Quark sea and quantum chromodynamics

John F. Donoghue*
Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213

Eugene Golowich†
University of Massachusetts, Amherst, Massachusetts 01003
(Received 18 February 1977)

Employing a quantum-chromodynamic framework, we study the existence of quark-antiquark pairs, in addition to the usual valence quarks, within the rest frame of baryons. The structure of such a quark sea is discussed and an estimate of its strength is made using the MIT bag model. A significant amount of sea is found, and its effect on calculated properties of hadrons is 20–30%.

I. INTRODUCTION

In recent years a picture of hadrons as extended objects composed of confined fermion quarks[1] has emerged. At the same time, the most promising candidate for the theory of quark interaction is the strong-interaction gauge theory referred to as quantum chromodynamics (QCD).[2] We present an analysis of one consequence of such a picture, the presence of quark-antiquark pairs in hadrons.

Our study is motivated not only by the natural desire to understand more deeply hadronic wave functions, but also because such knowledge might provide an important clue as to the nature of nonleptonic processes. For example, a potentially important contributor to $|\Delta S|=1$ nonleptonic transitions is the four-quark operator[3]: $\bar{d}\,\Gamma_1\chi\bar{\chi}\,\Gamma_2 s$:, where Γ_1, Γ_2 describe the spin structure and χ is some heavy quark. One method for calculating matrix elements of such an operator between physical hadron states is to have an explicit model for the $\chi\bar{\chi}$ content of the hadron. We shall present results arising from this line of research in a separate publication.

The quark model may be discussed in various stages. At the most basic level one is concerned with the quantum numbers of the quarks, such as quark identity (flavor), spin, and color. The model used here contains quarks of four types (u,d,s,c), with spin $\frac{1}{2}$ and three colors. Baryons are, in the first approximation, color-singlet combinations of three quarks, in the appropriate spin and flavor combination. At the next level one constructs the space-time structure of the hadronic wave functions. Experience has taught us that quarks in the lowest-lying hadrons appear to be confined to a volume with a characteristic dimension of 1 fm and to be in S-wave states. The spin structure of the proton, for example, has the form

$$|p\rangle_0=\frac{1}{\sqrt{18}}\epsilon^{\alpha\beta\gamma}\left[b^\dagger(u,\uparrow,\alpha)b^\dagger(d,\downarrow,\beta)-b^\dagger(u,\downarrow,\alpha)b^\dagger(d,\uparrow,\beta)\right]b^\dagger(u,\uparrow,\gamma)\,|0\rangle\,, \tag{1}$$

where $b^\dagger(f,m,\alpha)$ is a creation operator for a quark of flavor f, spin m, and color α in the lowest-lying S-wave state. Several models have been developed which incorporate these features.[4-6] While the models differ as to the precise shape of the wave function, most appear to be reasonable approximations to the true hadronic wave function. In particular, the space-time structure of the ground state in the MIT bag model is given in Appendix A.[3] Various properties of hadrons, such as masses, magnetic moments, charge radii, and coupling constants may be calculated and in general are found to be in agreement with experiment to within 20–30%. In addition, higher-mass hadrons may be formed by radial and orbital excitation from the ground states. Again these are found to be in general agreement with experiment. These successes force us to take the quark model of hadron structure seriously.

More recently there have been attempts[6,7] to go a step beyond the above by looking at the quark dynamics in greater detail, within the framework of QCD. The quarks interact with spin-one gluons coupled to the color degree of freedom via the Hamiltonian density

$$H(x)=g\bar{\psi}(x)\gamma^\mu\frac{\lambda^A}{2}\psi(x)A^A_\mu(x)\,, \tag{2}$$

where λ^A is an SU(3) matrix $(A=1,\ldots,8)$ in color space. The spin-dependent interaction of quarks exchanging gluons splits the mass degeneracy of the N-Δ, ρ-π, Λ-Σ, etc. systems, resulting in a reasonable mass spectrum.

The presence of the interaction of Eq. (2) necessarily has other consequences as well. For example, quark-antiquark pairs are produced in addition to the usual three valence quarks. These pairs, which we shall call the quark sea, change the hadron structure and modify the properties of hadrons. The analysis of the quark sea follows in a straightforward way from Eq. (2). It is important to study such effects in order to understand the consequences of the sea and to check whether the dynamical picture of hadrons is self-consistent. That sea quarks do in fact exist appears to have been verified in deep-inelastic scattering.[8] We shall restrict our study of their properties to that of the hadron rest frame. Much of our work depends only on the interaction Hamiltonian of QCD, Eq. (2). However, to present a complete analysis we shall need explicit hadron wave functions. When necessary, we shall use those of the MIT bag model.

The outline of the paper is as follows. In Sec. II we discuss the general framework of the quark sea and its structure. Section III is devoted to a study of the amounts of the various components in the sea. Then we examine the effects of the sea on earlier quark-model calculations in Sec. IV. Section V contains a summary of results and our conclusion. We have two appendixes, the first presenting portions of the MIT bag theory that are needed in the text, and the second listing assorted cumbersome formulas on the effects of the sea.

II. STRUCTURE OF THE SEA

We study the properties of a three-quark state propagating under the influence of the quark-gluon coupling, Eq. (2). For the proton, which we shall use as an example throughout

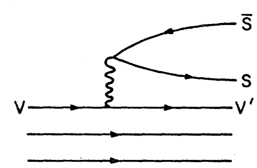

FIG. 1. Quark-sea pairs produced by the action of a gluon. V is a valence particle which the gluon transforms into V', producing a sea quark S and an antiquark $\bar{S}$.

$$|p\rangle = U(0,-\infty)\,|p\rangle_0\,, \tag{3}$$

where $|p\rangle_0$ is the three-quark state Eq. (1) and $U(0,-\infty)$ describes the time development in the presence of the interaction.

$$U(0,-\infty) = T\,\exp\left[-ig\int_{-\infty}^{0} dt \int d^3x\,\bar{\psi}(x)\gamma^\mu \frac{\lambda^A}{2}\psi(x)A_\mu^A(x)\right]. \tag{4}$$

To second order in g quark pairs may be produced via the mechanism illustrated in Fig. 1. At this point, we urge the reader to acquaint himself with the notation of Fig. 1, where the quarks V', S, and $\bar{S}$ are defined. We shall refer repeatedly to these in the following discussion. There are other effects to order g^2; however, we are concentrating here on the quark sea only. The relevant portion of Eq. (4) is

$$U_{\text{pair}} = -g^2\int_{-\infty}^{0} dt \int_{-\infty}^{0} dt' \int d^3x \int d^3x'\, iD(x',x) :\bar{\psi}(x)\gamma^\mu \frac{\lambda^A}{2}\psi(x)\bar{\psi}(x')\gamma_\mu \frac{\lambda^A}{2}\psi(x'):\,, \tag{5}$$

where $D(x',x)$ is the gluon propagator. The time integration may be performed to obtain

$$U_{\text{pair}} = \frac{g^2}{E_S + E_{\bar{S}} + E_{V'} - E_V + i\epsilon}\int d^3x\, d^3x'\,\bar{\psi}(\vec{x})\gamma_\mu \frac{\lambda^A}{2}\psi(\vec{x}')G(\vec{x}',\vec{x})\,\bar{\psi}(\vec{x})\gamma^\mu \frac{\lambda^A}{2}\psi(\vec{x})\,, \tag{6}$$

where

$$G(\vec{x}',\vec{x}) = \int_{-\infty}^{\infty} d\tau\, D(\tau, x', x) \tag{7}$$

and E_k is the energy of quark k, as labeled in Fig. 1. Equations (3) and (6) are just the first terms of the expansion of the Lippmann-Schwinger equation

$$|\psi\rangle = |\psi_0\rangle + \frac{1}{E - H_0 + i\epsilon} V\,|\psi\rangle\,. \tag{8}$$

When U_{pair} acts on the three-quark state $|p\rangle_0$ it produces a five-quark state. Two of the original three valence quarks in the proton remain unchanged in their space, spin, and color structure. The third valence quark is changed by its interaction with the gluon and may be transformed into any one of a variety of allowable states. Observe that, given the value of the quark-gluon coupling constant g, the sea operator of Eq. (6) contains no free parameters. We now examine the structure of the five-quark state $U_{\text{pair}}|p\rangle_0$.

The sea quarks, S and $\bar{S}$, and the final valence quark V' will be in various energy levels within the hadron. However, not all combinations of levels are allowed. Since S and $\bar{S}$ have opposite intrinsic parity, one (or three) of the quarks S, $\bar{S}$, and V' must be in a state of odd parity in order to give the proton the proper overall parity. The lowest-energy sea state then consists of all quarks but one in the S-wave ground state and the remaining quark in the first P-wave state. Higher-energy states can be constructed from other combinations of modes, such that the spin and parity of the proton remain unchanged.

The quark spins must also be constructed properly. The interaction Eq. (5) is a spin singlet, and this allows for two types of spin coupling. The initial valence-quark spin may remain unchanged, with the quark-antiquark pair forming a spin scalar, or the valence spin may change ($\Delta S = 1$) with the sea quarks also forming a spin vector such that the overall four-quark operator is a spin singlet. For obvious reasons we call these cases scalar and vector coupling. Finally, since the interaction is mediated by a color gluon, the colors will be rearranged by octet coupling. With this information we can write down the spin wave function, analogous to Eq. (1), for the quark-sea contribution. We shall employ subscripts i, j, k to label the energy modes of the individual quarks, subject to the constraints discussed above, and use the label f for the flavor of the sea quark-antiquark pair. For the scalar case, which we call ψ_A, we have

$$|\psi^f_{A(i,j,k)}\rangle = (-1)^{m-1/2} b_i^\dagger(S, m, \alpha)\lambda^A_{\alpha\beta} d_j^\dagger(\bar{S}, -m, \beta) b_k^\dagger(V', n, \gamma)\lambda^A_{\gamma\delta}\vec{b}(V, n, \delta)\,|p\rangle_0 \tag{9}$$

with $|p\rangle_0$ being given by Eq. (1). The arrow over the destruction operator $\vec{b}(V, n, \delta)$ indicates that it is to be contracted sequentially on the three creation operators that appear in $|p\rangle_0$:

$$\begin{aligned}\vec{b}(V,n,\delta)\,|p\rangle_0 = \frac{1}{\sqrt{18}}\epsilon^{\alpha\beta\gamma}\{&\delta_{Vu}\delta_{\delta\alpha}[\delta_{n\uparrow} b^\dagger(d,\downarrow,\beta)b^\dagger(u,\uparrow,\gamma) - \delta_{n\downarrow} b^\dagger(d,\uparrow,\beta)b^\dagger(u,\uparrow,\gamma)] \\ &- \delta_{Vd}\delta_{\delta\beta}[\delta_{n\downarrow} b^\dagger(u,\uparrow,\alpha)b^\dagger(u,\uparrow,\gamma) - \delta_{n\uparrow} b^\dagger(u,\downarrow,\alpha)b^\dagger(u,\uparrow,\gamma)] \\ &+ \delta_{Vn}\delta_{\delta\gamma}\delta_{n\uparrow}[b^\dagger(u,\uparrow,\alpha)b^\dagger(d,\downarrow,\beta) - b^\dagger(u,\downarrow,\alpha)b^\dagger(d,\uparrow,\beta)]\}\,|0\rangle\,. \end{aligned}\tag{10}$$

For the case of vector coupling we have the spin wave function, called ψ_B:

$$|\psi^f_{B(i,j,k)}\rangle = (-1)^{m'-1/2} b_i^\dagger(S, m, \alpha)\sigma^i_{mm'}\lambda^A_{\alpha\beta} d_j^\dagger(\bar{S}, -m', \beta) b_k^\dagger(V', n, \gamma)\sigma^i_{nn'}\lambda^A_{\gamma\delta}\vec{b}(V, n, \delta)\,|p\rangle_0\,. \tag{11}$$

The total wave function of a proton including sea quarks will then have the form

$$|p\rangle = |p\rangle_0 + \sum_{i,j,k}(A^f_{(k,j,k)}\,|\psi_{A(i,j,k)}\rangle + B^f_{(i,j,k)}\,|\psi_{B(i,j,k)}\rangle). \tag{12}$$

The $A^f_{(i,j,k)}$ and $B^f_{(i,j,k)}$ are constants (of order g^2) which give the admixture of the various wave functions for a sea-quark pair of flavor f in the modes labeled i, j, and k. They may be read off of Eq. (6) once we are given the quark wave functions. The sum is over all four flavors of quark and all allowable modes.

In the fixed-sphere MIT bag model only $j=\frac{1}{2}$ states satisfy the boundary condition, so that there are only S- and P-wave states. As above, one (or all three) of the quarks $S, \bar{S}, V'$ must be in a P state. It is convenient to classify states into groups defined by their total energy. For example, the three lowest-energy five-quark states each have energy $12/R$, where R is the bag radius ($R \cong 5$ GeV^{-1}). There are 10 states with energy approximately $17/R$ and 21 with $E \approx 20/R$, etc. In general the nth block will contain $n(2n+1)$ states, and will have an energy which asymptotically goes as $E \sim (n\pi + 10)/R$. Using notation defined in the first appendix, we find the A and B amplitudes to have the form,

$$\begin{aligned} A_{(i,j,k)} &= \frac{\alpha_c k}{4\Delta}\sum_{l=0,1,2}\int_0^1 u^2 du\int_0^1 u'^2 du'\, a_l(u,u')\, j_l(ku_<) n_l(ku_>)\,, \\ B_{(i,j,k)} &= \frac{\alpha_c k}{4\Delta}\sum_{l=0,1,2}\int_0^1 u^2 du\int_0^1 u'^2 du'\, b_l(u,u')\, j_l(ku_<) n_l(ku_>)\,, \end{aligned}\tag{13}$$

where $k=\frac{1}{2}(\omega_S + \omega_{\bar{S}} + \omega_V - \omega_{V'})$, $\Delta = \omega_S + \omega_{\bar{S}} + \omega_{V'} - \omega_V$, and $\alpha_c = g^2/4\pi$. The index l labels the angular momentum carried by the gluon. In the following, we exhibit the quantities a_l, b_l for the cases where the S, $\bar{S}$, or V' quark occupies a P-wave mode. The notation is defined in Appendix A, and in particular, a P-wave quark is denoted by means of a tilde. For and S quark occupying a P state, we have

$$\begin{aligned} a_0^S(u,u') &= [\tilde{f}(\omega_S u) f(\omega_{\bar{S}} u) - \tilde{g}(\omega_S u) g(\omega_{\bar{S}} u)][f(\omega_{V'} u') f(\omega_V u') + g(\omega_{V'} u') g(\omega_V u')]\,, \\ a_1^S(u,u') &= [\tilde{f}(\omega_S u) g(\omega_{\bar{S}} u) + \tilde{g}(\omega_S u) f(\omega_{\bar{S}} u)][g(\omega_{V'} u') f(\omega_V u') - f(\omega_{V'} u') g(\omega_V u')]\,, \\ b_1^S(u,u') &= \tfrac{2}{3}[\tilde{f}(\omega_S u) g(\omega_{\bar{S}} u) - \tilde{g}(\omega_S u) f(\omega_{\bar{S}} u)][g(\omega_{V'} u') f(\omega_V u') + f(\omega_{V'} u') g(\omega_V u')]\,, \\ a_2^S(u,u') &= b_0^S(u,u') = b_2^S(u,u') = 0\,. \end{aligned}\tag{14a}$$

The functional form of a_l and b_l is similar when the $\bar{S}$ quark is in the P state,

$$\begin{aligned}
a_0^{\bar{S}}(u,u') &= [f(\omega_S u)\tilde{f}(\omega_{\bar{S}} u) - g(\omega_S u)\tilde{g}(\omega_{\bar{S}} u)][f(\omega_{V'} u')f(\omega_V u') + g(\omega_{V'} u')g(\omega_V j')]\,,\\
a_1^{\bar{S}}(u,u') &= [f(\omega_S u)\tilde{g}(\omega_{\bar{S}} u) + g(\omega_S u)\tilde{f}(\omega_{\bar{S}} u)][g(\omega_{V'} u')f(\omega_V u') - f(\omega_{V'} u')g(\omega_V u')]\,,\\
b_1^{\bar{S}}(u,u') &= \tfrac{2}{3}[f(\omega_S u)\tilde{g}(\omega_{\bar{S}} u) - g(\omega_S u)\tilde{f}(\omega_{\bar{S}} u)][g(\omega_{V'} u)f(\omega_V u') + f(\omega_{V'} u')g(\omega_V u')]\,,\\
a_2^{\bar{S}}(u,u') &= b_0^{\bar{S}}(u,u') = b_2^{\bar{S}}(u,u') = 0\,,
\end{aligned} \tag{14b}$$

whereas for the case of the V' quark in a state of odd parity,

$$\begin{aligned}
a_l^{V'}(u,u') &= 0\,,\\
b_0^{V'}(u,u') &= \tfrac{1}{3} f(\omega_S u)f(\omega_{\bar{S}} u)\tilde{g}(\omega_{V'} u')g(\omega_V u') - \tfrac{1}{3} g(\omega_S u)g(\omega_{\bar{S}} u)\tilde{f}(\omega_{V'} u')f(\omega_V u)\\
&\quad - f(\omega_S u)f(\omega_{\bar{S}} u)\tilde{f}(\omega_{V'} u')f(\omega_V u') + \tfrac{1}{9} g(\omega_S u)g(\omega_{\bar{S}} u)\tilde{g}(\omega_{V'} u')g(\omega_V u')\,,\\
b_1^{V'}(u,u') &= \tfrac{1}{3}[f(\omega_S u)g(\omega_{\bar{S}} u) + g(\omega_S u)f(\omega_{\bar{S}} u)][f(\omega_V (u')g(\omega_V u') - \tilde{g}(\omega_{V'} u')f(\omega_V u')]\,,\\
b_2^{V'}(u,u') &= \tfrac{8}{9} g(\omega_S u)g(\omega_{\bar{S}} u)g(\omega_{V'} u')g(\omega_V u')\,.
\end{aligned} \tag{14c}$$

Finally, when all of the three quarks are in P states,

$$\begin{aligned}
a^{S,\bar{S},V'}(u,u') &= 0\,,\\
b_0^{S,\bar{S},V'}(u,u') &= -\tfrac{1}{3}\tilde{f}(\omega_S u)\tilde{f}(\omega_{\bar{S}} u)\tilde{g}(\omega_{V'} u')g(\omega_V u') + \tfrac{1}{3}\tilde{g}(\omega_S u)\tilde{g}(\omega_S u)\tilde{f}(\omega_{V'} u')f(\omega_V u')\\
&\quad + \tilde{f}(\omega_S u)\tilde{f}(\omega_{\bar{S}} u)\tilde{f}(\omega_{V'} u')f(\omega_V u') - \tfrac{1}{9}\tilde{g}(\omega_S u)\tilde{g}(\omega_{\bar{S}} u)\tilde{g}(\omega_{V'} u')g(\omega_V u')\,,\\
b_1^{S,\bar{S},V'}(u,u') &= -\tfrac{1}{3}[\tilde{f}(\omega_S u)\tilde{g}(\omega_{\bar{S}} u) + \tilde{g}(\omega_S u)\tilde{f}(\omega_{\bar{S}} u)][\tilde{f}(\omega_{V'} u')g(\omega_V u') - \tilde{g}(\omega_{V'} u')f(\omega_V u')]\,,\\
b_2^{S,\bar{S},V'}(u,u') &= -\tfrac{8}{9} g(\omega_S u)g(\omega_{\bar{S}} u)g(\omega_{V'} u')g(\omega_V u')\,.
\end{aligned} \tag{14d}$$

We have evaluated the A and B amplitudes numerically. The results for the first 13 states (the first two blocks in energy) are listed in Table I.

III. THE PROBABILITY OF THE SEA COMPONENT

We now discuss the overlap of the wave function Eq. (12) with itself, which we interpret as measuring the probability of finding the various configurations in the proton. The relative probability of the sea part compared to the valence part of the wave function is given by

$$\begin{aligned}
P = \sum_{f,i,j,k} &[S^A_{f,i,j,k}\,|A^f_{i,j,k}|^2 + S^B_{f,i,j,k}\,|^2 + B^f_{i,j,k}|^2\\
&+ S^{AB}_{f,i,j,k}(A^{f*\prime}_{i,j,k}B^f_{i,j,k} + B^{f*}_{i,j,k}A^f_{i,j,k})]\,.
\end{aligned} \tag{15}$$

The numbers $S_{f,i,j,k}$ give the overlap of the various states.

$$\begin{aligned}
S^A_{f,i,j,k} &= \langle \psi^f_{A,i,j,k} | \psi^f_{A,i,j,k}\rangle\,,\\
S^B_{f,i,j,k} &= \langle \psi^f_{B,i,j,k} | \psi^f_{B,i,j,k}\rangle\,,\\
S^{AB}_{f,i,j,k} &= \langle \psi^f_{B,i,j,k} | \psi^f_{A,i,j,k}\rangle\,.
\end{aligned} \tag{16}$$

These overlaps are not flavor independent. Because of the Pauli exclusion principle, a sea quark cannot have the same quantum numbers as a valence quark. This will be relevant only if the sea quark is either u or d. One particular consequence of the Pauli principle is that the probabilities of

TABLE I. Quark-sea amplitudes in the bag theory.

State			u or d		s		c	
V'	S	$\bar{S}$	A/α_s	B/α_s	A/α_s	B/α_s	A/α_s	B/α_s
1P	1S	1S	⋯	1.28×10^{-2}	⋯	7.28×10^{-3}	⋯	-1.13×10^{-3}
1S	1P	1S	-9.85×10^{-4}	1.98×10^{-3}	1.87×10^{-3}	-3.85×10^{-5}	2.45×10^{-5}	-3.40×10^{-4}
1S	1S	1P	-9.85×10^{-4}	-1.98×10^{-3}	1.87×10^{-3}	3.85×10^{-5}	2.45×10^{-5}	3.40×10^{-4}
2P	1S	1S		-1.73×10^{-3}	⋯	-1.55×10^{-3}	⋯	-3.65×10^{-4}
1S	2P	1S	-3.53×10^{-5}	-5.20×10^{-5}	4.95×10^{-4}	-7.70×10^{-4}	-5.13×10^{-6}	1.55×10^{-4}
1S	1S	2P	-3.53×10^{-5}	5.20×10^{-5}	4.95×10^{-4}	7.70×10^{-4}	-5.13×10^{-6}	-1.55×10^{-4}
2S	1P	1S	-7.48×10^{-3}	6.35×10^{-4}	-7.52×10^{-3}	1.19×10^{-3}	1.97×10^{-3}	-9.55×10^{-4}
2S	1S	1P	-7.48×10^{-3}	-6.35×10^{-4}	-7.52×10^{-3}	-1.19×10^{-3}	1.97×10^{-3}	9.55×10^{-4}
1P	2S	1S	⋯	3.80×10^{-3}	⋯	3.28×10^{-3}	⋯	-9.38×10^{-4}
1P	1S	2S	⋯	3.80×10^{-3}	⋯	3.28×10^{-3}	⋯	-9.38×10^{-4}
1S	2S	1P	1.18×10^{-3}	-2.00×10^{-3}	7.45×10^{-4}	-1.62×10^{-3}	-4.75×10^{-5}	-2.95×10^{-4}
1S	1P	2S	1.18×10^{-3}	2.00×10^{-3}	7.45×10^{-4}	1.62×10^{-3}	-4.75×10^{-5}	2.95×10^{-4}
1P	1P	1P	⋯	1.33×10^{-3}	⋯	9.88×10^{-4}	⋯	-3.00×10^{-4}

TABLE II. The spin overlaps of five-quark states. The notation is such that $q = G$ means that quark q (as labeled in Fig. 1) is in the ground state and $V' = S$ implies that quark V' is in the same mode as the sea quark S. The sea antiquark $\bar{S}$ may be in an arbitrary mode in all cases. The Kronecker delta δ^{Su} is unity if the sea quark S is a u quark, and zero otherwise.

Description of state	S^A	S^B	S^{AB}
$V' = S = G$	...	$4608 + 1792\delta^{Su} - 1024\delta^{Sd}$	...
$V' = G, S \neq G$	...	4608	...
$V' \neq G, S = G$	$1152 + 640\delta^{Su} + 320\delta^{Sd}$	$3456 + 1280\delta^{Su} - 320\delta^{Sd}$	$-320\delta^{Su} - 640\delta^{Sd}$
$V' = S \neq G$	$1152 + 128\delta^{Su} + 64\delta^{Sd}$	$3456 - 384\delta^{Su} - 192\delta^{Sd}$	$384\delta^{Su} + 192\delta^{Sd}$
$V' \neq S, V' \neq G, S \neq G$	1152	3456	...

finding a u and a d quark in the sea are not identical even though the interaction is isospin invariant. The calculation of the overlaps is straightforward but tedious, and we list the formulas for all possible cases in Table II.

With our A and B amplitudes in the bag model, we can determine the probability [Eq. (15)] numerically. A published fit[6] within the bag model to the hadron masses has determined the effective coupling constant to be $\alpha_S' \equiv \frac{1}{4}\alpha_S = 0.55$. Using this we list in Table III the probability in the first several modes for the four flavors of quark. It is immediately obvious that the quark sea contributes to a considerable degree. The largest contribution occurs when the sea quarks are in the 1S state with the V' quark in the 1P state. Indeed, this configuration alone gives almost half the sea probability among the modes which we have examined. Observe that the large charmed-quark mass suppresses dramatically the charmed-quark content of the nucleon. Likewise SU(3) symmetry is broken by the strange-quark mass, resulting in a reduced $s\bar{s}$ component.

We are not able to analytically sum over all modes to obtain a total probability. However, it is possible to test whether the sum converges or diverges. The analysis presented below shows that for the bag-model wave functions, the sum is in fact finite. We do not know whether this result is unique to the bag model or is more general. Let us now consider in some detail how the question of convergence is resolved. In the following, we shall use the simplified notation $P = \omega_{V'}$, $S = \omega_S$, $\bar{S} = \omega_{\bar{S}}$, and thus $k = \frac{1}{2}(S + \bar{S} - P)$ and $\Delta = S + \bar{S} + P$. In order to spare the reader an overly tedious analysis, we consider the case where the V' quark is restricted to any fixed mode whereas the $S, \bar{S}$ quarks are free to occupy any mode. It turns out that not only is the sum over all modes convergent for the squared amplitude, it even converges for the amplitude itself. As we shall see in Sec. IV, this behavior is crucial in order to obtain finite corrections to two-quark operator matrix elements. Consider a quantity characteristic of the kinds of integrals [appearing in Eq. (13)] which serve to define the sea amplitudes A and B:

TABLE III. The probability of finding various sea configurations within the proton in the bag theory.

State			Probability			
V'	S	$\bar{S}$	u	d	s	c
1P	1S	1S	0.209	0.138	0.049	0.0012
1S	1P	1S	0.005	0.005	2×10^{-6}	0.0001
1S	1S	1P	0.005	0.004	2×10^{-6}	0.0001
2P	1S	1S	0.004	0.003	0.002	0.0001
1S	2P	1S	3×10^{-6}	3×10^{-6}	0.001	3×10^{-5}
1S	1S	2P	5×10^{-6}	2×10^{-6}	0.001	3×10^{-5}
2S	1P	1S	0.018	0.018	0.020	0.0020
2S	1S	1P	0.027	0.021	0.020	0.0020
1P	2S	1S	0.013	0.013	0.010	0.0008
1P	1S	2S	0.018	0.012	0.010	0.0008
1S	2S	1P	0.005	0.005	0.003	0.0001
1S	1P	2S	0.005	0.005	0.003	0.0001
1P	1P	1P	0.001	0.002	0.001	0.0001
Total block I (3 states)			0.220	0.147	0.049	0.0014
Total block II (10 states)			0.091	0.078	0.071	0.0060
Total block III (21 states)			0.024	0.025	0.024	0.0055
Total block IV (36 states)			0.024	0.021	0.023	0.0070
Total blocks I–IV			0.360	0.271	0.167	0.020

$$I=\int_0^1 u^2du\, j_0(Su)j_0(\bar{S}u)\left[n_0(ku)\int_0^u v^2dv\, j_0(kv)+j_0(ku)\int_u^1 v^2dv\, n_0(kv)\right]. \tag{17}$$

For convenience, we have taken $\omega_V=\omega_{V'}=0$. This does not affect in any way the convergence property under discussion. Exact integration of Eq. (17) yields

$$I=\frac{1}{S\bar{S}}\frac{2}{(S+\bar{S})^3}[2j_0(S-\bar{S})-2j_0(S+\bar{S})]$$
$$-\left(\frac{2\cos[\frac{1}{2}(S+\bar{S})]}{S+\bar{S}}+\sin[\tfrac{1}{2}(S+\bar{S})]\right)\{\mathrm{Si}[\tfrac{1}{2}(3S-\bar{S})]+\mathrm{Si}[\tfrac{1}{2}(3\bar{S}-S)]+\mathrm{Si}[\tfrac{1}{2}(S+\bar{S})]-\mathrm{Si}[\tfrac{1}{2}(3S+3\bar{S})]\}, \tag{18}$$

where $\mathrm{Si}(x)$ is the standard sine-integral function. We must next multiply the quantity I by both the ratio k/Δ and also the normalizing factors which accompany field operators in the bag model. By employing the asymptotic expressions $S\sim\pi m$ and $\bar{S}\sim\pi n$, where m,n label the bag modes, we can bound the resulting sum over modes by

$$K\sum_{m,n}^{\infty}(m+n)^{-3}, \tag{19}$$

where K is some constant. The double sum in (19) is easily seen to converge. The squared amplitudes converge even more rapidly. In like manner, we have studied the convergence of all contributions to Eq. (15) and after varying amounts of tedious analysis, we have found them to converge.

A conceivable criticism of our numerical results concerns the possibility that we are overestimating the contribution when the sea quarks are in the higher-energy modes. Quantum chromodynamics is a non-Abelian gauge theory and is asymptotically free.[1] Throughout our calculation we use a rather large effective coupling constant determined from low-energy phenomenology. However, in some of the modes studied in the fourth block the gluon carries roughly 3 GeV of energy. In asymptotically free theories the effective coupling constant decreases as the energy scale increases (Ref. 9):

$$\alpha_S(Q^2)=\frac{12\pi}{27\ln Q^2/\Lambda^2} \tag{20}$$

with Λ being a constant. We have not taken this effect into account in our numerical analysis, sticking to a strictly perturbative framework. Its inclusion would certainly increase the rate of convergence of our sum over modes, although in view of its logarithmic nature, it would not have a drastic effect upon the set of modes studied here.

Finally, it is useful to explore the model dependence of our results within the bag model. Several bag-model fits have been performed, yielding somewhat different parameters.[2,3] In our evaluation we have used the parameters of Ref. 6, i.e., $m_u=m_d=0$, $m_s=0.279$ GeV, $m_c=1.55$ GeV, and $R=5\ \mathrm{GeV}^{-1}$. The $m=0$ results are independent of the bag radius R and bag pressure B, depending only on α_c. When $m\neq0$ the only model dependence enters through the dimensionless quantity mR. The variation with mR of the numerically large amplitude for which the $V',S,\bar{S}$ quarks occupy the $1P, 1S, 1S$ modes is given in Fig. 2. We notice that the $m=0$ model has the largest sea amplitude, but, since all reasonable models proposed so far have $mR\lesssim1$, the variation is only 30% in amplitude. While this would reduce the probability by a factor of 2, it appears hard to significantly reduce the sea probability beyond this in any bag model. We need the large coupling constant, and hence the large sea, if we are to use Eq. (2) to provide mass splittings of the appropriate size.

IV. EFFECT OF THE SEA UPON TWO-QUARK OPERATORS

The probability discussed in the last section is of limited importance since it is not empirically observable. Of greater interest is the effect that the sea has on various observables which can be calculated within the quark model. We first review these quantities as calculated in the valence-quark model and then discuss the corrections due to the sea.

The magnetic-moment operator is defined by

$$\vec{\mu}=\tfrac{1}{2}\int d^3x\,\vec{r}\times\bar{\psi}(x)\vec{\gamma}Q\psi(x), \tag{21}$$

where Q is the electric-charge matrix. As is well

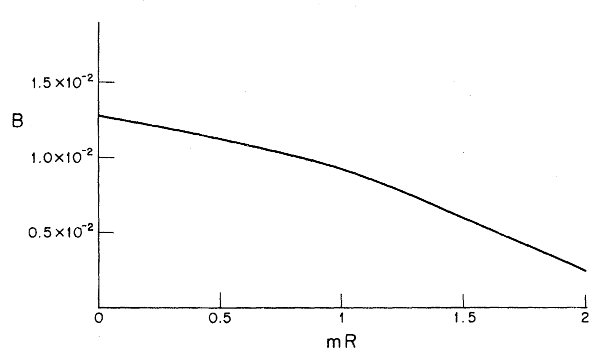

FIG. 2. The variation of the largest sea amplitude (V' in the $1P$ state, S and $\bar{S}$ in the $1S$ state) with mR.

known, the valence-quark wave functions give a satisfactory relation between the neutron and proton magnetic moments $\mu_n/\mu_p = -\frac{2}{3}$. The analytic bag-model expression for the proton magnetic moment is given by

$$\mu_p = \frac{R}{6}\frac{4\omega + 2mR - 3}{2\omega^2 - 2\omega + mR}. \tag{22}$$

With the parameters of Ref. 6 this yields $\mu = 1.01$ GeV^{-1}, implying a gyromagnetic ratio of $g_p = 2M_p\mu_p = 1.9$. The absolute magnitude of the proton magnetic moment is the main failure of the fit of Ref. 6. Other fits[4] yield a more reasonable value, $g_p = 2.6$, compared with the experimental value $g_p = 2.79$.

The vector and axial-vector coupling constants

$$g_V = \int d^3x\bar{\psi}(x)\gamma_0 Q\psi(x), \tag{23}$$

$$g_A = \int d^3x\bar{\psi}(x)\gamma_3\gamma_5\tau_3\psi(x) \tag{24}$$

and measurable charge radii

$$\langle r^2\rangle_{\mathrm{em}} = \int d^3x\, x^2\bar{\psi}(x)\gamma_0 Q\psi(x), \tag{25}$$

$$\langle r^2\rangle_{\mathrm{ax}} = \int d^3x\, x^2\bar{\psi}(x)\gamma_3\gamma_5\tau_3\psi(x) \tag{26}$$

may be calculated for a proton. The vector coupling constant is of course $g_V = 1$ (and $g_V = 0$ for a neutron), whereas the bag-model formulas for the remaining quantities are

$$g_A = \frac{5}{9}\left(\frac{2\omega^2 + 4mR\omega - 3mR}{2\omega^2 - 2\omega + mR}\right), \tag{27}$$

$$\langle r^2\rangle_{\mathrm{em}} = \frac{R^2}{6}\left(\frac{F}{(\omega^2 - m^2R^2)(2\omega^2 - 2\omega + mR)}\right), \tag{28a}$$

where

$$\begin{aligned} F = {} & 4\omega^4 - 4\omega^3 + \omega^2(8 + 6mR - 4m^2R^2) \\ & + \omega(-6 - 8mR + 4m^2R^2) \\ & + mR(9 - 6mR - 6m^2R^2), \end{aligned} \tag{28b}$$

$$\langle r^2\rangle_{\mathrm{ax}} = \frac{10R^2}{27}\left(\frac{G}{(\omega^2 - m^2R^2)(2\omega^2 - 2\omega + mR)}\right), \tag{29a}$$

where

$$\begin{aligned} G = {} & \omega^4 + 2\omega^3(1 + mR) - \omega^2(4 + mR/2 + m^2R^2) \\ & + \omega(3 - mR - 2m^2R^2 - 2m^3R^3) \\ & + mR(-\tfrac{3}{2} + mR + m^2R^2)/2. \end{aligned} \tag{29b}$$

Using the parameters of Ref. 6, we obtain numerical values $g_A = 1.09$, $\langle r^2\rangle_{\mathrm{em}} = 0.52\ \mathrm{fm}^2$, and $\langle r^2\rangle_{\mathrm{ax}} = 0.48\ \mathrm{fm}^2$.

Finally, the σ term in πN scattering defined in terms of quark fields as

$$\sigma = m_u(\bar{u}u + \bar{d}d) \tag{30}$$

is given in the bag model by

$$\sigma = 3m_u\left(\frac{\omega^2(1 + 2mR) + \omega mR(2mR - 1) - 2(mR)^2}{(\omega + mR)(2\omega^2 - 2\omega + mR)}\right). \tag{31}$$

For the $m_u = 0$ model σ of course is also zero. The experimental value $\sigma = 70$ MeV was used in Ref. 4 to fix the u, d quark mass, and a value $m_u = m_d = 44$ MeV was found.

All the operators just discussed are bilinear in quark fields. This means that they can connect a five-quark state and a three-quark state, leading to a correction term of order α_S. However, it is not possible for a two-quark operator to destroy a quark-antiquark sea pair since the operator is a color singlet and the pair forms a color octet. Instead, the operator must act on the antiquark and a valence quark, as in Fig. 3. This limits the relevant sea states to those with u or d quarks, and with the sea quark occupying the 1S state.

Upon inclusion of the pair contributions to the hadron wave function, the matrix elements will have the form

$$\langle O\rangle = \langle O\rangle_{\mathrm{valence}} + \sum_{\substack{ijk \\ f=u,d}}' (A^f_{i,j,k}T^A_{i,j,k} + B^f_{i,j,k}T^B_{i,j,k})\langle O_{ijk}\rangle_{\mathrm{sea}}. \tag{32}$$

$\langle O\rangle_{\mathrm{valence}}$ corresponds to the contributions listed

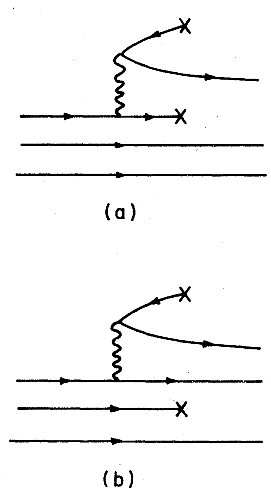

FIG. 3. Diagrams for the action of a two-quark operator taken between a three-quark and a five-quark state. An × indicates the action of a quark-field operator.

in Eqs. (27)–(31). $\langle O_{ijk}\rangle_{\text{sea}}$ is the spatial part of the sea-corrected matrix element, while T_{ijk} is the spin and color sum, analogous to S_{ijk} in the last section. As mentioned above, only the u, d pairs in the sea contribute in this situation. We reserve the presentation of the full form of the corrections for Appendix B because of their length. We have evaluated the effects numerically and these are given in Table IV. We see that, despite the large amount of sea observed in the results of Sec. II, the effect of the sea on the two quark operators (except for σ) is on the order of 20–30%. That σ/m_μ is changed dramatically by the sea is not very important phenomenologically because a modest change in the parameter m_u will allow us to regain a reasonable value for σ. Our result for σ/m_μ implies $m_u \approx 10$ MeV, consistent with zero. For the other matrix elements the effect of the sea is reasonable and does not destroy the successes of the valence-quark model. One interesting case is that of the neutron's charge radius. In the valence-quark model this is identically zero since u and d quarks have the same spatial wave function. With the sea, however, the Pauli principle requires that some states by populated differently for u and d quarks. This generates a charge radius of the appropriate sign, although the magnitude is still too small (see Table IV). Another important feature to be noted is that, although in principle the sea could markedly alter the result $\mu_n/\mu_p = -\frac{2}{3}$, in fact this result is changed by only 1%. Because of the accuracy and importance of this prediction, this is quite reassuring. Observe that this is not a chance result, owing to some lucky combination of the many contributions in the bag model. Instead, it is mainly due to the spin structure of the wave functions $|\psi_A\rangle$ and $|\psi_B\rangle$ [see Eqs. (9) and (11)] such that most of the contributions in the sum over modes satisfy $\langle\mu_n\rangle_i / \langle\mu_p\rangle_i = -\frac{2}{3}$ for each individual mode i. That some modes do not satisfy this will cause some deviation from the original prediction, but these modes are never important.

As in our discussion of the sea probability density, we must examine whether a sum over bag modes causes the sea corrections to the valence-quark results to diverge. The issue is somewhat more delicate than in the case of the probability densities because the correction amplitudes are not to be squared. Alleviating this is the fact that only two $(V', \bar{S})$ and not all three $(S, \bar{S}, V)$ quarks are allowed to occupy arbitrary modes. Thus we are confronted with a double sum instead of a triple sum. It suffices to say that employing the methods discussed in the previous section, we find all correction terms to be convergent in the bag model.

V. DISCUSSION AND CONCLUSION

In order to remove unwanted SU(6) mass degeneracies, such as between the nucleon and $\Delta(1236)$ in quark models of hadrons, spin-spin interactions between constituent quarks can be introduced. Evidently, the most rational way to accomplish this is perturbatively through the exchange of color gluons. To the same order g^2 in quark-gluon coupling, there inevitably appear quark-antiquark pairs, which constitute a nonvalence component to the hadron wave function. We have presented an analysis of the structure and effect of these pairs. In doing so, we have attempted to make our discussion general but have used the MIT bag model to estimate various numerical quantities associated with this phenomenon.

The pair contributions not only must exist, but they enter with no additional free parameters. Moreover, in the bag model these effects can be shown to provide a bounded contribution to the overall probability density. Upon taking the 70 lowest-energy sea components into account, we find the relative probabilities (see Table III) valence: sea(u): sea(d): sea(s): sea(c):: 1:0.36: 0.27: 0.17: 0.02 for the proton. The importance of the Pauli exclusion principle and the damping effect of large quark mass are immediately discernible from these numbers. The sea contributions from nonstrange pairs can certainly not be considered insignificant. However, it is also true that by far the single largest probability is associated with the three-quark valence configuration. Can the remaining infinite sum over allowed modes have a qualitative effect upon these conclusions? We think not. Although it is hard to estimate, the rate of convergence with increasing mode energy leads us to conjecture that the sea probability in its entirety is no more than 50% larger than the effects which we have been able to explicitly calculate. We have nothing to offer as to the magnitude of $O(g^4)$ contributions to the hadronic wave function.

Quark-antiquark pairs in the hadron wave function generate contributions to matrix elements of operators associated with various observables. These $O(g^2)$ corrections to the valence model are summarized in Table IV and depicted in Fig. 3. We have been able to prove that they are finite in the bag model. Moreover, to the extent that we have been able to pursue the numerical analysis up to a fairly large but finite number of allowed bag modes, we find the net effect of the corrections to be appreciably less than the valence-model contribution. All this can and should be viewed as satisfactory. It is consistent with the standard feeling that the valence model provides a good first

TABLE IV. The effects of the sea in the bag model on calculated properties of hadrons.

V'	S	$\bar{S}$	μ_p (GeV^{-1})	μ_n (GeV^{-1})	g_A	$\langle r_{em}^2\rangle_p$ (fm^2)	$\langle r_{em}^2\rangle_n$ (fm^2)	$\langle r^2\rangle_{ax}$ (fm^2)	σ/m_u
1P	1S	1S	0.199	−0.133	0.235	0.14	···	0.06	1.90
1S	1S	1P	0.031	−0.010	−0.029	−0.02	−0.007	−0.01	0.39
2P	1S	1S	0.004	−0.002	0.012	0.01	···	0.01	0.10
1S	1S	2P	10^{-4}	-3×10^{-5}	-3×10^{-4}	-1.5×10^{-4}	-4×10^{-5}	-4×10^{-4}	0.004
2S	1S	1P	0.105	−0.070	0.111	-1×10^{-3}	···	0.01	0.41
1P	1S	2S	−0.058	0.039	0.062	-1.5×10^{-4}	···	0.01	0.50
Total sea (4 blocks)			0.314	−0.199	0.43	0.12	−0.007	0.07	3.71
Valence			1.01	−0.67	1.09	0.52	···	0.48	1.44
Sea + valence			1.32	−0.87	1.52	0.64	−0.007	0.55	5.15
Experiment			1.49	−1.02	1.25	0.66 ± 0.03	−0.12 ± 0.01	0.67 ± 0.1	57 ± 12 MeV/m_u

approximation to hadron phenomena. Beyond this, it is hard to gauge the success of the quantitative bag-model results we have calculated here. The $O(g^2)$ corrections considered in this paper modify the valence-model results in a reasonable manner for magnetic moments and charge radii, but the net axial-charge prediction overshoots the experimental value by a serious amount. Of course, it must be remembered that other $O(g^2)$ corrections occur,[10] and must be included before the bag model can be judged fairly. Finally, the σ term analysis has forced us to reduce our previous estimate of the nonstrange-quark mass from 44 MeV down to a mass near zero. It would be of interest to analyze the σ term in competing quark models to see what bounds one obtains.

It is common practice to interpret the structure functions of deep inelastic lepton scattering in terms of quark content of hadrons in the infinite-momentum frame.[8,11] Unfortunately, since we deal with particles at rest, we know of no way to make a comparison directly between our results and those of the infinite-momentum frame. The infinite-momentum boost is expected to change the structure of valence and sea quarks, but so far only qualitative arguments have been put forth.[12] At any rate, to the extent that in our model the three valence quarks dominate any other single-quark configuration, we are consistent with the fact that structure functions peak at $x\cong\frac{1}{3}$.

As mentioned in the Introduction, one future application of our model concerns the role of heavy quarks in nonleptonic processes. In light of this proposed line of investigation, it is particularly relevant that the sea contribution to the hadron wave function enters with no free parameters. Thus, the nonleptonic analysis can proceed unhindered by *ad hoc* parameters which would otherwise be needed to characterize the admixture of heavy quarks in hadrons. Work on this project is now under way.

ACKNOWLEDGMENTS

One of us (E.G.) wishes to thank Professor B. W. Lee for the hospitality extended him by the Theory Group at Fermilab, where part of the analysis described in this paper was carried out. We also thank Dr. Barry R. Holstein for useful discussions.

APPENDIX A: THE MIT BAG MODEL

The general framework of the fixed-sphere bag model has been presented elsewhere and will not be repeated here.[4,6] Rather we present and develop the notation that we will need in the text. A quark of mass m is contained within a sphere of radius R and satisfies the Dirac equation

$$(i\partial\!\!\!/ - m)\psi(x) = 0\,, \tag{A1}$$

with a boundary condition

$$-i\vec{\gamma}\cdot\hat{r}\,\psi(x) = \psi(x) \tag{A2}$$

at $r=R$. Only $j=\frac{1}{2}$ solutions are allowed by Eq. (A2). The S-wave solution ($\kappa=-1$) has the form

$$\psi(x) = (4\pi)^{-1/2}\sum_{\text{modes}}\left[\begin{pmatrix} if(\omega r/R)\chi_m \\ g(\omega r/R)\vec{\sigma}\cdot\hat{r}\,\chi_m\end{pmatrix} b^\dagger(m,\alpha)\,e^{-i\omega t/R} + \begin{pmatrix} ig(\omega r/R)\vec{\sigma}\cdot\hat{r}\,\chi_m \\ f(\omega r/R)\chi_m\end{pmatrix} d^\dagger(m,\alpha)\,e^{+i\omega t/R}\right]. \tag{A3}$$

Here, χ_m is a Pauli spinor for spin m and we use α to label other quantum numbers. The energy is $E=\omega/R$, and

$$f\left(\frac{\omega r}{R}\right) = Nj_0(pr)\,,\quad g\left(\frac{\omega r}{R}\right) = -N\left(\frac{\omega - mR}{\omega + mR}\right)^{1/2} j_1(pr)\,, \tag{A4}$$

where $p=(\omega^2-m^2R^2)^{1/2}/R$ is the momentum and N is a normalization factor such that $\int d^3x\,\psi^\dagger(x)\,\psi(x)=1$. Also, $b^\dagger$ ($d^\dagger$) is a creation operator for a quark (antiquark) in a given bag mode and obeys the anticommutation relations

$$\{b^\dagger(m,\alpha),b(m',\alpha')\}=\{d^\dagger(m,\alpha),d(m',\alpha')\}=\delta_{mm'}\delta_{\alpha\alpha'}\,. \tag{A5}$$

P-wave solutions ($\kappa=+1$) have the form

$$\psi(x)=(4\pi)^{-1/2}\sum_{\text{modes}}\left[\begin{pmatrix}-i\tilde g(\omega r/R)\vec\sigma\cdot\hat r\,\chi_m\\ \tilde f(\omega r/R)\,\chi_m\end{pmatrix}b^\dagger(m,\alpha)\,e^{-i\omega t/R}+\begin{pmatrix}i\tilde f(\omega r/R)\,\chi_m\\ -\tilde g(\omega r/R)\vec\sigma\cdot\hat r\,\chi_m\end{pmatrix}d^\dagger(m,\alpha)\,e^{+i\omega t/R}\right] \tag{A6}$$

with

$$\begin{aligned}\tilde f(\omega r/R)&=\tilde N j_0(pr)\,,\\ \tilde g(\omega r/R)&=-\tilde N\left(\frac{\omega+mR}{\omega-mR}\right)^{1/2}j_1(pr)\,.\end{aligned} \tag{A7}$$

The boundary condition (A2) becomes

$$\tan[(\omega^2-m^2R^2)^{1/2}]=-\frac{\kappa(\omega^2-m^2R^2)^{1/2}}{\omega+\kappa mR-\kappa} \tag{A8}$$

which, given mR, determines the allowable ω. A second, nonlinear, boundary condition is not relevant for us here. Fits have been performed to determine the various bag parameters, the most elaborate being Ref. 6 which yields $m_u=m_d=0$, $m_s=0.279$ GeV, $m_c=1.55$ GeV, $R=5\ \text{GeV}^{-1}$, and $\alpha_s'=\frac{1}{4}\alpha_s=0.55$. These are the numbers that are used in the text.

The gluon propagator defined in Eq. (7) obeys

$$(\nabla_x{}^2+k^2)\,G(\vec x,\vec x')=-\delta^3(\vec x-\vec x') \tag{A9}$$

plus a boundary condition on the surface of the bag

$$G(\vec x,\vec x')=-(\vec x,\nabla_x)\,G(x,x')\,|_{x=R}\,. \tag{A10}$$

The solution, using standard techniques, is

$$G(\vec x,\vec x')=-k\sum_{l,m}[j_l(kr_<)\,n_l(kr_>)-\gamma_l j_l(kr)\,j_l(kr')\,Y_{lm}(r)\,Y_{lm}^*(r')]\,, \tag{A11}$$

with

$$\gamma_l=\frac{n_l(kR)+kRn_l'(kR)}{j_l(kR)+kRj_l'(kR)}\,. \tag{A12}$$

A numerical study of Eq. (A12) quickly reveals that in some modes γ_l is extremely large. For example, in the notation of Fig. 1, if $n_S=3$, $n_{\bar S}=4$, $n_{V'}=8$, $n_V=1$ and $\kappa_S=\kappa_{\bar S}=\kappa_{V'}=+1$, $\kappa_V=-1$, we find $\gamma_2=-1.26\times10^5$. In the bag model, the bag surface acts as a source and sink of gluons. With a fixed-sphere model, the gluons resonate at particular frequencies, giving this behavior. Clearly this is an unphysical characteristic of the fixed-sphere geometry. We remove this behavior by using $\gamma_l=0$. This corresponds to equal amounts of incoming and outgoing waves, as is reasonable for confined gluons. Other prescriptions would change the details of our results somewhat, but not the general conclusions.

APPENDIX B: ANALYTIC EXPRESSIONS FOR CORRECTION TERMS

We list here the analytic form of the order α_s corrections to the static properties of nucleons, as discussed in Sec. IV. In all cases the S quark is in the 1S state (which we will label G), and we sum over the modes of the V' and $\bar S$ quarks. The situation where the V' quark is (is not) in the 1S state will be labeled $V'=G$ ($V'\neq G$), and we include a subscript on the A or B amplitudes to indicate which quark is in a P-wave state.

The sea correction to the proton's magnetic moment μ_p^{sea} is

$$\begin{aligned}\mu_p^{\text{sea}}=\tfrac{32}{9}\sum_{S,V'}B_{V'}R\int_0^1u^3du\,[f(\omega_{\bar S}u)\,\tilde g(\omega_{V'}u)-g(\omega_{\bar S}u)\tilde f(\omega_{V'}u)]\\ +\tfrac{32}{9}\sum_{\substack{\bar S\\ V'=G}}B_{\bar S}R\int_0^1u^3du\,[\tilde f(\omega_{\bar S}u)\,g(\omega_{V'}u)-\tilde g(\omega_{\bar S}u)f(\omega_{V'}u)]\\ +\tfrac{32}{9}\sum_{\substack{\bar S\\ V'\neq G}}(B_{\bar S}-A_{\bar S})R\int_0^1u^3du\,[\tilde f(\omega_{\bar S}u)\,g(\omega_{V'}u)-\tilde g(\omega_{\bar S}u)f(\omega_{V'}u)]\,;\end{aligned} \tag{B1}$$

for the neutron's magnetic moment

$$\mu_n^{\text{sea}} = -\tfrac{64}{27}\sum_{S,V'} B_{V'}R\int_0^1 u^3du\,[f(\omega_S u)\tilde g(\omega_{V'}u) - g(\omega_{\bar S}u)\tilde f(\omega_{V'}u)]$$
$$-\tfrac{32}{27}\sum_{\substack{\bar S\\ V'=G}} B_{\bar S}R\int_0^1 u^3du\,[\tilde f(\omega_{\bar S}u)g(\omega_{V'}u) - \tilde g(\omega_{\bar S}u)f(\omega_{V'}u)]$$
$$-\tfrac{64}{27}\sum_{\substack{\bar S\\ V'\neq G}} (B_{\bar S} - A_{\bar S})R\int_0^1 u^3du\,[\tilde f(\omega_{\bar S}u)g(\omega_{V'}u) - \tilde g(\omega_{\bar S}u)f(\omega_{V'}u)]\,. \quad \text{(B2)}$$

The axial-vector coupling-constant correction is

$$g_A^{\text{sea}} = \tfrac{160}{9}\sum_{\bar S,V'} B_{V'}\int_0^1 u^2du\,[\tilde f(\omega_{V'}u)f(\omega_{\bar S}u) + \tfrac{1}{3}\tilde g(\omega_{V'}u)g(\omega_{\bar S}u)]$$
$$+\tfrac{128}{9}\sum_{\substack{\bar S\\ V'=G}} B_{\bar S}\int_0^1 u^2du\,[f(\omega_{V'}u)\tilde f(\omega_{\bar S}u) + \tfrac{1}{3}g(\omega_{V'}u)\tilde g(\omega_S u)]$$
$$+\tfrac{160}{9}\sum_{\substack{\bar S\\ V'\neq G}} (B_{\bar S} - A_{\bar S})\int_0^1 u^2du\,[f(\omega_{V'}u)\tilde f(\omega_{\bar S}u) + \tfrac{1}{3}g(\omega_{V'}u)\tilde g(\omega_{\bar S}u)]\,; \quad \text{(B3)}$$

The various charge-radii corrections are

$$\langle r_{\text{em}}^2\rangle_p^{\text{sea}} = -32R^2\sum_{\bar S,V'} B_{V'}\int_0^1 u^4du\,[\tilde f(\omega_{V'}u)f(\omega_{\bar S}u) - \tilde g(\omega_{V'}u)g(\omega_{\bar S}u)]$$
$$-32R^2\sum_{\substack{\bar S\\ V'=G}} B_{\bar S}\int_0^1 u^4du\,[f(\omega_{V'}u)\tilde f(\omega_{\bar S}u) - g(\omega_{V'}u)\tilde g(\omega_{\bar S}u)]$$
$$-\tfrac{32}{3}R^2\sum_{\substack{\bar S\\ V'\neq G}} (3B_{\bar S} + A_{\bar S})\int_0^1 u^4du\,[f(\omega_{V'}u)\tilde f(\omega_{\bar S}u) - g(\omega_{V'}u)\tilde g(\omega_{\bar S}u)]\,, \quad \text{(B4)}$$

$$\langle r_{\text{em}}^2\rangle_n^{\text{sea}} = -\tfrac{32}{3}R^2\sum_{\substack{\bar S\\ V'=G}} B_{\bar S}\int_0^1 u^4du\,[f(\omega_{V'}u)\tilde f(\omega_{\bar S}u) - g(\omega_{V'}u)\tilde g(\omega_{\bar S}u)]\,, \quad \text{(B5)}$$

$$\langle r_{\text{ax}}^2\rangle_p^{\text{sea}} = \tfrac{160}{9}R^2\sum_{\bar S,V'} B_{V'}\int_0^1 u^4du\,[\tilde f(\omega_{V'}u)f(\omega_{\bar S}u) + \tfrac{1}{3}\tilde g(\omega_{V'}u)g(\omega_{\bar S}u)]$$
$$+\tfrac{128}{9}R^2\sum_{\substack{\bar S\\ V'=G}} B_{\bar S}\int_0^1 u^4du\,[f(\omega_{V'}u)\tilde f(\omega_{\bar S}u) + \tfrac{1}{3}g(\omega_{V'}u)\tilde g(\omega_{\bar S}u)]$$
$$+\tfrac{160}{9}R^2\sum_{\substack{\bar S\\ V'\neq G}} (B_{\bar S} - A_{\bar S})\int_0^1 u^4du\,[f(\omega_{V'}u)\tilde f(\omega_{\bar S}u) + \tfrac{1}{3}g(\omega_{V'}u)\tilde g(\omega_{\bar S}u)]\,. \quad \text{(B6)}$$

Finally, for the σ term

$$\frac{\sigma^{\text{sea}}}{m_u} = 96\sum_{\bar S,V'} B_{V'}\int_0^1 u^2du\,[\tilde f(\omega_{V'}u)f(\omega_{\bar S}u) + \tilde g(\omega_{V'}u)g(\omega_{\bar S}u)]$$
$$-128\sum_{\substack{\bar S\\ V'=G}} B_{\bar S}\int_0^1 u^2du\,[f(\omega_{V'}u)\tilde f(\omega_{\bar S}u) + g(\omega_{V'}u)\tilde g(\omega_{\bar S}u)]$$
$$-32\sum_{\substack{\bar S\\ V'\neq G}} (3B_{\bar S} + A_{\bar S})\int_0^1 u^2du\,[f(\omega_{V'}u)\tilde f(\omega_{\bar S}u) + g(\omega_{V'}u)\tilde g(\omega_{\bar S}u)]\,. \quad \text{(B7)}$$

There is no change to order α_s in the vector-coupling constant g_V, as must be the case since g_V is just the total charge of the particle.

*Work supported by the U. S. Energy and Research and Development Administration.
†Work supported in part by the National Science Foundation.

[1]M. Gell-Mann, Phys. Lett. 8, 214 (1964); G. Zweig, CERN Report No. TH 401, 1964 (unpublished).
[2]D. Gross and F. Wilczek, Phys. Rev. D 8, 3633 (1973).
[3]Among authors who have suggested the importance of heavy quarks for nonleptonic processes, see R. K. Ellis, Nucl. Phys. B108, 239 (1976); F. Wilczek and A. Zee, Phys. Rev. D 15, 2660 (1977).
[4]A. Chodos, R. Jaffe, K. Johnson, C. Thorn, and V. Weisskopf, Phys. Rev. D 7, 3971 (1974). Also see A. Chodos, R. Jaffe, K. Johnson, and C. Thorn, *ibid.* 10, 2599 (1974); E. Golowich, *ibid.* 12, 2108 (1975); J. Donoghue, E. Golowich, and B. Holstein, *ibid.* 12, 2875 (1975).
[5]R. P. Feynman, M. Kisslinger, and F. Ravndal, Phys. Rev. D 3, 2706 (1971); W. A. Bardeen, M. Chanowitz, S. Drell, M. Weinstein, and T. Yan, *ibid.* 11, 1094 (1975); A. De Rújula, H. Georgi, and S. L. Glashow, *ibid.* 12, 147 (1975); Y. S. Kim and M. E. Noz, *ibid.* 8, 3521 (1973).
[6]T. Degrand, R. Jaffe, K. Johnson, and J. Kiskis, Phys. Rev. D 12, 2060 (1975).
[7]J. Donoghue, E. Golowich, and B. Holstein, Phys. Rev. D 15, 1341 (1977).
[8]See, for example, the numbers quoted by E. Witten, Nucl. Phys. B104, 445 (1976). Of course, one should keep in mind that the relation between a quark-antiquark pair in a rest-frame hadron and what is commonly referred to as the "sea" in deep inelastic processes is far from clear.
[9]As quoted, for example, A. De Rújula, H. Georgi, and H. D. Politzer, Ann. Phys. (N.Y.) 103, 315 (1977).
[10]A. Halprin and P. Sorba, Phys. Lett. 66B, 177 (1977).
[11]H. Georgi and H. D. Politzer, Phys. Rev. D 14, 1829 (1976).
[12]See, for example, J. D. Bjorken, SLAC Report No. 1756, 1976 (unpublished), especially Sec. V A.
[13]The numbers quoted are from μ_p and μ_n, Particle Data Group, Phys. Lett. 50B, 1 (1974); g_A, K. Kleinknecht, in *Proceedings of the XVII International Conference on High Energy Physics, London 1974,* edited by J. R. Smith (Rutherford Laboratory, Chilton, Didcot, Berkshire, England, 1974); $\langle r_{\rm em}{}^2\rangle_p$, F. Borkowski *et al.*, Nucl. Phys. A222, 269 (1975); $\langle r_{\rm em}{}^2\rangle_n$, R. W. Berard *et al.*, Phys. Lett. 47B, 355 (1973); $\langle r_{\rm ax}{}^2\rangle$, M. Derrick, in *Proceedings of the Sixth International Symposium on Electron and Photon Interactions at High Energy, Bonn, Germany, 1973,* edited by H. Rollnik and W. Pfeil (North-Holland, Amsterdam, 1974); σ, Y. A. Chao, R. E. Cutkosky, R. L. Kelly, and J. W. Alcock, Phys. Lett. 57B, 150 (1975).