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Cabibbo-suppressed nonleptonic decays of charmed mesons*

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Tests of the Glashow-Iliopoulos-Maiani Hamiltonian based on SU(3) symmetry are presented that do not depend on any enhancement assumptions. The significance of possible failures of these predictions are discussed with emphasis on alternative models involving right-handed currents.

I. INTRODUCTION

The charm-changing nonleptonic Hamiltonian in the theory of Glashow, Iliopoulos, and Maiani (GIM)[1] has the form

$$\begin{aligned} H_1 \sim\ & \cos^2\theta_C(\bar{c}s)_L(\bar{d}u)_L \\ &+ \sin\theta_C\cos\theta_C[(\bar{c}s)_L(\bar{s}u)_L - (\bar{c}d)_L(\bar{d}u)_L] \\ &+ \sin^2\theta_C[(\bar{c}d)_L(\bar{s}u)_L] + \text{H.c.} \end{aligned} \tag{1}$$

A major prediction of this theory, that the non-strange charmed particles D^0 and D^+ decay predominantly into strange particles, appears to have been verified in the observed D^0 and D^+ decays[2] at SPEAR. Another prediction of this theory is the existence of Cabibbo-suppressed decays arising from the second term in Eq. (1) in which charm does not convert into strangeness. The purpose of this note is to discuss the quantitative predictions that follow from Eq. (1), with particular emphasis on the Cabibbo-suppressed decays, and the significance of possible failures of these predictions.

The predictions, based on the assumption of perfect SU(3) symmetry but without any assumed enhancement mechanism, are summarized in Sec. II. Most of these have been presented before.[3,4] In Sec. III we consider an alternative to GIM involving the addition of right-handed charm-changing currents.[5,6] On the basis of SU(3) considerations alone, we discuss the consequences if these additional terms dominate the charm-changing decays or at least contribute significantly to them. The possible consequences of SU(3) violation are briefly discussed in Sec. IV.

II. PREDICTIONS BASED ON GIM

Many results follow from the U-spin transformation properties of Eq. (1), the first two terms of which can be written

$$H_1 \sim \cos^2\theta_C U_1^{-1} + \sqrt{2}\,\sin\theta_C\cos\theta_C U_1^0, \tag{2}$$

where U_1^m transforms as a U-spin vector with $U_3 = m$. The charmed state D^0 transforms as a U-spin scalar and (D^+, F^+) as a U-spin spinor, while the final pseudoscalar mesons consist of two U-spin spinors (π^+, K^+) and $(\pi^- K^-)$ plus a U-spin vector and scalar containing the neutral mesons.

For any final state with strangeness S containing n mesons, of which n_c are charged, it follows from Eq. (2) that[7]

$$\frac{\Gamma(D^0\to n, n_c, S=+1)}{\Gamma(D^0\to n, n_c, S=0)} = \frac{1}{2\tan^2\theta_C}. \tag{3}$$

For the Cabibbo-suppressed D^0 decays, for which the final state has $U=1$, $U_3=0$, it follows from U-spin reflection for a set of decays with given (n, n_c) that

$$\langle N(K^0)\rangle = \langle N(\bar{K}^0)\rangle, \tag{4a}$$

$$\langle N(K^+)\rangle = \langle N(\pi^+)\rangle = \langle N(K^-)\rangle = \langle N(\pi^-)\rangle, \tag{4b}$$

where $\langle N\rangle$ is the average number. For final states without π^0 or η there is also the amplitude relation

$$\begin{aligned} &A[D^0\to \mathfrak{F}(K^+, K^-, \pi^+, \pi^-, K^0, \bar{K}^0)] \\ &\qquad = -A[D^0\to \mathfrak{F}(\pi^+, \pi^-, K^+, K^-, \bar{K}^0, K^0)]. \end{aligned} \tag{5a}$$

For example, among the three-body decays,

$$\Gamma(D^0\to K^+\bar{K}^0\pi^-) = \Gamma(D^0\to \pi^+K^0K^-). \tag{5b}$$

Equations (3) and (5a) applied to the four-body decays give

$$\frac{\Gamma(D^0\to K^-\pi^-\pi^+\pi^+) + \Gamma(D^0\to K^-K^-K^+\pi^+)}{2\Gamma(D^0\to \pi^+\pi^+\pi^-\pi^-) + \Gamma(D^0\to K^+K^-\pi^+\pi^-)} = \frac{1}{2\tan^2\theta_C}. \tag{6}$$

Similarly it follows from U-spin reflection for the Cabibbo-suppressed decays that

$$\begin{aligned} &A[F^+\to \mathfrak{F}(K^+K^-\pi^+\pi^-K^0\bar{K}^0)] \\ &\qquad = -A[D^+\to \mathfrak{F}(\pi^+\pi^-K^+K^-\bar{K}^0K^0)]. \end{aligned} \tag{7}$$

For example, among the three-body decays,

$$\Gamma(F^+\to K^+\pi^+\pi^-) = \Gamma(D^+\to \pi^+K^+K^-), \tag{8a}$$

$$\Gamma(F^+\to K^0\bar{K}^0K^+) = \Gamma(D^+\to \bar{K}^0K^0\pi^+). \tag{8b}$$

We now turn to a more detailed consideration of the two-body decays. Using the full SU(3) symmetry the $\Delta C = -1$ piece of the Hamiltonian (1) tranforms as $\underline{\bar{6}} + \underline{15}$. Operating on the initial SU(3) triplet $\underline{\bar{3}}$ of charmed mesons

TABLE I. Reduced matrix elements M_n for Cabibbo-favored decays where n is the representation of SU(3) in H and M (O=octet, $S=\underline{27}$) is the final-state SU(3) representation. Results are the same for H_2 and H_1.

	O_6	$O_{15'}$	S_{15}
$A(D^0 \to \pi^+ K^-)$	-1	-1	1
$A(D^0 \to \pi^0 \overline{K}^0)$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$\frac{3}{2\sqrt{2}}$
$A(D^0 \to n \overline{K}^0)$	$-\frac{1}{\sqrt{6}}$	$-\frac{1}{\sqrt{6}}$	$-\frac{1}{2}(\frac{3}{2})^{1/2}$
$A(D^+ \to \pi^+ \overline{K}^0)$	0	0	$\frac{5}{2}$
$A(F^+ \to K^+ \overline{K}^0)$	1	-1	1
$A(F^+ \to \pi^+ n)$	$(\frac{2}{3})^{1/2}$	$-(\frac{2}{3})^{1/2}$	$-(\frac{3}{2})^{1/2}$

$$\underline{\overline{6}} \times \underline{\overline{3}} = \underline{\overline{10}} \oplus \underline{8},$$

$$\underline{15} \times \underline{\overline{3}} = \underline{27} \oplus \underline{10} \oplus \underline{8}.$$

For the final state of two pseudoscalar mesons in an S state there is one $\underline{8}$ and one $\underline{27}$ so that there exists one reduced matrix element O_6 for the $\underline{\overline{6}}$ and two, O_{15} and S_{15}, for the $\underline{15}$. The expansion of the decay amplitudes in terms of these reduced matrix elements is given in Tables I and II.[8] The following rate relations are obtained:

$$\Gamma(D^0 \to \pi^+ K^-) = \cot^2\theta_C \Gamma(D^0 \to \pi^+\pi^-)$$
$$= \cot^2\theta_C \Gamma(D^0 \to K^+K^-), \qquad (10)$$

$$\Gamma(D^0 \to K^0\overline{K}^0) = 0, \qquad (11)$$

$$\Gamma(D^0 \to \pi^0\overline{K}^0) = 3\Gamma(D^0 \to \eta K^0)$$
$$= \cot^2\theta_C \Gamma(D^0 \to \pi^0\pi^0)$$
$$= \cot^2\theta_C \Gamma(D^0 \to \eta\eta)$$
$$= \tfrac{3}{2} \cot^2\theta_C \Gamma(D^0 \to \pi^0\eta), \qquad (12)$$

$$\Gamma(F^+ \to K^0K^+) = \Gamma(D^+ \to \overline{K}^0K^+), \qquad (13)$$

$$\Gamma(F^+ \to \pi^+\eta) = \tfrac{2}{3} \cot^2\theta_C \Gamma(F^+ \to K^0\pi^+), \qquad (14a)$$

$$\Gamma(F^+ \to K^+\overline{K}^0) = 2\cot^2\theta_C \Gamma(F^+ \to K^+\pi^0), \qquad (14b)$$

$$\Gamma(D^+ \to \pi^+\overline{K}^0) = 2\cot^2\theta_C \Gamma(D^+ \to \pi^+\pi^0), \qquad (15)$$

$$\Gamma(F^+ \to \pi^+\pi^0) = 0. \qquad (16)$$

Equations (10)–(13) follow from U-spin invariance alone and Eq. (16) follows from the $\Delta I = 1$ character of the Cabibbo-favored decays.

III. AN ALTERNATIVE TO GIM

The main alternatives to GIM as far as charm-changing decays are concerned consist of models in which there exists a charm-changing right-hand-

TABLE II. Reduced matrix elements M_n for Cabibbo-suppressed decays, where n is the representation of SU(3) in H and M (O=octet, $S=\underline{27}$, $U=\underline{1}$) is the final-state SU(3) representation.

	H_1			H_2				
	$O_{\overline{6}}$	O_{15}	S_{15}	$O_{\overline{6}}$	O_{15}	S_{15}	O_3	U_3
$A(D^0 \to \pi^+\pi^-)\cot\theta_C$	-1	-1	1	$-\frac{1}{2}$	$-\frac{3}{4}$	$\frac{1}{8}$	1	1
$A(D^0 \to K^+K^-)\cot\theta_C$	1	1	-1	$\frac{1}{2}$	$\frac{1}{4}$	$-\frac{7}{8}$	1	1
$A(D^0 \to \pi^0\pi^0)\cot\theta_C$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$\frac{3}{2\sqrt{2}}$	$\frac{1}{2\sqrt{2}}$	$\frac{3}{4\sqrt{2}}$	$-\frac{1}{8\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$
$A(D^0 \to \pi^0\eta)\cot\theta_C$	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2\sqrt{3}}$	$-\frac{1}{4\sqrt{3}}$	$\frac{\sqrt{3}}{2}$	$\sqrt{3}$	0
$A(D^0 \to \eta\eta)\cot\theta_C$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	$-\frac{3}{2\sqrt{2}}$	$-\frac{1}{2\sqrt{2}}$	$-\frac{3}{4\sqrt{2}}$	$-\frac{9}{8\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$
$A(D^0 \to K^0\overline{K}^0)\cot\theta_c$	0	0	0	0	$-\frac{1}{2}$	$-\frac{1}{8}$	2	-1
$A(D^+ \to K^+\overline{K}^0)\cot\theta_c$	1	-1	$-\frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{4}$	-1	3	0
$A(D^+ \to \pi^+\pi^0)\cot\theta_C$	0	0	$\frac{5}{2\sqrt{2}}$	0	0	0	0	0
$A(D^+ \to \pi^+\eta)\cot\theta_C$	$(\frac{2}{3})^{1/2}$	$-(\frac{2}{3})^{1/2}$	$+\frac{3}{2}(\frac{3}{2})^{1/2}$	$\frac{1}{\sqrt{6}}$	$-\frac{1}{2\sqrt{6}}$	$(\frac{3}{2})^{1/2}$	$\sqrt{6}$	0
$A(F^+ \to \pi^+K^0)\cot\theta_C$	-1	$+1$	$\frac{3}{2}$	$-\frac{1}{2}$	$\frac{3}{4}$	$\frac{1}{2}$	3	0
$A(F^+ \to K^+\pi^0)\cot\theta_C$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2\sqrt{2}}$	$-\frac{3}{4\sqrt{2}}$	$-\frac{1}{2\sqrt{2}}$	$-\frac{3}{\sqrt{2}}$	0
$A(F^+ \to K^+\eta)\cot\theta_C$	$\frac{1}{\sqrt{6}}$	$-\frac{1}{\sqrt{6}}$	$\sqrt{6}$	$\frac{1}{2\sqrt{6}}$	$-\frac{1}{4}(\frac{3}{2})^{1/2}$	$\frac{3}{2}(\frac{3}{2})^{1/2}$	$-(\frac{3}{2})^{1/2}$	0

ed current of the form $(\bar{c}s)_R$. The most widely advertized models of this kind[5] are the six-quark vectorlike models, which may possibly be ruled out by the inequality of ν and $\bar{\nu}$ neutral-current cross sections. Independent of other details of the model we wish to consider here how the addition of this current affects the SU(3) properties of nonleptonic charmed-particle decay.[6] The Hamiltonian of interest now has the form $H = H_1 + H_2$ with

$$H_2 \sim \cos\theta_C(\bar{c}s)_R(\bar{d}u)_L + \sin\theta_C(\bar{c}s)_R(\bar{s}u)_L. \tag{17}$$

From the point of view of U spin H_2 differs from Eq. (2) in the Cabibbo-suppressed piece

$$H_2 \sim \cos\theta_C\, U_1^{-1} + \frac{1}{\sqrt{2}}\sin\theta_C(U_1^0 + U_0), \tag{18}$$

where U_0 is a U-spin scalar.

We consider first the possibility that all charmed-particle decays are predominantly due to H_2. This can be argued on the basis of the short-distance analysis of the current product in the framework of quantum chromodynamics, which indicates that the product of a right-handed current with a left-handed current is considerably enhanced over those combinations involving only left-handed currents.[9] With the dominance of H_2 we do not expect any of the relations Eqs. (3)–(16) to hold except for Eq. (16) and

$$\Gamma(D^0 \to \pi^0\bar{K}^0) = 3\Gamma(D^0 \to \eta K^0), \tag{19}$$

both of which involve the Cabibbo-favored decays only. Neither of these is very useful; in particular, Eq. (19) is likely to be particularly affected by the SU(3)-breaking η-η' mixing. The Cabibbo-suppressed decays due to H_2 satisfy a $\Delta I = \frac{1}{2}$ rule; for the two-body decays this gives

$$\Gamma(D^0 \to \pi^+\pi^-) = 2\Gamma(D^0 \to \pi^0\pi^0), \tag{20}$$

$$\Gamma(F^+ \to K^0\pi^+) = 2\Gamma(F^+ \to K^+\pi^0), \tag{21}$$

$$\Gamma(D^+ \to \pi^+\eta) = 2\Gamma(D^0 \to \pi^0\eta), \tag{22}$$

$$\Gamma(D^+ \to \pi^+\pi^0) = 0. \tag{23}$$

It should be noted that the Hamiltonian H_1 also leads to a $\Delta I = \frac{1}{2}$ rule for the Cabibbo-suppressed decays if the $\underline{6}$ piece of H_1 is dominant. The signature for the dominance of H_2 lies in the failure of some or all of Eqs. (3)–(15) [except for Eq. (19)] coupled with the $\Delta I = \frac{1}{2}$ rule for the Cabibbo-suppressed decays. A particular signature that follows from Eq. (23) is

$$\frac{2\cot^2\theta_C\Gamma(D^+ \to \pi^+\pi^0)}{\Gamma(D^+ \to \pi^+\bar{K}^0)} = 0. \tag{24}$$

This quantity equals unity [Eq. (15)] in the case of H_1 dominance. The U-spin structure given in Eq. (18) does yield some inequalities obtained by eliminating the interference term between the $U=1$ and $U=0$ pieces

$$\Gamma(D^0 \to K^+K^-) + \Gamma(D^0 \to \pi^+\pi^-) \geq \tfrac{1}{2}\tan^2\theta\,\Gamma(D^0 \to K^-\pi^+), \tag{25}$$

$$\Gamma(D^0 \to \eta\eta) + \Gamma(D^0 \to \pi^0\eta) + \Gamma(D^0 \to \pi^0\pi^0) - \tfrac{1}{2}\Gamma(D^0 \to \bar{K}^0K^0) \geq \tfrac{2}{3}\tan^2\theta\,\Gamma(D^0 \to \bar{K}^0\pi^0). \tag{26}$$

The SU(3) decomposition of the matrix elements of H_2 for two-body final states is shown in Tables I and II. In this case, in addition to the $\underline{6}$ and $\underline{15}$ there is for the Cabibbo-suppressed decays a $\underline{\bar{3}}$, which can lead to either an $\underline{8}$ or $\underline{1}$ final state; correspondingly there are two additional reduced matrix elements O_3 and U_3.

This is as far as straight SU(3)-invariance arguments will take us. To proceed further we consider the assumption that the Cabibbo-suppressed piece of H_2 $[(\bar{c}s)_R(\bar{s}u)_L]$ must always produce strange quarks in the final state. In a short-distance analysis the effective interaction in which the strange quarks annihilate is proportional to the s-quark mass.[10] From the U-spin point of view this means the $U=0$ and $U=1$ matrix elements must interfere destructively for decays with all pions; this yields the predictions with H_2 dominance

$$\begin{aligned}
&\Gamma(D^0 \to \pi^+K^-) = \cot^2\theta_C\Gamma(D^0 \to K^+K^-),\\
&\Gamma(D^0 \to \pi^+\pi^-) = \Gamma(D^0 \to \pi^0\pi^0) = 0,\\
&\Gamma(D^0 \to \pi^0\bar{K}^0) = \cot^2\theta_C[2\Gamma(D^0 \to \pi^0\eta^0) - \tfrac{9}{2}\Gamma(D^0 \to \eta\eta) + \tfrac{1}{3}\Gamma(D^0 \to \bar{K}^0K^0)].
\end{aligned} \tag{27}$$

These are to be contrasted with Eqs. (10)–(12) which hold for H_1 dominance. While this assumption does not involve SU(3) violation, it does appear unnatural from the point of view of SU(3) since it involves coherent superpositions of different SU(3) representations in the final state. Therefore final-state interactions may modify these results.

IV. DISCUSSION

Quantitative tests of the GIM Hamiltonian H_1 of Eq. (1) are possible if the rates of the Cabibbo-suppressed decays of the charmed mesons can be observed. These tests summarized in Sec. II in Eqs. (3)–(16) are all based on the assumption of perfect SU(3) symmetry. Failure of these tests may indicate failure of SU(3) or the presence of some additional term such as H_2. While some tests involving η decays may have large SU(3) corrections[11] we may expect that other tests such as Eq. (10) involving two-body decays may have only small corrections. In this case large failures of these tests would signal the presence of additional terms in the weak Hamiltonian. However, it is not possible using only general symmetry considerations to distinguish the case of H_1 with large SU(3) breaking from the case of comparable contributions of H_1 and H_2. The alternative that H_2 is dominant is discussed in Sec. III.

To indicate the possible pattern of failures of these tests we consider more specific dynamical assumptions. A possible assumption for SU(3) violation is that the weak interaction obeys U-spin symmetry of Eq. (2) but that it is more difficult to produce an $\bar{s}s$ pair than a $\bar{d}d$ pair from the vacuum. With this assumption Eqs. (10) and (15) as well as

$$\Gamma(D^0 \to \pi^0 \overline{K}^0) = \cot^2\theta_C \Gamma(D^0 \to \pi^0\pi^0)$$

from Eq. (12) still hold for H_1. On the other hand the decay $D^0 \to \overline{K}^0 K^0$ is no longer forbidden. If we assume that SU(3) violation is small but that there is a significant contribution from H_2, then we expect in general that both the equalities in Eq. (10) will fail. However, if we make the dynamical assumption discussed in Sec. III, that H_2 yields strange quarks in the Cabibbo-suppressed final state, then for any combination of H_1 and H_2 we still have

$$\Gamma(D^0 \to \pi^+ K^-) = \cot^2\theta_c \Gamma(D^0 \to K^+ K^-)\,.$$

However, $\Gamma(D^0 \to \pi^+\pi^-)$ would be expected to be quite different from $\Gamma(D^0 \to K^+K^-)$.

In our discussion we have avoided making assumptions about the relative sizes of different reduced matrix elements. In the case of the GIM Hamiltonian H_1, arguments have been given[12] that the $\underline{6}$ matrix elements may be enhanced in analogy with the $\underline{8}$ enhancement of $|\Delta S| = 1$ decays. An alternative possibility is that final octet states are preferred as a result of final-state interactions. As can be noted from Tables I and II, in the case of two-body decays these alternatives give the same result for branching ratios of a single charmed meson. The alternatives can be distinguished only by comparing the absolute decay rates of different charmed mesons. This is no longer true when three-body decays are considered and the observation of $D^+ \to K^-\pi^+\pi^+$ suggests that there is not an overwhelming octet enhancement in all channels.

In the case of H_2 the assumption of final-state octet enhancement is seen from the tables not to be equivalent to $\underline{6}$ enhancement as far as the ratio of Cabibbo-suppressed to Cabibbo-enhanced decays for a single meson. If H_2 were sextet dominated, the relationships among the Cabibbo-favored decays and among the Cabibbo-suppressed decays would be the same as for $\underline{6}$ dominance of H_1, but the ratio of the suppressed to the favored decays would be down by a factor 4 from the case of H_1. If H_2 were $\underline{3}$ dominated, an idea which follows from the assumption of the dominance[13] of the $\underline{15}$ piece in the SU(4) decomposition of H_2, the Cabibbo-suppressed decays might have a rate comparable to the Cabibbo-favored ones. As far as we know, however, there are no strong arguments for the dominance of the $\underline{3}$ or $\underline{6}$ or $\underline{15}$ pieces of H_2.

In summary, we have emphasized how the observation of the Cabibbo-suppressed decay modes can yield information about the structure of the charm-changing Hamiltonian that is not available from the Cabibbo-favored decays. From the SU(3) point of view, H_1 and H_2 are equivalent as far as Cabibbo-favored decays but very different with respect to the Cabibbo-suppressed. It is fortunate that those modes which are most favorable for experimental observation (i.e., $D^0 \to \pi^+\pi^-, K^+K^-$) also provide useful tests of the form of the interaction, and may be least affected by SU(3) breaking. Already upper limits on these modes exist. If these experimental results are improved they may confirm or rule out the original GIM model.

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[7]Γ stands for the sum of the squares of the matrix elements to all final states of the category indicated; this would be proportional to the decay rate if mass differences could be ignored. Similarly $\langle N \rangle$ in Eq. (4) is the average weighted with the square of the matrix elements.

[8]This table is equivalent to Tables I–III of Ref. 3 with $O_6 = (1/\sqrt{2}\,(C-B)$, $S_{15} = -(2\sqrt{2}/5)\,(A+B)$, $O_{15} = (\sqrt{2}/10)$ $(6A+B-5C)$. Note these quantities are in general complex with phases determined by the strong-inter-

action S matrix.

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