PHYSICAL REVIEW D VOLUME 15, NUMBER 1 1 JANUARY 1977

# Radiative decay of the $J$ ($\psi$) in a spectrum-generating SU(4) *

A. Bohm, M. Hossain, B. A. Kagali, P. Moylan, R. B. Teese, P. M. Valanju, and H. Alhendi
*Center for Particle Theory, Physics Department, University of Texas at Austin, Austin, Texas 78712*
(Received 15 December 1975; revised manuscript received 7 April 1976)

The radiative decay rates of the "old" vector mesons are used to determine the suppression factor for the spectrum-generating SU(4) group. From the suppression factor the decay rate for $J$ ($\psi$)$\to\chi(2.75)+\gamma$ is predicted.

In view of the recent discovery of the radiative decay of the $J$ ($\psi$) (see Refs. 1 and 2), which may be interpreted as $J$ ($\psi$)$\to$ pseudoscalar $\chi(2.75)$ $+\gamma$, it is interesting to obtain an estimate of the decay rate for this process. We shall do this under the assumption that SU(4) is the spectrum-generating group for the vector and pseudoscalar mesons, where $J$ ($\psi$) belongs to an ideally mixed 16-plet with the other "old" vector mesons and $\chi(2.75)$ belongs to a 15-plet with the "old" pseudoscalar mesons.[3] Since the process $J$ ($\psi$)$\to\chi+\gamma$ is not suppressed as the consequence of an SU(4) property, it is a major task to explain the smallness of the decay rate. We will show in this communication that the suppression is explained naturally in a framework which has already been applied successfully to other processes in which the deviation from the symmetry limit is essential.

The small deviation from ideal mixing by a few degrees in mixing angle, which in fact exists, has an effect of only a few percent upon the radiative decay data, except for the cases $\phi\to\pi\gamma$, $\psi\to\pi\gamma$, and $\psi\to\eta\gamma$, which have zero amplitude in the ideal mixing limit. This small deviation will explain the existence of these decays [e.g. an increase by 4°–5° from the ideal $\phi$-$\omega$ mixing angle will predict a decay rate of $\Gamma(\phi\to\pi\gamma)\approx 10$ keV (see Ref. 4)]. At the present time the experimental data are not sufficient to warrant consideration of these small deviations, and as they have no effect upon the problem of our principal interest in this paper we shall ignore them.

As we shall see below, the assumption that SU(4) be a symmetry group of the amplitude, i.e., that the mass differences are only taken into account in the phase space, will not be able to explain the experimental data even of the old mesons for any reasonable assumption about the electromagnetic current operator. Therefore, SU(4) must be a genuine spectrum-generating group. We had already arrived at the same conclusion from the leptonic decays of the vector mesons $\rho, \omega, \varphi, \psi$,[8a] and in still earlier papers[8b] it was shown that there are considerable advantages to treating also SU(3) in the leptonic and semileptonic weak decays as a genuine spectrum-generating group. Then, in addition to the Clebsch-Gordan coefficients, the amplitude contains a symmetry-breaking factor (suppression factor) $\phi(M_V, M_P)$.

This suppression factor arises from the following theoretical considerations, which are described in full detail in Ref. 8b and which are also applied in Ref. 8a: As SU(4) [or SU(3)] is not a symmetry group the momentum operator $P_\mu$ cannot commute with it, but we assume that $[\hat{P}_\mu, \mathrm{SU}(4)]=0$, where $\hat{P}_\mu=P_\mu M^{-1}$ with $M$ the mass operator. Then the vector currents $V^\beta_\mu$ ($\beta=1,\ldots,16$) need not be regular tensor operators, but it may as well be that functions of them which also contain the mass operator, e.g., $\{V^\beta_\mu, M^P\}$, are. As a consequence of these assumptions one has to use eigenvectors of $\hat{P}_\mu$ (velocity eigenvectors) instead of the usual momentum eigenvectors. The Wigner-Eckart theorem for matrix elements of regular tensor operators between velocity eigenvectors holds exactly, and the usual amplitude which is not SU(4) invariant is related to the SU(4)-invariant amplitude through a symmetry-breaking factor $\phi$. The precise form of $\phi$ depends upon the particular transformation property assumed for the currents, as is discussed in detail in Ref. 8. One aim of our research program is to conjecture these transformation properties from the experimental data. We shall therefore proceed by determining the symmetry-breaking factor $\varphi(m_V, m_P)$ (see Ref. 8) from the radiative decay rates of the old vector mesons [essentially from $\Gamma(\omega\to\pi\gamma)=(870\pm 61)$ keV,[5a] $\Gamma(\phi\to\eta\gamma)=(74\pm 15)$ keV,[5b] $\Gamma(\rho\to\pi\gamma)=(35\pm 10)$ keV,[6] $\Gamma(K^{0*}\to K^0\gamma)=(75\pm 35)$ keV[7]], and then use it to predict $\Gamma(\psi\to\chi(2.75)+\gamma)$.

The decay rate for the process $V\to P+\gamma$ is written as

$$\Gamma(V\to P\gamma)=|g_{VP}|^2\tfrac{1}{24}\alpha\, m_V{}^3[1-(m_P/m_V)^2]^3, \tag{1}$$

with

$$g_{VP}=g\langle P|V^{\mathrm{el}}|V\rangle\phi(m_V, m_P). \tag{2}$$

Here $g$ is an overall constant expressing the strength of the electromagnetic decay [the Sommerfeld constant $\alpha$ has been used in (1) just for convenience], $\langle P|V^{\rm el}|V\rangle$ is the SU(4) matrix element of the SU(4) part of the electromagnetic current $V_\mu^{\rm el}$, and $\phi(m_V, m_P)$ is a symmetry-breaking factor. In the SU(4)-symmetry limit $\phi(m_V, m_P)=1$ and (1) is the familiar expression for the radiative decay rate.

For ideal mixing the SU(4) part of the vector-meson states $|V\rangle$ is given by

$$\begin{aligned}
|\varphi\rangle &= (\tfrac{2}{3})^{1/2}|\eta_V\rangle + (12)^{-1/2}|\chi_V\rangle - \tfrac{1}{2}|\sigma_V\rangle,\\
|\omega\rangle &= -3^{-1/2}|\eta_V\rangle + 6^{-1/2}|\chi_V\rangle - 2^{-1/2}|\sigma_V\rangle,\\
|\psi\rangle &= (\tfrac{3}{4})^{1/2}|\chi_V\rangle + \tfrac{1}{2}|\sigma_V\rangle,\\
|\rho\rangle &= |\pi_V^0\rangle.
\end{aligned} \tag{3}$$

Here $|\pi_V^0\rangle$ denotes the SU(3)-octet $SU(2)_I$-triplet state with $I_3 = Y = \text{charm} = 0$, $|\eta_V\rangle$ denotes the SU(3)-octet $SU(2)_I$-singlet state with $I_3 = Y = \text{charm} = 0$, $|\chi_V\rangle$ denotes the SU(3)-singlet state with $I_3 = Y = \text{charm} = 0$, and $|\sigma_V\rangle$ denotes the SU(4)-singlet state.

The corresponding pseudoscalar-meson states are denoted $|\pi^0\rangle$, $|\eta\rangle$, and $|\chi\rangle$, respectively.

For the electromagnetic current operator we take

$$V_\mu^{\rm el} = V_\mu^{\pi^0} + \frac{1}{\sqrt{3}} V_\mu^{\eta} - X V_\mu^{\chi} - V_\mu^{\sigma}, \tag{4}$$

where $V_\mu^{\pi^0}, V_\mu^{\eta}, V_\mu^{\chi}$ are the $\pi^0, \eta, \chi$ components of the SU(4) regular tensor operator, $V_\mu^\sigma$ is an SU(4)-scalar operator, and $X$ is a number for which the values $X = (\frac{2}{3})^{1/2}$, $X = -1/\sqrt{6}$, and $X = -2(\frac{2}{3})^{1/2}$ have obtained favorable consideration in the literature.[9-12].

If the $F$-type reduced matrix element of the SU(4) regular tensor operator $V^\alpha$ between the pseudoscalar- and vector-meson states is taken to be zero (as a consequence of the transformation property of the pseudoscalar- and vector-meson states under charge conjugation), then all SU(4) matrix elements $\langle P|V^{\rm el}|V\rangle$ can be expressed in terms of two reduced matrix elements of the regular tensor operator,

$$\begin{aligned}
D &= \langle P, 15||V^{15}||15, V\rangle,\\
A &= \langle P, 15||V^{15}||1, V\rangle\\
&= -(15)^{-1/2}\langle V, 1||V^{15}||15, P\rangle,
\end{aligned}$$

the matrix element of the scalar operator $S = \langle P, 15||V^s||15, V\rangle$, and the SU(4) Clebsch-Gordan coefficients.[13] The results are the following:

$$\langle\pi^0|V^{\rm el}|\omega\rangle = D/2\sqrt{3} - A/\sqrt{2}, \tag{5a}$$

$$\langle\eta|V^{\rm el}|\phi\rangle = (\sqrt{2}/9)[\tfrac{5}{4} - (\tfrac{3}{2})^{1/2}X]D - (\tfrac{2}{3})^{1/2}S - A/2\sqrt{3}, \tag{5b}$$

$$\langle\pi^{0,\pm}|V^{\rm el}|\rho^{0,\pm}\rangle = -(27)^{-1/2}(1 + (\tfrac{3}{2})^{1/2}X)D - S, \tag{5c}$$

$$\langle K^0|V^{\rm el}|K^{0*}\rangle = (27)^{-1/2}[2 - (\tfrac{3}{2})^{1/2}X]D - S, \tag{5d}$$

$$\langle K^+|V^{\rm el}|K^{+*}\rangle = (27)^{-1/2}[-1 - (\tfrac{3}{2})^{1/2}X]D - S, \tag{5e}$$

$$\langle\eta|V^{\rm el}|\omega\rangle = -(18)^{-1}(1 - \sqrt{6}X)D + S/\sqrt{3} - A/\sqrt{6}, \tag{5f}$$

$$\langle\eta|V^{\rm el}|\rho^0\rangle = D/3, \tag{5g}$$

$$\langle\chi|V^{\rm el}|\psi\rangle = (D/\sqrt{6} - \tfrac{1}{2}A)X - \sqrt{3}\,S/2, \tag{5h}$$

$$\begin{aligned}
\langle\pi^0|V^{\rm el}|\psi\rangle &= -\langle\pi^0|V^{\rm el}|\phi\rangle\\
&= \langle\eta|V^{\rm el}|\psi\rangle\sqrt{3}\\
&= \tfrac{1}{2}(D/\sqrt{6} + A).
\end{aligned} \tag{5i}$$

Without any further assumption we have the three arbitrary parameters $D, A, S$ to fit the experimental decay rates. Such a fit will lead to the empirical relation $A \approx -D/\sqrt{6}$ because $\Gamma(\phi \to \pi^0\gamma)$ is orders of magnitude lower than the decay rate for other

TABLE I. The strength constant $gd$, with $g$ given by Eq. (2), $d = D/\sqrt{3}$, and $X = (\frac{2}{3})^{1/2}$, for various decays and theoretical assumptions. Experimental values are shown in the form $|g_{VP}|$ using Eq. (1). The percentage errors for $gd$ are roughly the same as those for $g_{VP}$.

| Decay (experimental $\Gamma$ in keV) | $\lvert g_{VP}\rvert$ (GeV$^{-1}$) | SU(4) matrix element $\langle P\lvert V^{\rm el}\rvert V\rangle$ | | $gd$ | | | |
|---|---|---|---|---|---|---|---|
| | | | | $\phi=1$ SU(4) symmetry | $p=0$ | $p=\frac{1}{2}$ | $p=1$ |
| | | $S=S_+$ | $S=S_-$ | $S=S_+$ | $S=S_-$ | $S=S_-$ | $S=S_-$ |
| $\omega \to \pi\gamma$ (870 ± 61) | 2.56 ± 0.10 | $d$ | $d$ | 2.56 | 0.14 | 0.22 | 0.29 |
| $\phi \to \eta\gamma$ (74 ± 15) | 0.80 ± 0.08 | $\frac{4}{5}(\frac{2}{3})^{1/2}d$ | $\frac{6}{5}(\frac{2}{3})^{1/2}d$ | 1.23 | 0.23 | 0.26 | 0.29 |
| $\rho \to \pi\gamma$ (35 ± 10) | 0.51 ± 0.07 | $-d/5$ | $d/5$ | 2.56 | 0.13 | 0.21 | 0.29 |
| $K^{0*} \to K^0\gamma$ (75 ± 35) | 1.03 ± 0.24 | $4d/5$ | $6d/5$ | 1.30 | 0.19 | 0.23 | 0.27 |

radiative decays.[4,5] Therefore, we make the assumption that the reduced matrix elements of $V^{15}$ are not independent but fulfill $A = -D/\sqrt{6}$ and describe the deviation of $\Gamma(\phi \to \pi\gamma)$ [and $\Gamma(\psi \to \pi\gamma)$, $\Gamma(\psi \to \eta\gamma)$] from zero by a deviation from the ideal mixing situation. This condition is equivalent to assuming the continuity of quark lines.[14]

With the assumption $A = -D/\sqrt{6}$ all of Eqs. (5) are not independent, and they impose with (2) conditions such as

$$|\varphi^{-1}(m_\phi, m_\eta) g_{\phi\eta}| = (\tfrac{2}{3})^{1/2} |\varphi^{-1}(m_\rho, m_\pi) g_{\rho\pi} \pm \phi^{-1}(m_\omega, m_\pi) g_{\omega\pi}| . \tag{6}$$

For the symmetry limit $\phi(m_V, m_P) = 1$ using the $g_{VP}$ data of Table I, (6) predicts a value of $g_{\phi\eta}$ about five standard deviations above the new world average, and over ten above the Orsay average.[5b] This is evidence that (for any $X$), the symmetry limit is inadequate, and so the symmetry-breaking suppression factor appears necessary. This can also be seen in particular symmetry limit fits. The best such fit is shown in Table I.

We have tried various possible suppression factors, according to the considerations in Ref. 8. The most reasonable appears to be

$$\phi^p(m_V, m_P) = (m_V^{\,p} + m_P^{\,p})/m_V m_P, \quad p = 0, \tfrac{1}{2}, 1, \tfrac{3}{2} \ldots \tag{7}$$

which is also supported by the results of Ref. 8. The results are in Table I. The first column gives $g_{VP}$ calculated from the experimental rates and Eq. (1). As $\phi(m_\rho, m_\pi) \approx \phi(m_\omega, m_\pi)$, we use $|g_{\omega\pi}|/|g_{\rho\pi}|$ to determine the relation between $D$ and $S$, and then express all the matrix elements in terms of $d = D/\sqrt{3}$. Two values of $S$ are allowed for each $D$:

$$S_\pm = [-(\tfrac{1}{3} + X/\sqrt{6}) \pm |g_{\rho\pi}|/|g_{\omega\pi}|] d. \tag{8}$$

We have calculated $gd$ [using (2)] from the values of $|g_{VP}|$ in the first column of Table I for various values of $p$ and $X$, and the two possibilities $S_\pm$. The best fits we obtained are shown in columns 6–8 of Table I. The value of $gd$ must be the same constant for all the known processes. We see that this is the case within the experimental errors only for the $p = \frac{1}{2}$ and $p = 1$ suppression factors. With the value of $gd$ determined in this way for $X = (\frac{2}{3})^{1/2}$ we calculate the predictions for the unknown decay rates which are listed in Table II. For the old vector mesons our predictions are well within the experimental upper limits: $\Gamma(K^{+*} \to K^+\gamma) < 80$ keV,[15] $\Gamma(\omega \to \eta\gamma) < 50$ keV,[16] and $\Gamma(\rho \to \eta\gamma) < 152$ keV.[17] For $\psi \to \chi\gamma$ we predict

$$\Gamma(\psi \to \chi\gamma) = 1.73 \pm 0.17 \text{ keV for } p = \tfrac{1}{2}, \quad X = (\tfrac{2}{3})^{1/2}, \tag{9}$$

$$\Gamma(\psi \to \chi\gamma) = 9.35 \pm 0.93 \text{ keV for } p = 1, \quad X = (\tfrac{2}{3})^{1/2}. \tag{10}$$

TABLE II. Predictions of $\Gamma$ in keV.

| Decay | SU(4) matrix elements $\langle P\|V_{el}\|V\rangle$ $S = S_-$ $X = (\frac{2}{3})^{1/2}$ | $p = \frac{1}{2}$ Best $\Gamma$ $S = S_-$ $X = (\frac{2}{3})^{1/2}$ | $p = 1$ Best $\Gamma$ $S = S_-$ $X = (\frac{2}{3})^{1/2}$ |
|---|---|---|---|
| $K^{+*} \to K^+\gamma$ | $d/5$ | $1.82 \pm 0.55$ | $2.47 \pm 0.74$ |
| $\omega \to \eta\gamma$ | $-d/5\sqrt{3}$ | $0.172 \pm 0.052$ | $0.21 \pm 0.06$ |
| $\rho \to \eta\gamma$ | $d/\sqrt{3}$ | $3.75 \pm 0.37$ | $4.64 \pm 0.46$ |
| $\psi \to \chi\gamma$ | $\frac{14}{15}\sqrt{3}d$ | $1.73 \pm 0.17$ | $9.35 \pm 0.93$ |

*Note added.* We have been informed that the DESY-Heidelberg collaboration has doubled their statistics and obtained the result that the accumulation of events observed at a mass $m_X = 2.75$ GeV in $(3.1) \to \gamma X$, $X \to \gamma\gamma$, is not a statistical fluctuation. We are grateful to J. Heintze for his communication.

*Work supported by the U. S. Energy Research and Development Administration under Contract No. AT(40-1)3992.

[1]DASP collaboration: W. Braunschweig *et al.*, Phys. Lett. 57, 407 (1975); J. Heintze, in *Proceedings of the 1975 International Symposium on Lepton and Photon Interactions at High Energies, Stanford, California*, edited by W. T. Kirk (SLAC, Stanford, 1976), p. 97.
[2]J. W. Simpson *et al.*, Phys. Rev. Lett. 35, 699 (1975).
[3]This is a generalization of the assumption that the old vector and pseudoscalar mesons belong to the 35-plet of SU(6) in which the SU(4) of nonstrange spin and isospin is diagonal. Besides the esthetic reason, there is a practical reason for this assumption: If we allow appreciable mixing of the pseudoscalar mesons, we have too many parameters to do anything useful with the sparse experimental data.
[4]The Orsay experiment gave $\Gamma(\phi \to \pi\gamma) = 6.5$ keV $\pm 30\%$ (see Ref. 5b).
[5a]Particle Data Group, Phys. Lett. 50B, 1 (1974).
[5b]For $\Gamma(\phi \to \eta\gamma)$ we use the average of the Orsay results: D. Benaksas, *et al.*, Phys. Lett. 42B, 511 (1972); C. Bemporad, in *Proceedings of the 1975 International Symposium on Lepton and Photon Interactions at High Energies, Stanford, California*, edited by W. T. Kirk (SLAC, Stanford, 1976), p. 113.
[6]B. Gobbi *et al.*, Phys. Rev. Lett. 33, 1450 (1974).
[7]W. C. Carithers *et al.*, Phys. Rev. Lett. 35, 349 (1975).
[8a]A. Bohm, Phys. Rev. D 13, 2110 (1976).
[8b]A. Bohm, Phys. Rev. D 13, 2656 (1976); A. Bohm and

J. Werle, Nucl. Phys. B106, 165 (1976).

[9]S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D 2, 1285 (1970).

[10]T. Goto and V. S. Mathur, Phys. Lett. 59B, 193 (1975).

[11]E. Takasugi and S. Oneda, Phys. Rev. Lett. 34, 1205(E) (1975).

[12]D. H. Boal, R. H. Graham, and J. W. Moffat, Phys. Rev. D 13, 3107 (1976).

[13]V. Rabl, G. Campbell, Jr., and K. C. Wali, J. Math. Phys. 16, 2494 (1975). In order to get the phases correct, one has to take for what is denoted by $\eta$ (and $\chi$) in this reference our states $-|\eta\rangle$ (and $-|\chi\rangle$) and our tensor operator components $-V^{\eta}$ (and $-V^{\chi}$).

[14]G. Zweig, CERN Report No. TH-412 (unpublished); J. Mandula, J. Weyers, and G. Zweig, Annu. Rev. Nucl. Sci. 20, 289 (1970).

[15]C. Bemporad *et al.*, Nucl. Phys. B51, 1 (1973).

[16]Particle Data Group, Rev. Mod. Phys. 48, S1 (1976).

[17]M. E. Nordberg *et al.*, Phys. Lett. 51B, 106 (1974).