Radiative $\eta''(2.8)$ decay and the charmed-quark content of ω and ϕ

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We propose a sum rule connecting radiative decay modes of η'' (the charmed partner of η and η', and presumably the 2γ resonance at 2.8 GeV). The sum rule should allow a determination of the percentage of charmed quarks in the ω and ϕ mesons. If the percentage of charmed quarks in the η and η' is large enough, the percentage of charmed quarks in the ω and ϕ can also be determined by study of the decay modes $\psi\to(\omega$ or $\phi)+(\eta$ or $\eta')$; this possibility is discussed. A phenomenological analysis suggests that the familiar octuplet-singlet pseudoscalar mixing angle θ_P for η and η' varies from $-8°$ at the η energy to approximately $-38°$ at the η' energy. The latter angle yields predictions $\Gamma(\eta'\to\text{all})=0.11$ MeV and $\Gamma(A_2\to\eta'\pi)/\Gamma(A_2\to\text{all})=0.12\%$; both predictions are smaller than those obtained from either the quadratic or linear mass formula values for θ_P. Finally, a relation $\Gamma(\psi\leftarrow\eta\gamma)/\Gamma(\psi\to\eta'\gamma)=$ (phase-space factors) $\times\,\Gamma(\psi'\to\eta\gamma)/\Gamma(\psi'\to\eta'\gamma)$ is shown to follow from the assumption that the $c\bar{c}$ content of ψ and $\psi'(3684)$ is negligible compared to the $c\bar{c}$ content of η and η'.

In this note we propose a sum rule to measure the amplitudes $\langle c\bar{c}|\omega\rangle$ and $\langle c\bar{c}|\phi\rangle$ which give the $c\bar{c}$ content of the ω and ϕ. Our notation is

$$\begin{pmatrix}|\omega\rangle\\|\phi\rangle\\|\psi\rangle\end{pmatrix}\cong\begin{pmatrix}1 & \langle s\bar{s}|\omega\rangle & \langle c\bar{c}|\omega\rangle\\ \langle N|\phi\rangle & 1 & \langle c\bar{c}|\phi\rangle\\ \langle N|\psi\rangle & \langle s\bar{s}|\psi\rangle & 1\end{pmatrix}\begin{pmatrix}|N\rangle\\|s\bar{s}\rangle\\|c\bar{c}\rangle\end{pmatrix} \tag{1}$$

where

$$|N\rangle=(|u\bar{u}\rangle+|d\bar{d}\rangle)/\sqrt{2}\ . \tag{2}$$

Because of the smallness of decays which violate the Okubo-Zweig-Iizuka rule, the mixing matrix (1) is an infinitesimal orthogonal transformation, and the off-diagonal elements therefore form an antisymmetric matrix, provided they are all evaluated at the same energy. However, the elements $\langle Q\bar{Q}|X\rangle$ are believed to depend significantly on the energies of resonant states X; therefore, we cannot use, say, $\langle c\bar{c}|\phi\rangle=-\langle s\bar{s}|\psi\rangle$ to infer the $c\bar{c}$ content of ϕ from measurements on ψ but rather must measure $\langle c\bar{c}|\phi\rangle$ directly. Although they are not equal, $\langle c\bar{c}|\phi\rangle$ and $-\langle s\bar{s}|\psi\rangle$ are analytic continuations of each other, and a measurement of $\langle c\bar{c}|\phi\rangle$ would yield information on the energy dependence of $-\langle s\bar{s}|\psi\rangle$, a subject currently of considerable theoretical interest.[1-5]

We now consider possible ways to measure the quantities $\langle\omega$ or $\phi|c\bar{c}\rangle$. Usually a selection rule forbids a reaction where these amplitudes would otherwise be expected to make the major contribution. [E.g., $\psi\to(\omega$ or $\phi)+\gamma$ and $\psi(3095)$ or $\psi'(3684)\to\phi+\eta''(2.8)$ are forbidden by charge-conjugation invariance or energy-momentum conservation, respectively.[6]] Sometimes a decay mechanism involving the $\langle c\bar{c}|\omega$ or $\phi\rangle$ is swamped by a competing mechanism expected to be much more probable. (E.g., in $\psi\to\omega\eta$ decay, the mechanism where the initial ψ turns into noncharmed quarks is expected to be much more probable than the mechanism where both final particles turn into charmed quarks.[7]]

We therefore consider the decays $\eta''(2.8)\to\omega$ or $\phi+\gamma$; here the term in each matrix element proportional to $\langle c\bar{c}|\omega$ or $\phi\rangle$ stands a reasonable chance of equaling or bettering its competition. The "competition" in this case comes from terms proportional to mixing amplitudes $\langle\text{n.c.}|\eta''\rangle$ for the charmed member of the $\eta\eta'\eta''$ system, where $\langle\text{n.c.}|$ denotes a noncharmed $Q\bar{Q}$ state $\langle N|$ or $\langle s\bar{s}|$. From the absence of a $\gamma\phi\phi$ or $\gamma\rho^0\rho^0$ signal in ψ decay, Kugler has concluded $\langle\text{n.c.}|\eta''\rangle\leq10^{-2}$ (Ref. 8). Because the η'' width is poorly known at present, we cannot pinpoint these amplitudes more closely than this from the data; however, theoretical calculations agree in favoring $\langle\text{n.c.}|\eta''\rangle$ amplitudes an order of magnitude smaller than the Kugler limit[2-3]; and we shall estimate $\langle\text{n.c.}|\eta''\rangle\lesssim$ order 10^{-3}. As for the amplitudes $\langle c\bar{c}|\omega$ or $\phi\rangle$, on theoretical grounds we expect these to be larger than their analytic continuations—$\langle\text{n.c.}|\psi\rangle$; both gluon annihilation and S-matrix arguments predict that the $\langle Q\bar{Q}|X\rangle$ should decrease as the energy of X increases. Since the $\langle\text{n.c.}|\psi\rangle$ are order 10^{-4} (from elementary comparisons of two-body decay rates of ψ to the corresponding decay rates for noncharmed mesons), and there is a large change in energy between ω or ϕ and ψ, one reasonably expects $\langle c\bar{c}|\omega$ or $\phi\rangle=$ order 10^{-3}. (The $\langle c\bar{c}|\omega$ or $\phi\rangle$ cannot be much larger than this, since then $\psi\to\phi\eta$, etc., rates would become too large; also, the Gell-Mann–Okubo mass-splitting formula would be affected by the $c\bar{c}$ content of the ω and ϕ.) Thus it is reasonable that $\langle \text{n.c.}|\eta''\rangle \lesssim \langle c\bar{c}|\omega \text{ or } \phi\rangle$, implying that the $\langle c\bar{c}|\omega \text{ or } \phi\rangle$ mechanism will hold its own in $\eta'' \to \eta\gamma$ and $\eta'\gamma$ decays.

We now consider the following sum rule, which should disentagle the two mechanisms even if $\langle \text{n.c.}|\eta''\rangle \simeq \langle c\bar{c}|\omega \text{ or } \phi\rangle$[9]:

$$[\bar{\Gamma}(\omega)^{1/2} \pm r\bar{\Gamma}(\phi)^{1/2}]^2 = \bar{\Gamma}(\rho)\left(\frac{1}{3} + \frac{\sqrt{2}}{3}\frac{m_u}{m_s} r\right)^2 , \tag{3}$$

where

$$r = \langle c\bar{c}|\omega\rangle / \langle c\bar{c}|\phi\rangle . \tag{4}$$

$\bar{\Gamma}(X)$ denotes the rate for $\eta'' \to \gamma X$ divided by (photon three-momentum),[3] and the ratio of quark masses m_u/m_s is a standard quark "Bohr-magneton" correction, which is known from fits to the magnetic moments of baryons.[10-11] The $\bar{\Gamma}(\rho)$ on the right involves only $\langle N|\eta''\rangle$, so that if $\langle N|\eta''\rangle \ll \langle c\bar{c}|\omega \text{ or } \phi\rangle$, then $\bar{\Gamma}(\omega \text{ or } \phi) \gg \bar{\Gamma}(\rho)$ and no sum rule is needed: r is just $[\bar{\Gamma}(\omega)/\bar{\Gamma}(\phi)]^{1/2}$. Of course, if we are out of luck and $\langle N|\eta''\rangle \gg \langle c\bar{c}|\omega \text{ or } \phi\rangle$, then Eq. (3) with the plus sign becomes an identity for all r. For orientation, Rosenzweig's predicted value for r is $4.5/2.5 = 1.8$.[2]

In deriving Eq. (3) we have assumed for simplicity that η'' mixing angles obey SU(3) symmetry, i.e., u, d, and s quarks are mixed into the η'' wave function in equal numbers:

$$\sqrt{2}\,\langle s\bar{s}|\eta''\rangle = \langle N|\eta''\rangle . \tag{5}$$

[This assumption is convenient but not essential: The $\sqrt{2}$ in Eq. (3) would be replaced by $\langle N|\eta''\rangle / \langle s\bar{s}|\eta''\rangle$ if Eq. (5) were not correct.] The evidence for the analogous relation in ψ decays,

$$\sqrt{2}\,\langle s\bar{s}|\psi\rangle = \langle N|\psi\rangle , \tag{6}$$

is mixed: the absence of a $K\bar{K}$ ψ-decay mode suggests Eq. (6) and SU(3) symmetry are good for ψ decays, while the observed $\Gamma(\psi\to\rho\pi)/\Gamma(\psi\to K^*K)$ ratio suggests the opposite.[12-13] On intuitive grounds the symmetries (5) and (6) are very appealing. If one sets up a simple model in which the unperturbed Hamiltonian is a diagonal matrix of small mass-splitting corrections acting upon the ideal states, while the off-diagonal perturbation is an even smaller "cylinder" correction, approximately SU(4)-symmetric, then the mass-splitting which breaks the SU(3) symmetry of the $\langle \text{n.c.}|\psi\rangle$ or $\langle \text{n.c.}|\eta''\rangle$ is $m_c - m_{\text{n.c.}}$, so that equating the two sides of (5) and (6) is like equating $m_c - m_s \cong m_c - m_u$, i.e., like neglecting $(m_s - m_u)$ compared to m_c. Thus there need be no contradiction between assuming Eqs. (5) and (6) on the one hand, and keeping Bohr-magneton corrections m_u/m_s on the other. [For that matter, there is no contradiction between Eq. (6) and the violation of SU(3) symmetry represented by the observed fact that $\langle s\bar{s}|\omega\rangle \cong \langle N|\phi\rangle \cong 0$.]

Because the $\langle Q\bar{Q}|X\rangle$ are now recognized as energy dependent, it becomes more difficult than previously to predict a rate using only mixing-angle arguments. [E.g., Eq. (3) becomes a method of measuring r rather than a rate prediction.] Though more difficult, prediction is not impossible: we present an example of a rate prediction which should hold whatever the energy dependence of the $\langle Q\bar{Q}|X\rangle$:

$$\frac{\bar{\Gamma}(\psi'\to\eta\gamma)}{\bar{\Gamma}(\psi'\to\eta'\gamma)} \cong \frac{\bar{\Gamma}(\psi\to\eta\gamma)}{\bar{\Gamma}(\psi\to\eta'\gamma)} . \tag{7}$$

Harari has argued convincingly that each ψ rate on the right-hand side should be proportional to the percentage of $c\bar{c}$ in the η and η', and largely independent of the (much smaller) percentage of noncharmed quarks in the ψ.[8] Since the ψ' is expected to have even less noncharmed quark content than the ψ, his arguments should work even better for the ψ'; and Eq. (7) then follows.

From Eq. (7) and Feldman's compilation of branching ratios for ψ decays we can compute the percentage of $c\bar{c}$ in the η and η'[13]:

$$\frac{|\langle c\bar{c}|\eta\rangle|^2}{|\langle c\bar{c}|\eta'\rangle|^2} \cong \frac{\bar{\Gamma}(\psi\to\eta\gamma)}{\bar{\Gamma}(\psi\to\eta'\gamma)} = \frac{(0.10\pm0.02)/3.37\ \text{GeV}^3}{(0.24\pm0.06)/2.74\ \text{GeV}^3} = 0.34\pm0.07 . \tag{8}$$

Harari's original estimate of this ratio, based on older data, was $\frac{1}{7}$.[8]

For completeness we list some additional rate sum rules which, like Eq. (7), should be independent of any energy dependence of the mixing amplitudes. These sum rules are probably contained (at least implicitly) in everybody's table of quark-model-amplitude predictions, but it is worth emphasizing here that they should hold independently of detailed dynamical assumptions about energy dependence. Let $\Gamma(XY)$ denote the rate for $\psi\to XY$, and measure $r_\psi \equiv \langle N|\psi\rangle / \langle s\bar{s}/\psi\rangle$ using $\bar{\Gamma}(K^*\bar{K}, \text{all})/\bar{\Gamma}(\rho\pi, \text{all}) = \frac{1}{3}(1+\sqrt{2}/r_\psi)^2$. Then

$$\bar{\Gamma}(\omega\eta) + (\tfrac{1}{2}r_\psi^2)\bar{\Gamma}(\phi\eta) = \bar{\Gamma}(\omega\eta') + (\tfrac{1}{2}r_\psi^2)\bar{\Gamma}(\phi\eta') = \tfrac{1}{3}\bar{\Gamma}(\rho\pi, \text{all}) . \tag{9}$$

In contrast, a prediction such as $\bar{\Gamma}(\omega\eta')\bar{\Gamma}(\phi\eta') = \bar{\Gamma}(\omega\eta)\Gamma(\phi\eta)$ would hold only if the $\langle \text{n.c.}|\eta \text{ or } \eta'\rangle$ were energy-independent.

One caveat about sum rules (9): In deriving them, we assumed that the amplitude for both final particles to turn into charmed quarks was negligible compared to the amplitude for the initial ψ to turn into noncharmed quarks, i.e.,

$$\langle \text{n.c.} | \psi \rangle \gg |\langle c\bar{c} | \omega \text{ or } \phi \rangle \langle c\bar{c} | \eta \text{ or } \eta' \rangle| . \qquad (10)$$

The situation where this does not hold is discussed in the Appendix.

APPENDIX: CALCULATION OF $\psi \to V + \pi$ DECAYS

Since the parameters on the right-hand side of Eq. (10) are poorly determined at present,[14] it is of interest to calculate matrix elements for $\psi \to V(1^-) + \pi(0^-)$ decays without assuming Eq. (10). We shall see that, if assumption (10) breaks down, we lose sum rules (9) but gain new sum rules as well as new ways to measure the $c\bar{c}$ content of ω, ϕ, η, and η'. From the quark-model matrix element

$$M(\psi \to V\pi) = G \operatorname{Tr}[\psi(V\pi + \pi V)] \qquad (A1)$$

it is straightforward to calculate

$$M(\psi \to \rho^+ \pi^-) = \sqrt{2}\, \langle N | \psi \rangle , \qquad (A2a)$$

$$M(\psi \to K^{*+} K^-) = \langle N | \psi \rangle / \sqrt{2} + \langle s\bar{s} | \psi \rangle , \qquad (A2b)$$

$$M(\psi \to \omega\eta) = \sqrt{2}\, \langle N | \psi \rangle \langle N | \eta \rangle + 2 \langle c\bar{c} | \omega \rangle \langle c\bar{c} | \eta \rangle, \qquad (A2c)$$

$$M(\psi \to \phi\eta) = 2 \langle s\bar{s} | \psi \rangle \langle s\bar{s} | \eta \rangle + 2 \langle c\bar{c} | \phi \rangle \langle c\bar{c} | \eta \rangle, \qquad (A2d)$$

$$M(\psi \to \omega\eta') = \sqrt{2}\, \langle N | \psi \rangle \langle N | \eta' \rangle + 2 \langle c\bar{c} | \omega \rangle \langle c\bar{c} | \eta' \rangle, \qquad (A2e)$$

$$M(\psi \to \phi\eta') = 2 \langle s\bar{s} | \psi \rangle \langle s\bar{s} | \eta' \rangle + 2 \langle c\bar{c} | \phi \rangle \langle c\bar{c} | \eta' \rangle , \qquad (A2f)$$

up to a common factor of G which we suppress. Since the $\langle c\bar{c} | \eta \text{ or } \eta' \rangle$ are infinitesimal, we also have

$$1 \cong |\langle N | \eta \rangle|^2 + |\langle s\bar{s} | \eta \rangle|^2, \qquad (A3a)$$

$$1 \cong |\langle N | \eta' \rangle|^2 + |\langle s\bar{s} | \eta' \rangle|^2 . \qquad (A3b)$$

If we assume Eq. (10), then the matrix elements (A2) simplify considerably, and Eqs. (9) follow immediately from Eqs. (A2) and (A3). However, if the $\langle c\bar{c} | \eta \text{ or } \eta' \rangle$ turn out to be as large as 10^{-1}, then Eq. (10) is probably incorrect. How can we disentangle Eqs. (A2), and perhaps obtain a value for r, when Eq. (10) does not hold? The key seems to lie in finding reliable values for the amplitudes $\langle \text{n.c.} | \eta \text{ or } \eta' \rangle$. If one knew these, then one could check the following sum rules which follow readily from Eqs. (A2)[15]:

$$\left(\frac{\bar{\Gamma}(\omega\eta')^{1/2} - \bar{\Gamma}(\rho\pi, \text{all})^{1/2} |\langle N | \eta' \rangle| / \sqrt{3}}{\bar{\Gamma}(\phi\eta')^{1/2} - \bar{\Gamma}(\rho\pi, \text{all})^{1/2} |\langle s\bar{s} | \eta' \rangle r_\psi| / \sqrt{6}} \right)^2 = \left(\frac{\bar{\Gamma}(\omega\eta)^{1/2} - \bar{\Gamma}(\rho\pi, \text{all})^{1/2} |\langle N | \eta \rangle| / \sqrt{3}}{\bar{\Gamma}(\phi\eta)^{1/2} - \bar{\Gamma}(\rho\pi, \text{all})^{1/2} |\langle s\bar{s} | \eta \rangle r_\psi| / \sqrt{6}} \right)^2 = r^2 . \qquad (A4)$$

Alternatively, one could solve Eqs. (A2) for the ratio $\langle c\bar{c} | \eta \rangle / \langle c\bar{c} / \eta' \rangle$, which is known from Eq. (1.8):

$$\frac{|\langle c\bar{c} | \eta \rangle|^2}{|\langle c\bar{c} | \eta' \rangle|^2} = \left(\frac{\Gamma(\omega\eta)^{1/2} - \bar{\Gamma}(\rho\pi, \text{all})^{1/2} |\langle N | \eta \rangle| / \sqrt{3}}{\bar{\Gamma}(\omega\eta')^{1/2} - \bar{\Gamma}(\rho\pi, \text{all})^{1/2} |\langle N | \eta' \rangle| / \sqrt{3}} \right)^2 = \left(\frac{\bar{\Gamma}(\phi\eta)^{1/2} - \bar{\Gamma}(\rho\pi, \text{all})^{1/2} |r_\psi \langle s\bar{s} | \eta \rangle| / \sqrt{6}}{\bar{\Gamma}(\phi\eta')^{1/2} - \bar{\Gamma}(\rho\pi, \text{all})^{1/2} |r_\psi \langle s\bar{s} | \eta' \rangle| / \sqrt{6}} \right)^2 . \qquad (A5)$$

The question therefore becomes: how can one obtain the $\langle \text{n.c.} | \eta \text{ or } \eta' \rangle$? In a pinch we could estimate these from the value $\theta_P \cong -11^\circ$ for the usual octuplet-singlet pseudoscalar mixing angle, since the $\langle \text{n.c.} | \eta \text{ or } \eta' \rangle$ are simply related to θ_P:

$$\langle N | \eta \rangle = \sin[\theta_I - \theta_P(\eta)] ,$$

$$\langle s\bar{s} | \eta \rangle = -\cos[\theta_I - \theta_P(\eta)] ,$$

$$\langle N | \eta' \rangle = \cos[\theta_I - \theta_P(\eta')] ,$$

$$\langle s\bar{s} | \eta' \rangle = \sin[\theta_I - \theta_P(\eta')] .$$

θ_I is the "ideal" mixing angle; $\tan\theta_I = \sqrt{1/2}$, $\theta_I \cong 35.3^\circ$; and presumably -11° is some average of $\theta_P(\eta)$ and $\theta_P(\eta')$. However, there is a more direct approach to the $\langle \text{n.c.} | \eta \text{ or } \eta' \rangle$ via the quark-model predictions,[16-17]

$$\bar{\Gamma}(\eta \to 2\gamma) / 3\bar{\Gamma}(\pi^0 \to 2\gamma) = \cos^2[\theta_P(\eta) + \delta] , \qquad (A7a)$$

$$\bar{\Gamma}(\eta' \to 2\gamma) / 3\bar{\Gamma}(\pi^0 \to 2\gamma) = \sin^2[\theta_P(\eta') + \delta] , \qquad (A7b)$$

$$\Gamma(A_2^+ \to \eta\pi^+) / \Gamma(A_2^+ \to K^+ \bar{K}^0) = 2(P_\eta / P_k)^5 |\langle N | \eta \rangle|^2, \qquad (A7c)$$

$$\Gamma(A_2^+ \to \eta'\pi^+) / \Gamma(A_2^+ \to \eta\pi^+) = (P_{\eta'} / p_\eta)^5 |\langle N | \eta' \rangle / \langle N | \eta \rangle|^2, \qquad (A7d)$$

where $\delta = \arctan 2\sqrt{2} = 70.5^\circ$. Equation (A7a) plus the Particle Data Group tables[18] yields $\theta_P(\eta) = (-7 \pm 2)^\circ$, in reasonable agreement with the quadratic-mass-formula value of -11°. Similarly, Eq. (A7c) yields $\theta_P(\eta) = (-12.5 \pm 4.3)^\circ$. Averaging the values obtained from Eqs. (A7a) and (A7c), we get

$$\theta_P(\eta) = (-8 \pm 2)^\circ. \qquad (A8)$$

Values for $\theta_P(\eta')$ cannot be extracted from Eqs. (A7b) and (A7d) at present, because the necessary η' rates are not yet known. However, from the data of Bloodworth *et al.* on the cross sections for $\pi^+ p \to (\eta \text{ or } \eta') \Delta^{++}$,[19]

$$|\langle N | \eta \rangle / \langle N | \eta' \rangle| = \begin{cases} 2.10 \pm 0.41 & (\text{A}) , \\ 2.36 \pm 0.40 & (\text{B}) . \end{cases} \qquad (A9)$$

[Result (A) is obtained by comparing the η and η' cross sections at the same c.m. total energy; result (B) is obtained by comparing them at the same c.m. kinetic energy.] Equation (A9) plus Eqs. (A8) and (A6) yield a $\theta_P(\eta')$ of $(-35.6+4°)$(method A) or $(-37.8\pm 4)°$ (method B). This approach already determines $\theta_P(\eta')$ and therefore $\langle \text{n.c.}|\eta'\rangle$ quite well, and the situation can only improve as the decays $\eta'\to 2\gamma$ and $A_2\to\eta'\pi$ become better measured. We note that, with $\theta_P(\eta)=-8°$ and $\theta_P(\eta')\cong 36.7°$, Eqs. (A7b) and (A7d) predict

$$\Gamma(\eta'\to 2\gamma)=2.3\text{ keV} \qquad \text{(A10a)}$$

[therefore $\Gamma(\eta'\to\text{all})=0.11$ MeV],

$$\Gamma(A_2\to\eta'\pi)/\Gamma(A_2\to\text{all})=0.12\%. \qquad \text{(A10b)}$$

The current upper limits are $\Gamma(\eta'\to\text{all})<1$ MeV, and $<1\%$ for the A_2 branching ratio.[18]

From Eqs. (A6)–(A10) we conclude that the parameters $\langle \text{n.c.}|\eta \text{ or } \eta'\rangle$ are reasonably well known at present. From this we further conclude that a breakdown of approximation (10) would not be a disaster; one would still have sum rules (A4) and (A5) to test; in fact, from them one could gain much additional information about the quantities $\langle c\bar{c}|\omega\rangle/\langle c\bar{c}|\phi\rangle$ and $\langle c\bar{c}|\eta\rangle/\langle c\bar{c}|\eta'\rangle$.

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[5] A. De Rújula, H. Georgi, and S. L. Glashow, Phys. Rev. D 12, 147 (1975).

[6] The decay $\psi'\to\omega\eta''$ is not forbidden; however, the theoretical interpretation of the decay rate is complicated by the presence of a "radial suppression factor" due to lack of overlap of $\omega\eta''$ wave functions with the wave function of the radially excited state ψ'. See Ref. 2 and L. Clavalli and S. Nussinov, Phys. Rev. D 13, 125 (1976).

[7] This expectation could be wrong, since it is based on some very tentative estimates that $\langle c\bar{c}|\eta\rangle$ and $\langle c\bar{c}|\eta'\rangle$ are quite small (order 10^{-2}). Accordingly, we devote an appendix to a discussion of ways to extract $\langle c\bar{c}|\omega \text{ or } \phi\rangle$ from $\psi\to\omega\eta$ and related decays, assuming $\langle c\bar{c}|\eta \text{ or } \eta'\rangle$ is more like order 10^{-1}.

[8] H. Harari, Phys. Lett. 60B, 172 (1976), and Kugler quote therein.

[9] H. Harari [in *Proceedings of the Fourteenth International Conference on High Energy Physics*, edited by J. Prentki and J. Steinberger (CERN, Geneva, 1968), p. 199] has proposed sum rules of this type for the noncharmed mesons.

[10] From SU(3), $\mu(\Lambda)=-(m_u/m_s)\mu(p)/3$, implying $(m_u/m_s)=0.72\pm 0.06$.

[11] Fits to meson masses tend to require values of (m_u/m_s) in the region $.62\pm 0.06$. See Ref. 5 and Table 1 of J. G. Wills, D. B. Lichtenberg, and J. T. Kiehl, Phys. Rev. D 15, 3358 (1977). However, we favor the value of m_u/m_s obtained from magnetic-moment data, Ref. 10, since Eq. (13) is a sum rule connecting electromagnetic $M1$ transitions, not meson masses.

[12] W. Braunschweig *et al.*, Phys. Lett. 57B, 297 (1975); F. Vannucci *et al.*, SLAC Report No. SLAC-PUB-1724, 1976 (unpublished).

[13] Gary Feldman, SLAC Report No. SLAC-PUB-1851 (unpublished).

[14] E. g., Harari (Ref. 8), estimates $\langle c\bar{c}|\eta \text{ or } \eta'\rangle$ = order 10^{-1} from a mass-matrix argument; whereas Rosenzweig (Ref. 2) estimates order 10^{-2} in order to suppress certain $\eta\gamma$ and $\eta'\gamma$ radiative decay modes.

[15] In taking the square root of each rate in Eqs. (A4) and (A5), one must resolve a sign ambiguity. The signs in Eqs. (A4) and (A5) are correct if the $\langle c\bar{c}|\eta \text{ or } \eta'\rangle$ terms in Eqs. (A2c) to (A2f) are smaller in magnitude than the $\langle \text{n.c.}|\eta \text{ or } \eta'\rangle$ terms.

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