Three simple classes of mixing models*

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Three classes of mixing models are examined with respect to mass constraints and Okubo-Zweig-Iizuka rule violation.

In the limit of ideal mixing and the Okubo ansatz for trilinear couplings, ψ and ϕ mesons (and their partners in other J^P multiplets) are stable against strong decay into mesons composed only of u, dquarks. Duality diagrams are a graphic realization of the static notion of pure wave functions as well as the dynamical notion of continuous quark line interaction implicit in the Okubo ansatz. This general picture for the new narrow resonances has been very persistent; we continue to refer to it, following Freund and Nambu,² as the Okubo-Zweiglizuka (OZI) rule.¹ In this limit, masses within the U(3) multiplets obey ideal-mixing mass formulas [or U(4) extensions] with ω degenerate with ρ and K^* the arithmetric mean of ϕ and ω .

An understanding, then, of the decay and production systematics of the new particles must involve theories of mixing and the breaking of the OZI rule. In fact, numerous authors have written numerous papers on this subject, couched in various languages and based on various dynamical theories.²⁻¹³ Despite the number and variety, we find certain unifying properties. In this note we would like to make some simple observations concerning three classes of models (MI, MII, and MIII) which we believe span many approaches and which probably exhaust the simplest possibilities.

For simplicity, we restrict ourselves first to a U(3) scheme, with ϕ initially $s\overline{s}$ and ω initially $\frac{1}{2}(u\overline{u} + d\overline{d})$. Extension to U(4) is straightforward. (ω and ϕ refer generically to the quark-species states without regard to spin-parity.) Common to all schemes, one has "bare" masses m_{ω} and m_{ϕ} (physical masses are denoted μ_{ω} and μ_{ϕ} , etc.) which satisfy the ideal-mixing mass formulas

$$m_{\omega}^{2} = m_{\rho}^{2},$$
 (1a)
 $m_{\omega}^{2} + m_{\phi}^{2} = 2m_{K}*^{2}.$

In all schemes it is a dynamical assumption that the interaction which mixes the ideally mixed states contributes only to isosinglet masses. This is the case, for example, when the interaction is unitaryflavor-singlet gluon exchange¹⁴ or when the interaction is the Pomeron continued to timelike momentum transfer.^{15,16,17} Thus

$$m_{\omega}^{2} = \mu_{\rho}^{2},$$
 (1b)
 $m_{\rho}^{2} = 2\mu_{K*}^{2} - \mu_{\rho}^{2},$

and one has the luxury of knowing one's bare masses, which play a quite physical role in these models.

It is only fair to comment at this point that the considerations of this short note lie within the narrow context of a zero-width approximation, ignoring the mixing of a broad ρ with the $\pi\pi$ *p*-wave continuum. An improved description of the ρ will possibly alter the results of Eq. (1b), shifting the ρ mass from its ideal "zero-width" position. This effect will not be considered in this short note, and all conclusions we draw from these simple models must be interpreted accordingly.

The remaining choice lies in the nature of the isosinglet mixing interaction and the size of the channel space in which it is to be realized.

In models of class MI the interaction is purely off-diagonal in the 2×2 system. In models of class MII the interaction proceeds through a third, independent, "quarkless" channel or intermediary, again with only off-diagonal elements, in a 3×3 system. In models of class MIII the interaction remains in a 2×2 system, is rank unity, and possibly energy-dependent. Apart from the strikingly different physical interpretations, models of class MII are simply class-MI models in a higher dimension, whereas class-MIII models, as will be seen, are closely related to those of class MII, with the intermediary no longer an independent state.

A few recent examples in the literature of these classes of models are Harari³ (M I, pseudoscalar mixing), Freund and Nambu² (M II, vector mixing, with the "O"-meson or bound-gluon state), and the present authors⁹⁻¹³ (M II, vector, scalar, pseudoscalar, and tensor mixings; M III, pseudo-scalar mixings), as well as Chew and Rosensweig⁴ (M III, cylinder corrections in various channels).

We discuss the models in a simple propagatormass matrix formalism, in the channel-space basis [ideally mixed $s\overline{s}$ and $(1/\sqrt{2})(u\overline{u} + d\overline{d})$ states], with "bare" propagator

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$$P = \begin{pmatrix} (s - m_{\omega}^{2})^{-1} & 0\\ 0 & (s - m_{\phi}^{2})^{-1} \end{pmatrix}.$$
 (2)

Class MI. In models of class MI, the transition between ω and ϕ channel states is given by

$$H = \begin{pmatrix} 0 & c \\ c & 0 \end{pmatrix}, \tag{3}$$

$$\pi = P \sum_{n} (HP)^{n} = (P^{-1} - H)^{-1}, \qquad (4)$$

$$\pi^{-1} = \begin{pmatrix} s - m_{\omega}^{2} & -c \\ -c & s - m_{\phi}^{2} \end{pmatrix}.$$
 (5)

Solving the eigenvalue problem we have

$$\pi = \sum_{\alpha=\omega,\phi} \frac{V_{\alpha} V_{\alpha}^{\dagger}}{s - \mu_{\alpha}} = \sum_{\alpha} \frac{\begin{pmatrix} (\mu_{\alpha}^{2} - m_{\omega}^{2})^{-1} & [(\mu_{\alpha}^{2} - m_{\omega}^{2})(\mu_{\alpha}^{2} - m_{\phi}^{2})]^{-1/2} \\ [(\mu_{\alpha}^{2} - m_{\omega}^{2})(\mu_{\alpha}^{2} - m_{\phi}^{2})]^{-1/2} & (\mu_{\alpha}^{2} - m_{\phi}^{2})^{-1} \\ (s - \mu_{\alpha}^{2})[(\mu_{\alpha}^{2} - m_{\omega}^{2})^{-1} + (\mu_{\alpha}^{2} - m_{\phi}^{2})^{-1}] \end{pmatrix}$$
(6)

with μ_{α} the roots of

$$(\mu^2 - m_{\omega}^2)(\mu^2 - m_{\phi}^2) - c^2 = 0 \tag{7}$$

and

$$V_{\alpha} = \left(\frac{\frac{c}{\mu_{\alpha}^{2} - m_{\omega}^{2}}}{1}\right) \left[\frac{c^{2}}{(\mu_{\alpha}^{2} - m_{\epsilon}^{2})} + 1\right]^{-1/2}.$$
 (8)

Defining the mixing angle by

$$|\omega\rangle = \cos\theta |\overline{\omega}\rangle + \sin\theta |\overline{\phi}\rangle ,$$

$$|\phi\rangle = -\sin\theta |\overline{\omega}\rangle + \cos\theta |\overline{\phi}\rangle ,$$

$$(9)$$

where $|\overline{\omega}\rangle$, $|\overline{\phi}\rangle$ represent channel states, we have

$$\tan\theta = \frac{\mu_{\omega}^2 - m_{\omega}^2}{\mu_{\omega}^2 - m_{\phi}^2}.$$

For respectable multiplets (vector, tensor, or pseudoscalars) the masses are all known and the mixing is completely determined, and one can calculate all OZI-rule-forbidden processes. The model, of course, can easily fail because the two physical masses must be fit by a single parameter c. This class of models can, however, be enlarged to allow c to be a function of s; the complications encountered with this generalization will become clear when models M II and M III are discussed.

Class M II. Let us now expand the system to include a third "quarkless" or "pure gluon"¹⁸ chan-

nel,

$$P = \begin{pmatrix} (s - m_{\omega}^{2})^{-1} & 0 & 0\\ 0 & (s - m_{\phi}^{2})^{-1} & 0\\ 0 & 0 & h^{-1}(s) \end{pmatrix}, \quad (10)$$

where $h^{-1}(s)$ is a linear function of s when the bare gluon resonates, but we allow for more general behavior. If this third channel now interacts with ω and ϕ as an SU(3) singlet [this condition is appropriate to models in which pure gluon or quarkless states (perhaps multiparticle) are unitary singlets; it may easily be relaxed], then

$$H = \begin{pmatrix} 0 & 0 & \sqrt{2}f \\ 0 & 0 & f \\ \sqrt{2}f^* & f^* & 0 \end{pmatrix}$$
(11)

and

$$\pi^{-1} = \begin{pmatrix} s - m_{\omega}^{2} & 0 & -\sqrt{2}f \\ 0 & s - m_{\phi}^{2} & -f \\ \sqrt{2}f^{*} & -f^{*} & h(s) \end{pmatrix}.$$
 (12)

Solving again the eigenvalue problem, we find

$$\pi = \sum_{\alpha} \frac{V_{\alpha}(s) V_{\alpha}^{\dagger}(s)}{s - M_{\alpha}^{2}(s)}, \qquad (13)$$

where the sum is over ω, ϕ , and gluon channels, $M_{\alpha}^{\ 2}(s)$, are the solutions of

$$(M^{2} - m_{\omega}^{2})(M^{2} - m_{\phi}^{2})[M^{2} + h(s) - s] - |f|^{2}(M^{2} - m_{\omega}^{2}) - 2|f|^{2}(M^{2} - m_{\phi}^{2}) = 0$$
(14)

with $V_{\alpha}(s)$ the associated eigenvectors

$$V_{\alpha}(s) = \begin{pmatrix} \sqrt{2}f [M_{\alpha}^{2}(s) - m_{\omega}^{2}]^{-1} \\ f[M_{\alpha}^{2}(s) - m_{\phi}^{2}]^{-1} \\ 1 \end{pmatrix} \left\{ 2f^{2} [M_{\alpha}^{2}(s) - m_{\omega}^{2}]^{-2} + f^{2} [M_{\alpha}^{2}(s) - m_{\phi}^{2}]^{-2} + 1 \right\}^{-1/2}.$$
(15)

For the "standard" case in which h(s) is linear in s, $M_{\alpha}^{2}(s)$ and $V_{\alpha}(s)$ are independent of s; otherwise they depend on s, and the poles of the theory in that case are solutions of

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$$\mu_{\alpha}^{2} = M_{\alpha}^{2} (\mu_{\alpha}^{2}) , \qquad (16)$$

which may be multivalued (in which case, below, an extra summation is implied). Then the propagator evaluated at the poles is

$$\pi \approx \sum_{\alpha} \frac{V_{\alpha}(\mu_{\alpha}^{2}) V_{\alpha}^{\dagger}(\mu_{\alpha}^{2})}{(s - \mu_{\alpha}^{2}) [1 - dM_{\alpha}(\mu_{\alpha}^{2})/ds]}.$$
(17)

To obtain a rank-one normalized propagator-residue matrix, the term $1 - (dM_{\alpha}/ds)(\mu_{\alpha}^{2})$ must be either absorbed into a coupling-constant renormalization

$$G_{\alpha} - G_{\alpha} \left[1 - \frac{dM_{\alpha}^{2}}{ds} (\mu_{\alpha}^{2}) \right]^{-1/2},$$
(18)

or we may require that $(dM_{\alpha}^2/ds)(\mu_{\alpha}^2) \equiv 0$, implying the condition

$$\frac{dh(\alpha^2)}{ds} = 1$$

on h(s), satisfied by the MII "standard" case. In either case the remaining propagator $\overline{\pi}$ has the form

$$\overline{\pi} = \sum_{\alpha} \left(\begin{array}{ccc} \frac{2}{(\mu_{\alpha}^{2} - m_{\omega}^{2})^{2}} & \frac{\sqrt{2}}{(\mu_{\alpha}^{2} - m_{\omega}^{2})(\mu_{\alpha}^{2} - m_{\phi}^{2})} & \frac{\sqrt{2}f^{-1}}{\mu_{\alpha}^{2} - m_{\omega}^{2}} \\ \frac{\sqrt{2}}{(\mu_{\alpha}^{2} - m_{\omega}^{2})(\mu_{\alpha}^{2} - m_{\phi}^{2})} & \frac{1}{(\mu_{\alpha}^{2} - m_{\phi}^{2})^{2}} & \frac{f^{-1}}{\mu_{\alpha}^{2} - m_{\phi}^{2}} \\ \frac{\sqrt{2}f^{*-1}}{\mu_{\alpha}^{2} - m_{\omega}^{2}} & \frac{f^{*-1}}{\mu_{\alpha}^{2} - m_{\phi}^{2}} & |f|^{-2}}{(s - \mu_{\alpha}^{2})[2/(\mu_{\alpha}^{2} - m_{\omega}^{2})^{2} + 1/(\mu_{\alpha}^{2} - m_{\phi}^{2})^{2} + |f|^{-2}]} \end{array} \right)$$
(19)

In the M II "standard" case with $h(s) = s - m_0^2$ and m_0 the "bare" quarkless states' mass, the two free parameters f and m_0 are determined by the physical ϕ and ω masses, with μ_0 an output (to be compared, naturally, with masses of quarkless states determined in simple calculable models). It is remarkable that this model can fail; the determinantal equation may require $|f|^2 < 0$. This cannot be overcome with a Hermitian interaction matrix. This problem arises for certain multiplets in the Freund and Nambu² scheme and has led us to consider models of class M III.

Class MIII. Consider a reordering of the perturbation sum in MII, as follows:

$$\pi = (1 + PH)(P^{-1} - Q)^{-1}, \qquad (20)$$

with

$$Q = HPH = \begin{bmatrix} \frac{2f^2}{h(s)} & \frac{\sqrt{2}f^2}{h(s)} & 0\\ \frac{\sqrt{2}f^2}{h(s)} & \frac{f^2}{h(s)} & 0\\ 0 & 0 & \frac{2f^2}{s - m_{\omega}^2} + \frac{f^2}{s - m_{\phi}^2} \end{bmatrix}.$$
(21)

Now Q [thus $(P^{-1} - Q)^{-1}$] has the form of a direct sum, with block-diagonal separation of the quark and quarkless channels, with all communication

between these channel species in the PH term, which has only off-block-diagonal terms.

If one were persuaded, by experiment or prejudice, that an extra quarkless channel simply does not materialize (i.e., that completeness is realized within quark channels) then one is led to the following theory (M III) which is a kind of projection or truncation of the 3×3 theory:

$$\pi = (P^{-1} - Q)^{-1} = P \sum_{n} (QP)^{n}, \qquad (22)$$

where P is as in M I and

$$Q = g(s) \begin{pmatrix} 2 & \sqrt{2} \\ \sqrt{2} & 1 \end{pmatrix}.$$
 (23)

It is significant that Q is rank one; the results which follow can easily be extended to any rankone interaction. As before we have

$$\pi = \sum_{\alpha} \frac{V_s(s) V_{\alpha}^{\alpha}}{s - M_{\alpha}^2(s)},$$
(24)

where now $M_{\alpha}(s)$ are solutions of

$$(M^2 - m_{\omega}^2)(M^2 - m_{\phi}^2) - 2g(s)(M^2 - m_{\phi}^2)$$

 $-g(s)(M^2 - m_{\omega}^2)$ (25)

and

$$V_{\alpha}(s) = \begin{bmatrix} \frac{\sqrt{2}}{M_{\alpha}^{2}(s) - m_{\omega}^{2}} \\ \frac{1}{M_{\alpha}^{2}(s) - m_{\phi}^{2}} \end{bmatrix} \times \left[\frac{2}{(M_{\alpha}^{2}(s) - m_{\omega}^{2})^{2}} + \frac{1}{(M_{\alpha}^{2}(s) - m_{\phi}^{2})^{2}} \right]^{-1/2}$$
(26)

with the physical-particle-poles solution of

$$\mu_{\alpha}^{2} = M_{\alpha}^{2} (\mu_{\alpha}^{2}), \qquad (27)$$

which again may be multivalued. As before, we have the propagator evaluated at the pole in the form Eq. (17), with a similar treatment for the $1 - dM_{\alpha}^{2}/ds$ term. Setting this term to unity puts the condition on g(s),

$$\frac{dg}{ds}(\mu_{\alpha}^{2})=0.$$

Whether handled this way or through vertex renormalization, the remaining propagator for $\overline{\pi}$ is

$$\overline{\pi} = \sum_{\alpha} \left[\frac{2(\mu_{\alpha}^{2} - m_{\omega}^{2})^{-2} \qquad \sqrt{2} \ (\mu_{\alpha}^{2} - m_{\omega}^{2})^{-1} (\mu_{\alpha}^{2} - m_{\phi}^{2})^{-1}}{\sqrt{2} \ (\mu_{\alpha}^{2} - m_{\phi}^{2})^{-1} \qquad (\mu_{\alpha}^{2} - m_{\phi}^{2})^{-2}}{(s - \mu_{\alpha}^{2}) \left[2(\mu_{\alpha}^{2} - m_{\omega}^{2})^{-2} + (\mu_{\alpha}^{2} - m_{\phi}^{2})^{-2} \right]} \right].$$
(28)

It is to be noted that, in contrast to MI, the eigenvectors here are not orthogonal, and physical states cannot be represented as a single unitary transformation of the channel states. This is due to the *s* dependence of the interaction. It is clear that there is no *a priori* requirement that ϕ and ω be orthogonal in channel space (even ignoring mixing with other isosinglets); that would be necessary only if they were degenerate. Thus a single angle does not suffice here to describe the mixing.

In the particular case $(d/ds)g(s) \equiv 0$ (M III "standard") the eigenvectors *are* orthogonal, and there is no coupling-constant normalization. Interestingly enough, that simple one-parameter theory [g(s) = constant] is distinct from the one-parameter M I theory and leads to different mixing.

The specific SU(3)-singlet nature of the interaction can be relaxed, keeping the rank-one structure. This implies

$$Q = g(s) \begin{pmatrix} \cot^2 \beta & \cot \beta \\ \cot^2 \beta & 1 \end{pmatrix}.$$
 (29)

Then the determinantal condition becomes

$$(M^{2} - m_{\omega}^{2})(M^{2} - m_{\phi}^{2}) - \cot^{2}\beta g(s)(M^{2} - m_{\omega}^{2}) = 0$$
(30)

and the eigenvectors are

$$V_{\alpha}(s) = \begin{pmatrix} \frac{\cot\beta}{M_{\alpha}^{2}(s) - m_{\omega}^{2}} \\ \frac{1}{M_{\alpha}^{2}(s) - m_{\phi}^{2}} \end{pmatrix} \times \left[\frac{\cot^{2}\beta}{(M_{\alpha}^{2}(s) - m_{\omega}^{2})^{2}} + \frac{1}{(M_{\alpha}^{2} - m_{\phi}^{2})^{2}} \right]^{-1/2}.$$
(31)

Generalizations to U(4) of M II and M III are straightforward. We briefly mention some possibly useful results: For M II the generalization of Eq. (15) is

$$V_{\alpha} = \begin{pmatrix} \frac{\sqrt{2}}{M_{\alpha}^{2}(s) - m_{\omega}^{2}} \\ \frac{1}{M_{\alpha}^{2}(s) - m_{\phi}^{2}} \\ \frac{1}{M_{\alpha}^{2}(s) - m_{\phi}^{2}} \\ \frac{1}{f^{-1}} \end{pmatrix} \left\{ \frac{2}{\left[M_{\alpha}^{2}(s) - m_{\omega}^{2}\right]^{2}} + \frac{1}{\left[M_{\alpha}^{2}(s) - m_{\phi}^{2}\right]^{2}} + \frac{1}{\left[M_{\alpha}^{2}(s) - m_{\phi}^{2}\right]^{2}} + \left|f\right|^{-2} \right\}^{-1/2},$$
(32)

and the generalization for the eigenvalue condition is $(M^{2} - m_{\omega}^{2})(M^{2} - m_{\phi}^{2})(M^{2} - m_{\phi}^{2})[M^{2} + h(s) - s] - |f|^{2}(M^{2} - m_{\omega}^{2})(M^{2} - m_{\phi}^{2}) - |f|^{2}(M^{2} - m_{\phi}^{2})(M^{2} - m_{\phi}^{2})(M^{2} - m_{\phi}^{2})(M^{2} - m_{\phi}^{2}) = 0.$ (33)

For MIII, the corresponding results are

$$V_{\alpha}(s) = \begin{pmatrix} \frac{\sqrt{2}}{M_{\alpha}^{2}(s) - m_{\omega}^{2}} \\ \frac{1}{M_{\alpha}^{2}(s) - m_{\phi}^{2}} \\ \frac{1}{M_{\alpha}^{2}(s) - m_{\phi}^{2}} \end{pmatrix} \left\{ \frac{2}{[M_{\alpha}^{2}(s) - m_{\omega}^{2}]^{2}} + \frac{1}{[M_{\alpha}^{2}(s) - m_{\phi}^{2}]^{2}} + \frac{1}{[M_{\alpha}^{2}(s) - m_{\phi}^{2}]^{2}} \right\}^{-1/2}$$
(34)

and

$$(M^{2} - m_{\omega}^{2})(M^{2} - m_{\phi}^{2})(M^{2} - m_{\phi}^{2}) - g(s)(M^{2} - m_{\omega}^{2})(M^{2} - m_{\phi}^{2}) - g(s)(M^{2} - m_{\phi}^{2})(M^{2} - m_{\phi}^{2}) - 2g(s)(M^{2} - m_{\phi}^{2})(M^{2} - m_{\phi}^{2}) = 0.$$
 (35)

Finally, how does one choose among the classes of mixing models? If one possesses a fundamental theory, then a particular mixing is implied and there is no need to read on. Alternatively, one can test the various models to see if one is preferred in fitting OZI-rule-violation data, thus limiting the class of fundamental theories to those which lead to a correct mixing model.

In Ref. 12, we studied the pseudoscalar mesons in the context of models M II and M III. At the present level of experimental data they both are acceptable, but improved data should shortly provide a definitive test. Model M I, we believe, clearly fails (but see also Ref. 3).

For the vector mesons one cannot, in models of class MII ("standard" variety) satisfy the mass constraints with $|f|^{2>0}$. We conclude therefore that the strict Freund and Nambu² model is wrong for the case to which it was originally applied.

For purposes of illustration, let us show how a corresponding model of class M III fares within this multiplet. The rate $\phi \rightarrow \rho \pi$ is given by

$$\Gamma(\phi - \rho \pi) = P_{\rho}^{3} \frac{G_{\omega \rho \pi}^{2}}{4\pi} \mathfrak{K}_{\omega \omega}^{\phi}, \qquad (36)$$

where $\mathfrak{R}^{\phi}_{\omega\omega}$ is the residue of the ϕ pole in the ω channel.

$$\mathfrak{K}_{\omega\omega}^{\phi} = \frac{2/(\mu_{\phi}^{2} - m_{\omega}^{2})^{2}}{\left[2/(\mu_{\phi}^{2} - m_{\omega}^{2})^{2} + 1/(\mu_{\phi}^{2} - m_{\phi}^{2})^{2} + 1/(\mu_{\phi}^{2} - m_{\phi}^{2})^{2}\right]}$$
(37)

and $G_{\omega\rho\pi}^{2}/4\pi$ is rather well determined from several other processes.¹² If a D^* or F^* meson were

discovered, then m_{ψ} , the bare mass of the ψ would be known [through analogies of Eq. (1b)] and $\Re^{\phi}_{\omega\omega}$ would be completely determined. However, since the ψ is known to be extremely pure relative to ϕ , $\mu_\phi^{\ 2}-m_\psi^{\ 2}\approx \mu_\phi^{\ 2}-\mu_\psi^{\ 2}$ is a very good approximation in $\mathfrak{K}^{\phi}_{\omega\omega}$, which depends insensitively on the bare ψ mass, with very sensitive dependence on the mass $m_{\omega}^{2} = \mu_{\rho}^{2}$; we obtain the experimental width $\Gamma(\phi)$ $\rightarrow \rho \pi$) = 660 keV for $\mu_{\rho} = 760 \pm 2$ MeV [error reflecting uncertainty in $G_{\omega\rho\pi}(\text{Ref. 12})$ while the experimental mass is 770 ± 10 . The simple M III model appears to survive this test. Note no positivity requirements such as $|f|^2 > 0$ plague M III. Alternatively, if one uses the energy-independent rank-one M III interaction of Eq. (29), relaxing the SU(3)-singlet nature of the interaction, one finds the (now quite simple) mass constraints imply $\tan^2\beta = 1.77$ and $\Gamma(\phi - \rho \pi) = 660$ keV for $\mu_{\rho} = 773 \pm 2$ MeV. Thus this simple form of M III also fares quite well so far as $\phi \rightarrow \rho \pi$ suppression is concerned, but there would appear to be difficulty with the non-SU(3)-symmetric interaction $[\tan^2\beta = \frac{1}{2}$ in SU(3) limit]. Finally, in the simple M I model we have $\Gamma(\phi \rightarrow \rho \pi)$ = 660 keV for $\mu_o = 763 \pm 2$ MeV. It is clear that in the nearly ideally mixed vector multiplet, no one class of models seems sharply distinguished as the best. We take the point of view, however, that there should be a single class of models for all multiplets. Then, in view of MI problems with the pseudoscalars (see Refs. 3 and 12) we believe that models of class MII and MIII, with interactions suggested by the gluon-exchange picture, are those more likely to succeed.

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