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Broken SU(4) symmetry and the baryon resonances*

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The $J^P = 3/2^+$ baryon-resonance mass spectrum, including the charmed states, and the resonance widths are calculated using a broken-SU(4)-symmetric one-baryon-exchange model of the baryon–pseudoscalar-meson forces. The dispersion-theoretic multichannel N/D matrix method is employed to construct the scattering amplitude for each state. The parameters of the theory are a mixing parameter analogous to the SU(3) F to D ratio, the mass of the exchanged baryon, a subtraction-point energy in the dispersion integrals, and the $1/2^+$ baryon and pseudoscalar-meson masses including those with nonzero values of charm. The positions and widths are determined for three different sets of predicted masses for those charmed particles.

I. INTRODUCTION

S-matrix theories of strong-interaction physics, which employed dispersion relations, hadronic "particle democracy," and bootstrap mechanisms while denying the validity of field theory in the realm of the strong interactions, were vigorously pursued in the late 1950's and early 1960's; however, the proliferation of hadrons, together with the requirement that all known particles which can participate in a particular bootstrap operation be included as input "forces" in the formalism, quickly carried the calculations to the point of unmanageability, while the results they yielded were insufficiently gratifying to justify themselves.[1] Furthermore, current analysis of data from deep-inelastic lepton scattering experiments lends little credence to the "elementary" nature of the observed hadrons.[2,3]

The dispersion theories now have been all but abandoned, to be superseded by the constituent view of hadrons both because of esthetic reasons and because of current interpretations of the evidence. It further appears that the concept of charm and the consequent significance of SU(4) in hadron spectroscopy has become entrenched in high-energy physics, although Han-Nambu–type models with broken color symmetry in lieu of charm are not completely untenable.[4]

The most attractive picture of hadrons which emerges, then, is of particles constructed from the more primal quarks, which come in at least four flavors and exactly three colors along with their other physical characteristics and which are cemented together with colored gluons. Four-color models have also been introduced with lepton number being the fourth color.[5]

On the other hand, the moderate success of the SU(3) bootstrap models in predicting parts of the hadronic mass spectrum in the correct channels and with the correct quantum numbers in the pre-charm era, together with their substantial physical content and incorporation of correct overall features such as the analyticity, unitarity, and crossing-symmetric structure of the S-matrix, suggests that the approach must have at least a validity on the phenomenological level. In fact, if indeed quarks are confined, there may be no inconsistencies between the quark theories and the bootstrap theory, particularly in the low-energy region, since the latter theory deals only with observable particles. Hence, in spite of the attractiveness of the quark model on the fundamental theoretical level, there is ample impetus for reexamining the bootstrap idea in the context of approximate-SU(4)-symmetric strong interactions.

The extension from SU(3) to SU(4) is, of course, not expected to alter significantly the severely limited range of applicability encountered in the earlier models, and most of the criticisms of them within their range of validity persist. They are meaningful only when applied to low- and sometimes medium-energy baryons whose strong decay is to several two-body channels only, since at higher energies many-body channels become significant. It is impossible to include all particle exchanges which could participate in the bootstrap process. They are inconsistent with heavy-free-quark theories, whether or not such theories include quark channels, because of the possibility of heavy-quark–heavy-diquark systems interacting to form a light bound state. Finally, the widths of the resonances usually turn out to be quantitatively incorrect.

Thus, accepting the fact that the bootstrap approach is inadequate for yielding a complete understanding of low-energy strong-interaction phenomena, we proceed with the hope that, as in the past, it will be useful in the determination of the low-energy baryon spectrum, or at least as a test of the consistency of strong-interaction theories with approximate SU(4) symmetry.

The objective in what follows is to obtain the $J^P = \frac{3}{2}^+$ baryon-resonance positions and widths using a model, the basis of which is the broken-SU(3)-symmetry model of Martin and Wali.[6] Actually we employ a simplified version of this model similar to that of Wali and Warnock.[7] It is imperative that we use a simple model because of the large number of two-particle channels which are introduced by the addition of charm. The resulting numerical analysis is rather prodigious even in the simple model.

The model intuits that the low-lying baryon resonances are due only to one-baryon exchanges in pseudoscalar-meson–baryon scattering and employs an N/D representation of the partial-wave multichannel scattering amplitudes. We assume that the nondegeneracy of the pseudoscalar-meson and baryon multiplets, i.e., the symmetry breaking, directly affects only the centrifugal barrier. Only two-particle channels are considered.

One difficulty of our calculation which is distinct from the similar calculations done during the reign of SU(3) symmetry is that some results depend strongly on the masses of those charmed pseudoscalar mesons and $J^P = \frac{1}{2}^+$ baryons which have not yet been seen. All of the octet meson and baryon masses were known during the time when many of the SU(3) bootstrap calculations were done. In fact, the decuplet (charm-zero resonances) masses and widths were also completely known soon after the SU(3) classification scheme was proposed, so the parameters of the theory could have been adjusted to give the correct results. More ministerially, since several schemes which predict the properties of the charmed baryon and pseudoscalar mesons have been proposed,[8,9] the possibility arises that our calculation could be exploited to select which scheme is most viable in the context of broken-symmetry bootstrap theories by comparing the various decuplet-state positions and widths obtained to their experimental values.

Unfortunately our *a priori* expectations are that in all cases the decuplet states will not differ appreciably from the results obtained in SU(3) models, since the new channels made possible by the new internal degree of freedom involve high-mass charm-anticharm couplings wherein, assuming short-range interactions are not significant, the high-threshold theorem becomes applicable. For example, in the octet model reasonable results for the Σ^* position, width, and partial decay widths into the $\Sigma\pi$ and $\Lambda\pi$ channels are obtained when the $\Sigma\eta$ and ΞK channels are ignored.[10]

II. OUTLINE OF THE MODEL

The general features of the theory to be employed here—the kinematic structure of spin-$\frac{1}{2}$-baryon–spin-0-meson scattering, partial-wave analysis, analyticity, unitarity, crossing symmetry, and the ND^{-1} matrix formalism—have been developed extensively in great detail in the literature in the context of SU(2) and SU(3) symmetry[1]; hence in what follows we merely give a brief outline of the theoretical ideas, providing details only where it is necessary to introduce new considerations.

We construct an ND^{-1} matrix to represent the $J = \frac{3}{2}$ partial-wave T matrix which exhibits the correct threshold behavior and from which all of the kinematic singularities have been removed. The latter requirement demands that we work in the center-of-mass energy, i.e., the complex ω plane, rather than in the square of the center-of-mass energy.[11] The N matrix, which comprises all of the physical or dynamical input, is assumed to be analytic on the right half-plane and incorporates the left-hand cut, $(-\omega_0, -\infty)$, where ω_0 is the threshold energy in the center-of-mass frame. Since we are regarding single-baryon exchange as the primary force responsible for generating the resonances, we define the N matrix to be the Born approximation for one-baryon exchange assuming exact SU(4) symmetry:

$$N(\omega) = h(\omega)N_0\,, \tag{2.1}$$

where N_0 is the symmetric matrix determined by the isospin factors, the SU(3) isoscalar factors, and the Yukawa-type meson-baryon coupling constants, which, in turn, depend on the pion-nucleon coupling constant and a mixing parameter. The dimensionality of N for a particular channel is just the number of coupled two-particle states in that channel. These two-particle states are given explicitly in Table I. $h(\omega)$ is the usual energy-dependent factor for the $J = \frac{3}{2}$ channel:

$$h(\omega) = \frac{1}{16\pi\omega}\left\{\frac{1}{E-M}\left[A_1 + (\omega - M)B_1\right] + \frac{1}{E+M}\left[-A_2 + (\omega + M)B_2\right]\right\}\,, \tag{2.2}$$

where

$$\begin{aligned} A_l &= -g^2\,\frac{M - M_x}{q^2}\,Q_l\left(1 + \frac{y}{2q^2}\right), \\ B_l &= \frac{g^2}{q^2}\,Q_l\left(1 + \frac{y}{2q^2}\right)\,, \\ y &= 2(M^2 + m^2) - M_x{}^2 - \omega^2\,, \\ E \pm M &= \left[(\omega \pm M)^2 - m^2\right]/2\omega\,. \end{aligned} \tag{2.3}$$

TABLE I. Coupled two-particle states for each channel.

Quantum numbers C	I	Y	Coupled two-particle states
0	$\frac{3}{2}$	1	$N\pi, \Sigma K, C_1\overline{D}$
	1	0	$\Lambda\pi, \Sigma\pi, N\overline{K}, \Sigma\eta, \Xi K, \Sigma\eta_c, A\overline{D}, C_1F^-, S\overline{D}$
	$\frac{1}{2}$	-1	$\Xi\pi, \Lambda\overline{K}, \Sigma\overline{K}, \Xi\eta, \Xi\eta_c, AF^-, SF^-, T\overline{D}$
	0	-2	$\Xi\overline{K}, TF^-$
1	1	1	$ND, \Sigma F^+, C_0\pi, AK, C_1\pi, C_1\eta, SK, C_1\eta_c, X\overline{D}$
	$\frac{1}{2}$	0	$\Lambda D, \Sigma D, \Xi F^+, A\pi, C_0\overline{K}, A\eta, S\pi, C_1\overline{K}, S\eta, TK, A\eta_c, S\eta_c, XF^-, X_s\overline{D}$
	0	-1	$\Xi D, A\overline{K}, S\overline{K}, T\eta, T\eta_c, X_sF^-$
2	$\frac{1}{2}$	1	$C_0D, AF^+, C_1D, SF^+, X\pi, X\eta, X_sK, X\eta_c$
	0	0	$AD, SD, TF^+, X\overline{K}, X_s\eta, X_s\eta_c$
3	0	1	XD, X_sF^+

Q_l is the Legendre function of the second kind; M_x, M, and m are the masses of the exchanged baryon, the incoming baryon, and the incoming meson, respectively; E is the baryon energy; and q is the center-of-mass momentum.

The D matrix incorporates the right-hand cut, (ω_0, ∞), and is assumed to be analytic on the left half-plane. A once-subtracted dispersion relation is written for D in terms of the N matrix

$$D(\omega) = 1 - \frac{\omega - \hat{\omega}}{\pi} \int_P \frac{\rho(\omega')N(\omega')d\omega'}{(\omega' - \hat{\omega})(\omega' - \omega)} \ . \tag{2.4}$$

P denotes the branch cuts and $\hat{\omega}$, the subtraction point, is smaller than the lowest threshold for the entire problem. The ρ matrix is given by

$$\rho_{ij} = \delta_{ij} q_i{}^3(\omega)\theta(\omega - \omega_{0_i}) \ , \tag{2.5}$$

where q_i and ω_{0_i} are, respectively, the i-channel momentum and threshold energy. Here symmetry breaking is introduced for the first time by evaluating ρ at the physical masses.

The problem of locating the positions of the resonances and bound states amounts to simply determining the positions of the poles of the kinematically reduced T matrix given by

$$\begin{aligned} A &= ND^{-1} \\ &= h(\omega)N_0\left[1 - \frac{\omega - \hat{\omega}}{\pi} \int_P d\omega' \frac{\rho(\omega')h(\omega')}{(\omega' - \hat{\omega})(\omega' - \omega)} N_0\right]^{-1} \\ &= h(\omega)[N_0{}^{-1} - I(\omega)]^{-1} \ , \end{aligned} \tag{2.6}$$

where

$$I(\omega) = \frac{\omega - \hat{\omega}}{\pi} \int \frac{\rho(\omega')h(\omega')}{(\omega' - \hat{\omega})(\omega' - \omega)} d\omega' \ . \tag{2.7}$$

The calculations are greatly simplified by transforming to the representation in which N_0 is diagonal. Each two-particle state, specified by isospin, hypercharge, and charm quantum numbers, can be expressed as a linear combination of irreducible representations of SU(4) through the SU(4) Clebsch-Gordan coefficients[12]:

$$(BM)_i = U_{ij}\psi_j(\{R, \mu\}, I, Y, C), \quad i,j = 1, 2, \ldots, n \tag{2.8}$$

corresponding to an n-channel case. I, Y, and C denote isospin, hypercharge, and charm, respectively, while R and μ label the SU(4) and SU(3) $\subset$SU(4) representations, respectively. The U_{ij} form a unitary matrix. Thus, for a set of n coupled two-particle channels, the elements in the matrix of amplitudes which constitute N_0 can be expressed as a linear combination of pure-SU(4)-symmetric amplitudes. Then the matrix can be diagonalized by the unitary matrix U. The eigenvalues of N_0 are the appropriate crossed-channel SU(4)-invariant amplitudes, F_i, corresponding to one-baryon exchange in the direct channel. These amplitudes, which depend only on the overall coupling constant and the mixing parameter, have been calculated in a previous work.[13] It is not necessary, then, to calculate the full Born matrix using the Yukawa couplings; but for the purposes of later discussion we give the full N_0 matrix for the eight-channel case corresponding to Ξ^* in Table II. In this and all other decuplet states the submatrix corresponding to the uncharmed channels reduces to the SU(3) matrices given in Ref. 6. In Table III we list the diagonalized N_0 matrices, $\mathfrak{N}$, for each channel:

TABLE II. Full N_0 matrix for Ξ^*. The notation is that of Ref. 6. (AB, C, DE) is the isospin amplitude for $A+B \rightarrow D+E$, where C denotes the exchanged particle. The values of the amplitudes in terms of the mixing parameter, α, are written below the exchange symbols. g^2 has been factored out of the matrix.

	$\Xi\pi$	$\Lambda\bar{K}$	$\Sigma\bar{K}$	$\Xi\eta$	$\Xi\eta_c$	AF^-	SF^-	$T\bar{D}$
$\Xi\pi$	$-(\Xi\pi, \Xi, \Xi\pi)$ $-4\alpha^2+4\alpha-1$	$\sqrt{3}(\Xi\pi, \Sigma, \Lambda\bar{K})$ 2α	$2(\Xi\pi, \Sigma, \Sigma K)$ $+(\Xi\pi, \Lambda, \Sigma K)$ $\frac{8}{3}\alpha^2-6\alpha+4$	$\sqrt{3}(\Xi\pi, \Xi, \Xi\eta)$ $4\alpha^2-8\alpha+3$	$\sqrt{3}(\Xi\pi, \Xi, \Xi\eta_c)$ $\frac{1}{\sqrt{2}}(8\alpha^2-10\alpha+3)$	$\sqrt{3}(\Xi\pi, S, AF^-)$ $+\sqrt{3}(\Xi\pi, A, AF^-)$ $-\frac{16}{3}\alpha^2+8\alpha-3$	$-\sqrt{3}(\Xi\pi, S, SF^-)$ $+\sqrt{3}(\Xi\pi, A, SF^-)$ $\frac{1}{\sqrt{3}}(-8\alpha^2+12\alpha-3)$	0 0
$\Lambda\bar{K}$		$(\Lambda\bar{K}, N, \Lambda\bar{K})$ $\frac{4}{3}\alpha^2-4\alpha+3$	$-\sqrt{3}(\Lambda\bar{K}, N, \Sigma\bar{K})$ $-4\alpha^2+8\alpha-3$	$(\Lambda\bar{K}, \Lambda, \Xi\eta)$ $\frac{8}{3}\alpha^2-2\alpha$	$-(\Lambda\bar{K}, \Lambda, \Xi\eta_c)$ $\frac{1}{\sqrt{2}}(-\frac{16}{3}\alpha^2+8\alpha-3)$	$(\Lambda\bar{K}, C_0, AF^-)$ $\frac{20}{9}\alpha^2-\frac{14}{3}\alpha+2$	$(\Lambda\bar{K}, C_0, SF^-)$ $\frac{1}{\sqrt{3}}(-\frac{4}{3}\alpha^2+2\alpha)$	$(\Lambda\bar{K}, S, T\bar{D})+(\Lambda\bar{K}, A, T\bar{D})$ $\frac{1}{\sqrt{6}}(-\frac{40}{3}\alpha^2+20\alpha-6)$
$\Sigma\bar{K}$			$-(\Sigma\bar{K}, N, \Sigma\bar{K})$ $-4\alpha^2+4\alpha-1$	$\sqrt{3}(\Sigma\bar{K}, \Sigma, \Xi\eta)$ -2α	$-\sqrt{3}(\Sigma\bar{K}, \Sigma, \Xi\eta_c)$ $-\frac{1}{\sqrt{2}}(4\alpha-3)$	$\sqrt{3}(\Sigma\bar{K}, C_1, AF^-)$ $4\alpha^2-2\alpha$	$\sqrt{3}(\Sigma\bar{K}, C_1, SF^-)$ $\frac{1}{\sqrt{3}}(-12\alpha^2+18\alpha-6)$	$\sqrt{3}(\bar{K}, S, T\bar{D})$ $+\sqrt{3}(\Sigma K, A, T\bar{D})$ $\frac{1}{\sqrt{6}}(8\alpha^2-12\alpha+6)$
$\Xi\eta$				$(\Xi\eta, \Xi, \Xi\eta)$ $\frac{4}{3}\alpha^2-4\alpha+3$	$(\Xi\eta, \Xi, \Xi\eta_c)$ $\frac{1}{\sqrt{2}}(\frac{8}{3}\alpha^2-6\alpha+3)$	$-(\Xi\eta, S, AF^-)$ $-(\Xi\eta, A, AF^-)$ $\frac{8}{9}\alpha^2+\frac{4}{3}\alpha-1$	$-(\Xi\eta, S, SF^-)$ $-(\Xi\eta, A, SF^-)$ $-\frac{1}{\sqrt{3}}$	$-(\Xi\eta, T, T\bar{D})$ $\frac{1}{\sqrt{6}}(16\alpha^2-24\alpha+8)$
$\Xi\eta_c$					$(\Xi\eta_c, \Xi, \Xi\eta_c)$ $\frac{8}{3}\alpha^2-4\alpha+\frac{3}{2}$	$(\Xi\eta_c, A, AF^-)$ $\frac{1}{\sqrt{2}}(-\frac{8}{9}\alpha^2+\frac{2}{3}\alpha+1)$	$(\Xi\eta_c, S, SF^-)$ $\frac{1}{\sqrt{6}}(8\alpha^2-6\alpha+1)$	$(\Xi\eta_c, T, T\bar{D})$ $\frac{1}{\sqrt{3}}(-8\alpha^2+6\alpha-1)$
AF^-						(AF, X, AF^-) $\frac{16}{3}\alpha^2-8\alpha+3$	$-(AF^-, X, SF^-)$ $\frac{1}{\sqrt{3}}(-4\alpha+3)$	$-(AF^-, X_s, T\bar{D})$ $\frac{1}{\sqrt{6}}(8\alpha-6)$
SF^-							(SF, X, SF^-) 1	$(SF^-, X_s, T\bar{D})$ $\sqrt{2}$
$T\bar{D}$								0 0

TABLE III. Diagonalized N_0 matrices, $\mathfrak{N}$. The $F(X)$, where X refers to the relevant SU(4) irreducible representation, are tabulated in Ref. 13.

Quantum numbers C	I	Y	$\mathfrak{N}_{ii}$
0	$\frac{3}{2}$	1	$[F(140), F(140), F(20')]$
	1	0	$[F(140), F(140), F(140), F(\overline{60}), F(\overline{60}), F(\overline{36}), F(20'), F(20_1)F(20_2)]$
	$\frac{1}{2}$	−1	$[F(140), F(140), F(140), F(\overline{60}), F(\overline{36}), F(20'), F(20_1), F(20_2)]$
	0	−2	$[F(140), F(20')]$
1	1	1	$[F(140), F(140), F(140), F(\overline{60}), F(\overline{60}), F(\overline{36}), F(20'), F(20_1), F(20_2)]$
	$\frac{1}{2}$	0	$[F(140), F(140), F(140), F(140), F(\overline{60}), F(\overline{60}), F(\overline{36}), F(\overline{36}),$ $F(20'), F(20_1), F(20_2)F(20_1)F(20_2), F(\overline{4})]$
	0	−1	$[F(140), F(140), F(\overline{60}), F(20'), F(20_1), F(20_2)]$
2	$\frac{1}{2}$	1	$[F(140), F(140), F(140), F(\overline{60}), F(\overline{36}), F(20')F(20_1)F(20_2)]$
	0	0	$[F(140), F(140), F(\overline{60}), F(20'), F(20_1), F(20_2)]$
3	0	1	$[F(140), F(20')]$

$$\mathfrak{N}_{ij} \equiv (U^\dagger N_0 U)_{ij} = \delta_{ij} F_i \,. \tag{2.9}$$

The amplitude reduces to

$$\begin{aligned} U^\dagger A U &= h(\omega)[U^\dagger N_0{}^{-1} U - U^\dagger I(\omega) U]^{-1} \\ &= h(\omega)[\mathfrak{N}^{-1} - U^\dagger I(\omega) U]^{-1} \equiv h(\omega)\mathfrak{D}^{-1}(\omega)\,. \end{aligned} \tag{2.10}$$

The positions of the bound states and resonances are now defined to be the zeros of $\det\mathfrak{D}$ on the appropriate sheets in the complex ω plane. The locations of the zeros of $\det\mathfrak{D}$ are the same as those of the determinant of the untransformed matrix.

In the exact-SU(4)-symmetry limit the diagonal ρ matrix reduces to a constant times the unit matrix; that is,

$$\rho_i = \rho_j = \rho \quad \text{for all } i, j \,, \tag{2.11}$$

where we have defined

$$\rho_i \equiv \rho_{ii} \,. \tag{2.12}$$

Then $U^\dagger \rho U$ is also diagonal,

$$(U^\dagger \rho U)_{ij} = \delta_{ij}\rho \,. \tag{2.13}$$

If the symmetry breaking is gradually introduced, we may hope that the off-diagonal elements of $U^\dagger \rho U$ remain small, because

$$(U^\dagger \rho U)_{ij} = \sum_k U^\dagger_{ik} U_{kj} \rho_k \equiv \sum_k a_k(i,j)\rho_k \,, \tag{2.14}$$

where the coefficients, a_k, add up to 0 for the off-diagonal case, $i \neq j$, and to 1 for $i = j$. The numerical results of Wali and Warnock for the SU(3) model support the conclusion that the nondiagonal elements may be neglected in a rough first approximation. To treat them as ignorable perturbations greatly simplifies the problem, for then the determinant factors into a product of the diagonal terms, each of which corresponds to an irredicible representation of SU(4).

However, in spite of the fact that corrections to the diagonal approximation appear as products of off-diagonal elements, it is certainly possible, particularly in the SU(4) scheme, where the mass degeneracy is much more badly broken than in the SU(3) scheme, that off-diagonal elements will have at least a significant quantitative effect on our results. Thus it is necessary to evaluate numerically all of the matrix elements and compare them over the energy ranges of interest before arriving at any conclusions regarding their relative significance.

The calculations are also simplified by utilizing the obvious fact that the $\mathfrak{D}$ matrix is symmetric, as it should be to accommodate time-reversal invariance.

The widths of the resonances, assuming they are narrow, are estimated by employing a linear approximation for $\mathrm{Re}\det\mathfrak{D}(\omega + i\epsilon)$ near the resonance position, ω_R:

$$\begin{aligned} \mathrm{Re}\det\mathfrak{D}(\omega + i\epsilon) &= (\omega - \omega_R)\left[\frac{d}{d\omega}(\mathrm{Re}\det\mathfrak{D}(\omega + i\epsilon)\right]_{\omega=\omega_R} \\ &\quad + O((\omega - \omega_R)^2)\,. \end{aligned} \tag{2.15}$$

Then, letting k denote the derivative in Eq. (2.15),

$$\det\mathfrak{D} \propto k(\omega - \omega_R) + i\,\mathrm{Im}\det\mathfrak{D}(\omega_R + i\epsilon) \tag{2.16}$$

and

$$\Gamma/2 = \mathrm{Im}[\det\mathfrak{D}(\omega + i\epsilon)]/k\,. \tag{2.17}$$

III. INPUT MASSES AND PARAMETERS

The $\frac{1}{2}^+$ baryon and pseudoscalar-meson mass spectra have been predicted in group-theoretical and quark-model calculations providing several sets of possible input masses for our calculation. The predictions which are the most theoretically satisfying are those of the quark model in the calculation of De Rújula, Georgi, and Glashow. Their model uses a specific quark-quark interaction and symmetry breaking, the general forms of which they are constrained to use in order to satisfy the underlying renormalizable color gauge theory.

To incorporate asymptotic freedom and infrared slavery they use the lattice theory as a guide,[14] i.e., they assume that the principal binding energy of the hadrons is due to the long-range interaction responsible for infrared slavery and that the spin-spin and spin-orbit energies are exponentially suppressed. Asymptotic freedom suggests that the effective short-range forces are determined by one-gluon exchange, and a Fermi-Breit interaction is used.

The hadrons appear in supermultiplets of $SU(8)_q \otimes SU(8)_{\bar{q}} \otimes O(3)$ and symmetry breaking is due strictly to differences in quark masses. Without charm $[SU(6)_q \otimes SU(6)_{\bar{q}} \otimes O(3)]$ the relatively small mass differences within supermultiplets permit a perturbative approach where the expectation value of the perturbation term in the Hamiltonian is parametrized to fit the observed particle masses. New mass relations as well as the usual ones emerge from the theory and are experimentally satisfied, lending further credence to the theory.

Charmonium spectroscopy yields for the charmed quark an approximate mass which is much higher than that of the other quarks, so that SU(4) symmetry is much more badly broken than SU(3) symmetry and there is a significant uncertainty in the charmed-quark mass. Thus, when charm is added to the model it suffers to some extent from the same malady as the calculation based on the Gell-Mann–Okubo type formula done by Okubo, Mathur, and Borchardt.[9] In both cases the mass splitting between members of a multiplet due to the electromagnetic interaction and to strangeness are expected to be much more reliable than the absolute mass of the charmed hadrons.

We calculate the $\frac{3}{2}^+$ baryon mass spectrum using three sets of input masses for the charmed pseudoscalar mesons and $\frac{1}{2}^+$ baryons—those given by De Rújula, Georgi, and Glashow (DGG) and those given by Okubo, Mathur, and Borchardt (OMB). The latter includes two sets of masses for the baryons depending on whether the Gell-Mann–Okubo formulas are interpreted as being linear (OMB_L) or quadratic (OMB_Q) in the masses. The numerical values for each set is given in Tables IV and V. The notation corresponds to that of Ref. 12 but, since the names of the particles have not been standardized, we give the quark content of the leading term in each isospin multiplet.

TABLE IV. Masses of the charmed pseudoscalar mesons in GeV. In this and other tables, the quark content of the leading particle of its isospin multiplet is given in parentheses. DGG and OMB denote, respectively, the masses given in Refs. 8 and 9.

Pseudoscalar mesons	DGG	OMB	Experimental
$D(c\bar{\mathfrak{N}})$	1.830–1.860	2.171	1.870
$F(c\bar{\lambda})$	1.975	2.222	···
$\eta_c(q\bar{q})$	···	2.755	···

It is important to note the qualitative similarities in, as well as the quantitative differences between, the several schemes. In particular each scheme, of course, exhibits approximately equal spacings between states differing in strangeness by one unit and between states differing in charm by one unit. In both the DGG and OMB_L schemes, the strangeness spacings are about 170 MeV, which coincides with the observed octet and decuplet spacings, but in the OMB_Q case, the spacings are about 60 MeV. The charm spacings are of the order of 1 GeV in DGG and OMB_Q and 4 GeV in OMB_L.

The parameters of our theory are the mass of the exchanged baryon, a mixing parameter, the pion-nucleon coupling constant, and a subtraction energy. As in the Wali and Warnock calculation we take $10.5m_\pi$ as the mass of the exchanged baryon. It is chosen to be higher than the masses of the octet states in order to move the real-axis branch points to the left of the lowest threshold of the

TABLE V. Masses of the charmed $\frac{1}{2}^+$ baryons in GeV. Subscripts Q and L denote quadratic and linear formula, respectively.

Baryon	DGG	OMB_Q	OMB_L
$C_1(\mathcal{P}\mathcal{P}c)$	2.360	3.479	6.202
$S(\mathcal{P}\lambda c)$	2.510	3.542	6.416
$T(\lambda\lambda c)$	2.680	3.600	6.581
$A(\mathcal{P}\lambda c)$	2.420	2.976	4.814
$C_0(\mathcal{P}\mathfrak{N}c)$	2.200	2.898	4.597
$X(\mathcal{P}cc)$	3.550	4.313	8.790
$X_s(\lambda cc)$	3.730	4.375	9.044

problem, thus transforming the symmetric N matrix into one which has the same analytic behavior as the Born matrix evaluated at the broken-symmetry masses throughout.

The mixing parameter, α, arises because there are two ways to couple the antibaryon-baryon combination to the pseudoscalar mesons to get an SU(4) singlet. In the case of SU(3) we distinguish the two by writing an F-type (antisymmetric) and a D-type (symmetric) coupling in the Langrangian with their relative strengths defined by α, i.e., α is the strength of the symmetric coupling and $(1-\alpha)$ is the strength of the antisymmetric coupling. In the case of SU(4) the baryons are not represented by the self-adjoint representation, and we are not able to maintain the antisymmetric and symmetric identification of the couplings in the same way.[15] The direct product of the baryon representation and the pseudoscalar-meson representation, $\underline{20} \otimes \underline{15}$, gives a $\underline{20}_1$ and a $\underline{20}_2$ along with the other irreducible representations (IR's). Here we define the mixing parameter, α, so that $\frac{44}{39}\alpha$ measures the strength of the $\underline{20}_1$ couplings and $(1-\frac{44}{39}\alpha)$ is the strength of the $\underline{20}_2$ couplings.[13] In Ref. 13 we calculated the value of α which ensures the bootstrap relationship between the IR's of SU(4) which represent the $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons, the $\underline{20}$ and $\underline{20}'$ representation, respectively. We expect the same value of α, $\alpha \simeq 0.675$, to give the bootstrap relationship in the appropriate channels here, and we fix the mixing parameter at that value.

The dispersion relations for the D matrix elements are once-subtracted to ensure convergence of the integrals. The value of the subtraction-point energy is taken to be five times the pion mass throughout. Changes in this parameter simply shift $I(\omega)$, defined by Eq. (2.7) vertically.[16]

Finally, the pion-nucleon coupling constant is fixed at the experimental value.

The degenerate mass of the baryon and of the pseudoscalar meson used in the calculation of the Born matrix might also be regarded as parameters, and, indeed, the calculations are sensitive to values of these quantities. In the octet model the choices were[17]

$$M = \frac{M_N + M_\Xi}{2} = \frac{3M_\Lambda + M_\Sigma}{4}$$

and

$$m^2 = \frac{3m_\eta^{\,2} + m_\pi^{\,2}}{4} = m_k^{\,2}, \tag{3.1}$$

or alternatively, in order to facilitate parametrization of the mass breaking,[7]

$$M = M_\Lambda \text{ and } m = m_\eta. \tag{3.2}$$

In either case the baryon and pseudoscalar-meson masses lie in the neighborhood of the average masses of the octets.

We again choose Eqs. (3.1) to define our degenerate masses even though they can no longer be regarded as average masses of the new SU(4) multiplets. As before, the choice is convenient for investigating the behavior of the amplitudes when the symmetry breaking is gradually increased from the degenerate to the physical values, where the amount of breaking for a particular state depends on the isospin, strangeness, and charm quantum numbers as well as on some symmetry-breaking parameter.

IV. RESULTS AND CONCLUSIONS

As expected, there appear poles at well-defined energies in the ND^{-1} matrices for each channel defined by isospin, strangeness, and charm. The numerical values for the resonance positions and widths are given in Tables VI and VII, respectively. The determinants of the $\mathfrak{D}$ matrices for the $C=0$ decuplet states, the $C=1$ sextuplet states, the

TABLE VI. $\frac{3}{2}^+$ baryon masses in GeV. I_1, I_2, and I_3 correspond to our calculations using the input masses of DGG, OMB_Q, and OMB_L, respectively. EXP denotes the observed value.

Baryon	DGG	OMB_Q	OMB_L	I_1	I_2	I_3	EXP
$\Delta(\mathcal{P}\mathcal{P}\mathcal{P})$	···	···	···	1.205	1.215	1.230	1.233
$\Sigma^*(\mathcal{P}\mathcal{P}\lambda)$	···	···	···	1.334	1.385	1.310	1.385
$\Xi^*(\mathcal{P}\lambda\lambda)$	···	···	···	1.430	1.480	1.440	1.530
$\Omega^-(\lambda\lambda\lambda)$	···	···	···	1.476	1.485	1.500	1.672
$C_1^*(\mathcal{P}\mathcal{P}c)$	2.420	3.215	4.261	2.500	2.200	2.781	2.5
$S^*(\mathcal{P}\lambda c)$	2.560	3.277	4.414	2.800	2.400	2.900	···
$T^*(\lambda\lambda c)$	2.720	3.342	4.562	3.000	2.460	2.950	···
$X^*(\mathcal{P}cc)$	3.610	4.377	7.291	4.230	3.380	3.630	···
$X_s^*(\lambda cc)$	3.770	4.422	7.444	4.420	4.350	3.990	···
$C_3(ccc)$	4.810	5.289	10.320	5.680	5.970	5.780	···

TABLE VII. Baronic widths in MeV. b indicates that the resonance position is below the threshold.

Baryon	DGG′	OMB′$_Q$	OMB′$_L$	EXP
Δ	71	74	82	110
Σ^*	30	81	25	42 ±4
Ξ^*	4	15	4	9.1±5
Ω^-	b	b	b	stable
C_1^*	0.0	b	b	···
S^*	b	b	b	···
T^*	b	b	b	···
X^*	3.3	b	b	···
X_s^*	34.5	b	b	···
C_3	89.4	b	b	···

$C=2$ triplet states, and the $C=3$ singlet state are plotted with arbitrary units along the ordinate in Figs. 1–4, respectively, for OMB$_L$ input masses. The behavior is similar for the other input masses. n in all of the figures denotes the dimensionality of $\mathfrak{D}$.

Numerically, for each case the calculated positions of the decuplet states are within a few percent of the observed values, with the exception of the Ω^- which exhibits an error of 10–12%. The positions of the nonzero-charm states are qualitatively very reasonable. The SU(4) mass formulas given in Ref. 9.

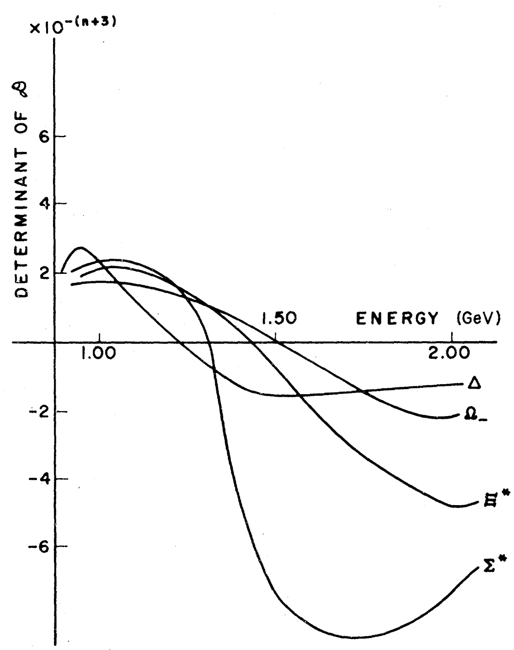

FIG. 1. The positions of the $C=0$ (I_3) decuplet state resonances as determined by the zeros of det$\mathfrak{D}$ as functions of energy.

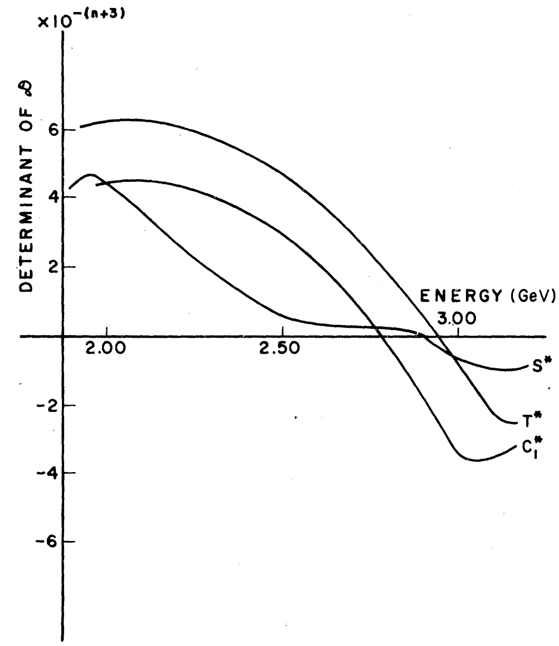

FIG. 2. The positions of the $C=1$ (I_3) sextuplet state resonances.

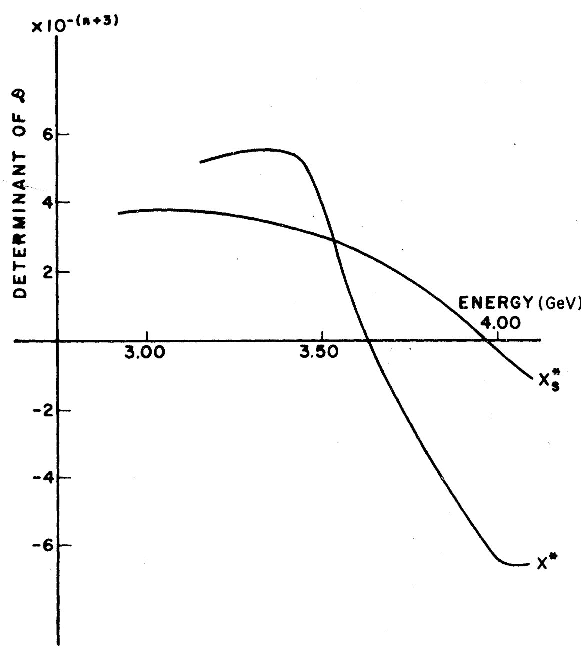

FIG. 3. The positions of the $C=2$ (I_3) triplet state resonances.

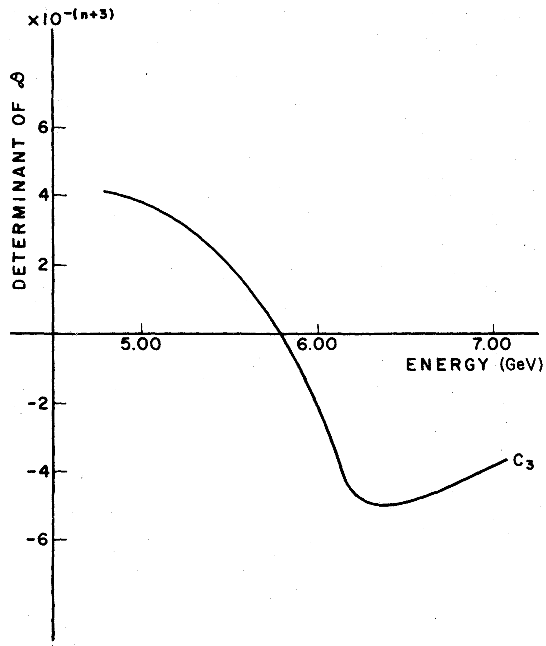

FIG. 4. The position of the $C=3$ (I_3) singlet state resonance.

$$C_1^* - \Delta = S^* - \Sigma^* = T^* - \Xi^* , \qquad (4.1)$$

$$X^* - \Delta = X_s^* - \Sigma^* , \qquad (4.2)$$

$$\Sigma^* - \Delta = \Xi^* - \Sigma^* = \Omega^- - \Xi^* , \qquad (4.3)$$

$$C_3 - \Delta = 3y\delta , \qquad (4.4)$$

where

$$y = 20.7 , \qquad (4.5)$$

$$\delta = (\Omega^- - \Delta)/3 \qquad (4.6)$$

are very well satisfied for each case, with the exception of the decuplet equal-spacing rule of Eq. (4.3), which is rather badly violated.

Virtually the same masses are obtained from each of the alternative definitions of the resonance position:

$$\mathrm{Re}[\det \mathcal{D}(\omega)] = 0 ,$$
$$\det[\mathrm{Re}\mathcal{D}(\omega)] = 0 , \qquad (4.7)$$
$$\mathrm{Re}[d_{20'}(\omega)] = 0 ,$$

where $d_{20'}(\omega)$ is the eigenvalue of the $\mathcal{D}$ matrix corresponding to the $\underline{20}'$ irreducible representation of SU(4) in the exact-symmetry limit.

The widths for the charm-zero decuplet states are, as usual in bootstrap models, not very accurate, although they are of the correct order of magnitude.

Interestingly, the positions of all of the nonzero-charm resonances appear below the thresholds for the reactions with OMB_L and OMB_Q input masses. With the DGG masses only the C_1^*, X^*, X_s^*, and the C_3 appear above threshold, but they occur very close to the threshold with correspondingly small phase space for decay. Hence all of the charmed states appear as bound states or very narrow-width resonances; that is all are nearly stable particles.

We have heretofore assumed that the resonant states unambiguously comprise the $\underline{20}'$-plet of SU(4); however, the reciprocal bootstrap mechanism in the case of exact SU(4) symmetry yields attraction in both the $\underline{140}$ channel and the $\underline{20}'$ channel. The magnitude of the attractive force in the former, however, is only about 25% of that in the latter channel, thus we expect the physical resonances to be primarily identifiable with the $\underline{20}'$ IR of SU(4) while exhibiting a small mixture of the $\underline{140}$ IR. The tensor representation of the $\underline{140}$ has the identical symmetry structure as the $\underline{27}$ IR of SU(3), which exhibits the largest mixing with the decuplet in broken-SU(3) models.

The mixing problem for broken SU(4) is complicated by the multiple occurrence of amplitudes corresponding to the various IR's and by SU(3) symmetry breaking which gives rise to mixing between the SU(3) $\subset$ SU(4) states. For example, when the N_0 matrix for Ξ^*, given in Table II is transformed to the unitary representation defined in Eq. (2.9), $F(140)$ appears three times with the same strength corresponding to states

$$\psi(\{R, \mu\}; I, Y, C) = \psi(\{140, \mu\}; \tfrac{1}{2}, -1, 0) ,$$

where the SU(3) label, μ, takes on values 27, 10, and 8. As shown in Table III multiple occurrence of $F(140)$ exists whenever the dimensionality of the $\mathfrak{N}$ matrix exceeds two. Then, when the $\mathfrak{N}_{ij}$ matrix elements together with the mass differences are introduced into the dispersion equations for the $\mathcal{D}_{ij}$ matrix elements, mixing occurs between the $\underline{20}'$ and each of the $\underline{140}$ amplitudes. Furthermore, since the zero-charm SU(3) content of $\underline{140}$ consists of a $\underline{27}$, a $\underline{10}$, and an $\underline{8}$, and the zero-charm SU(3) content of $\underline{20}'$ consists of a $\underline{10}$, mixing may occur at the SU(3) level. Similar circumstances prevail for the charmed cases. The $\underline{140}$ contains singly charmed IR's $\underline{24}$, $\overline{\underline{15}}$, $\underline{6}$, and $\overline{\underline{3}}$ and doubly charmed IR's $\underline{15}$, $\overline{\underline{6}}$, and $\underline{3}$ while the $\underline{20}'$ contains a singly charmed $\overline{\underline{6}}$ and a doubly charmed $\underline{3}$.

In the unitary representation, the off-diagonal elements of the transition-amplitude matrix, $U^\dagger A U$, at the resonance position provide a measure of the degree of symmetry breaking that emerges as a consequence of the input of nondegenerate masses in the D matrix, that is, the relative magnitudes of the off-diagonal elements correspond to the degree of mixing of the SU(4) IR's in the resonant state.

Numerically, the squared absolute value of the diagonal element $(U^\dagger AU)_{20',20'}$ is consistently larger by at least an order of magnitude than the other elements, while the largest off-diagonal elements are consistently the $(U^\dagger AU)_{140,20'}$ elements commensurate with an expectation from exact-symmetry considerations.

Also not unexpected is the fact that the discrepancies in the calculated decuplet masses discussed earlier are all manifested in a lowering of the values compared to those obtained in SU(3)-symmetric calculations. The parameters of the theory have nearly the same values as those of Ref. 7, and from quantum mechanics we know that simply introducing new channels, i.e., expanding the vector space of particle states, cannot raise the resonance position, but may lower it.

Furthermore, the new decuplet masses obtained corresponding to the different sets of input masses are all affected similarly, and the calculation is insufficiently sensitive to the charmed-particle masses to provide a means of selecting the correct ones.

The violation of the equal-spacing rule for the decuplet is most significant in the $\Omega^- - \Xi^*$ mass difference, which can be attributed to the rather high error in the Ω^- mass. In broken SU(3) symmetry the Ω^- is produced in a single-channel reaction as a bound state of Ξ and $\bar{K}$. The charm hypothesis introduces another two-particle state with the quantum numbers of the Ω thereby doubling the number of channels and necessitating a change to the multichannel formalism. The masses T and F^- as well as the other input parameters might be expected to have a disproportionate effect on the position of the resonance compared to the effects on other decuplet states.

There is no doubt that, through manipulation of the parameters of the theory, the numerical deviations from the observed decuplet masses may be removed. For example, it may be more reasonable to use a higher exchange mass and to use a different subtraction point for each dispersion integral. However, the goal here is not to reproduce accurately the decuplet masses or to make accurate predictions of the charmed-baryon resonances. To carry out such a program with our model at this time is unwarranted, since most of the masses of the charmed $\frac{1}{2}^+$ baryons are not yet known experimentally.

Our objective has been simply to determine whether or not dispersion theory and the bootstrap mechanism along with the simple dynamics of one-baryon exchange give meaningful and consistent results when charm is introduced. As was mentioned in the Introduction, we do not expect any formalism which regards the hadrons as elementary to lead to a fundamental theory. However, the ideas embodied in the original Chew-Low[18] dispersion theory for the P_{33} resonance in π^+p scattering, in the bootstrap approach of Chew and Frautschi,[19] and in the reciprocal bootstrap mechanism of Chew[20] and Cutkosky[21] and expanded upon in the SU(3) model of Martin and Wali[6] continue to yield, at least qualitatively, the correct observed features of low-energy baryon–pseudoscalar-meson scattering. Well-defined resonances do appear in the correct channels with the correct quantum numbers. Such consistency must somehow be accounted for in any complete theory of hadron spectroscopy and strong-interaction physics.

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[1]For a review and detailed references see, for example, B. M. Udgaonkar, in *Proceedings of the Seminar on High-Energy Physics and Elementary Particles, Trieste, Italy, 1965* (IAEA, Vienna, 1965).

[2]R. P. Feynman, *Photon-Hadron Interactions* (Benjamin, Reading, Mass., 1972).

[3]J. Kuti and V. F. Weisskopf, Phys. Rev. D 4, 3418 (1971).

[4]M. Y. Han and Y. Nambu, Phys. Rev. 139, B1006 (1965); G. Feldman and P. T. Mathews, Phys. Rev. Lett. 20, 604 (1968).

[5]J. Pati, A. Salam, and J. Strathdee, Phys. Lett. 59B, 265 (1975).

[6]A. W. Martin and K. C. Wali, Phys. Rev. 130, 2455 (1963).

[7]K. C. Wali and Robert L. Warnock, Phys. Rev. 135, B1358 (1964).

[8]A. De Rújula, Howard Georgi, and S. L. Glashow, Phys. Rev. D 12, 147 (1975).

[9]S. Okubo, V. S. Mathur, and S. Borchardt, Phys. Rev. Lett. 34, 236 (1975).

[10]R. E. Cutkosky, J. Kalckar, and Tarjanne, Phys. Lett. 1, 93 (1962).

[11]S. C. Frautschi and J. D. Walecka, Phys. Rev. 120, 1486 (1960).

[12]SU(4) Clebsch-Gordan coefficients are tabulated in V. Rabl, George Campbell, Jr., and K. C. Wali, J. Math.

Phys. 16, 2494 (1975).
[13]George Campbell, Jr., Phys. Rev. D 13, 662 (1976).
[14]K. Wilson, Phys. Rev. D 10, 2445 (1974); J. Kogut and L. Susskind, *ibid.* 9, 3501 (1974).
[15]In Ref. 13 this point is not clearly stated. In particular, the $F^{\rho}_{\mu\nu}$ and $D^{\rho}_{\mu\nu}$ matrices with $\mu,\nu=1, \ldots, 20$ and $\rho=1, \ldots, 15$ of Eq. (3) can be identified with the structure constants as in Eqs. (4) and (5) only for the SU(3) subspace, i.e., for $\mu,\nu,\rho=1,\ldots,8$. The matrices are defined by the equation

$$\begin{aligned}\mathcal{L}&=-g\left[\tfrac{44}{39}\alpha\bar{B}^{\mu}D^{\rho}_{\mu\nu}B^{\nu}+(1-\tfrac{44}{39}\alpha)\bar{B}^{\mu}F_{\mu\nu}B^{\nu}\right]P_{\rho}\\&=-g\left[\tfrac{44}{39}\alpha B(BP)_{201}+(1-\tfrac{44}{39}\alpha)\bar{B}(BP)_{202}\right],\end{aligned}$$

where B and P denote the baryon and pseudoscalar-meson multiplets, respectively.
[16]A. W. Martin and K. C. Wali, Nuovo Cimento 31, 1324 (1964).
[17]M. Gell-Mann, Phys. Rev. 125, 1067 (1962); Y. Ne'eman, Nucl. Phys. 26, 222 (1961); S. Okubo, Prog. Theor. Phys. 27, 949 (1962).
[18]G. F. Chew and F. E. Low, Phys. Rev. 101, 1570 (1956).
[19]G. F. Chew and S. C. Frautschi, Phys. Rev. Lett. 7, 394 (1961).
[20]G. F. Chew, Phys. Rev. Lett. 9, 233 (1962).
[21]K. E. Cutkosky, Ann. Phys. (N.Y.) 23, 415 (1963).