Magnetic moments of charmed baryons in the quark model*

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We compute the magnetic moments of charmed baryons using the gauge-theory quark model of De Rújula, Georgi, and Glashow. We find that these moments are quite different (sometimes differing in sign as well as magnitude) from those computed by Choudhury and Joshi, who used U(8) symmetry. The reason for the difference is that in our calculation the heavy mass of the charmed quark badly breaks U(8) symmetry. Our results reduce essentially to those of Choudhury and Joshi when all quark masses are set equal to one another.

In two recent papers, Choudhury and Joshi[1,2] have used U(4) and U(8) symmetries to calculate the magnetic moments of charmed baryons in terms of the magnetic moments of uncharmed baryons. In the first paper, which makes use of U(4) symmetry, the authors obtain expressions for the charmed-baryon magnetic moments in terms of the moments of the proton, neutron, and Λ. In the second paper, which uses U(8) symmetry, the magnetic moments of all charmed baryons are given in terms of the proton magnetic moment.

In this note, we calculate the magnetic moments of charmed baryons in the quark model. We assume, following the gauge-theory model of De Rújula, Georgi, and Glashow,[3] that the quark magnetic moments are proportional to their charge-to-mass ratios.[4] (For a discussion of the justification for this assumption, we refer the reader to Ref. 3.) We obtain values of the charmed-baryon magnetic moments which are quite different from those found by Choudhury and Joshi. The difference between their results and ours arises from the fact that in the model of De Rújula, Georgi, and Glashow, U(4) and U(8) symmetries are badly broken by the heavy mass of the charmed quark. If we let all the quark masses be equal, our results essentially reduce to those in the U(8) work of Choudhury and Joshi. We should remark that the groups U(8) and SU(8) lead to the same results.

It will probably be a long time before there is any hope that the magnetic moments of charmed baryons can be measured. Nevertheless, we present our results now because there may be other calculations more accessible to experimental tests in which the charmed-baryon magnetic moments are needed. It therefore seems to us valuable to point out that there exists a quite different alternative to the charmed-baryon magnetic moments found by Choudhury and Joshi.

To calculate the magnetic moments, we need to assume something about the symmetry of the baryon wave functions. In the usual gauge-theory model,[3] quarks have color, and the baryon wave functions are antisymmetric in the color indices of any two quarks. It is usually assumed that, as a consequence, the baryon wave functions are symmetric under the interchange of all other indices of any two quarks. However, we obtain the same results with the weaker assumption that the wave functions are invariant under isospin rotations and are symmetric under the interchange of the noncolor indices of any two identical quarks.[5] As is usual in quark-model calculations of baryon magnetic moments, we assume that the entire contribution to these moments comes from the intrinsic quark moments, neglecting any contribution from any orbital angular momentum of the quarks.

We use the symbols, u, d, s, c to denote the quarks and m_u, m_d, m_s, m_c to denote their masses. Our notation for the charmed baryons is not the same as that used by Choudhury and Joshi, but instead follows that of our earlier work.[6] Specifically, we keep the usual symbol for a baryon of a given isospin except that we use a subscript to denote the number of charmed quarks which have replaced strange quarks in the baryon. Thus, for example, the Ξ^0 is composed of uss, the Ξ_1^+ of usc, and the Ξ_2^{++} of ucc. In this note, we shall neglect the mass difference between the u and d quarks.

Once the wave functions are given, it is straightforward to write down expressions for the baryon magnetic moments in terms of the proton magnetic moment and the two mass ratios x and y given by

$$x = m_u/m_s, \quad y = m_u/m_c.$$

We compute the magnetic moments of the members of the $\underline{20}_s$ and $\underline{20}_m$ baryon multiplets as well as the nonvanishing transition moments. The $\underline{20}_s$ and $\underline{20}_m$ refer to dimensionality and symmetry of the representation (s = symmetric, m = mixed symmetries) in an SU(4) classification scheme, although SU(4) is of course badly broken. We write down all these magnetic moments to facilitate comparison with the U(8) paper of Choudhury and Joshi.[2] We group the baryons for convenience into SU(3) multiplets of given charm. For example, (8, 0) means an SU(3) octet of charm zero—in other words, the usual baryon octet.

With this notation, the magnetic moments of the members of the $\underline{20}_m$ are (in units in which the proton moment is unity)

$$(8,0):\quad \mu(n) = -\tfrac{2}{3},$$
$$\mu(\Sigma^+) = \tfrac{1}{9}(8+x),\quad \mu(\Sigma^-) = \tfrac{1}{9}(-4+x),$$
$$\mu(\Sigma^0) = \tfrac{1}{9}(2+x),\ \mu(\Lambda) = -\tfrac{1}{3}x\ ,$$
$$\mu(\Xi^0) = -\tfrac{2}{9}(1+2x),\quad \mu(\Xi^-) = \tfrac{1}{9}(1-4x)\ ,$$

$$(6,1):\quad \mu(\Sigma_1^{++}) = \tfrac{2}{9}(4-y),\quad \mu(\Sigma_1^+) = \tfrac{2}{9}(1-y)\ ,$$
$$\mu(\Sigma_1^0) = -\tfrac{2}{9}(2+y)\ ,$$
$$\mu(\Xi_1^+) = \tfrac{2}{9}(2-x-y),\quad \mu(\Xi_1^0) = -\tfrac{2}{9}(1+x+y)\ ,$$
$$\mu(\Omega_1^0) = -\tfrac{2}{9}(2x+y)\ ,$$
$$(\bar{3},1):\quad \mu(\Lambda_1^+) = \mu(\Xi_1'^+) = \mu(\Xi_1'^0) = \tfrac{2}{3}y\ ,$$
$$(3,2):\quad \mu(\Xi_2^{++}) = \tfrac{2}{9}(-1+4y),\quad \mu(\Xi_2^+) = \tfrac{1}{9}(1+8y)\ ,$$
$$\mu(\Omega_2^+) = \tfrac{1}{9}(x+8y)\ ,$$

The absolute values of the nonvanishing transition moments among baryons of the $\underline{20}_m$ are

$$(\Lambda\,|\,\mu\,|\,\Sigma^0) = (\Lambda_1^+\,|\,\mu\,|\,\Sigma_1^+) = 1/\sqrt{3}\,,$$
$$(\Xi_1'^+\,|\,\mu\,|\,\Xi_1^+) = (2+x)/(3\sqrt{3}),$$
$$(\Xi_1'^0\,|\,\mu\,|\,\Xi_1^0) = (1-x)/(3\sqrt{3}).$$

Note that for two baryons B and B' we have $(B\,|\,\mu\,|\,B') = (B'\,|\,\mu\,|\,B)$.

Likewise, the magnetic moments of the members of the $\underline{20}_s$ are (again in units of the proton moment)

$$(10,0):\quad \tfrac{1}{2}\mu(\Delta^{++}) = \mu(\Delta^+) = -\mu(\Delta^-) = 1,\quad \mu(\Delta^0) = 0\ ,$$
$$\mu(\Sigma^{*+}) = \tfrac{1}{3}(4-x),\quad \mu(\Sigma^{*0}) = \tfrac{1}{3}(1-x),\quad \mu(\Sigma^{*-}) = -\tfrac{1}{3}(2+x)\ ,$$
$$\mu(\Xi^{*0}) = \tfrac{2}{3}(1-x),\quad \mu(\Xi^{*-}) = -\tfrac{1}{3}(1+2x)\ ,$$
$$\mu(\Omega^-) = -x,$$
$$(6,1):\quad \mu(\Sigma_1^{*++}) = \tfrac{2}{3}(2+y),\quad \mu(\Sigma_1^{*+}) = \tfrac{1}{3}(1+2y),\quad \mu(\Sigma_1^{*0}) = -\tfrac{2}{3}(1-y)\ ,$$
$$\mu(\Xi_1^{*+}) = \tfrac{1}{3}(2-x+2y),\quad \mu(\Xi_1^{*0}) = -\tfrac{1}{3}(1+x-2y)\ ,$$
$$\mu(\Omega_1^{*0}) = -\tfrac{2}{3}(x-y)\ ,$$
$$(3,2):\quad \mu(\Xi_2^{*++}) = \tfrac{2}{3}(1+2y),\quad \mu(\Xi_2^{*+}) = -\tfrac{1}{3}(1-4y),$$
$$\mu(\Omega_2^{*+}) = -\tfrac{1}{3}(x-4y)\ ,$$
$$(1,3):\quad \mu(\Omega_3^{++}) = 2y.$$

Lastly, in the same units, the absolute values of the nonvanishing transition moments are

$$(p\,|\,\mu\,|\,\Delta^+) = (n\,|\,\mu\,|\,\Delta^0) = \tfrac{2}{3}\sqrt{2}\ ,$$
$$(\Sigma^+\,|\,\mu\,|\,\Sigma^{*+}) = \tfrac{2}{9}\sqrt{2}\,(2+x),\quad (\Sigma^0\,|\,\mu\,|\,\Sigma^{*0}) = \tfrac{1}{9}\sqrt{2}\,(1+2x)\ ,$$
$$(\Sigma^-\,|\,\mu\,|\,\Sigma^{*-}) = \tfrac{2}{9}\sqrt{2}\,(1-x)\ ,$$
$$(\Lambda\,|\,\mu\,|\,\Sigma^{*0}) = (\Lambda_1^+\,|\,\mu\,|\,\Sigma_1^{*+}) = (\tfrac{2}{3})^{1/2}\ ,$$
$$(\Xi^0\,|\,\mu\,|\,\Xi^{*0}) = \tfrac{2}{9}\sqrt{2}\,(x+2),\quad (\Xi^-\,|\,\mu\,|\,\Xi^{*-}) = \tfrac{2}{9}\sqrt{2}\,(1-x)\ ,$$
$$(\Sigma_1^{++}\,|\,\mu\,|\,\Sigma_1^{*++}) = \tfrac{4}{9}\sqrt{2}\,(1-y),\quad (\Sigma_1^+\,|\,\mu\,|\,\Sigma_1^{*+}) = \tfrac{1}{9}\sqrt{2}\,|1-4y|,$$
$$(\Sigma_1^0\,|\,\mu\,|\,\Sigma_1^{*0}) = \tfrac{2}{9}\sqrt{2}\,(1+2y)\ ,$$
$$(\Xi_1^+\,|\,\mu\,|\,\Xi_1^{*+}) = \tfrac{1}{9}\sqrt{2}\,|2-x-4y|,\quad (\Xi_1^0\,|\,\mu\,|\,\Xi_1^{*0}) = \tfrac{1}{9}\sqrt{2}\,(1+x+4y),$$
$$(\Xi_1'^+\,|\,\mu\,|\,\Xi_1^{*+}) = \sqrt{2}\,(2+x)/(3\sqrt{3}),\quad (\Xi_1'^0\,|\,\mu\,|\,\Xi_1^{*0}) = \sqrt{2}\,(1-x)/(3\sqrt{3}),$$
$$(\Omega_1^0\,|\,\mu\,|\,\Omega_1^{*0}) = (\Omega_2^+\,|\,\mu\,|\,\Omega_2^{*+}) = \tfrac{2}{9}\sqrt{2}\,(x+2y)\ ,$$
$$(\Xi_2^{++}\,|\,\mu\,|\,\Xi_2^{*++}) = \tfrac{4}{9}\sqrt{2}\,(1-y),\quad (\Xi_2^+\,|\,\mu\,|\,\Xi_2^{*+}) = \tfrac{2}{9}\sqrt{2}\,(1+2y)\ .$$

In obtaining the transition moments, we have assumed that the overlap integrals of the spatial wave functions are all equal to unity. This is a good approximation in the model, but is not strictly true. Therefore, the transition moments ought to be slightly smaller than the values given here.

If we set $x = y = 1$ in these expressions, we obtain SU(8)-invariant magnetic moments. These are for the most part the same as the magnetic moments in the U(8) paper of Choudhury and Joshi.[2] However, we do not get the same results as in their original paper for the Λ and Σ_1^{++} magnetic moments as well as for some transition moments. These differences are attributable to misprints in the original paper of Choudhury and Joshi, corrected in the Erratum. With $x = y = 1$ our results for the moments of uncharmed baryons reduce to the SU(6) results of Bég, Lee, and Pais[7] and Thirring.[8]

But the main purpose of our note is not to point out the few differences between our SU(8)-invariant results and those of Choudhury and Joshi, but to discuss the more important differences which occur from the breaking of SU(8). Because we are using the model of De Rújula, Georgi, and Glashow, we use their calculated quark masses. They obtained[3]

$$m_u = m_d = 336 \text{ MeV}, \quad m_s = 540 \text{ MeV}, \quad m_c = 1660 \text{ MeV}.$$

The mass m_u was obtained from the experimental value of the proton magnetic moment, with the assumption that quarks have Dirac moments. The masses m_s and m_c were obtained from relations among the baryon masses derived in the gauge-theory model. These values of the masses give

$$x = 0.62, \quad y = 0.20.$$

With the above value of x, we obtain for the baryon-octet moments approximately the same values as those given in Table I of the paper of De Rújula, Georgi, and Glashow.[3] In particular, the value of the Λ magnetic moment is predicted to be $\mu(\Lambda) = -0.58\ \mu_0$, where μ_0 is the nucleon magneton. On the other hand, if $x = 1$, $\mu(\Lambda) = -0.93\ \mu_0$. The experimental value $\mu(\Lambda) = (-0.67 \pm 0.06)\mu_0$ is somewhat closer to the predicted value with $x = 0.62$.

Turning to the charmed baryons, we see that, because $y = \frac{1}{5}$, the predictions change quite drastically from the SU(8) results. In fact, according to the model, the approximation $y = 0$ is much better than $y = 1$. Let us point out some examples of the differences in the two approximations. If $y = 1$, the charmed baryons belonging to the $(\bar{3}, 1)$ representation have moments $\frac{2}{3}$ as large as the proton moment, whereas in the model of De Rújula, Georgi, and Glashow, these moments are close to zero ($\frac{2}{15}$ the proton moment). Another example is the moment of the Ξ_2^{++}. If $y = 1$, this moment is positive and large; with $y = \frac{1}{5}$, it is negative and small. Also a number of transition moments which vanish in the SU(8) limit are now quite large. This could have an appreciable effect on γ-decay rates of charmed baryons. The reader can easily obtain other examples from our equations, where setting $y = \frac{1}{5}$ leads to quite different results from setting $y = 1$.

In conclusion, in the model of De Rújula, Georgi, and Glashow, SU(8) symmetry is badly broken by the heavy mass of the charmed quark. This symmetry breaking qualitatively changes the static and transition magnetic moments of charmed baryons compared to the values they would have in an SU(8)-invariant theory.

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