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Possible three-photon couplings

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One of the simplest modifications to quantum electrodynamics, if charge-conjugation invariance is abandoned, is the occurrence of direct coupling of three photons. This paper generalizes and extends Dolgov's phenomenological analysis of the consequences of such interactions and confirms that gauge invariance and Bose symmetry permit such interactions only if at least two of the photons are virtual. The effects of such couplings are considered for processes in which the third photon is real. The best present limit on the possible strength of such couplings is set by experiments on bremsstrahlung from e^+e^- collisions.

I. INTRODUCTION

Modifications[1] of quantum electrodynamics (QED) owing to the occurrence of interactions which are not invariant under space inversion and charge conjugation acquire additional interest from the proposals[2] to unify weak and electromagnetic interactions. In the context of such theories, renormalizability is no longer a concern since one of the favorable features of the unified gauge theories is precisely that they are renormalizable. The question of physical interest is: "What are the simplest qualitative changes which must be made in QED (by which we mean the quantized theory of interacting Maxwell and Dirac fields) as a consequence of relaxing the conditions of charge-conjugation invariance and reflection invariance?" One of the first modifications is the appearance of a possible three-photon vertex, and this paper is devoted to a phenomenological study of the consequences of such an interaction. At the present uncertain state of our knowledge concerning unified gauge theories, we consider such a conservative approach to be more appropriate than explicit calculation of such effects in any particular gauge model.

In Sec. II of this paper, we present arguments leading to the consideration of three-photon couplings and find the simplest phenomenological representation for such an amplitude, which is a generalization of the form given earlier by Dolgov.[3,4] In Sec. III, we examine the consequences of such an hypothetical interaction. Section IV summarizes the limits on the strength of three-photon interactions established by various experiments and how those limits could be further improved.

II. THE THREE-PHOTON VERTEX

The constraints of Lorentz invariance and gauge invariance severely restrict the modifications that can be introduced into QED. The standard form[5] of the photon propagator is the most general one that satisfies these constraints. Similarly, it is well known that by suitable redefinition[6] of the Dirac field, the fermion propagator can also always be cast into the standard form. Therefore, if parity and charge-conjugation invariance are relaxed, the most primitive changes that one expects in QED are modification of the electron-photon vertex, which has been discussed previously,[7] and the occurrence of a possible three-photon vertex. The latter clearly violates Furry's theorem, which is a direct consequence of charge-conjugation invariance, and one's first thought is that the occurrence of such an intereaction should give rise to clear signatures in purely photonic processes, which could unmistakeably announce the presence of three-photon coupling. Unfortunately, gauge invariance and the requirement of Bose statistics also restrict the form of the three-photon coupling, requiring at least two of the photons to be virtual, which makes the search for three-photon couplings considerably more difficult.

Dolgov considered a parity-conserving C-violating three-photon interaction as a source of CP violation and found the corresponding phenomenological Hamiltonian density with the least possible number of derivatives, representing a cubic interaction of the electromagnetic field with itself, to have the unique form[3]

$$\mathcal{H}_v = \tfrac{1}{2}\lambda\mu^{-6}(\partial_\gamma F_{\alpha\beta})(\partial^\beta F_{\sigma\tau})(\partial^\gamma\partial^\alpha F^{\sigma\tau}), \tag{1}$$

where $F_{\alpha\beta}=\partial_\alpha A_\beta - \partial_\beta A_\alpha$, λ is a real dimensionless parameter, and μ is a scale mass which we take to be the nucleon mass. Likewise, the corresponding CP-invariant C-violating cubic interaction of the electromagnetic field has the unique form

$$\mathcal{H}_c = \tfrac{1}{2}\lambda'\mu^{-6}(\partial_\gamma F_{\alpha\beta})(\partial^\beta \tilde{F}_{\sigma\tau})(\partial^\gamma\partial^\alpha F^{\sigma\tau}), \tag{2}$$

where $\tilde{F}_{\sigma\tau}=\epsilon_{\sigma\tau\alpha\beta}\partial^\alpha A^\beta$ and λ' is another real dimensionless parameter. When these interactions contribute a three-photon vertex to a Feynman graph, in which the photons are real or attached to conserved currents, the factor associated with the vertex is

$$\lambda\mu^{-6}[k_1^{\,2}(k_2\cdot\epsilon_1)(k_3^{\,2}-k_2^{\,2})(k_2\cdot k_3)(\epsilon_2\cdot\epsilon_3) + \text{cyclic permutations}] \quad (3)$$

for the CP-violating interaction (1), and

$$\lambda'\mu^{-6}[k_1^{\,2}(k_2\cdot\epsilon_1)(k_3^{\,2}-k_2^{\,2})\epsilon_{\alpha\beta\rho\sigma}k_3^{\,\alpha}\epsilon_3^{\,\beta}k_2^{\,\rho}\epsilon_2^{\,\sigma} + \text{cyclic permutations}] \quad (4)$$

for the CP-invariant interaction (2). $\epsilon_1, \epsilon_2, \epsilon_3$ are the polarization vectors of the photons and their momenta have been chosen such that $k_1+k_2+k_3=0$. It is clear from (3) and (4) that the three-photon interaction vanishes if more than one participating photon is real. For a timelike photon coupled to two real photons, this simply expresses the fact that two real photons cannot have total angular momentum of 1.[8]

Since at least two of the photons must be virtual, the three-photon interactions take place only in the neighborhood of the currents which generate the virtual photons, and (1) and (2) may be interpreted as interactions which allow two currents to cooperatively emit or absorb a photon. In general, the coefficients λ and λ' in the matrix elements (3) and (4) should be invariant functions of k_1, k_2, and k_3, falling to zero for large values of the arguments. Otherwise, the insertion of three-photon vertices in higher-order virtual processes involving loop integrations would lead to highly divergent expressions. For processes in which no loops are present, it may not be too unreasonable an approximation to replace λ and λ' by average values, at least for "small" values of the photon momenta. We wish to investigate the effects of the interactions (1) and (2) for such processes and find what existing experiments can tell us about the possible magnitudes of λ and λ'. A further point to note is that λ and λ' will in general develop imaginary parts when any of the photons is timelike with k^2 exceeding[9] some physical threshold, in our case $4m^2$ corresponding to electron-pair production.

III. EXPERIMENTAL CONSEQUENCES

We restrict our attention to processes in which one of the photons involved in a three-photon vertex is real. We shall separately consider the various possibilities that the two other photons are both spacelike, both timelike, or have one of each type.

An example of the first type, for which λ and λ' are necessarily real, is bremsstrahlung by an electron in the field of a nucleus. The diagram of lowest order in the three-photon coupling, shown in Fig. 1(a), has the corresponding matrix-element

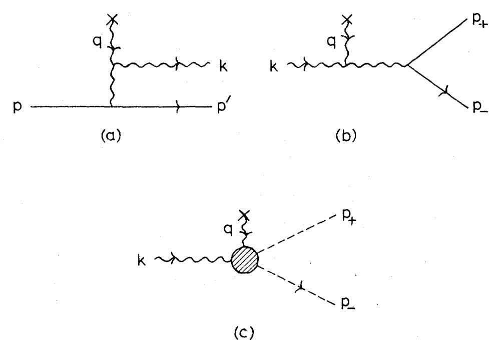

FIG. 1. Feynman diagrams for (a) bremsstrahlung and (b) pair production through three-photon coupling; (c) shows pion pair production in quantum electrodynamics.

$$M_{3\gamma}^{B}=\tfrac{1}{2}e\mu^{-6}(Q^2-q^2)(\lambda f_{\alpha\beta}+\lambda'\tilde{f}_{\alpha\beta})j_{\text{ex}}^{\alpha}\bar{u}(p')\gamma^{\beta}u(p), \quad (5)$$

where $f_{\alpha\beta}=k_\alpha\epsilon_\beta-k_\beta\epsilon_\alpha$, $\tilde{f}_{\alpha\beta}=\epsilon_{\alpha\beta\sigma\tau}k_\sigma\epsilon_\tau$, and $j_{\text{ex}}^{\alpha}=ZeF(q^2)\delta_{\alpha\sigma}$ in the rest system of the nucleus, which is treated as infinitely massive. p, p', and k are the momenta of the incident and outgoing electrons and of the radiated photon, respectively. ϵ is the polarization vector of the photon, $Q=p-p'$ and $q=k-Q$. An interesting feature of Eq. (5) is that the matrix element does not vanish when Q^2 or q^2 separately go to zero, even though the three-photon vertices vanish in those limits. This is because the factors Q^2, q^2 arising from the three-photon couplings (3), (4) are cancelled by corresponding factors $1/Q^2, 1/q^2$ from the propagators of the virtual photons involved. The three-photon contribution (5) to bremsstrahlung thus has the form of a contact interaction. Consequently, in contrast to the QED contribution, which favors production of low-energy photons and small angles of emission, the three-photon interaction preferentially produces energetic photons, and large angles are not disfavored. Because of this, one must take care to include the form factor $F(q^2)$ of the nucleus in evaluating the contribution of this process. There is no interference between $M_{3\gamma}^{B}$ and the lowest-order QED (Bethe-Heitler) amplitude M_{BH} if one sums over the polarizations of the final particles and averages over the polarization of the incident electron, if final-state interactions are neglected. The three-photon vertex which is CP- (and T-) invariant is odd under P, therefore such an interference between the amplitude involving this vertex, Fig. 1(a), and the parity-conserving Bethe-Heitler amplitude, must be proportional to $[\vec{k}\cdot(\vec{p}\times\vec{p}')]$, the only pseudoscalar at our disposal. But the occurrence of such a term also requires, in the absence of final-state interactions, a failure of time-reversal invariance. Consequently, $d\sigma_{\rm int}=0$ for the CP-conserving three-photon vertex. For the parity-conserving, CP- (and T-) noninvariant vertex, any term which is even under space inversion, e.g., $(\vec{k}\cdot\vec{p})$ will also be unchanged under time reversal. Consequently, in both cases, the interference term must vanish if all spins are summed over and final-state interactions are ignored. Also, if both CP-conserving and CP-violating three-photon interactions are present, the interference between the two vanishes after summation over polarizations. This is because the radiated photon does not "know" about the separate existence of p and p', and it is impossible to form a pseudoscalar from the pairs of vectors $(\vec{p},\vec{p}')$, $(\vec{k},\vec{Q})$, or $(\vec{k},\vec{q})$. Therefore, to lowest order in the three-photon couplings, these only make an additive contribution to the Bethe-Heitler cross section which, in the extreme relativistic limit, is

$$d\sigma_{3\gamma}^{B}=\tfrac{1}{4}\pi^{-3}[Z\alpha\mu^{-6}F(q^2)]^2(\lambda^2+\lambda'^2)\delta(E-w-E')\times w^3[w(1-\hat{p}'\cdot\hat{k})+E(\hat{p}'-\hat{p})\cdot\hat{k}]^2\times[1-(\hat{p}\cdot\hat{k})(\hat{p}'\cdot\hat{k})]\,d^3p'\,d^3k \tag{6}$$

for radiation of a photon of energy w by an incident electron of energy E. With the simple approximation $F(q^2)=\Theta(q^2+\Lambda^2)$, the total cross section for bremsstrahlung, when $\Lambda\ll E\ll M$ where M is the nuclear mass, is given by

$$\sigma_{3\gamma}^{B}=\frac{1}{90\pi}[Z\alpha\mu^{-6}]^2(\lambda^2+\lambda'^2)E^4\Lambda^6[1+O(\Lambda/E)]. \tag{7}$$

As is well known, crossing symmetry relates the amplitude for bremsstrahlung to that for electron pair production by a photon. The contribution of three-photon couplings to the latter process, shown in Fig. 1(b), involve one spacelike and one timelike photon coupled to the real incident photon. If the imaginary parts of λ, λ' are neglected, the lowest-order effect of three-photon couplings is an additive contribution to the pair-production cross section which is given, in the extreme relativistic limit, by[10,11]

$$d\sigma=\tfrac{1}{4}\pi^{-3}[Z\alpha\mu^{-6}F(q^2)]^2(\lambda^2+\lambda'^2)(w-\epsilon_+\hat{p}_+\cdot\hat{k}-\epsilon_-\hat{p}_-\cdot\hat{k})^2\times[1-(\hat{p}_+\cdot\hat{k})(\hat{p}_-\cdot\hat{k})]w^3\delta(w-\epsilon_+-\epsilon_-)d^3p_+d^3p_- \tag{8}$$

for production by a photon of energy w of a positron and an electron with energies ϵ_+ and ϵ_-. Note that, analogous to the corresponding result for bremsstrahlung, the cross section does not vanish when the invariant mass of the pair goes to zero. Therefore, in contrast to the QED (Bethe-Heitler) con tribution which strongly favors production of low-mass pairs (at small angles), the three-photon interaction tends to produce electrons and positrons at large angles. With the simple nuclear form factor chosen earlier, the total contribution to the pair production cross section from the three-photon mechanism, for $\Lambda\ll w\ll M$, is found to be

$$\sigma_{\rm tot}=\frac{1}{144\pi}(Z\alpha\mu^{-6})^2(\lambda^2+\lambda'^2)w^4\Lambda^6[1+O(\Lambda/w)]. \tag{9}$$

We now consider effects which receive contributions in first order of the three-photon coupling. Firstly, in addition tothe effects considered by Dolgov[3] when the incident photon is circularly polarized or the longitudinal polarization of the outgoing electron is measured, there will be corresponding interference effects proportional to $\mathrm{Re}\lambda'$ between the contribution of the parity-violating three-photon coupling and the Bethe-Heitler amplitude,

$$2\,\mathrm{Re}M_{\rm BH}M_{3\gamma}^{*}=\frac{2\,\mathrm{Re}\lambda' e^3}{\mu^6q^2}(j_{\rm ex}^0)^2(P^2-q^2)\xi_2\times\left\{\left\langle w\frac{p_+\cdot p_-}{k\cdot p_-}+\epsilon_+\frac{k\cdot q}{k\cdot p_+}-\frac{\epsilon_+}{k\cdot p_-}[\vec{p}_-\cdot(\vec{p}_+-\vec{p}_-)-\vec{p}_-\cdot\hat{k}(\vec{p}_+-\vec{p}_-)\cdot\hat{k}]\right\rangle-\langle p_+\leftrightarrow p_-\rangle\right\}, \tag{10}$$

when the incident photon has circular polarization described by the Stokes parameter ξ_2. If instead, using unpolarized photons, the helicity χ of the outgoing electron is measured, there will be interference of the form

$$2\,\mathrm{Re}M_{\rm BH}M_{3\gamma}^{*}=\frac{2\,\mathrm{Re}\lambda' e^3}{\mu^6q^2}(j_{\rm ex}^0)^2(P^2-q^2)\chi\left\{\left\langle -q\cdot p_++\frac{\epsilon_-}{p_-\cdot k}[w(p_-\cdot q)-\epsilon_-(k\cdot q)]\right\rangle+\langle p_+\leftrightarrow p_-\rangle\right\} \tag{11}$$

Furthermore, there will be interference effects in pair production even when no polarizations are measured, because λ and λ' are no longer purely real. An example of such an effect will be illustrated for the more general case of pion-pair production, where first-order interference effects can arise either from the imaginary parts which λ and λ' can develop above physical thresholds or from the complex factors associated with the pion couplings, representing the effect of strong interactions.

For the QED amplitude, Fig. 1(c), the possible effects of strong interactions of the pions are taken into account by writing the amplitude in the general form[12]

$$M_{\mathrm{QED}} = -i(q^2)^{-1}\{M_{\mathrm{B}}^{\alpha\beta}(A(p_-, p_+, k) + [g^{\alpha\beta}(kq) - q^{\alpha}k^{\beta}]B(p_-, p_+, k)\}\epsilon_{\alpha} j_{\beta}^{\mathrm{ex}}, \tag{12}$$

where M_{B} is the Born amplitude for $\gamma\gamma \to \pi^+\pi^-$, while for the three-photon contribution, Fig. 1(b), all one needs to do is insert, at the electromagnetic vertex which produces the pair, the pion form factor $F_\pi(P^2)$ for the invariant mass $P^2 = (p_+ + p_-)^2$ of the pion pair. The interference between the C-conserving and C-nonconserving amplitudes then yields

$$2\,\mathrm{Re}M_{\mathrm{QED}}M_{3\gamma}^{*} = \frac{+e^3}{\mu^6 q^2}(P^2 - q^2)(j_{\mathrm{ex}}^{0})^2$$
$$\times\left\{\left\langle\left[k\cdot p_- + \frac{\epsilon_+}{k\cdot p_-}(\epsilon_- k\cdot p_+ - wq\cdot p_-)\right]\mathrm{Im}(\lambda^* AF_\pi^*) + \tfrac{1}{2}[w^2 k\cdot p_- + (k\cdot q)(w\epsilon_- - k\cdot p_-)]\,\mathrm{Im}(\lambda'^* BF_\pi^*)\right.\right.$$
$$\left.\left. + \vec{\mathrm{k}}\cdot\vec{\mathrm{p}}_-\times\vec{\mathrm{p}}_+\left[\frac{\epsilon_+}{k\cdot p_-}\,\mathrm{Im}(\lambda'^* AF_\pi^*) - \tfrac{1}{2}w\,\mathrm{Im}(\lambda'^* BF_\pi^*)\right]\right\rangle - \langle p_- \leftrightarrow p_+\rangle\right\}. \tag{13a}$$

If λ, λ' are considered to be essentially real, the terms proportional to λ represent effects which are reflection invariant while those proportional to λ' violate parity. Correspondingly, they yield correlations which are often (loosely) described as time-reversal invariant and noninvariant, respectively.

It is clear from Eq. (13) that C-odd correlations will persist, with unpolarized incident photons, even if $F_\pi = 1$, and A and B are purely real, provided that $\mathrm{Im}\lambda$ and $\mathrm{Im}\lambda'$ are non-negligible. Such effects will therefore also occur in electron pair production, even after summation over all polarizations.[13] The interference between the Bethe-Heitler amplitude and the parity-violating three-photon contribution has the form

$$2\,\mathrm{Re}M_{\mathrm{BH}}M_{3\gamma}^{*} = -2\,\frac{e^3\,\mathrm{Im}\lambda'}{\mu^6 q^2}(j_{\mathrm{ex}}^{0})^2(P^2 - q^2)\left(\frac{\epsilon_+}{k\cdot p_+} + \frac{\epsilon_-}{k\cdot p_-}\right)$$
$$\times(\vec{\mathrm{k}}\cdot\vec{\mathrm{p}}_-\times\vec{\mathrm{p}}_+) \tag{13b}$$

Purely leptonic interactions

Electron-electron and electron-positron collisions possess the advantage that they are not complicated by strong interactions. In particular, there are no form factors, as far as we know, to suppress high momentum transfers.

The matrix element for the three-photon contribution to e^--e^- bremsstrahlung, arising from Figs. 2(a) and 2(b), is[14]

$$M = e^2\mu^{-6}(\lambda f_{\alpha\beta} - \lambda'\tilde{f}_{\alpha\beta})$$
$$\times[(p_1\cdot p_4 - p_2\cdot p_3)\bar{u}_4\gamma^{\alpha}u_1\bar{u}_3\gamma^{\beta}u_2$$
$$-(p_1\cdot p_3 - p_2\cdot p_4)\bar{u}_3\gamma^{\alpha}u_1\bar{u}_4\gamma^{\beta}u_2]. \tag{14}$$

As in the case of bremsstrahlung in the nuclear potential, the three-photon graphs will not interfere with the lowest-order QED graphs after summation over polarizations. Also, the contribution of the three-photon amplitude is independent of the parity structure of the three-photon vertex and there will be no interference between parity-violating and parity-conserving interactions under the same conditions. The bremsstrahlung cross section calculated from Eq. (14) is

$$\frac{d\sigma}{dw\,d\cos\theta} = \frac{1}{2\pi}(\alpha\mu^{-6})^2(\lambda^2 + \lambda'^2)E^3 w^5$$
$$\times\left[\tfrac{3}{5}(E - \tfrac{4}{9}w) + \tfrac{14}{5}(E - \tfrac{6}{7}w)\cos^2\theta\right.$$
$$\left. - \tfrac{7}{3}(E - \tfrac{8}{7}w)\cos^4\theta\right], \tag{15}$$

where the photon is emitted with energy w at an angle θ relative to the beam direction in the c.m. system and E is the beam energy. The integrated cross section is

$$\sigma_{\mathrm{tot}}^{B} = \frac{32}{315\pi}(\alpha\mu^{-6})^2(\lambda^2 + \lambda'^2)E^{10}. \tag{16}$$

Thus the cross section for bremsstrahlung in electron-electron collisions through the three-photon

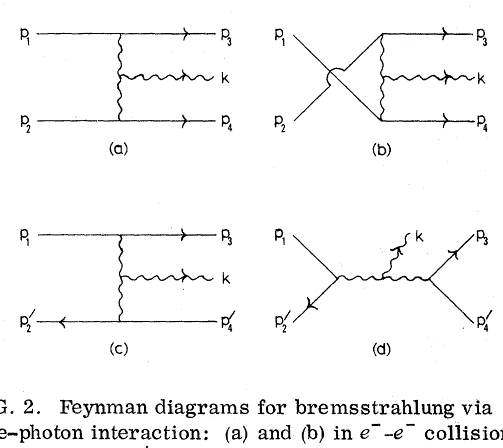

FIG. 2. Feynman diagrams for bremsstrahlung via three-photon interaction: (a) and (b) in e^--e^- collisions, and (c) and (d) in e^+-e^- collisions.

mechanism increases rapidly with energy,[15] and since the spectral distribution (15) favors high photon energies without the strong forward peaking of the QED contribution, the large-angle production of hard photons should be an extremely sensitive test for the presence of three-photon couplings.[3]

Bremsstrahlung in electron-positron collisions is related to the above by crossing. Since one of the graphs, Fig. 2(d), now couples two timelike photons to the radiated photon, the corresponding vertex functions λ, λ' are in general complex and there can be first-order C-asymmetric effects owing to interference between this graph and the usual QED amplitudes. If significant asymmetry occurs, the departure of the observed *rate* of bremsstrahlung provides a similarly sensitive test for three-photon interactions. The bremsstrahlung arising from three-photon couplings, Figs. 2(c), and 2(d), is given by

$$\frac{d\sigma}{dw\, d\cos\theta} = \frac{2}{\pi}(\alpha\mu^{-6})^2(\lambda^2+\lambda'^2)E^3w^5 \times\left[\tfrac{38}{15}(E-\tfrac{109}{152}w)+\tfrac{1}{5}(E+7w)\cos^2\theta -\tfrac{1}{3}(E-\tfrac{5}{4}w)\cos^4\theta\right], \tag{17}$$

which has the integrated cross section

$$\sigma_{\text{tot}} = \frac{304}{315\pi}(\lambda^2+\lambda'^2)(\alpha\mu^{-6})^2E^{10}. \tag{18}$$

In Eqs. (17) and (18), we have neglected the possible variation of λ, λ' over the range of spacelike and timelike momenta accessible to the virtual photons in Figs. 2(c) and 2(d). With this approximation, the cross section for e^+-e^- bremsstrahlung through three-photon coupling is an order of magnitude larger than the corresponding cross section for the e^--e^- case. The annihilation graph, Fig. 2(d), contributes significantly more than the other graphs because the momentum of one of the timelike photons is held fixed at the e^+-e^- total energy (in the c.m. system) whereas for the other graphs in Fig. 2, the (spacelike) momenta vary over the entire kinematic range.

IV. COMPARISON WITH EXPERIMENT

The strongest limit on the strength of three-photon couplings that Dolgov could deduce was from the production of wide-angle electron pairs by high-energy photons. Expressed in terms of our formation, he obtained the limit

$$(|\lambda|^2+|\lambda'|^2) \lesssim 400 \tag{19}$$

by considering data on pair production by 5.5-GeV photons. Systematic studies of pair production have not been made at appreciably higher energies, consequently we cannot significantly improve the bound (19) by using more recent data on pair production.

The high-energy bremsstrahlung from e^+e^- storage rings provides the best present limit on the strength of possible three-photon interactions. Simpson *et al.*[16] have reported a search for energetic γ rays from e^+e^- collisions at 3684 MeV. Their result corresponds to an upper limit of about 1 nb sr^{-1} GeV^{-1} for the bremsstrahlung cross section at 90° for $w = 1$ GeV. Comparison of this experimental limit with Eq. (17) yields the upper limit

$$(|\lambda|^2+|\lambda'|^2) < 5\times10^{-2} \tag{20}$$

for the possible strength of three-photon couplings, which is a few orders of magnitude below the limit (19).

The construction of e^+-e^- storage rings with higher c.m. energies offers an obvious opportunity to search for possible three-photon couplings with higher sensitivity.[17] It may also be worth examining the data from production of electron and pion pairs by high-energy photons, to search for the more complex types of asymmetries shown in Eqs. (13a) and (13b), in addition to anomalies in large-angle production which are usually sought as indicators of deviations from quantum electrodynamics.

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[1]K. Sekine, Nuovo Cimento 11, 87 (1959); C. H. Albright, R. Haag, and S. B. Treiman, *ibid.* 13, 1282 (1959); K. Hiida, Phys. Rev. 132, 1239 (1963).

[2]For a review, see S. Weinberg, Rev. Mod. Phys. 46, 255 (1974).

[3]A. D. Dolgov, Yad. Fiz. 7, 394 (1968) [Sov. J. Nucl. Phys. 7, 255 (1968)].

[4]Cf. F. Behrends, Phys. Lett. 16, 178 (1965).

[5]See, e.g., J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965).

[6]G. Feinberg, P. Kabir, and S. Weinberg, Phys. Rev. Lett. 3, 527 (1959); N. Cabibbo, R. Gatto, and C. Zemach, Nuovo Cimento 16, 1192 (1960).

[7]For a review, see Ya. B. Zel'dovich and A. M. Perelomov, Zh. Eksp. Teor. Fiz. 39, 1115 (1960) [Sov. Phys.—JETP 12, 777 (1961)].

[8]L. D. Landau, Dokl. Akad. Nauk USSR 60, 207 (1948); C. N. Yang, Phys. Rev. 77, 242 (1950).

[9]We use the metric $k^2 = k_0^2 - (\vec{k})^2$.

[10]For any detection arrangement which is symmetric under exchange of e^+ and e^- the effects of Imλ and Imλ' are included if $(\lambda^2+\lambda'^2)$ in Eq. (9) is replaced by

$(|\lambda|^2+|\lambda'|^2)$.

[11]Our formula differs from Dolgov's (Ref. 3) by the replacement of $(\hat{p}_1\cdot\hat{p}_2)$ by $(\hat{k}\cdot\hat{p}_1)(\hat{k}\cdot\hat{p}_2)$.

[12]See, e.g., S. Brodsky, T. Kinoshita, and H. Terazawa, Phys. Rev. D 4, 1532 (1971).

[13]An example of such a correlation in lepton-pair production was found, in an explicit model including Z^0 bosons, by R. F. Cahalan and K. O. Mikaelian, Phys. Rev. D 10, 3769 (1974).

[14]Our matrix-element and cross-section formulas disagree with Dolgov's (Ref. 3), which apparently use Bose symmetrization.

[15]This conclusion is valid for the regime in which the vertex functions λ and λ' vary relatively little.

[16]J. W. Simpson *et al.*, Phys. Rev. Lett. 35, 699 (1975).

[17]After this paper was written we found a report by R. Delbourgo, J. Phys. G2, 787 (1976), in which the strength λ' of the *CP*-conserving three-photon vertex is estimated in a unified gauge theory to be of order $e^3G\mu^2(\mu/m_w)^4$, with a numerical value of the order of 10^{-20}.