. It is also interesting to observe that a nonet model with octet symmetry breaking is likewise unable to fit all the data, as shown by Edwards and Kamal.[3] Thus, an effort should be made to measure ω and ρ transition to $\eta\gamma$, the ratio of $\eta' \to \omega\gamma$ to $\eta' \to \rho\gamma$, and the η' width, in order to check the general validity of the present model.

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