Comments and Addenda

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Covariant harmonic oscillators and the parton picture

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It is shown that the covariant-harmonic-oscillator wave function exhibits the peculiarities of the Feynman parton picture in the infinite-momentum frame.

In our previous publications,[1,2] we discussed both the conceptual and phenomenological aspects of the covariant-harmonic-oscillator formalism. Based on the Lorentz-invariant differential equation proposed by Yukawa in connection with Born's reciprocity hypothesis,[3] our starting point was a technical innovation over the work of Feynman *et al.*[4] Our solutions to the same oscillator equation satisfy all the requirements of nonrelativistic quantum mechanics in a given Lorentz frame, and satisfy the requirement of Lorentz-contracted probability interpretation for different Lorentz frames. We contend that our oscillator model is the first formalism since the invention of quantum mechanics in which the wave functions carry a covariant probability interpretation.[5]

The real strength of our oscillator model lies in the fact that one and the same wave function can provide the languages for both slow and fast hadrons. Our formalism can be applied to the quark-model calculations in the low-, intermediate-, and high-energy regions.[1,2,6] However, one of the most challenging questions in high-energy physics has been how to explain Feynman's parton[7,8] picture in terms of a formalism which can also describe the static properties of the hadron.

Another approach to this problem has been to explain Bjorken scaling in terms of the light-cone commutators and the initial hadron in its rest frame.[9] Here, one promising line of reasoning has been that the hadron is a composite particle and that its distribution function eliminates all the singularities which cause deviations from Bjorken scaling.[10] Our oscillator model does not contradict this physical picture.

Perhaps the most puzzling and irritating questions in Feynman's parton picture[7] have been the following problems:

(a) The picture is valid only in the infinite-momentum frame.

(b) Partons behave as free independent particles.

(c) While the hadron moves fast, there are wee partons.

(d) The longitudinal parton momenta are light-like.

(e) The number of partons seems much larger than the number of quarks inside the hadron.

The purpose of this paper is to provide qualitative answers to all of the above questions. Our starting point is the system of two bound quarks in the rest frame which can be described by a covariant-harmonic-oscillator wave function. We shall then boost this covariant bound system to an infinite-momentum frame and show that the peculiarities of the covariant oscillator coincide exactly with the parton properties mentioned above.

Following Feynman *et al.*[4] we call these two quarks a and b. In the harmonic-oscillator formalism,[1,2,3] the quark momenta p_a and p_b are not on the mass shell, but the total hadronic momentum

$$P = p_a + p_b \tag{1}$$

is on the mass shell. It is convenient to use the four-momentum difference

$$q=\sqrt{2}\,(p_b-p_a)\ . \tag{2}$$

We can also assign space-time coordinates to these quarks. Let us denote their coordinates by x_a and x_b, and introduce the relative coordinate

$$x=\frac{1}{2\sqrt{2}}\,(x_b-x_a)\ . \tag{3}$$

The transverse variables play only trivial roles in the harmonic-oscillator formalism and also in the parton picture. For this reason, we shall omit the transverse part of the wave function in the following discussion.

If the hadron moves along the z axis with velocity β, the ground-state wave function for this two-quark system can be written as[1,11]

$$\psi(x,\beta)=\frac{\omega}{2\pi}\exp\left\{-\frac{\omega}{2}\left[\left(\frac{1-\beta}{1+\beta}\right)\xi_+{}^2+\left(\frac{1+\beta}{1-\beta}\right)\xi_-{}^2\right]\right\}\ , \tag{4}$$

where

$$\xi_+=\frac{1}{\sqrt{2}}(t+z),\quad \xi_-=\frac{1}{\sqrt{2}}(t-z)\ .$$

ω is the "spring constant" of the oscillator system. We can construct the momentum wave function by taking the Fourier transform of the above expression,

$$\phi(q,\beta)=\left(\frac{1}{2\pi}\right)^2\int d^4x e^{-iq\cdot x}\psi(x,\beta)\ . \tag{5}$$

This momentum wave function takes the form

$$\phi(q,\beta)=\frac{1}{2\pi\omega}\exp\left\{-\frac{1}{2\omega}\left[\left(\frac{1-\beta}{1+\beta}\right)q_+{}^2+\left(\frac{1+\beta}{1-\beta}\right)q_-{}^2\right]\right\}, \tag{6}$$

where

$$q_+=\frac{1}{\sqrt{2}}\,(q_0+q_z),\quad q_-=\frac{1}{\sqrt{2}}\,(q_0-q_z)\ .$$

According to Eq. (5), we have

$$q_z=-i\frac{\partial}{\partial z},\quad q_0=i\frac{\partial}{\partial t}\ . \tag{7}$$

Because of the above asymmetry in sign,

$$q_-=i\frac{\partial}{\partial \xi_+},\quad q_+=i\frac{\partial}{\partial \xi_-}\ . \tag{8}$$

This means that q_+ is conjugate to ξ_-, and q_- is conjugate to ξ_+. In terms of these variables, the above β-dependent wave functions generate the following β-independent (Lorentz-invariant) minimum-uncertainty product.[11]

$$\langle\xi_+{}^2\rangle\langle q_-{}^2\rangle=\tfrac{1}{4}\ ,$$
$$\langle\xi_-{}^2\rangle\langle q_+{}^2\rangle=\tfrac{1}{4}\ . \tag{9}$$

Let us go back to the wave functions of Eqs. (4) and (5). If $\beta=0$, the wave functions correspond to those in the rest frame. As the momentum of the hadron becomes large,

$$\left(\frac{1+\beta}{1-\beta}\right)\to\left(\frac{2P_0}{M}\right)^2\ , \tag{10}$$

where M is the mass of the hadron. As $P_0\to\infty$, the width of the ξ_- (and q_-) distribution becomes vanishingly small. Consequently,

$$\xi_-=0\ \text{ and }\ q_-=0\ . \tag{11}$$

This means that both ξ and q are lightlike vectors, and

$$\xi_+=\sqrt{2}\,z=\sqrt{2}\,t\ ,$$
$$q_+=\sqrt{2}\,q_z=\sqrt{2}\,q_0\ . \tag{12}$$

In the infinite-momentum limit, the effective spring constant associated with the ξ_+ motion becomes vanishingly small. The motion along the ξ_+ axis therefore becomes like that of a free lightlike particle.

The behavior of the q_- distribution and that of the q_+ distribution are illustrated in Fig. 1. The width of the q_+ distribution becomes large when P_0 becomes large. This may appear as a violation of the uncertainty relation, but it is not. q_+ and ξ_+ are not conjugate variables. The precise uncertainty relation was derived in Ref. 11 and is stated in Eq. (9) of the present paper.

We can now associate the above-mentioned pecularity with the puzzling features of Feynman's parton picture. Let us first observe that the hadronic four-momentum P becomes lightlike in the infinite-momentum limit, and consider the four-

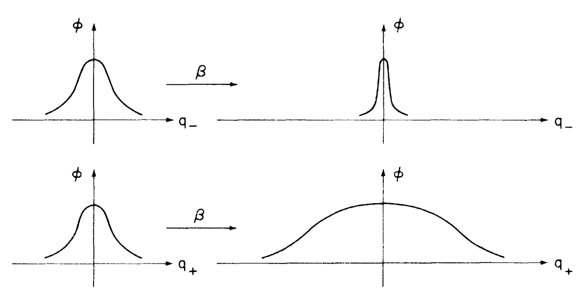

FIG. 1. Momentum wave functions in the rest frame and in a large-momentum frame. In the rest frame where $\beta=0$, the q_+ and q_- distributions take the same form. When the hadron moves fast and $\beta\to1$, the q_- distribution becomes narrower while the q_+ distribution becomes wide spread. This wide-spread q_+ distribution corresponds to the parton distribution.

momentum of the constituent quark

$$p_a = \frac{1}{2} P - \frac{1}{2\sqrt{2}} q \quad . \tag{13}$$

Since the four-vector q is lightlike, and we are considering here only longitudinal momenta, p_a is also lightlike.

Considering the width of the Gaussian function for the q_+ distribution, which is also the $\sqrt{2}\, q_0$ distribution, we can say that the momentum of the constituent quark mostly lies in the interval defined by the following limits:

$$p_{\max} = P_0\left(\frac{1}{2} + \frac{\sqrt{\omega}}{2M}\right) ,$$
$$p_{\min} = P_0\left(\frac{1}{2} - \frac{\sqrt{\omega}}{2M}\right) . \tag{14}$$

The quantity $(\sqrt{\omega}/2M)$ is of the same order of magnitude as $\frac{1}{2}$. For this reason, the lightlike four-momentum p_a can be written as

$$p_a = \alpha P \ , \tag{15}$$

with α ranging approximately from zero to one. This wide-spread distribution and division of the four-momentum are exactly like those of the parton model.

Let us go back to the ξ_+ distribution, which is also the $\sqrt{2}z$ distribution. We noted above that the motion along this axis should be almost free. Then the momentum has to be sharply defined, and the momentum cannot have a wide-spread distribution. Therefore the momentum distribution we noted in Eqs. (14) and (15) should be regarded as a distribution of free particles which are lightlike. This is exactly what we have in the original form of Feynman's parton model, as well as being characteristic of the quantum-mechanical picture of blackbody radiation. In both cases, the number of lightlike particles is not conserved.

Finally, let us consider the time interval during which the above-mentioned partons behave as free particles. According to Eq. (12), the ξ_+ axis is also the $\sqrt{2}t$ axis. Therefore, the time duration is of the order of $(P_0/M\sqrt{\omega})$. This interval increases as P_0 becomes large. If this interval is much larger than the characteristic time of electromagnetic interaction, then the partons of the present paper will indeed behave as Feynman's partons.

We have shown above qualitatively how the covariant oscillator produces Feynman's parton picture in the infinite-momentum limit. The next question then is how we can use this formalism to carry out the parton-model calculations.

In order to answer this question, we note first of all that the above two-body formalism can be easily generalized to the three-quark nucleon system.[4] In performing the parton-model calculations, we have to square the wave function to get the probability-density function. The Gaussian form remains Gaussian during the squaring process. The Lorentz-contraction property of the Gaussian probability distribution is identical to that of the wave function except for the factor of 2 in the exponent. In fact, the width quoted in Eq. (14) is derived from the width of the probability function.

As was noted earlier in this paper, the probability function exhibits a δ function in the $(q_z - q_0)$ variable in the infinite-momentum limit. We can now eliminate the q_0 dependence by integrating over this variable. The resulting function becomes the parton distribution function in the three-dimensional space.

The immediate calculations we can do using the above-mentioned procedure have already been carried out by Le Yaouanc *et al.*[12] Starting from the three-dimensional parton distribution function which we could obtain by following the procedure outlined above, Le Yaouanc *et al.* indeed carried out a comprehensive phenomenological analysis of all interesting physical quantities in the inelastic electron-nucleon scattering.

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