Comment on right-handed-current restrictions*

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Current-algebra, PCAC (partial conservation of axial-vector current) restrictions on the structure of the $\Delta S = 1$ nonleptonic Hamiltonian are reexamined. The result ($\Delta I = 1/2$ chirality) = ($\Delta I = 3/2$ chirality) is strengthened, while the conclusion that the chirality is left-handed survives but is weakened slightly by more careful analysis.

A recent Letter[1] explored the consequences of current algebra with partial conservation of axial-vector current (PCAC) on the structure of the nonleptonic Hamiltonian, specifically with respect to right-handed currents.[2] The conclusion may be stated succinctly as follows:

(i) From the relation of $\Delta I = \frac{1}{2}$ and $\Delta I = \frac{3}{2}$ effects in nonleptonic kaon and hyperon decays, we require

$$\frac{[F_a{}^5, H_w{}^{1/2}]}{[F_a, H_w{}^{1/2}]} = \frac{[F_a{}^5, H_w{}^{3/2}]}{[F_a, H_w{}^{3/2}]}, \quad a = 1, 2, 3\,. \tag{1}$$

(ii) From the relative signs of S- and P-wave amplitudes in nonleptonic hyperon decays we have

$$\frac{[F_a{}^5, H_w{}^{1/2}]}{[F_a, H_w{}^{1/2}]} = +1\,. \tag{2}$$

The analysis can be (and has been) criticized, however, on both counts. Some of the objections are:

(a) The $K \to 3\pi$ analysis avoided the unitarity question by assuming the decay amplitude to be real at each stage of the calculation. Perhaps final-state interactions can change conclusion (i).[3]

(b) It has been claimed that the standard current-algebra derivation of kaon decay depends critically upon the $\langle K|\sigma|K\rangle$ matrix element, and that if this matrix element has the value predicted by Gell-Mann, Oakes, and Renner (GMOR)[4] conclusion (i) is no longer valid.[5]

(c) The hyperon analysis is strictly valid for models wherein the octet piece satisfies (p.v. = parity-violating)

$$H_w^{\text{p.v.}} \sim \lambda_6$$

so that

$$\langle B'|H_w^{\text{p.v.}}|B\rangle = 0 \tag{3}$$

in the SU(3) limit.[6] Right-handed-current models, however, have the transformation property

$$H_w^{\text{p.v.}} \sim i\lambda_7\,.$$

It has been stated that this upsets conclusion (ii).[7]

(d) The current-algebra analysis of hyperon decays avoided K^*-pole diagrams, which vanish in the soft-pion limit. A recent calculation suggested that such terms may completely alter conclusion (ii).[8]

It is the purpose of this note to tighten both of the above conclusions [Eqs. (1), (2)] by addressing each of these points.

In order to study objection (a), we have unitarized our current-algebra calculation of $K \to 3\pi$. We utilized a generalization of the method of Neveu and Scherk,[9] who employ a version of the Khuri-Treiman equations[10] to find a unitary solution up to terms of order $\delta_{\pi\pi}{}^2$, where $\delta_{\pi\pi}$ is the s-wave $I = 0$ $\pi\pi$ phase shift at (low) energies relevant to the 3π final state. The Neveu-Scherk analysis, however, neglects $I = 2$ $\pi\pi$ scattering. We have modified their procedure to include this omitted effect. We find that unitarity corrections increase the $\Delta I = \frac{3}{2}$ amplitude by roughly 30%, while having little effect on the dominant $\Delta I = \frac{1}{2}$ terms. The important point is that the relative *sign* of the amplitudes are unchanged, so that the good qualitative and reasonable quantitative fits obtained for $K \to 3\pi$ amplitudes via the current-algebra approach are retained in the unitarized version. Thus confidence in conclusion (i) is strengthened.

Objection (b) is based upon a misunderstanding of the current-algebra approach to nonleptonic decays. It has been pointed out that if the $\langle K|\sigma|K\rangle$ matrix element has its GMOR value, the standard "derivation" of the $\Delta I = \frac{1}{2}$ rule in terms of dominance of the

$$\langle 0|H_w{}^{1/2}|K\rangle$$

matrix element is no longer valid.[11] However, this has no major effect on the relationship between $K \to 3\pi$ and $K \to 2\pi$ amplitudes; only changes of $O(m_\pi{}^2/m_K{}^2)$ are involved. What *is* mysterious in this case is why

$$\frac{\langle 2\pi|H_w{}^{1/2}|K\rangle}{\langle 2\pi|H_w{}^{3/2}|K\rangle} \gg 1\,.$$

Objection (c) is quite correct in asserting that

$\langle B' | H_w^{\text{p.v.}} | B\rangle$ matrix elements should be included in the analysis. Thus, defining (p.c. = parity-conserving)

$$\langle B' | H_w^{\text{p.v.}} | B\rangle = \bar{u}_{B'}\gamma_5 u_B T_{B'B} ,$$
$$\langle B' | H_w^{\text{p.c.}} | B\rangle = \bar{u}_{B'} u_B S_{B'B} , \quad (4)$$

we expect

$$\frac{T_{B'B}}{S_{B'B}} \sim O(1)$$

in right-handed-current models. However, a correct current-algebra, PCAC analysis requires the use of pseudovector coupling for the strong $\pi B'B$ vertices.[12] Thus we can write the hyperon-decay amplitudes as

$$\langle \pi\beta_{p'} | H_w | \alpha_p\rangle = \bar{u}_\beta(p')(A + B\gamma_5)u_\alpha(p) , \quad (5)$$

with

$$A = -\frac{i}{F_\pi}(I^a_{\beta\gamma} S_{\gamma\alpha} - I^a_{\rho\alpha} S_{\beta\rho}) - \frac{i}{2F_\pi}(m_\alpha - m_\beta)\left(\frac{G_{\alpha\beta\delta} T_{\delta\alpha}}{m_\alpha + m_\delta} - \frac{G_{a\gamma\alpha} T_{\beta\gamma}}{m_\beta + m_\gamma}\right) \equiv A_S + A_T ,$$

$$B = -\frac{i}{F_\pi}(I^a_{\beta\tau} T_{\tau\alpha} - I^a_{\rho\alpha} T_{\beta\rho}) - \frac{i}{2F_\pi}(m_\alpha + m_\beta)\left(\frac{G_{\alpha\beta\delta} S_{\delta\alpha}}{m_\alpha - m_\delta} + \frac{G_{a\gamma\alpha} S_{\rho\gamma}}{m_\beta - m_\gamma}\right) \equiv B_T + B_S , \quad (6)$$

where we have defined

$$\langle \beta | A^a_\mu | \alpha\rangle = G_{a\beta\alpha}\bar{u}_\beta\gamma_\mu\gamma_5 u_\alpha \quad (7)$$

and F_π is the pion-decay constant. Now, conclusion (ii) was derived in Ref. 1 neglecting A^T, B^T. From Eq. (6), we see that this assumption is justified in that if $T_{\beta\alpha} \sim S_{\beta\alpha}$

$$\frac{A_T}{A_S} \sim \frac{\Delta}{2M}, \quad \frac{B_T}{B_S} \sim \frac{\Delta}{2M} , \quad (8)$$

where Δ is a typical baryon mass splitting (~0.2 GeV), while M is a baryon mass (~1 GeV). Thus unless $T_{\beta\alpha} \gg S_{\beta\alpha}$ the dominant contribution to the weak hyperon decays comes from the parity-conserving weak spurion, so that conclusion (ii) remains valid.

Finally, objection (d) assumes a value for the

$$\langle \pi | H_w | K^*\rangle$$

matrix element three times as large as that used by previous workers.[13] From a strictly phenomenological standpoint one might take the view that any value can be used, but it *is* fair to ask whether a particular value is reasonable. Although it is not possible at present to provide a rigorous theoretical estimate for this quantity, we shall outline a calculation which addresses this point.

Using the effective Hamiltonian

$$H_{wK} = \lambda' m_{K_V}{}^2 f_{7ij}\partial_\mu P^i V^j_\mu , \quad (9)$$

where P (V_μ) represents the pseudoscalar (vector) nonet, the value used in Ref. 8 is

$$\tfrac{1}{2}\lambda' m_{K_V}{}^2 g_r = 7.56 \times 10^{-3}\ \text{MeV} , \quad (10)$$

where g_r gives the coupling strength between vector mesons and baryons. In order to decide whether this is justifiable, one may integrate by parts, rewriting the Hamiltonian as

$$H_{wK} = -\lambda' m_{K_V}{}^2 f_{7ij} P^i \partial_\mu V^j_\mu . \quad (11)$$

Using vector dominance and partial conservation of vector current (PCVC)[14] one can relate the Hamiltonian to the matrix element

$$\langle \pi | H_w | \kappa\rangle$$

which has been estimated using the MIT bag model.[15] Finally one can estimate

$$\langle 0 | \partial^\mu V_\mu | \kappa\rangle$$

from K_{13} decay data, using subtracted dispersion relations.[16] We leave details of the calculation for a future publication, but the result is

$$\tfrac{1}{2}\lambda' g_r m_{K_V}{}^2 \sim 7 \times 10^{-5}\ \text{MeV} . \quad (12)$$

This number is nearly two orders of magnitude smaller than the calculated value used in Ref. 8, thus reinforcing the analysis underlying conclusion (ii). Of course, employment of the PCVC hypothesis and the bag model[17] make this the most model-dependent of our results. However, it is our intent here only to point out the K^*-pole dominance is far from obvious and, in light of our admittedly crude numerical analysis, even questionable. An additional argument which reinforces this conclusion can be formulated within the framework of dispersion theory, wherein no K^*-pole term exists: the pion couples only to spin-zero systems. It then seems rather unlikely that exchange of an s-wave system with $m^2 \geq (m_K + m_\pi)^2$ can produce a contribution to the decay amplitude which varies rapidly for $0 \leq q^2 \leq m_\pi{}^2$.

None of our arguments are definitive, of course. However, the weight of evidence seems rather clearly to support conclusions (i) and (ii), while suggestions to the contrary are based on phenomenologically allowed but perhaps unreasonable values for various parameters.

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