Sum rules for inclusive decays of charmed hadrons

T. Das and V. Gupta
Tata Institute of Fundamental Research, Homi Bhabha Road, Bombay 400005, India
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Inclusive nonleptonic and semileptonic weak decays of the type $c \to h +$ anything and $c \to h + l^+ +$ anything are considered, where c is a charmed hadron and h is an ordinary hadron belonging to the $\underline{3}^*$ and $\underline{8}$ representation of SU(3), respectively. The SU(3) relations for the inclusive decay rates are presented. These would provide tests of the isospin and SU(3) transformation properties of the weak charm-changing current and $\Delta C = \Delta S$ nonleptonic interaction Hamiltonian in the Glashow-Iliopoulos-Maiani scheme. We also point out that it may be possible to check these relations in the near future.

I. INTRODUCTION

Among the various theoretical attempts to understand the systematics of the new particles[1,2] $\psi(3.1)$ and $\psi(3.7)$, a particularly popular scheme[3] is one in which the extremely narrow width of these particles is understood in terms of a new quantum number, C, called charm. In this scheme the symmetry group SU(4), which contains the usual SU(3) symmetry, underlies the strong interactions. A direct test of the underlying SU(4) would be to study relations between strong-interaction quantities such as masses[4,5] and coupling constants.[6] However, the low-lying charmed hadrons with $C=1$ are expected to be stable with respect to strong interaction and will manifest themselves through their weak-interaction decays.

The recent discovery[7] of a neutral and a charged boson of mass around 1.87 GeV heralds the discovery of the lowest charmed mesons D^0 and D^+ with $C=1$ belonging to the $\underline{3}^*$ representation of SU(3). If (D^+, D^0) form an isodoublet and since $D^0 \to K^-\pi^+$ while $D^+ \to K^-\pi^+\pi^+$, then their decays violate parity assuming that they are pseudoscalars, i.e., $J^P = 0^-$. Though one has to await experimental confirmation of the J^P of the D^0 and D^+, it is very striking that their observed decays obey the selection rules of the elegant weak interaction proposed by Glashow, Iliopoulos, and Maiani,[8] referred to as the GIM scheme for short. Many authors[5,9] have explored the consequences of the GIM scheme for the two- and three-body nonleptonic and simple semileptonic decays of the low-lying charmed mesons and baryons. However, since the mass of the charmed hadrons is large, one expects many multibody decay modes and, moreover, that an exclusive decay channel will have a branching ratio of only a few percent. Consequently, it is important to have tests for the GIM scheme for the inclusive decays of the charmed hadrons.

In the present paper we consider the following inclusive semileptonic and nonleptonic decays:

$$c(\underline{3}^*) \to l^+ + \text{anything} \tag{1a}$$

$$\to l^+ + h(\underline{8}) + \text{anything} \tag{1b}$$

and

$$c(\underline{3}^*) \to h(\underline{8}) + \text{anything}\,, \tag{2}$$

where $c(\underline{3}^*)$ represents the charmed mesons or baryons with $C=1$ belonging to the $\underline{3}^*$ representation of SU(3), $l^+ = e^+$ or μ^+, and $h(\underline{8})$ stands for an ordinary meson or baryon octet. We give relations for the above inclusive decay processes which will provide tests of the isospin and SU(3) properties of the $\Delta C = \Delta S$ weak current and the $\Delta C = \Delta S$ nonleptonic interaction Hamiltonian in the GIM scheme.

In Sec. II we briefly discuss the I-spin, V-spin, and U-spin transformation properties of the weak interaction in the GIM scheme. These transformation properties are exploited and the relations so obtained for the processes (1) and (2) above are presented in Secs. III and IV, respectively. In Sec. V we briefly indicate the extension of our results to the case when $h(\underline{8})$ is replaced by a nonet $h(\underline{9})$ with arbitrary mixing. The final section is devoted to discussion and possible experimental verification of our results.

II. THE WEAK INTERACTION IN GIM SCHEME

The hadronic weak current in the GIM scheme[8] is given by (suppressing space-time properties)

$$J_h = \cos\theta\, J_2^1 + \sin\theta\, J_3^1 + \cos\theta\, J_3^4 - \sin\theta\, J_2^4\,, \tag{3}$$

where θ is the Cabibbo angle and the indices indicate the transformation properties under SU(4). The charm-changing currents J_3^4 and J_2^4 satisfy the selection rules $\Delta C = \Delta S$ and $\Delta C = 1$, $\Delta S = 0$, respectively, where S refers to strangeness.

The current J_h will effect the semileptonic transitions and the leading contribution to the decay of charmed hadrons will come from J_3^4 since the angle θ is small. The current J_3^4 transforms as a triplet under SU(3) and under its SU(2) subgroups

I, U, and V spins it behaves as a singlet ($I=0$), doublet ($U=\frac{1}{2}$, $U_3=\frac{1}{2}$), and doublet ($V=\frac{1}{2}$, $V_3=\frac{1}{2}$), respectively. It is these transformation properties which we exploit in order to present relations among various inclusive semileptonic decays in Sec. III below.

The leading nonleptonic decays of charmed hadrons are those which obey the $\Delta C=\Delta S$ selection rule. The current $\times$ current Hamiltonian for these is

$$H_{CS}=H_{-}+H_{+}\,, \tag{4a}$$

where

$$H_{\pm}=\frac{G}{2\sqrt{2}}\cos^2\theta[\{J_2^1,J_4^3\}\pm\{J_2^3,J_4^1\}]+\text{H.c.} \tag{4b}$$

It is easy to see that the part H_- transforms as $(\underline{6}\oplus\underline{6}^*)$ under SU(3) and as the self-conjugate 20-dimensional representation of SU(4) and is denoted by $\underline{20}''$. On the other hand, H_+ transforms as the $(\underline{15}\oplus\underline{15}^*)$ representation of SU(3), which is contained in the 84-plet of SU(4). Furthermore, H_+ and H_- individually satisfy the selection rules $|\Delta\vec{\text{I}}|=1$, $|\Delta I_3|=1$, $|\Delta\vec{\text{U}}|=1$, $|\Delta U_3|=1$, while under V spin H_+ and H_- transform like $V=1$, $V_3=0$, and $V=0$ objects, respectively. It is these transformation properties which we exploit in order to obtain the SU(3) relations for the inclusive nonleptonic decays given in (2) above. As it turns out, the full H_{CS} does not lead to any relations. However, since the generalization of the arguments based on short-distance operator-product expansion, which attempt to give an understanding of the $|\Delta\vec{\text{I}}|=\frac{1}{2}$ for the usual nonleptonic interaction, suggests[10] that the SU(4) representation $\underline{20}''$ (i.e., H_-) is enhanced relative to $\underline{84}$ (i.e., H_+), for H_- enhancement one can obtain simple SU(3) relations which are given in Sec. IV below.

III. INCLUSIVE SEMILEPTONIC DECAYS

We now consider relations for the inclusive decays given in (1a) and (1b). For definiteness, we take the $c(3^*)$ to be the charmed pseudoscalars (D^+,D^0,F^+) and $h(\underline{8})$ to be the usual pseudoscalar octet, though as pointed out earlier, the relations obtained will be valid for any charmed hadron $\underline{3}^*$ with $C=1$ and any usual hadron octet. The inclusive decay rate for the process (1a) is given by

$$p_{10}\frac{d^3\Gamma}{d^3p_1}\equiv R(c\to l^+)=\frac{G^2\cos^2\theta}{(2\pi)^6}\int\frac{d^3p_2}{p_{20}}\,L_{\mu\nu}R_{\mu\nu}\,, \tag{5}$$

where

$$L_{\mu\nu}=p_{1\mu}p_{2\nu}+p_{1\nu}p_{2\mu}-\delta_{\mu\nu}(p_1\cdot p_2)-\epsilon_{\mu\nu\lambda\sigma}p_{1\lambda}p_{2\sigma} \tag{6}$$

and

$$R_{\mu\nu}=V\int d^4x\,e^{i(p_1+p_2)\cdot x}\langle c(p)|J_\nu(x)J_\mu^\dagger(0)|c(p)\rangle\,. \tag{7}$$

Here p_1, p_2, and p refer respectively to the four-momentum of the detected charged lepton l^+, the accompanying neutrino, and the decaying charmed particle c. V is the normalization volume and J is the leading charm-carrying hadronic current, viz., J_3^4. Since the current J is an isosinglet one immediately has the relation

$$R(D^+\to l^+)=R(D^0\to l^+)\,. \tag{8}$$

To use the U-spin property of J we note that $R_{\mu\nu}$ is the norm of a vector which transforms like $|\phi(c)\rangle=J^\dagger|c\rangle$ under U spin. Since $J^\dagger$ is a U-spin doublet with $U_3=-\frac{1}{2}$ one has $|\phi(D^+)\rangle=|\phi(1,-1)\rangle$, while $\sqrt{2}\,|\phi(F^+)\rangle=|\phi(1,0)\rangle+|\phi(0,0)\rangle$, where $|\phi(a,a_3)\rangle$ is a vector with $U=a$ and $U_3=a_3$. Since the two vectors with $U=1$ and $U=0$ will not interfere and their norms will be positive, one arrives at the inequality

$$2R(F^+\to l^+)\geq R(D^+\to l^+)\,. \tag{9}$$

By integrating over the lepton momentum one can convert (8) and (9) into relations for the total semileptonic decay rates $\Gamma_l(D^+)$ for D^+, etc.

We now consider the inclusive decays of the type (1b), the transition rate for which is given by

$$\frac{p_{10}q_0d^6\Gamma}{d^3p_1d^3q}\equiv S(c\to l^+h)=\frac{G^2\cos^2\theta}{(2\pi)^6}\int\frac{d^3p_2}{p_{20}}\,L_{\mu\nu}S_{\nu\mu}\,, \tag{10}$$

where

$$S_{\nu\mu}=N_hV\int d^4x\,d^4y\,d^4z\,e^{-iq\cdot(x-y+z)-i(p_1+p_2)\cdot z}\,\theta(x_0)\theta(y_0)\times\langle c(p)|[J_\nu(0),j_h^\dagger(y)][j_h(x+z),J_\mu^\dagger(z)]|c(p)\rangle\,. \tag{11}$$

Here q is the four-momentum and j_h is the source function of the detected hadron h, while p, p_1, and p_2 have the same meaning as in (7) above. The factor N_h in (11) is $\frac{1}{2}$ (or m_h) if the detected hadron is a boson (or fermion). Let $S(c\to h)$ be obtained from (10) after integrating over the charged lepton momentum. We note the relations

$$\begin{aligned}\int\frac{d^3q}{q_0}S(c\to h)&=\int\frac{d^3q}{q_0}\frac{d^3p_1}{p_{10}}S(c\to l^+h)\\&=\langle h\rangle_c\int\frac{d^3p_1}{p_{10}}R(c\to l^+)\\&=\langle h\rangle_c\Gamma_l(c)\,,\end{aligned} \tag{12}$$

where $\langle h\rangle_c$ is the average multiplicity of the hadron

h in the semileptonic decay of the charmed hadron c and $\Gamma_l(c)$ is its total semileptonic decay rate. For compactness we give the relations for $S(c\to h)$, though they are clearly valid for $S(c\to l^+h)$. Since the weak current J is an isosinglet one has the relations

$$S(D^+\to\pi^+)=S(D^0\to\pi^-),\quad S(D^+\to\pi^-)=S(D^0\to\pi^+), \tag{13a}$$

$$2S(D^+\to\pi^0)=2S(D^0\to\pi^0)=S(D^+\to\pi^+)+S(D^+\to\pi^-), \tag{13b}$$

$$S(D^+\to K^-)=S(D^0\to\bar K^0),\quad S(D^+\to\bar K^0)=S(D^0\to K^-), \tag{13c}$$

$$S(D^+\to K^+)=S(D^0\to K^0),\quad S(D^+\to K^0)=S(D^0\to K^+), \tag{13d}$$

$$S(F^+\to\pi^+)=S(F^+\to\pi^-)=S(F^+\to\pi^0), \tag{13e}$$

$$S(F^+\to K^+)=S(F^+\to K^0),\quad S(F^+\to K^-)=S(F^+\to\bar K^0), \tag{13f}$$

$$3S(D^+\to\pi^+)\geq S(D^+\to\pi^-), \tag{13g}$$

$$2S(D^+\to\bar K^0)\geq S(D^+\to K^-),\quad 2S(D^+\to K^+)\geq S(D^+\to K^0). \tag{13h}$$

Use of the U-spin and V-spin properties of the current leads to one equality and a number of inequalities. The more interesting ones are

$$6S(F^+\to\pi^+)+6S(D^+\to\pi^+)\geq 3S(F^+\to K^+)\geq S(D^+\to K^+), \tag{14a}$$

$$6S(F^+\to K^-)+6S(D^+\to K^-)\geq 3S(F^+\to\pi^-)\geq S(D^+\to\pi^-), \tag{14b}$$

$$6S(F^+\to\bar K^0)+6S(D^0\to\bar K^0)\geq 3S(F^+\to\pi^+)\geq S(D^0\to\pi^+), \tag{14c}$$

$$6S(F^+\to\pi^-)+6S(D^0\to\pi^-)\geq 3S(F^+\to K^0)\geq S(D^0\to K^0), \tag{14d}$$

$$2S(D^+\to\bar K^0)\geq S(D^+\to\pi^+),\quad 2S(D^+\to\pi^-)\geq S(D^+\to K^0), \tag{14e}$$

$$2S(D^0\to K^-)\geq S(D^0\to\pi^-),\quad 2S(D^0\to\pi^+)\geq S(D^0\to K^+), \tag{14f}$$

$$3S(D^+\to K^-)\geq S(D^+\to K^+),\quad 3S(D^0\to\bar K^0)\geq S(D^0\to K^0). \tag{14g}$$

The other relations are

$$\sum_{c=D^+,D^0,F^+}[2S(c\to K^+)+2S(c\to K^-)-3S(c\to\eta)-S(c\to\pi^0)]=0, \tag{15}$$

$$S(D^+\to K^+)+S(D^+\to K^-)\leq\tfrac{1}{2}\{\sqrt{3}\,[S(D^+\to\eta)]^{1/2}+[S(D^+\to\pi^0)]^{1/2}\}^2, \tag{16}$$

$$\sum_{c=F^+,D^0}[S(c\to K^+)+S(c\to K^-)]\leq\tfrac{1}{2}\sum_{c=F^+,D^0}\{\sqrt{3}\,[S(c\to\eta)]^{1/2}+[S(c\to\pi^0)]^{1/2}\}^2. \tag{17}$$

In writing (15) we have used the isospin relations (13). From the V-spin inequalities (16) and (17) two U-spin inequalities can be obtained by the replacements $D^+\leftrightarrow D^0$, $K^+\leftrightarrow K^0$, and $K^-\leftrightarrow\bar K^0$. The relations (15), (16), and (17) may not be very useful as the observation of $S(c\to\eta)$ and $S(c\to\pi^0)$ will be difficult. The other relations using (12) can be converted into terms of the average multiplicities and total semileptonic rates and may be amenable to verification. The inequalities in (14) can be used to put a limit on the Γ_l's if one knows a particular $\langle h\rangle$; e.g., (14a) gives $3\langle K^+\rangle_{F^+}\Gamma_l(F^+)\geq\langle K^+\rangle_{D^+}\Gamma_l(D^+)$.

IV. INCLUSIVE NONLEPTONIC DECAYS

For the inclusive nonleptonic decay of the type $c(p)\to h(q)+$anything, the transition rate for the $\Delta C=\Delta S$ decays is given by

$$q_0\frac{d^3\Gamma}{d^3q}\equiv N(c\to h)$$

$$=\frac{VN_h}{(2\pi)^3}\int d^4x\,d^4y\,d^4z\,e^{iq\cdot(y-x-z)}\theta(x_0)\theta(y_0)\langle c(p)|[H_{CS}(0),j_h^\dagger(y)][j_h(x+z),H_{CS}(z)]|c(p)\rangle. \tag{18}$$

The full H_{CS} given in (4) does not lead to any relations; however, as pointed out earlier, one expects the enhancement of H_-, which means sextet dominance at the SU(3) level. The hypothesis of sextet dominance, i.e., $H_{CS}=H_-$ leads to simple SU(3) relations from V-spin considerations since H_- is a V-spin singlet. One obtains

$$N(D^+\to\pi^+)=N(D^+\to\bar{K}^0),\quad N(D^+\to\pi^-)=N(D^+\to K^0)\,, \tag{19a}$$

$$N(D^+\to K^+)=N(D^+\to K^-)\le\tfrac{1}{4}\{\sqrt{3}[N(D^+\to\eta)]^{1/2}+[N(D^+\to\pi^0)]^{1/2}\}^2\,, \tag{19b}$$

$$2N(D^0\to\bar{K}^0)=2N(F^+\to\pi^+)\ge N(F^+\to\bar{K}^0)=N(D^0\to\pi^+)\,, \tag{19c}$$

$$2N(D^0\to\pi^-)=2N(F^+\to K^0)\ge N(F^+\to\pi^-)=N(D^0\to K^0)\,, \tag{19d}$$

$$3N(F^+\to K^+)=3N(D^0\to K^-)\ge N(F^+\to K^-)=N(D^0\to K^+)\,, \tag{19e}$$

$$N(F^+\to\eta)+N(F^+\to\pi^0)=N(D^0\to\eta)+N(D^0\to\pi^0)\,, \tag{19f}$$

$$N(D^0\to K^+)+N(D^0\to K^-)\le\tfrac{1}{2}\{\sqrt{3}[N(D^0\to\eta)]^{1/2}+[N(D^0\to\pi^0)]^{1/2}\}^2\,. \tag{19g}$$

These relations can be converted in terms of the mean multiplicity $\bar{N}_c(h)$ of the hadron h in the nonleptonic decay of c by noting that

$$\int\frac{d^3q}{q_0}N(c\to h)=\bar{N}_c(h)\Gamma_{\rm NL}(c)\,, \tag{20}$$

where $\Gamma_{\rm NL}(c)$ is the width of c for nonleptonic transitions. To make fruitful use of our results, one has to await experimental data which we hope will be available in the near future.

V. MODIFICATIONS DUE TO A NONET WITH ARBITRARY MIXING

The results presented so far for the semileptonic and nonleptonic decays are valid when the detected hadron is an octet. We extend our results to the inclusive processes $c(\underline{3}^*)\to l^+ + h(\underline{9})$ + anything and $c(\underline{3}^*)\to h(\underline{9})$ + anything, so as to include the case of the usual vector nonet $V(9)$. We present the results for a pseudoscalar nonet with arbitrary mixing angle α. The physical states are

$$\eta=\cos\alpha\,\eta_8-\sin\alpha\,\eta_1\,,$$

$$\eta'=\sin\alpha\,\eta_8+\cos\alpha\,\eta_1\,,$$

where η_1 is the SU(3) singlet which mixes with η_8, the eighth component of the octet. It is clear this mixing will only affect the relations involving η given earlier. For the semileptonic case one has the additional relation

$$S(D^+\to\eta')=S(D^0\to\eta')\,,$$

while for the nonleptonic case, the relation (19f) is simply modified to

$$N(F^+\to\eta)+N(F^+\to\pi^0)+N(F^+\to\eta')$$
$$=N(D^0\to\eta)+N(D^0\to\pi^0)+N(D^0\to\eta')\,.$$

These two relations can be translated to the case when $h(\underline{9})$ is the vector nonet by the replacements $\pi^0\to\rho^0$, $\eta\to\phi$, $\eta'\to\omega$ as they are independent of the mixing angle. The modification of the other relations can be obtained by the replacement, e.g., of $S(c\to\eta)$ by $S(c\to\cos\alpha\,\eta+\sin\alpha\,\eta')$ and by making the resulting relation into an inequality involving $S(c\to\eta)$ and $S(c\to\eta')$. We do not give them explicitly as they do not seem very useful.

VI. DISCUSSION

The SU(3) relations in the foregoing sections have been given for the $\Delta C=\Delta S$ semileptonic and nonleptonic inclusive decays of a charmed pseudoscalar $\underline{3}^*$ into an octet (or a nonet with arbitrary mixing) of pseudoscalars. It is important to note that our results are also valid for the inclusive decay of any charmed hadron $\underline{3}^*$ (meson or baryon), where the detected octet (or nonet) can have any J^P. Even though the inclusive decay relations involve only matrix elements, the SU(3) relations (from V and U spin) need to be used with caution since the total energy available in a decay is fixed and not large so that large phase-space suppression effects can arise owing to the mass differences of the particles in the final state. These SU(3)-breaking effects can be important depending on the relation and the particular decay process, so that all the relations may not be useful. We briefly discuss this point below for various processes.

(i) $P(3^*)\to P(8)$ + anything. The integrated form of the inequality in (19b) gives $N(D^+\to K^-)=N(D^+\to K^+)$. However, to satisfy the decay selection rules the K^+ must be accompanied by at least a $\bar{K}^0\bar{K}^0$ pair while the K^- needs a $\pi^+\pi^+$ pair. Since the D^+ mass is 1.87 GeV it is clear, from the phase-space suppression for the $\bar{K}^0\bar{K}^0$ compared to $\pi^+\pi^+$ in each exclusive rate in the sum for the inclusive rate, that one would directly expect $N(D^+\to K^-)>N(D^+\to K^+)$. For the same reason one would expect $N(D^+\to\pi^+)>N(D^+\to\bar{K}^0)$ and $N(D^+\to\pi^-)>N(D^+\to K^0)$. This illustrates the *caveat emptor* mentioned above. However, all the results are not so seriously affected; for example, the second equality in (19c) leads to $N(F^+\to\bar{K}^0)=N(D^0\to\pi^+)$, which may hold reasonably well since the F^+ is expected to be about 200 MeV heavier than D^0 and this will circumvent the phase-space suppression of the kaon present in $F^+\to\bar{K}^0+\cdots$ compared to the pion in $D^0\to\pi^++\cdots$.

(ii) $B(3^*) \to B(8)$ or $P(8)$ + anything. Denote the numbers of $B(\underline{3}^*)$ by (A^+, A^0, C_0^+) following Gaillard *et al.*[5] The relations in (19) can be easily translated for these processes, and one finds that the SU(3)-breaking effects due to mass differences in general turn out to be smaller. For example, one expects (19a) to hold better, though one expects $N(A^+ \to \Xi^-) > N(A^+ \to p)$. The relations for A^0 and C_0^+ may work reasonably well, e.g. $N(A^+ \to \bar{K}^0$ (or Ξ^0)) $= N(A^+ \to \pi^+$ (or Σ^+)), but $N(A^0 \to \Sigma^+)) > N(C_0^+ \to \Xi^0)$ since the A^0 is expected to be heavier than C_0^+ by about 200 MeV.

(iii) Semileptonic modes. For the purpose of discussion here, we may leave out all isospin relations, which should all hold well. The other relations are all inequalities and the validity of each would depend upon the arrangement of the masses and the quantum numbers. The inequality in (9) would, we feel, be a useful constraint on the semileptonic decay rate of F^+ *vis-à-vis* that of D^+. Since F^+ should be more massive than $D^{+,0}$, we expect that the inequality (14a), and each of the second inequalities in (14a)–(14f) and (14g), should be satisfied quite well. The inequalities in (14) should also hold reasonably well for $S(B(\underline{3}^*) \to B(\underline{8}))$ and $S(B(\underline{3}^*) \to P(\underline{8}))$ because of the compensating effects of the various masses.

Thus in spite of the symmetry-breaking effects, the inclusive sum rules given earlier may be useful depending upon the particular decay processes and it may be worthwhile to test them.

Obtaining experimental information on the inclusive decays of the charmed hadrons may be in general difficult, particularly for the nonleptonic decays. In the case of process (1b) the detection of l^+h, particularly if h is a K meson, may be enough to decide that the decaying particle was a charmed hadron. Such signals for charmed particles are already being studied in the search for charmed hadrons in e^+e^- annihilation and other processes. One may also obtain information on the inclusive decays of the charmed hadron c, if its mass is known, by producing c and $\bar{c}$ nearly at rest in $p\bar{p}$ or $e\bar{e}$ annihilation and identifying the $\bar{c}$ by an exclusive decay mode. At present, one has to await experimental data to check our relations which provide some simple tests of the isospin and SU(3) structure of the weak $\Delta C = \Delta S$ current and nonleptonic interaction in the GIM scheme.

Note added. Apart from the relations obtainable for the average multiplicities from the relations in the text, there exist exact relations for the average multiplicities which are a consequence of the conservation of an additive quantum number, Q, in an inclusive process.[11] The result is simply

$$Q_{\text{in}} = \sum_h \langle h \rangle Q_h ,$$

where the sum is over the hadrons h in the final state which carry $Q = Q_h$, while Q_{in} is the value of Q for the incoming state. This may be applied to an inclusive decay, Q_{in} being determined by the decaying particle as well as by the weak-interaction spurion. We illustrate the result for the decays of $c = D^+, D^0$, and F^+ for which baryon-antibaryon decays may be ignored. The two independent sum rules corresponding to $Q = I_3$ and Y for $\Delta C = \Delta S$ semileptonic decays are

$$\langle \pi^+ \rangle_c - \langle \pi^- \rangle_c + \tfrac{1}{2}[\langle K^+ \rangle_c - \langle K^0 \rangle_c + \langle \bar{K}^0 \rangle_c - \langle K^- \rangle_c] = (I_3)_{\text{in}}^c ,$$

$$\langle K^+ \rangle_c + \langle K^0 \rangle_c - \langle K^- \rangle_c - \langle \bar{K}^0 \rangle_c = Y_{\text{in}}^c ,$$

where $(I_3)_{\text{in}}^c = \frac{1}{2}, -\frac{1}{2}$, and 0 and $Y_{\text{in}}^c = -1, -1$, and 0 for $c = D^+$, D^0, and F^+ respectively. For $\Delta C = \Delta S$ nonleptonic decays, $(I_3)_{\text{in}}^c = \frac{3}{2}, \frac{1}{2}$, and 1 while Y_{in}^c remains unchanged. These exact relations together with those in the text may be of use in analyzing inclusive days.

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