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ψ decays into baryon pairs*

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We discuss the decays of ψ into octet-baryon–antibaryon pairs using a model due to Okubo wherein ψ decays into uncharmed hadrons via its couplings to both hadronic and electromagnetic currents with appreciable interference effects. Relations are derived for the baryon form factors in terms of the nucleon electromagnetic form factors. As a model for the nucleon electromagnetic form factors we use a Veneziano-type dual current model subject to the Drell-Yan constraints and the Okubo-Zweig-Iizuka rule. Branching ratios are computed and comparisons are made to other form-factor models.

I. INTRODUCTION

Since the discovery of the ψ(3.095 GeV) and ψ'(3.684 GeV) particles,[1,2] much attention has been devoted to ascertaining their quantum numbers, describing their internal structure, and measuring and predicting their various decay rates into other kinds of particles. They are known to behave as hadrons with $J^P = 1^-$ and $I^{CG} = 0^{--}$,[3-5] and from a simultaneous fit of different measurements their total decay widths are found to be 69 ± 7 keV for the ψ and 228 ± 56 keV for the ψ'.[6]

Perhaps the most popular description of the internal structure of ψ and ψ' is that of the charmonium model.[7] For this model ψ and ψ' are assigned to the $1\,^3S_1$ and $2\,^3S_1$ (first and second radial excitations), respectively, of a $c\bar{c}$ (charmed-quark–charmed-antiquark) system bound by SU(3)-color gauge gluons. Hence, if the charmonium model is to be believed, ψ and ψ' should be pure SU(3) singlets. However, there are known to be deviations from this pure charmonium model[8]; in fact we shall see that these deviations may be important if one is to account for the experimentally determined branching ratios of ψ decays into baryon-antibaryon pairs and uncharmed pseudoscalar–vector-meson pairs.

There have been various theoretical speculations in the literature as to the dynamic mechanism for the decays of ψ and ψ' into ordinary (uncharmed) hadrons. One of the more notable models is due to Okubo[9,10]; he contends that the ψ and ψ' decay into ordinary hadrons via their couplings to both strong and electromagnetic currents. He further maintains that these couplings are of comparable strengths; consequently, interference effects between these two types of currents are very important. On the other hand, there are those who believe that the electromagnetic contributions are negligible when the strong, direct couplings of ψ and ψ' are present[11]; thus, interference effects are negligibly small and probably quite difficult to detect experimentally.

In this paper we discuss the decays of ψ into $B_8 + \bar{B}_8$ (octet-baryon–antibaryon) pairs within the context of the Okubo model, using a Veneziano-type dual current model for the baryon form factors[12,13] satisfying the Drell-Yan constraints.[14] Admittedly such phenomenological models of form factors probing timelike q^2 (momentum squared) are on precarious ground since most are models which have proved useful for describing data for spacelike q^2, and there is no sound reason to justify that they continue to properly describe the dynamics under analytic continuation into the timelike q^2 domain. Furthermore, experimental data points are practically nonexistent for large timelike q^2 for the processes of interest. For example, aside from upper bounds,[15] the only data point for $\sigma_{\text{total}}(e^+e^- \to p\bar{p})$ is the Frascati measurement, which needs further verification.[16] In spite of these shortcomings, however, we are able to calculate with reasonable accuracy the branching ratios of $\psi \to B_8 + \bar{B}_8$ by performing a least-squares fit of the Okubo-model coupling parameters to appropriate existing data points. The expected errors in the branching ratios are also calculated and correspond reasonably well to the experimental errors. In addition, we find that the SU(3)-nonsinglet piece of ψ makes a major contribution to the branching ratios. By using other models for the baryon form factors, such as the dipole parametrization, those exponentially falling in q^2 and $(q^2)^{1/2}$, and those violating the Drell-Yan relation by falling off like $1/q^2$, we find that the Okubo model is reasonably independent of the particular baryon-form-factor model provided the form-factor model itself satisfies reasonable constraints, such as having the appropriate falloff in q^2. Further, we find that our Veneziano-type form-factor model giving best agreement with the ψ decay data also predicts $\sigma_{\text{total}}(e^+e^- \to p\bar{p})$ to within nearly one standard deviation of the Frascati measurement at $q^2 = 4.41$ GeV^2.

In the next section we briefly review Okubo's model for ψ and ψ' decays into ordinary hadrons. Also, we derive relations which express all the form factors for the octet baryons in terms of the electromagnetic form factors of the proton and neutron. In Sec. III we discuss our Veneziano-type dual current model—or generalized vector-dominance model (GVDM)—for the baryon form factors and compare this model with some others, especially at the Frascati data point. In Sec. IV we determine the Okubo coupling parameters by performing a least-squares fit to appropriate data points, and we calculate all the branching ratios of $\psi \to B_8 + \bar{B}_8$. Comparisons with other form-factor models are made. Finally, in Sec. V we make concluding remarks.

II. THE OKUBO MODEL

A. Review

Ever since Sakurai's original statement of universality,[17] many authors have exploited the association of vector mesons with conserved currents, thus for example, proposing vector-meson dominance of electromagnetic form factors[18] and deriving sum rules.[19] Okubo's model can be thought of in the same vein. Since ψ and probably ψ' are more than 99% $c\bar{c}$ bound states,[20] the direct couplings of their uncharmed pieces to final-state ordinary hadrons, as dictated by the Okubo-Zweig-Iizuka (OZI) rule,[21] are greatly suppressed relative to ψ and ψ''s couplings first to their associated currents and these currents' transitions to the final state ordinary hadrons.

If ψ were a pure SU(3) singlet then we would expect it to couple only to an SU(3)-singlet current and the electromagnetic current; hence, from U-spin invariance we would have the following ratio of decay rates:

$$\frac{\Gamma(\psi \to K^- K^{+*})}{\Gamma(\psi \to \pi^- \rho^+)} = 0.85, \tag{2.1}$$

where the deviation from unity is due to phase-space considerations. But the experimental ratio is 0.41 ± 0.10,[6] so that Eq. (2.1) is very badly violated. This means that even though the uncharmed piece of ψ is less than 1%, it contributes significantly to ψ decays by coupling rather strongly to an SU(3)-nonsinglet current.

To account for the above discrepancies Okubo writes the following Hamiltonians for ψ and ψ':

$$\begin{aligned} \mathcal{H}_\psi(x) &= [g J_\mu(x) + f j_\mu^{\text{em}}(x)]\, \psi^\mu(x), \\ \mathcal{H}_{\psi'}(x) &= [g' J_\mu(x) + f' j_\mu^{\text{em}}(x)]\, \psi'^\mu(x) \end{aligned} \tag{2.2}$$

(see Refs. 9 and 10), where J_μ is a neutral hadronic current of negative charge conjugation parity and zero hypercharge and j_μ^{em} is the electromagnetic current. With the above Hamiltonians Okubo is able to predict an approximate universal decay ratio of ψ and ψ' into any ordinary hadronic channel n:

$$\begin{aligned} R(n) &= \frac{\Gamma(\psi' \to n)}{\Gamma(\psi \to n)} \times \frac{\Gamma(\psi \to \text{all})}{\Gamma(\psi' \to \text{all})} \\ &\simeq (0.12^{+0.13}_{-0.06}). \end{aligned} \tag{2.3}$$

He is also able to explain the puzzling ratio

$$\frac{\Gamma(\psi \to \phi\pi^+\pi^-)}{\Gamma(\psi \to \omega\pi^+\pi^-)} = 0.20 \pm 0.10 \tag{2.4}$$

(see Ref. 8), as well as discuss a wide class of ordinary hadronic decay modes of ψ and ψ'. So far, we have just given heuristic arguments as to why Eq. (2.2) seems to be a good model. But a more thorough field-theoretic justification is contained in Ref. 22.

Okubo discusses Eq. (2.2) under different assumptions about the SU(3) content of J_μ. For our purposes we find that from the usual nonet of SU(3) vector currents $j_\mu^{(\alpha)}(x)$ ($\alpha = 0, 1, \ldots, 8$), we must include both the singlet and eighth components so that

$$J_\mu(x) = \beta_0 j_\mu^{(0)}(x) + \beta_8 j_\mu^{(8)}(x), \tag{2.5}$$

and also we have the usual

$$j_\mu^{(\text{em})}(x) = j_\mu^{(3)}(x) + \frac{1}{\sqrt{3}} j_\mu^{(8)}(x), \tag{2.6}$$

so that finally we can write Eq. (2.2) as follows:

$$\begin{aligned} \mathcal{H}_\psi(x) &= [g_0 j_\mu^{(0)}(x) + g_8 j_\mu^{(8)}(x) + f j_\mu^{(3)}(x)]\, \psi^\mu(x), \\ \mathcal{H}_{\psi'}(x) &= [g_0' j_\mu^{(0)}(x) + g_8' j_\mu^{(8)}(x) + f' j_\mu^{(3)}(x)]\, \psi'^\mu(x), \end{aligned} \tag{2.7}$$

with

$$\begin{aligned} g_0 &= \beta_0 g, \\ g_8 &= \beta_8 g + \frac{1}{\sqrt{3}} f, \\ g_0' &= \beta_0 g', \\ g_8' &= \beta_8 g' + \frac{1}{\sqrt{3}} f'. \end{aligned} \tag{2.8}$$

For the decay rates of interest we discuss the results for ψ since those for ψ' should be exactly analogous. The simplest decay rate formula which we shall need in Sec. IV is that for the leptonic decay of ψ into a lepton pair:

$$\Gamma(\psi \to l\bar{l}) = \frac{f^2}{12\pi} M, \tag{2.9}$$

where l is either an electron or muon, M is the mass of the ψ, and only the electromagnetic current contributes. We shall also need the following two ratios of decay rates which are easily derivable from Eq. (2.7) (see Ref. 10):

$$\frac{\Gamma(\psi\to K^-K^{+*})}{\Gamma(\psi\to\pi^-\rho^+)}=\left(\frac{2\sqrt{2}\,g_0-g_8+\sqrt{3}f}{2\sqrt{2}\,g_0+2g_8}\right)^2\times 0.85, \quad (2.10)$$

$$\frac{\Gamma(\psi\to\overline{K}^0K^{0*})}{\Gamma(\psi\to\pi^-\rho^+)}=\left(\frac{2\sqrt{2}\,g_0-g_8-\sqrt{3}f}{2\sqrt{2}\,g_0+2g_8}\right)^2\times 0.85. \quad (2.11)$$

B. Baryon form factors

We turn our attention now to deriving relations which express all the octet-baryon form factors for ψ decays in terms of the usual Sachs electric and magnetic form factors $G_E^N(t)$ and $G_M^N(t)$, where N is the proton or neutron. If we set

$$j_\mu(x)=g_0 j_\mu^{(0)}(x)+g_8 j_\mu^{(8)}(x)+f j_\mu^{(3)}(x) \quad (2.12)$$

in Eq. (2.7), then the matrix element for $\psi\to B\overline{B}$ is proportional to

$$M_\mu=\langle B(p)\overline{B}(-p')|j_\mu(0)|0\rangle, \quad (2.13)$$

where p and $-p'$ are the momenta. In Ref. 23 Okubo writes, in the usual notation,

$$M_\mu^{B\overline{B}}=\left(\frac{m_B^{\,2}}{p_0|p_0'|V^2}\right)^{1/2}\overline{u}(p)\left\{i\gamma_\mu F_M^B(t)+\frac{2m_B}{t-4m_B^{\,2}}\left[F_M^B(t)-F_E^B(t)\right](p_\mu+p'_\mu)\right\}v(p'), \quad (2.14)$$

where m_B is the mass of B and $\overline{B}$ and $t=q^2$ is the square of the center-of-mass energy. We shall be interested in evaluating t at the square of the ψ mass. Also, for the process $\psi\to\Lambda^0\overline{\Sigma}^0+\Sigma^0\overline{\Lambda}^0$ we take m_B to be the average of the baryon masses. Based upon Eq. (2.14) the decay rate is given by

$$\Gamma(\psi\to B\overline{B})=\frac{M}{12\pi}\left[1-\left(\frac{2m_B}{M}\right)^2\right]^{1/2}\left(|F_M^B|^2+\frac{2m_B^{\,2}}{M^2}|F_E^B|^2\right), \quad (2.15)$$

where again M is the ψ mass and F_M^B and F_E^B are defined with respect to the total current $j_\mu=g_0 j_\mu^{(0)}+g_8 j_\mu^{(8)}+f j_\mu^{(3)}$ and are to be evaluated at $t=M^2$.

To derive the SU(3) relations for the form factors we can use the quark model as our guide and rewrite the current j_μ in SU(3) tensor notation (suppressing Lorentz indices) in terms of

$$j_a^b(x)=i\,\overline{q}_a(x)\gamma_\mu q_b(x), \quad (2.16)$$

where $q_a(x)$ and $q_b(x)$ $(a,b=1,2,3)$ are SU(3) quark fields. We can write

$$\langle B\overline{B}|j_a^b(0)|0\rangle=\kappa_1\overline{B}_\beta^b B_a^\beta+\kappa_2\overline{B}_a^\beta B_\beta^b+\kappa_3\delta_a^b\overline{B}_\alpha^\beta B_\beta^\alpha, \quad (2.17)$$

where B_α^β and $\overline{B}_\alpha^\beta$ are the traceless baryon- and antibaryon-octet matrices, respectively, κ_i are independent reduced matrix elements, and repeated Greek indices are to be summed over the values 1, 2, 3. We are interested in expanding Eq. (2.17) for the three cases $a=b=1,2,3$, where 1, 2, 3 correspond to the three quark flavors $\mathcal{P}$, $\mathcal{N}$, λ, respectively. Application of the OZI rule[21,24] demands that

$$\begin{aligned}&\langle p\overline{p}|j_\lambda^\lambda(0)|0\rangle=\langle n\overline{n}|j_\lambda^\lambda(0)|0\rangle=0,\\&\langle\Sigma^+\overline{\Sigma}^+|j_{\mathcal{N}}^{\mathcal{N}}(0)|0\rangle=\langle\Xi^0\overline{\Xi}^0|j_{\mathcal{N}}^{\mathcal{N}}(0)|0\rangle=0,\\&\langle\Sigma^-\overline{\Sigma}^-|j_{\mathcal{P}}^{\mathcal{P}}(0)|0\rangle=\langle\Xi^-\overline{\Xi}^-|j_{\mathcal{P}}^{\mathcal{P}}(0)|0\rangle=0,\end{aligned} \quad (2.18)$$

and hence

$$\kappa_3=-\kappa_2, \quad (2.19)$$

so that we are left with only two independent reduced matrix elements, κ_1 and κ_2. If we set $g_0=0$ and $g_8=(1/\sqrt{3})f$, then we find that we can write the nucleon electromagnetic-current matrix elements in terms of κ_1 and κ_2:

$$\begin{aligned}&\langle p\overline{p}|j^{\rm em}(0)|0\rangle=\tfrac{4}{3}\kappa_1-\tfrac{2}{3}\kappa_2,\\&\langle n\overline{n}|j^{\rm em}(0)|0\rangle=-\tfrac{2}{3}(\kappa_1+\kappa_2).\end{aligned} \quad (2.20)$$

Hence for the full current j_μ contained in Eq. (2.12) it is straightforward to express all form factors F_M^B and F_E^B defined by Eq. (2.14) in terms of G_M^N and G_E^N for the nucleons:

$$F_M^p=\frac{\sqrt{3}}{2}(\sqrt{2}\,g_0+g_8)(G_M^p+G_M^n)+\frac{f}{2}(G_M^p-G_M^n), \quad (2.21a)$$

$$F_M^n=\frac{\sqrt{3}}{2}(\sqrt{2}\,g_0+g_8)(G_M^p+G_M^n)-\frac{f}{2}(G_M^p-G_M^n), \quad (2.21b)$$

$$F_M^{\Lambda^0}=(\tfrac{3}{2})^{1/2}g_0(G_M^p+G_M^n)+\frac{\sqrt{3}}{2}g_8G_M^n, \quad (2.21c)$$

$$F_M^{\Sigma^0}=(\tfrac{3}{2})^{1/2}g_0(G_M^p+G_M^n)-\frac{\sqrt{3}}{2}g_8G_M^n, \quad (2.21d)$$

$$F_M^{\Lambda^0\overline{\Sigma}^0}=-\frac{\sqrt{3}}{2}fG_M^n, \quad (2.21e)$$

$$F_M^{\Sigma^+}=(\tfrac{3}{2})^{1/2}g_0(G_M^p+G_M^n)-\frac{\sqrt{3}}{2}g_8G_M^n+f(G_M^p+\tfrac{1}{2}G_M^n), \quad (2.21f)$$

$$F_M^{\Sigma^-}=(\tfrac{3}{2})^{1/2}g_0(G_M^p+G_M^n)-\frac{\sqrt{3}}{2}g_8G_M^n-f(G_M^p+\tfrac{1}{2}G_M^n), \quad (2.21g)$$

$$F_M^{\Xi^0}=(\tfrac{3}{2})^{1/2}g_0(G_M^p+G_M^n)-\frac{\sqrt{3}}{2}g_8G_M^p+f(G_M^n+\tfrac{1}{2}G_M^p), \quad (2.21h)$$

$$F_M^{\Xi^-} = (\tfrac{3}{2})^{1/2} g_0 (G_M^p + G_M^n) - \frac{\sqrt{3}}{2} g_8 G_M^p - f(G_M^n + \tfrac{1}{2} G_M^p), \tag{2.21i}$$

where again all the form factors are to be evaluated at $t = M^2$. The same relations are true for F_E^B; simply replace M by E above. Now that we know how to express all the baryon form factors for the ψ decay current in terms of the nucleon electromagnetic form factors, we turn our attention in the next section to specific models for the electromagnetic form factors.

III. FORM-FACTOR MODELS

We start this section by describing the model which we found to give the best fit to the ψ decay data. It is the Ademollo–Del Giudice Veneziano-type dual current form-factor model[13] in the zero-width approximation for the meson resonances. This zero-width approximation should be good for our purposes since we are interested in large t relative to the mass squared of the allowed resonant contributions. Also, in this approximation the form factors will be real as opposed to complex, as they should be in general for timelike t.

Since the ψ decay form factors can be expressed in terms of the nucleon electromagnetic form factors, we first focus our attention on $F_1^N(t)$ and $F_2^N(t)$, the Dirac and Pauli electromagnetic form factors, respectively. These are expressible in the form

$$\begin{aligned} F_1^p(t) &= \tfrac{1}{2}[F_1^S(t) + F_1^V(t)], \\ 1.793 F_2^p(t) &= \tfrac{1}{2}[-0.12 F_2^S(t) + 3.706 F_2^V(t)], \\ F_1^n(t) &= \tfrac{1}{2}[F_1^S(t) - F_1^V(t)], \\ -1.913 F_2^n(t) &= \tfrac{1}{2}[-0.12 F_2^S(t) - 3.706 F_2^V(t)], \end{aligned} \tag{3.1}$$

where F_i^S and F_i^V are isotopic scalar and vector form factors, respectively, normalized to the value $+1$ at $t = 0$. We take each of the isotopic form factors to have the form

$$F(t) = B[1 - \alpha(t), \gamma - 1], \tag{3.2}$$

where $B(x, y)$ is the Euler beta function

$$B(x, y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}. \tag{3.3}$$

γ controls the number of pole contributions since if γ is an integer, $F(t)$ has only the poles $\alpha(t) = 1, 2, \ldots, \gamma - 1$, corresponding to contributions from two degenerate Regge trajectories which we shall consider the same trajectory. Also, we assume linearly rising trajectories:

$$\alpha_i(t) = \alpha_i' t + \alpha_i(0), \tag{3.4}$$

where $i = \rho, \omega, \phi, \psi$, and we take $\alpha' \equiv \alpha_i' = 0.9\ \text{GeV}^{-2}$ for all i. Equation (3.4) allows us to write the following relationship between masses of neighboring poles on the same trajectory:

$$m_{j+1}^2 = m_j^2 + \alpha'^{-1}. \tag{3.5}$$

Again we assume validity of the OZI rule and allow meson dominance of the electromagnetic form factors only by ρ and ω since the direct couplings of ψ and ϕ to the nucleons would be highly suppressed. We further impose the following Drell-Yan constraints on the isotopic form factors at increasingly large $t = q^2$:[14]

$$\begin{aligned} &F_1^{S(V)}(q^2) \propto 1/q^4, \\ &F_2^{S(V)}(q^2)/F_1^{S(V)}(q^2) \text{ decreasing.} \end{aligned} \tag{3.6}$$

As usual the ω trajectory couples to F_1^S and F_2^S and the ρ to F_1^V and F_2^V. Also, the simplest way to satisfy the Drell-Yan constraints is to take $\gamma = 3$ in Eq. (3.2) for F_1^S and F_1^V and $\gamma = 4$ for F_2^S and F_2^V. Hence, using Eq. (3.5) we arrive at the following isotopic form factors:

$$F_1^S(q^2) = C_1^\omega \frac{(\alpha')^{-2}}{(m_\omega^2 - q^2)(m_{\omega'}^2 - q^2)}, \tag{3.7a}$$

$$F_1^V(q^2) = C_1^\rho \frac{(\alpha')^{-2}}{(m_\rho^2 - q^2)(m_{\rho'}^2 - q^2)}, \tag{3.7b}$$

$$F_2^S(q^2) = C_2^\omega \frac{(\alpha')^{-3}}{(m_\omega^2 - q^2)(m_{\omega'}^2 - q^2)(m_{\omega''}^2 - q^2)}, \tag{3.7c}$$

$$F_2^V(q^2) = C_2^\rho \frac{(\alpha')^{-3}}{(m_\rho^2 - q^2)(m_{\rho'}^2 - q^2)(m_{\rho''}^2 - q^2)}, \tag{3.7d}$$

where we take $m_\omega^2 = 0.613$ and $m_\rho^2 = 0.585$ and denote Veneziano recurrences by primes. The constants C_j^i contain information about both the coupling of the mesons to the electromagnetic current and their couplings to the baryons; however, they are determined by the form-factor normalizations at $q^2 = 0$ and thus we are finally led to the following nucleon form factors (hereafter called model I):

$$F_1^p(q^2) = \frac{1}{2}\left[\frac{m_\omega^2 m_{\omega'}^2}{(m_\omega^2 - q^2)(m_{\omega'}^2 - q^2)} + \frac{m_\rho^2 m_{\rho'}^2}{(m_\rho^2 - q^2)(m_{\rho'}^2 - q^2)}\right], \tag{3.8a}$$

$$F_2^p(q^2) = \frac{1}{2}\left[-\frac{0.12}{1.793}\frac{m_\omega^2 m_{\omega'}^2 m_{\omega''}^2}{(m_\omega^2 - q^2)(m_{\omega'}^2 - q^2)(m_{\omega''}^2 - q^2)} + \frac{3.706}{1.793}\frac{m_\rho^2 m_{\rho'}^2 m_{\rho''}^2}{(m_\rho^2 - q^2)(m_{\rho'}^2 - q^2)(m_{\rho''}^2 - q^2)}\right], \tag{3.8b}$$

$$F_1^n(q^2) = \frac{1}{2}\left[\frac{m_\omega^2 m_{\omega'}^2}{(m_\omega^2-q^2)(m_{\omega'}^2-q^2)} - \frac{m_\rho^2 m_{\rho'}^2}{(m_\rho^2-q^2)(m_{\rho'}^2-q^2)}\right], \quad (3.8c)$$

$$F_2^n(q^2) = \frac{1}{2}\left[\frac{0.12}{1.913}\frac{m_\omega^2 m_{\omega'}^2 m_{\omega''}^2}{(m_\omega^2-q^2)(m_{\omega'}^2-q^2)(m_{\omega''}^2-q^2)} + \frac{3.706}{1.913}\frac{m_\rho^2 m_{\rho'}^2 m_{\rho''}^2}{(m_\rho^2-q^2)(m_{\rho'}^2-q^2)(m_{\rho''}^2-q^2)}\right]. \quad (3.8d)$$

The Sachs form factors needed for Eqs. (2.21) are simply linear combinations of the Pauli and Dirac form factors given in Eq. (3.8):

$$G_E^N(q^2) = F_1^N(q^2) + \frac{q^2\kappa_N}{4m_N^2}F_2^N(q^2),$$
$$G_M^N(q^2) = F_1^N(q^2) + \kappa_N F_2^N(q^2), \quad (3.9)$$

where $\kappa_p = 1.793$, $\kappa_n = -1.913$, and m_N is the nucleon mass. At the ψ mass squared we get

$$G_E^p = -1.37\times10^{-2},$$
$$G_M^p = 4.13\times10^{-3},$$
$$G_E^n = 3.07\times10^{-2},$$
$$G_M^n = 1.16\times10^{-2}. \quad (3.10)$$

In the next section we shall use Eqs. (2.21) and (3.10) to calculate ψ decays into baryon pairs. But we shall also be interested in how independent the Okubo dynamics is from the particular model chosen for the baryon form factors. To check this independence we shall calculate decay rates for four other models for the nucleon electromagnetic form factors. For model II we use the same Ademollo-Del Giudice model discussed above but this time with one and two pole contributions to F_1^N F_2^N, respectively. This model badly violates the first Drell-Yan constraint so that we do not expect too good an agreement with the data.

For model III we use the well-known dipole parametrization

$$G_E^p(q^2) = \frac{G_M^p(q^2)}{\mu_p} = \frac{G_M^n(q^2)}{\mu_n} = \frac{1}{[1-(q^2/0.71)]^2},$$
$$G_E^n(q^2) = 0, \quad (3.11)$$

where $\mu_p = 2.793$ and $\mu_n = -1.913$ and are the total magnetic moments of the proton and neutron, respectively. Even though these form factors are consistent with the Drell-Yan constraints they do not satisfy the kinematic constraint

$$G_M^N(4m_N^2) = G_E^N(4m_N^2), \quad (3.12)$$

which is demanded by Eq. (3.9); however, since they have the right power falloff with q^2, we would suspect that agreement with the ψ decay data would not be too bad.

For models IV and V, we consider exponential falloffs with q^2 of the form

$$a_i \exp[-(q^2)^{1/2}/b_i]$$

and

$$a_i' \exp(-q^2/b_i').$$

But since there are four independent nucleon form factors, we introduce too many free parameters into the theory for the number of existing data points. However, since the dipole parametrization (model III) gives a reasonable fit to the ψ decay data (as we shall see in the next section), we take it as our guide and define model IV by

$$G_E^p(q^2) = \frac{G_M^p(q^2)}{\mu_p} = \frac{G_M^n(q^2)}{\mu_n} = \exp[-(q^2)^{1/2}/b_1], \quad (3.13)$$

and model V by

$$G_E^p(q^2) = \frac{G_M^p(q^2)}{\mu_p} = \frac{G_M^n(q^2)}{\mu_n} = \exp(-q^2/b_2). \quad (3.14)$$

Before applying models I-V to ψ decays, we first check their consistencies with the Frascati measurement of $\sigma_{\text{total}}(e^+e^-\to p\bar{p})$ at $q^2 = 4.41$ and the upper bounds for $\sigma_{\text{total}}(p\bar{p}\to e^+e^-)$ contained in Ref. 15. These are shown in Fig. 1. We find that models I and V are in reasonable agreement with Frascati, both lying nearly within one standard deviation of the data point. We should not, however, be too strict about making agreement with the Frascati measurement a constraint on our form-factor models since the Frascati data point needs further verification.

IV. DECAY RATES INTO BARYON PAIRS

Now that we have shown that all the form factors for ψ decays into baryon pairs can be expressed in terms of the nucleon electromagnetic form factors and since we have further proposed several models to represent these electromagnetic form factors, we now proceed to calculate the branching ratios of ψ into baryon pairs. But first we must determine the parameters g_0, g_8, and f of the Okubo Hamiltonian by making a least-squares fit to appropriate data points. Because model I is our primary form-factor model and the one in which we have the most faith since it satisfies the Drell-Yan constraints, we shall first focus our attention on it.

To determine the Okubo parameters we perform the χ^2 test,[25] setting

$$\chi^2 = \sum_{i=1}^{5}\frac{[D_i - \bar{\Gamma}_i(g_0,g_8,f)]^2}{(\delta D_i)^2}, \quad (4.1)$$

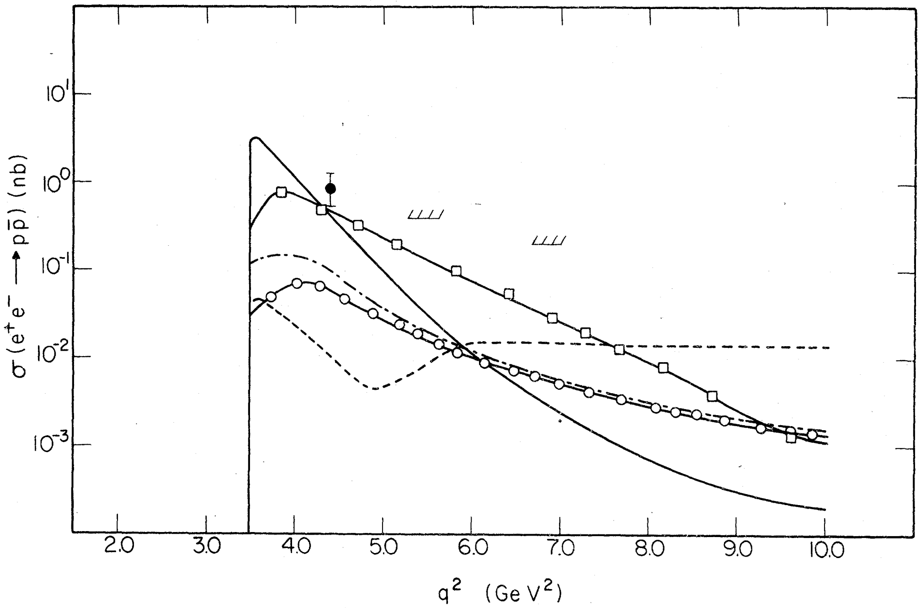

FIG. 1. $\sigma_{\text{total}}(e^+e^- \to p\bar{p})$. Model I: ——, model II: ---, model III: -•-•-, model IV: -∘-∘-, model V: —□—□—□—. We cannot obtain the electromagnetic contribution to the total cross section at the ψ resonance by simply reading this graph because of appreciable interference effects between the strong and electromagnetic currents. This graph is valid only for q^2 outside the ψ and ψ' resonances.

where for each i, D_i, δD_i, and $\tilde{\Gamma}_i$ correspond to the experimentally measured value, experimental error, and our theoretical fitting function for the following:

$$i=1:\ \frac{\Gamma(\psi \to K^- K^{+*})}{\Gamma(\psi \to \pi^- \rho^+)}\ , \tag{4.2a}$$

$$i=2:\ \frac{\Gamma(\psi \to \bar{K}^0 K^{0*})}{\Gamma(\psi \to \pi^- \rho^+)}\ , \tag{4.2b}$$

$$i=3:\ \Gamma(\psi \to p\bar{p})\ , \tag{4.2c}$$

$$i=4:\ \Gamma(\psi \to \Lambda^0 \bar{\Lambda}^0)\ , \tag{4.2d}$$

$$i=5:\ \Gamma(\psi \to l\bar{l})\ . \tag{4.2e}$$

The values for D_i and δD_i, obtained from Ref. 6, are a simultaneous fit to several measured values. $\tilde{\Gamma}_1$ is given by Eq. (2.10), $\tilde{\Gamma}_2$ by Eq. (2.11), $\tilde{\Gamma}_3$ and $\tilde{\Gamma}_4$ by Eq. (2.15), and $\tilde{\Gamma}_5$ by Eq. (2.9). Further, we see that Eq. (2.9) essentially determines f so that in actuality we have to fit four decay rates with two parameters.

In performing the χ^2 test it is useful to define the reduced χ^2:

$$\chi^2_r = \frac{\chi^2}{N_D - N_P}\ , \tag{4.3}$$

where N_D is the number of data points (which is 5 in our case), N_P is the number of parameters (which is 3 in our case), and $N_D - N_P$ is called the degrees of freedom.

The best fit to the data occurs for those values of the parameters such that χ^2 is a minimum and the theoretical fitting functions give good agreement with the data if $\chi^2 \sim N_D - N_P$, or $\chi^2_r \sim 1$.[26] Further, if we are at the minimum of χ^2, hold all the parameters but one constant, and vary the one until χ^2 changes by 1, then the change in the one

TABLE I. Branching ratios of ψ decay modes from model I and the data. The last two results are the indicated ratios of decay rates. Four events of $\psi \to \Xi^-\bar{\Xi}^-$ have been observed [see G. Goldhaber, SLAC-LBL internal report (unpublished)].

Decay mode	Model I	Data (Ref. 6)
$p\bar{p}$	$(2.1\pm 0.3)\times 10^{-3}$	$(2.2\pm 0.2)\times 10^{-3}$
$n\bar{n}$	$(2.5\pm 0.3)\times 10^{-3}$	
$\Lambda^0\bar{\Sigma}^0 + \Sigma^0\bar{\Lambda}^0$	$(2.9\pm 0.3)\times 10^{-5}$	$<4.0\times 10^{-4}$
$\Lambda^0\bar{\Lambda}^0$	$(2.2\pm 0.3)\times 10^{-3}$	$(1.6\pm 0.8)\times 10^{-3}$
$\Sigma^+\bar{\Sigma}^+$	$(7.2\pm 1.7)\times 10^{-4}$	
$\Sigma^-\bar{\Sigma}^-$	$(5.1\pm 1.4)\times 10^{-4}$	
$\Sigma^0\bar{\Sigma}^0$	$(6.1\pm 1.5)\times 10^{-4}$	
$\Xi^-\bar{\Xi}^-$	$(8.6\pm 1.6)\times 10^{-4}$	$\sim 4\times 10^{-4}$ (?)
$\Xi^0\bar{\Xi}^0$	$(1.4\pm 0.2)\times 10^{-3}$	
$l\bar{l}$	$(7.3\pm 0.9)\times 10^{-2}$	$(7.3\pm 0.5)\times 10^{-2}$
$\frac{\Gamma(\psi\to K^-K^{+*})}{\Gamma(\psi\to\pi^-\rho^+)}$	0.44 ± 0.10	0.41 ± 0.10
$\frac{\Gamma(\psi\to\bar{K}^0K^{0*})}{\Gamma(\psi\to\pi^-\rho^+)}$	0.29 ± 0.08	0.32 ± 0.09

TABLE II. Okubo parameters and χ_r^2 for form-factor models I–III.

Model	g_0	g_8	f	g_8/f	χ_r^2
I	$(5.64\pm 0.44)\times 10^{-2}$	$(2.40\pm 0.41)\times 10^{-2}$	$(7.82\pm 0.46)\times 10^{-3}$	3.07 ± 0.56	0.37
II	$(1.10\pm 0.11)\times 10^{-2}$	$(8.02\pm 1.18)\times 10^{-3}$	$(7.60\pm 0.49)\times 10^{-3}$	1.06 ± 0.17	5.89
III	$(1.44\pm 0.13)\times 10^{-1}$	$(5.79\pm 1.09)\times 10^{-2}$	$(7.83\pm 0.48)\times 10^{-3}$	7.40 ± 1.46	1.14

parameter is the theoretical prediction for its error. Since for our form-factor models IV and V we have additional parameters b_i so that $N_P = 4$, for a comparison of the models it is more convenient for us to use the condition $\chi^2_r \sim 1$ as determining a good theoretical fit to the data.

Our computations for model I yield the following best fit for the Okubo parameters and χ^2_r:

$$g_0 = (5.64 \pm 0.44)\times 10^{-2} , \quad (4.4a)$$

$$g_8 = (2.40 \pm 0.41)\times 10^{-2} , \quad (4.4b)$$

$$f = (7.82 \pm 0.46)\times 10^{-3} , \quad (4.4c)$$

$$\chi^2_r = 0.37 . \quad (4.4d)$$

First of all, χ^2_r tells us that we have a very good fit to the data. Second, the hadronic and electromagnetic coupling parameters are of comparable strengths so that interference effects should be appreciable. And third, $g_8/f \gg 1/\sqrt{3}$ so that the nonsinglet piece of ψ makes a significant contribution to the decay rates. Table I shows the results of our calculations and the experimental data for the branching ratios of interest. We find that we get very good agreement with both the measured branching ratios and the experimental errors. We also give predictions for the branching ratios of those baryon paris for which no data currently exist.

In Table II we compare the best fit for the Okubo parameters and χ^2_r for form-factor models I–III. In Table III we present several branching ratios calculated from models I–III along with the corresponding data. The value of χ^2_r for model III (dipole parametrization) indicates a reasonably good fit to the data. But this is what we suspected since model III is consistent with the Drell-Yan constraints. On the other hand, model II yields much too low a value for the ratio of the $\bar{K}^0 K^{0*}$ and $\pi^- \rho^+$ channels. But the three coupling parameters fit the other four channels surprisingly well for a model which badly violates the first Drell-Yan constraint. We further note that for all the models the hadronic and electromagnetic coupling parameters are of comparable strengths so that again interference effects are predicted to be appreciable. Also, for all the models $g_8/f > 1/\sqrt{3}$ so that the nonsinglet piece of ψ contributes appreciably.

Models IV and V yield the same results for the Okubo parameters, with $b_1 = 0.60 \pm 0.26$ and $b_2 = 1.9 \pm 0.81$. Also, they give essentially the same least-squares fit to the branching ratios shown in Table III as does model III but with $\chi^2_r = 2.28$. However, for both models IV and V the errors involved are large.

V. CONCLUSION AND DISCUSSION

We have investigated ψ decays into octet-baryon pairs within the context of the Okubo model, using SU(3) symmetry to derive relations expressing all the form factors for the ψ decay current in terms of the nucleon Sachs electromagnetic form factors and further using a Veneziano-type dual current model for the nucleon electromagnetic form factors satisfying the Drell-Yan constraints and consistent with the OZI rule as to which vector mesons

TABLE III. Branching ratios of ψ decay modes from models I–III and the data. The last two results are the indicated ratios of decay rates.

Decay mode	Model I	Model II	Model III	Data (Ref. 6)
$p\bar{p}$	$(2.1\pm 0.3)\times 10^{-3}$	$(2.3\pm 0.4)\times 10^{-3}$	$(2.2\pm 0.3)\times 10^{-3}$	$(2.2\pm 0.2)\times 10^{-3}$
$\Lambda^0\bar{\Lambda}^0$	$(2.2\pm 0.3)\times 10^{-3}$	$(1.6\pm 0.3)\times 10^{-3}$	$(4.0\pm 1.4)\times 10^{-4}$	$(1.6\pm 0.8)\times 10^{-3}$
$\Lambda^0\bar{\Sigma}^0+\Sigma^0\bar{\Lambda}^0$	$(2.9\pm 0.3)\times 10^{-5}$	$(1.6\pm 0.2)\times 10^{-4}$	$(1.1\pm 0.1)\times 10^{-5}$	$<4.0\times 10^{-4}$
$l\bar{l}$	$(7.3\pm 0.9)\times 10^{-2}$	$(6.9\pm 0.9)\times 10^{-2}$	$(7.3\pm 0.9)\times 10^{-2}$	$(7.3\pm 0.5)\times 10^{-2}$
$\dfrac{\Gamma(\psi\to K^-K^{+*})}{\Gamma(\psi\to\pi^-\rho^+)}$	0.44 ± 0.10	0.50 ± 0.13	0.41 ± 0.11	0.41 ± 0.10
$\dfrac{\Gamma(\psi\to\bar{K}^0K^{0*})}{\Gamma(\psi\to\pi^-\rho^+)}$	0.29 ± 0.08	0.038 ± 0.028	0.35 ± 0.10	0.32 ± 0.09

can dominate the form factors. We obtained reasonable agreement with the Frascati measurement and other upper bounds of the total cross section for $e^+e^- \to p\bar{p}$. We further obtained very good agreement with both the measured branching ratios and experimental errors for existing ψ decay data into baryon pairs. Predictions were made for those branching ratios for which no data currently exist. Also, we tested the Okubo dynamics with several other models for the baryon form factors and conclude that the Okubo model is reasonably independent of the particular form-factor model used, provided of course the model satisfies a set of reasonable constraints, such as the correct falloff in q^2.

We believe that these ideas can be easily extended to the study of $\psi' \to B_8\bar{B}_8$ when more data for these decays become available.

Note added. Körner and Kuroda[11] have proposed a model for the nucleon form factors similar to our model I; however, our usages are quite different. For example, for $\sigma_{\text{total}}(e^+e^- \to p\bar{p})$ at the ψ resonance they use their model to calculate the one-photon contribution from $e^+e^- \to \psi \to \gamma \to p\bar{p}$ neglecting interference effects with the direct process $e^+e^- \to \psi \to p\bar{p}$, whereas at the ψ resonance we use our model I to calculate the baryon form factors (and hence ψ decay rates) corresponding to a sum of both strong and electromagnetic currents with appreciable interference effects.

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[26] Here and hereafter we shall always mean the minimum value of χ^2 and χ_r^2 unless otherwise indicated.