Constraints on gauge theories with diagonal neutral currents*

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We derive constraints for the representation content of SU(2) × U(1) gauge theories, whose neutral weak interactions conserve all flavors. Among the results are the following: (a) For theories where all right-handed quarks are singlets under weak isospin, the existence of quarks with different values of T_L^2 and Y_L implies one or more conservation laws in addition to charge. (b) In models with quarks of charges 2/3 and −1/3 and no new conservation laws we find that there are only three possibilities: (i) a generalization of the standard model with left-handed doublets and right-handed singlets, (ii) its mirror image, and (iii) a pure vector model with all quarks in doublets on the right and on the left, with degenerate masses for quarks of the same charge.

I. INTRODUCTION AND SUMMARY

The striking fact that the neutral weak currents conserve strangeness to a high degree has led to speculation that all quark flavors, including those not yet discovered, might be conserved to the same degree of accuracy. The question of what restrictions this imposes on the structure of the theory has been considered by several authors.[1-5] In particular, Glashow and Weinberg[4] have used the concept of naturality to analyze this question. They have insisted that any unitary transformations of the quark fields (in flavor) which commute with the charge operator Q (and with any other *absolutely* conserved quantum numbers that may exist) must leave the neutral currents diagonal to order G and αG. As a consequence the weak isospin, T_3 and $\vec{T}^2$, for right- and left-handed quarks separately must be a function of Q (and the other absolutely conserved quantum numbers) alone. By demanding that all neutral Higgs couplings also be diagonal under the same kind of arbitrary transformations, they argue somewhat less conclusively that the only models satisfying these conditions are generalizations of the standard model: all quarks in left-handed doublets and right-handed singlets.

While the assumption of naturality in this form is very attractive, since it emphasizes structure and does not depend on the values of the parameters in the theory, it is also very strong. In this note we point out that many of Glashow and Weinberg's results can be obtained without it. We will follow them by dealing with SU(2) × U(1) gauge theories and making the following assumptions:

(i) Direct neutral-current couplings to order G conserve all quark flavors.

(ii) Neutral currents induced by one-loop radiative corrections conserve all quark flavors to order αG.

(iii) The coupling of each neutral Higgs meson conserves all quark flavors to lowest order.

We allow for an arbitrary number of quarks of various charges. If we arrange them in a multicomponent vector q, the coupling to the isovector weak boson takes the form[6]

$$\mathcal{L}_{\text{int}} = g\left(\bar{q}\gamma_\mu \frac{1-\gamma_5}{2} T_{L\,i}q + \bar{q}\gamma_\mu \frac{1+\gamma_5}{2} T_{R\,i}q\right) A_i^\mu, \tag{1.1}$$

where $T_{L,R\,i}$ form (reducible) representations of SU(2) and satisfy the usual relation

$$[T_i, T_j] = i\epsilon_{ijk}T_k. \tag{1.2}$$

For any gauge theory there exists a basis in which $T_{L\,i}$ and $T_{R\,i}$ are completely reduced; the quark fields then lie in multiplets of weak isospin characterized by their dimension or $\vec{T}^2$ and by their weak hypercharge $Y = 2(Q - T_3)$. There is also another basis, which is in general different, in which the quarks have definite flavors and the mass matrix is diagonal. We shall refer to this as the flavor basis. Assumption (i) implies that in the flavor basis

$$(T_{L,R\,3})_{mn} = (T_{L,R\,3})_m \delta_{mn}\,. \tag{1.3}$$

Assumption (ii) implies[4] that in the flavor basis

$$(\vec{T}_{L,R}{}^2)_{mn} = (T_{L,R}{}^2)_m \delta_{mn}\,. \tag{1.4}$$

This is true because order αG terms of the induced neutral currents are proportional to

$$T_\pm T_\mp = \vec{T}^2 - T_3{}^2 \pm T_3. \tag{1.5}$$

We note here that the naturality assumption is not needed in the derivation of Eq. (1.4).

To make our objective clear: We are studying the consequences of assumptions (i)–(iii). They may well not be true for other than $s \leftrightarrow d$ transitions. We simply wish to learn what structures are allowed if these assumptions hold for all flavors.

We will summarize our results in the remainder of this section and present the elementary calculations leading to them in the next section.

(1) Any quark field of a definite flavor is a linear combination of the fields which reduce $T_{L\,i}$ such that each term in the sum has the same value of $\vec{T}_L^{\,2}$ and Y_L. The same is true for L replaced by R. Thus every flavor carries a definite value of $\vec{T}_L^{\,2}$, Y_L, $\vec{T}_R^{\,2}$, and Y_R. This follows from only assumptions (i) and (ii). GW's result is stronger than this: They show, under stronger assumptions, that all multiplets on the left (or right) have the same value of $\vec{T}_L^{\,2}$ and Y_L (or $\vec{T}_R^{\,2}$ and Y_R), if charge is the only absolutely conserved quantum number. We shall show that the condition that there are no additional absolutely conserved quantum numbers in the quark sector, supplemented by assumption (iii), put very stringent constraints on the structure of allowed theories and in many interesting cases leads to the same conclusions in GW.

(2) If there are no right-handed currents (or more generally if $[\vec{T}_L^{\,2}, T_{R\,i}]=0$, $[Y_L, T_{R\,i}]=0$, and $L \leftrightarrow R$) then $\vec{T}_L^{\,2}$ and Y_L are absolutely conserved quantum numbers. Hence if there exists quarks of different values of $\vec{T}_L^{\,2}$ and Y_L the theory will have one or more conservation laws in addition to charge. Conversely, if there are no additional conservation laws, all quarks have the same value of $\vec{T}_L^{\,2}$ and Y_L, presumably $\vec{T}_L^{\,2}=T_L(T_L+1)=\frac{3}{4}$ and $Y_L=\frac{1}{3}$.

(3) In the case where both right-handed and left-handed currents exist, we have restricted our attention to quarks of charge $+\frac{2}{3}$ and $-\frac{1}{3}$. We then need to specify only whether they are in doublets under $\vec{T}_L$ and $\vec{T}_R$. We will use the notation $u_i(\frac{1}{2},\frac{1}{2})$ for a $\frac{2}{3}$ charge quark in a doublet on the right and the left, $u_i(0,\frac{1}{2})$ for a $\frac{2}{3}$ charge quark in a doublet on the right and a singlet on the left, $d_i(\frac{1}{2},0)$ for a charge $-\frac{1}{3}$ quark in a doublet on the left and a singlet on the right, etc. [We omit $d_i(0,0)$ and $u_i(0,0)$ since they are individually conserved and uninteresting.] We show that if

$$T^{+}_{R\,ij}\neq 0 \text{ or } T^{+}_{L\,ij}\neq 0$$

between

$$u_i(t_{L\,i}, t_{R\,i}) \text{ and } d_j(t'_{L\,j}, t'_{R\,j})$$

then

$$t_{L\,i}=t'_{L\,j}, \qquad t_{R\,i}=t'_{R\,j}. \tag{1.6}$$

This is not trivial because in general, while T^{+}_{L} has nonzero matrix elements only if $t_L=t'_L$, it changes t_R. Since the strong interactions, presumably quantum chromodynamics, conserve flavor we conclude that t_L and t_R are conserved absolutely. Thus if there are no new absolutely conserved quantum numbers we have only three possibilities: (a) the standard left-handed doublets and right-handed singlets,[7] (b) its mirror reflection, and (c) a pure vector model[8] with all quarks in doublets on the right and on the left. A model such as[9]

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L, \begin{pmatrix} c \\ s' \end{pmatrix}_L, (b)_L, (h)_L, \qquad \begin{pmatrix} u \\ b' \end{pmatrix}_R, \begin{pmatrix} c \\ h' \end{pmatrix}_R, (d)_R, (s)_R \tag{1.7a}$$

or such as

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L, \begin{pmatrix} c \\ s' \end{pmatrix}_L, \begin{pmatrix} c \\ s'' \end{pmatrix}_R, (u)_R, (d'')_R \tag{1.7b}$$

with nonzero Cabibbo angle will not satisfy all of our conditions.

(4) In the pure vector model, if the mass matrix is given by the Higgs mechanism, then if $T^{+}_{ij}\neq 0$ and $T^{+}_{ik}\neq 0$ for either right- or left-handed currents then the down quarks d_k and d_j are degenerate:

$$m_j^{\,2}=m_k^{\,2}. \tag{1.8}$$

The corresponding result is also true for the up quarks; if $T^{+}_{ij}\neq 0$ and $T^{+}_{kj}\neq 0$, then $m_i^{\,2}=m_k^{\,2}$. Thus the only mixing allowed in a multiplet is between degenerate quarks. As a special case, if there are no mass degeneracies, then every doublet is unmixed and only doublets containing definite flavors are allowed:

$$\begin{pmatrix} u_i \\ d_i \end{pmatrix}_L, \begin{pmatrix} u_i \\ d_i \end{pmatrix}_R.$$

Hence a realistic pure vector or vectorlike model must violate one or more of our assumptions.

II. DERIVATION OF RESULTS

We suppose the unbroken $SU(2)\times U(1)$ theory to be originally written in terms of quark fields q'_L in multiplets of definite $\vec{T}_L^{\,2}$, Y_L, and fields q'_R in multiplets of definite $\vec{T}_R^{\,2}$, Y_R. [We define $q'_{R,L}=\frac{1}{2}(1\pm\gamma_5)q'$.] In this basis the generators $T_{L,R\,i}$ are completely reduced, block-diagonal, Hermitian matrices. When the symmetry is broken, a mass term

$$-\bar{q}'_R M' q'_L - \bar{q}'_L M'^{\dagger} q'_R \tag{2.1}$$

is generated in the Lagrangian. M' is in general not diagonal or even Hermitian. However, as pointed out by GW, it can always be brought to diagonal form M by the transformation

$$M=U_L M' U_R^{-1}, \tag{2.2}$$

with U_L and U_R unitary. The quark fields q of definite flavor are related to the original fields by

$$q = U_L q_L' + U_R q_R'. \tag{2.3}$$

The generators $T_{L,R\,i}$ in the flavor basis are no longer block-diagonal, but they are Hermitian representations of the SU(2) algebra.

By definition the matrix

$$T_{L3}' = U_L^\dagger T_{L3} U_L \tag{2.4}$$

is diagonal and has as possible eigenvalues $0, \pm\frac{1}{2}, \pm 1, \pm\frac{3}{2}, \ldots$. Now if a q_L' quark of index j appears in the expansion of a definite flavor $q_{L\,i}$ then $U_{L\,ij} \neq 0$ and hence

$$T_{L3}' = T_{L3}. \tag{2.5}$$

The same is obviously true for $\vec{T}_L^2$ and for L replaced by R. This is a very simple result but not quite a tautology. The T_3 part alone is enough to rule out, as the simplest case, the old $\mathcal{P}, \mathfrak{N}, \lambda$ model with nonzero Cabibbo angle. The $\vec{T}^2$ part is needed to rule out triplet and singlet models,[10] the simplest prototype of which is

$$\begin{pmatrix} q_1 \\ \alpha q_2 + \beta q_3 \\ q_4 \end{pmatrix}_L, \quad (-\beta q_2 + \alpha q_3)_L, \tag{2.6}$$

with $\alpha \neq 0$ and $\beta \neq 0$.

This tells us that in the flavor basis, the matrix elements of T_L^+ between all states of charge Q and $Q+1$, for any fixed Q, form a block-diagonal matrix whose blocks are unitary matrices times the usual angular momentum factor:

$$T_L^+ q_{iL}(Q, \vec{T}_L^2, Y_L) = [(T - T_3)(T + T_3 + 1)]^{1/2} V_{L\,ij} \times q_{jL}(Q+1, \vec{T}_L^2, Y_L), \tag{2.7}$$

where V_L is a unitary matrix. This shows that universality, in the precise sense of Eq. (2.7), is a necessary as well as sufficient condition for diagonal neutral currents.[1-3]

The strong interactions, including the mass matrix, are diagonal in flavor and hence do not change $\vec{T}^2$ and Y. However, T_{Ri} will in general change $\vec{T}_L^2$ and Y_L and vice versa, unless they commute. If they do commute, in particular, if the right-handed multiplets are all singlets, then both the strong interaction and the weak boson couplings conserve $\vec{T}_L^2$ and Y_L. In order to establish our second result that $\vec{T}_L^2$ and Y_L are conserved, we need only consider what effect the Higgs couplings have. Suppose there are several quark multiplets with the same value of $\vec{T}_L^2$ and Y_L. We can obviously always arrange these multiplets so that for an arbitrary fixed value of charge within the multiplet, there is no mixing. For example, the usual way of arranging the multiplets in the standard model fixes the $Q = \frac{2}{3}$ quarks as unmixed. These multiplets can be coupled only to a Higgs field of the same dimension. Fix on a right-handed quark q_i of a particular Q, Y_L, and $\vec{T}_L^2$ and a Higgs field with $\vec{T}_L'^2$, Y_L'. Its neutral couplings have the form

$$\phi^0 \bar{q}_i(Q, \vec{T}_L^2, Y_L) \frac{1-\gamma_5}{2} q_j(Q, \vec{T}_L'^2, Y_L') \tag{2.8}$$

for each different multiplet having the same $\vec{T}_L'^2$ and Y_L' as the Higgs field. (We label the multiplets by the fixed charge Q.) The charged partners of ϕ^0 will cause transitions between $q_i(Q, \vec{T}_L^2, Y_L)$ and quarks of differing charges but all having the same values of $\vec{T}_L'^2$ and Y_L' as the Higgs fields. The diagonality of the neutral Higgs coupling assumed forces the two quarks in (2.8) to be the same, and hence

$$\vec{T}_L^2 = \vec{T}_L'^2,$$

$$Y_L = Y_L'.$$

Thus the Higgs couplings conserve $\vec{T}_L^2$ and Y_L and result (2) is established.

As soon as right-handed currents are allowed as well, it is easy to construct examples which satisfy all of our conditions but violate GW's conditions and do not have any new absolute conservation laws. For example, take the standard model, with the addition of a single new quark X of charge $+\frac{5}{3}$ and put the quarks into multiplets as shown:

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L, \begin{pmatrix} c \\ s' \end{pmatrix}_L, (X)_L, \begin{pmatrix} X \\ u \end{pmatrix}_R, (c)_R, (d)_R, (s)_R. \tag{2.9}$$

GW would forbid the equal-charge c and u from lying in multiplets of different dimension on the right. So far as we can see, the model has no new conservation laws. Furthermore, it is very easy to generate a mass matrix with nonzero masses via the Higgs mechanism; in addition to the usual doublet Higgs we need a triplet Higgs with $Y = 2$ coupled in the form

$$[\sqrt{2}\,\bar{X}_R \phi^{++} d_L' + \sqrt{2}\,\bar{u}_R \phi^0 u_L + \phi^+(\bar{X}_R u_L - \bar{u}_R d_L')]. \tag{2.10}$$

Because of examples such as this, we will restrict our attention from now on to models with left- and right-handed currents having quarks of charges $+\frac{2}{3}$ and $-\frac{1}{3}$ which *a priori* may lie in either doublets or singlets on the left and right independently.[11] Now the Higgs mechanism provides just enough freedom to construct an arbitrary mass matrix. For the standard model (and its mirror reflection) the mass matrix and the neutral Higgs coupling are proportional, so that both can be made diagonal. However, this proportionality does not hold in models containing quarks with $T_L=T_R=\frac{1}{2}$, and both can be made diagonal only under special (and rather unphysical) conditions.

The multiplet structure is specified by giving the values of T_L and T_R for each quark. We introduce the following notation:

	T_L	T_R
$u_1,\ldots,u_{l_1}$	$\frac{1}{2}$	$\frac{1}{2}$
$u_{l_1+1},\ldots,u_{l_1+l_2}$	$\frac{1}{2}$	0
$u_{l_1+l_2+1},\ldots,u_{l_1+l_2+l_3}$	0	$\frac{1}{2}$
$d_1,\ldots,d_{l_1'}$	$\frac{1}{2}$	$\frac{1}{2}$
$d_{l_1'+1},\ldots,d_{l_1'+l_2'}$	$\frac{1}{2}$	0
$d_{l_1'+l_2'+1},\ldots,d_{l_1'+l_2'+l_3'}$	0	$\frac{1}{2}$.

We omit as uninteresting any quarks with $T_L=T_R=0$. It is easy to see that

$$l_1+l_2=l_1'+l_2',\qquad l_1+l_3=l_1'+l_3'. \tag{2.11}$$

The basic doublets of the theory can be labeled by their u quarks, and we denote them by

$$\begin{pmatrix}u_{iL}\\ d'_{iL}\end{pmatrix}=\begin{pmatrix}u_{iL}\\ U^\dagger_{L\,ij}d_{jL}\end{pmatrix},\qquad \begin{pmatrix}u_{iR}\\ d''_{iL}\end{pmatrix}=\begin{pmatrix}u_{iR}\\ U^\dagger_{R\,ij}d_{jL}\end{pmatrix}, \tag{2.12}$$

with U_L and U_R as in Eq. (2.3). Consider the u quarks with $T_L=T_R=\frac{1}{2}$. They can acquire a mass from a scalar mass term or from an isotriplet Higgs meson. If we require in accordance with assumption (iii) that the isotriplet Higgs coupling is diagonal in u-quark flavors, then the mass matrix for these has the form

$$\mathcal{L}_m=-\sum_{\text{all } i}(+\bar u_{iR}M_iu_{iL}+\bar d''_{iR}M_id'_{iL}+\bar u_{iR}\Gamma_iu_{iL}-\bar d''_{iR}\Gamma_id'_{iL})+\text{H.c.} \tag{2.13}$$

with $M_i=0$ and $\Gamma_i=0$ for $i>l_1$. There are no other doublet-to-doublet couplings which do not change u-quark flavors. For $i\le l_1$, we assume that not both M_i and Γ_i are zero, i.e., no u quark is exactly massless. Now define matrices M and L by

$$M_{ij}=\delta_{ij}M_i,\qquad \Gamma_{ij}=\delta_{ij}\Gamma_j. \tag{2.14}$$

The requirement that the isotriplet neutral Higgs is diagonal in d-quark flavors then implies that

$$\tilde\Gamma=U_R\Gamma U_L^\dagger \tag{2.15a}$$

is diagonal. The mass matrix is, by definition, diagonal in d-quark flavors, so

$$\tilde M=U_RMU_L^\dagger \tag{2.15b}$$

is also diagonal. But then

$$\tilde\Gamma\tilde\Gamma^\dagger=U_R\Gamma\Gamma^\dagger U_R^\dagger=U_L\Gamma\Gamma^\dagger U_L^\dagger \tag{2.16}$$

so if $i>l_1$ and $j\le l_1$ or if $i\le l_1$, and $j>l_1$

$$(U_R^\dagger U_L)_{ij}=0. \tag{2.17}$$

[The step leading from (2.16) to (2.17) could fail if for some $j_0<l_1$, Γ_{j_0} "accidentally" vanished. However, the argument could just as well be given with M replacing Γ and since we assume that not both vanish for any given j, (2.17) follows.] Hence $U_R^\dagger U_L$ breaks into block-diagonal unitary matrices. This shows that the quarks d_i'', $i=1\cdots l_1$, are given by a unitary transformation on the quarks d_1', $i=1\cdots l_1$. Since the quarks d' have $T_L=\frac{1}{2}$ and the quarks d'' have $T_R=\frac{1}{2}$, they must all have $T_L=T_R=\frac{1}{2}$. By repeating the argument from Eq. (2.12) on with multiplets labeled by d's instead of u's we learn that

$$l_1=l_1',\qquad l_2=l_2',\qquad l_3=l_3'. \tag{2.18}$$

Hence the number of u and d quarks must be the same. Furthermore, every vertex—strong, weak L and R, and Higgs—conserves T_L and T_R and the space of quarks separates. The number of quarks in R and L doublets is conserved, the number in R doublets and L singlets is conserved, and the number in R singlets and L doublets is conserved. Hence, if there are no new quantum numbers, our assumptions allow only the extension of the standard model, its mirror reflection, and the pure vector model.

The standard-model extensions clearly have no problem with degeneracies. Consider now the pure vector model. We have two more equations like Eq. (2.16):

$$\tilde M\tilde M^\dagger=U_RMM^\dagger U_R^\dagger=U_LMM^\dagger U_L^\dagger, \tag{2.19}$$

$$(\tilde M-\tilde\Gamma)(\tilde M-\tilde\Gamma)^\dagger=U_R(M-\Gamma)(M-\Gamma)^\dagger U_R^\dagger=U_L(M-\Gamma)(M-\Gamma)^\dagger U_L^\dagger\,. \tag{2.20}$$

If d_k is in a multiplet with u_i, then either

$$U^\dagger_{L\,ik}\ne0 \text{ or } U^\dagger_{R\,ik}\ne0.$$

Since $(\tilde M-\tilde\Gamma)$ is assumed to be diagonal,

$$(\tilde M-\tilde\Gamma)_k{}^2=(M-\Gamma)_i{}^2. \tag{2.21}$$

Hence, every d_k in a multiplet with the same u_i has the same mass. Under the same assumption, obviously

$$\tilde{M}_k{}^2 = M_i{}^2, \quad \tilde{\Gamma}_k{}^2 = \Gamma_i{}^2, \tag{2.22}$$

and so

$$(\tilde{M}+\tilde{\Gamma})_k{}^2 = (M+\Gamma)_i{}^2; \tag{2.23}$$

i.e., every up quark u_i that contains d_k in its multiplet has the same mass. In particular, if there are no degeneracies only one element in each row and column of U_L and U_R is nonzero so they are essentially relabeling matrices; each doublet is composed of pure flavor quarks and each doublet is absolutely conserved.

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