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Rising pion inclusive cross section and $N\bar{N}$ cluster production*

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We argue that the reason for the rise in the π^- inclusive cross section over CERN ISR energies is the threshold production of $N\bar{N}$ clusters. We formulate and calculate such contributions at $y = 0$. Our results can account for the observed rise.

It has been argued by many authors[1-6] that the rise of the p-p total cross section σ_T over CERN ISR energies is due to the dynamical opening up of the nucleon-antinucleon ($N\bar{N}$) threshold (more precisely, the baryon-antibaryon thresholds). The delayed dynamical threshold may be attributed to the large mass of the $N\bar{N}$ cluster and the associated multiperipheral $t_{\min}$ effect. This correlation of the rise of σ_T and $N\bar{N}$ production is based on the experimental observation[7] that the $\bar{p}$ average multiplicity $\langle n_{\bar{p}}\rangle$ seems to show a rapid threshold rise starting around $s \approx 200$–300 GeV^2. At lower energies, there may be other thresholds corresponding to the production of mesons with strange quarks versus mesons without strange quarks and/or to the production of vector and tensor mesons versus pseudoscalar mesons. Because of the smaller masses of these mesons as compared to that of an $N\bar{N}$ cluster, these thresholds are not as distinct as that associated with the $N\bar{N}$ cluster. Furthermore, these thresholds may partially overlap each other. Thus it is a much more subtle problem to extract out the contributions to σ_T from the dynamical onset of these lower thresholds. However, it is definitely conceivable that a proper inclusion of these thresholds and the $N\bar{N}$ threshold allows one to explain the rise of σ_T over Serpukhov and Fermilab energies.[8]

It has also been pointed out[9] that the onset of $N\bar{N}$ production gives rise to a new contribution to the single-pion inclusive cross section and so can cause it to rise with energy. Recent ISR data[10-12] show that, from $\sqrt{s} = 23$ GeV to 53 GeV, single-particle inclusive cross sections ρ_{π^-} and $\rho_{\bar{p}}$ rise, by approximately 30% and 50%, respectively,[13] where $\rho_i = d\sigma^i/dy|_{y=0}$. The purpose of this paper is to examine these phenomena within a multiperipheral cluster model containing $N\bar{N}$ dynamical thresholds and to make a systematic quantitative calculation of the π^- inclusive cross section. As we will see, similar analysis also applies to the kaon case.

We first argue that the reason for $\rho_{\bar{p}}$ to rise over the ISR energies is different from the reason for ρ_{π^-} and ρ_{K^-} to rise over this energy range. For the purpose of this argument, we assume that π^- and K^- come predominantly from the decays of vector and tensor (V-T) mesons. These mesons, which shall also be referred to as meson clusters, are directly produced in the multiperipheral (MP) chain. From the 18 known vector and tensor mesons, one can easily show[14] that there is an average of 0.66 π^- and 0.17 K^- per V-T meson decay.[15] Furthermore, owing to dynamical SU(3)-symmetry breaking in the production process, there is a further suppression in the production of mesons with strange quarks than in the production of those without. The experimental data of $\langle n_{K^-}\rangle/\langle n_{\pi^-}\rangle \approx 0.1$ indicate that this suppression factor is roughly 2.5. This means each detected π^- and K^- corresponds to a production of approximately 1.5 and 15 V-T meson clusters, respectively. Then the $\langle n_{\pi^-}\rangle$ or $\langle n_{K^-}\rangle$ ISR data[7] imply that roughly 4–6 V-T meson clusters are being produced.

On the other hand, the $\bar{p}$'s come from $N\bar{N}$ clusters which are also directly produced in the MP chain. If the probability for an $N\bar{N}$ cluster to result in a $N\bar{N}$ asymptotic final state is $1/\eta$, then each detected $\bar{p}$ corresponds to a production of 2η $N\bar{N}$ clusters, where the factor of 2 comes from the fact that the $\bar{N}$ in an $N\bar{N}$ cluster can be either a $\bar{p}$ or an $\bar{n}$. In the next paragraph we will show that the total-cross-section data require $\eta \lesssim 1.5$ and preferably $\eta \approx 1$. To have an estimate of how many $N\bar{N}$ clusters are being produced in the ISR energies, we assume that the partial cross sections are given by a Poisson distribution. A simple calculation shows that the ISR $\langle n_{\bar{p}}\rangle$ data[7] imply that in all likelihood, in each hadron collision, at most one $N\bar{N}$ cluster is being produced, even at $\sqrt{s} = 53$ GeV. The high dynamical threshold for producing two $N\bar{N}$ clusters is actually not unexpected in the multiperipheral model.[1,16] Thus the rise of $\rho_{\bar{p}}$ over the ISR energies may be attributed to this threshold transition of producing one $N\bar{N}$ cluster which results in an increase of the effective coupling constant of the $N\bar{N}$ cluster in the MP chain. Because several V-T meson clusters are being produced, the rise of ρ_{π^-} or ρ_{K^-} over the ISR energies is not due to an in-

TABLE I. Determination of bare cross sections.

$\sqrt{s}$ (GeV)	σ_T (Ref. 18)	$\eta=1$ σ_N	$\eta=1$ σ_0	$\eta=1$ $\tilde{\sigma}_0$	$\eta=1.5$ σ_N	$\eta=1.5$ σ_0	$\eta=1.5$ $\tilde{\sigma}_0$
23	39.2	3.9	35.3	22.0	5.9	33.3	20.0
31	40.6	7.0	33.6	19.5	10.6	30.0	15.9
45	42.6	9.6	33.0	18.1	14.4	28.2	13.3
53	43.2	10.2	33.0	17.3	15.4	27.8	12.1

crease in the effective coupling constant of the V-T meson clusters; we will argue that it is due to the additional contribution from events containing an $N\bar{N}$ cluster.

We now show that the total-cross-section data require $\eta \lesssim 1.5$ and preferably $\eta \approx 1$. We denote by σ_0 and σ_N the contributions to σ_T from events containing zero and at least one $N\bar{N}$ cluster, respectively. They are related by

$$\sigma_0 = \sigma_T - \sigma_N . \tag{1}$$

The fact that in all likelihood at most one $N\bar{N}$ cluster is being produced over the ISR energies implies σ_N is given by[17]

$$\sigma_N = 2\eta \int \frac{d\sigma_{\bar{p}}}{dy}\, dy , \tag{2}$$

which can be calculated using the data of Ref. 7. The results are summarized in Table I, where all cross sections are in millibarns. In Table I we have also calculated the bare multiperipheral cross section $\tilde{\sigma}_0$ by subtracting from σ_0 the diffractive elastic cross section σ^D_{el} and the diffractive inelastic cross section σ^D_{in}, i.e.,

$$\tilde{\sigma}_0 = \sigma_0 - \sigma^D_{\text{el}} - \sigma^D_{\text{in}} , \tag{3}$$

where we used Morrison's estimate[19] of σ^D_{el} and σ^D_{in}.

If we parametrized $\sigma_0 = \beta_0 s^{\alpha_0 - 1}$, where α_0 is the bare Pomeron intercept, then $\eta = 1$ implies $\alpha_0 \approx 0.96$ and $\eta = 1.5$ implies $\alpha_0 \approx 0.89$. The first result is almost identical to the results of phenomenological fits[14,20] to total-cross-section data at lower energies; these phenomenological fits are based on solutions of dual bootstrap models. If $\eta > 1.5$, it gives unacceptably low values for α_0. Similarly, parametrizing the bare multiperipheral cross section by $\tilde{\sigma}_0 = \tilde{\beta}_0 s^{\tilde{\alpha}_0 - 1}$, we find $\tilde{\alpha}_0 \approx 0.85$ for $\eta = 1$ and $\tilde{\alpha}_0 \approx 0.69$ for $\eta = 1.5$. Again, $\eta > 1.5$ gives unacceptably low values for $\tilde{\alpha}_0$. Thus we conclude $\eta \approx 1$, which means that there is little, if any, probability for the $N\bar{N}$ cluster to result in mesons. This rules out the *a priori* attractive model of assuming the $p\bar{p}$ cluster to behave like $p\bar{p}$ annihilation near threshold, where η may be estimated from the data[21] to be approximately 2.1.

For the π^- inclusive cross section ρ_π, we can also express it as $\rho_\pi(y_\pi) = \rho_0(y_\pi) + \rho'(y_\pi)$, where ρ_0 and ρ' correspond, respectively, to events containing zero and (at least) one $N\bar{N}$ cluster. We are interested in the detected π^- being in the central region. In this region we expect $\rho_0(y_\pi) \approx \tilde{\rho}_0(y_\pi)$, where $\tilde{\rho}_0$ is the bare (no $N\bar{N}$) multiperipheral inclusive cross section, or

$$\rho_\pi(y_\pi) \approx \tilde{\rho}_0(y_\pi) + \rho'(y_\pi) . \tag{4}$$

Since $\eta \approx 1$, the contribution ρ' is then given by the convolution of the inclusive cross section of the $N\bar{N}$ cluster $2\eta\, d\sigma^{\bar{p}}/dy_N$ and the bare multiperipheral π^- inclusive differential multiplicity $(1/\tilde{\sigma}_0)\tilde{\rho}_0(y_\pi)$, together with a factor $f(y_\pi - y_N)$ which takes into account the suppression of π production near the rapidity of the $N\bar{N}$ cluster. Thus,

$$\rho'(y_\pi) = 2\eta \int_{-\infty}^{\infty} dy_N \frac{d\sigma^{\bar{p}}}{dy_N} f(y_\pi - y_N) \frac{1}{\tilde{\sigma}_0} \tilde{\rho}_0(y_\pi) . \tag{5}$$

We represent the suppression factor by

$$f(y_\pi - y_N) = 1 - \exp\left[- \frac{(y_\pi - y_N)^2}{\Delta^2} \right] , \tag{6}$$

where

$$\int_{-\infty}^{\infty} dy_N \exp\left(- \frac{y_N{}^2}{\Delta^2} \right) = \Delta\sqrt{\pi} \equiv \Delta' . \tag{7}$$

The quantity Δ' is the effective width in rapidity carved out by the $N\bar{N}$ cluster. We estimate Δ' by two different methods. The first is to set[22]

$$\Delta' \approx \sigma_{\text{in}} \left[\frac{\langle n_{\pi^-} \rangle}{\rho_\pi(y_\pi = 0)} - \frac{\langle n_{\bar{p}} \rangle}{\rho_{\bar{p}}(y_{\bar{p}} = 0)} \right] . \tag{8}$$

The data[7,10] in the ISR energies give $\Delta' \approx 1.8$ or $\Delta \approx 1.0$. The second is to make use of MP kinematics of $s \sim s_1 s_{N\bar{N}} s_3$, where $s_1 s_3$ is the effective energy squared for π production. Setting $s_{N\bar{N}} \approx 5$ GeV^2 gives $\Delta' \approx 1.6$ or $\Delta \approx 0.9$, which is very near the first estimate. We use $\Delta = 1.0$ in our calculation.

The function $d\sigma^{\bar{p}}/dy_N$ may be parametrized from the data as

$$\frac{d\sigma^{\bar{p}}}{dy_N} = \rho_{\bar{p}} \exp\left(- \frac{y_N{}^2}{B^2} \right) , \tag{9}$$

where $\rho_{\bar{p}}$ is obtained from Ref. 10[23] and B is estimated from Ref. 7.[24] Making use of Eqs. (6) and

(9), we can perform the integration in (5). Replacing the obtained $\rho_{\bar{p}} B\sqrt{\pi}$ factor by the experimental value of 32 mb $\times\langle n_{\bar{p}}\rangle$, we get

$$\rho'(y_\pi)=\frac{64\eta\langle n_{\bar{p}}\rangle\tilde{\rho}_0(y_\pi)}{\tilde{\sigma}_0}\times\left[1-\frac{\Delta}{(\Delta^2+B^2)^{1/2}}\exp\left(-\frac{{y_\pi}^2}{(\Delta^2+B^2)^{1/2}}\right)\right]. \tag{10}$$

At $y_\pi=0$, we have the simple result

$$\rho'=\tilde{\rho}_0\gamma\,, \tag{11a}$$

where

$$\gamma=\frac{64\eta\langle n_{\bar{p}}\rangle}{\tilde{\sigma}_0}\left[1-\frac{\Delta}{(\Delta^2+B^2)^{1/2}}\right]. \tag{11b}$$

Combining Eqs. (4) and (11) gives

$$\rho_\pi=\tilde{\rho}_0(1+\gamma)\,. \tag{12}$$

The quantity $(1+\gamma)$ can be calculated for the four ISR energies; the results for $\eta=1$ are listed in Table II. We see that $(1+\gamma)$ increases by about 26% from $\sqrt{s}=23$ GeV to $\sqrt{s}=53$ GeV. If the bare multiperipheral inclusive cross section $\tilde{\rho}_0$ does not fall appreciably over the ISR energies, then the bulk of the observed 30% increase of ρ_π can be considered to be due to ρ'.

We argue that since, for total cross sections, secondary trajectories are not negligible until $s\gtrsim 40$ GeV^2, within the Mueller-Regge formalism for inclusive cross sections secondary trajectories should not be negligible until $s\gtrsim 1600$ GeV^2. Therefore, a negative secondary internal Mueller-Regge coupling will result in a small rise of $\tilde{\rho}_0$ at the lower ISR energies; then when these secondary contributions become negligible at the higher ISR energies, $\tilde{\rho}_0$ will fall like $s^{\tilde{\alpha}_0-1}$. Thus, it is not unreasonable to approximate $\tilde{\rho}_0$ by a constant over the whole ISR energy range. Setting $\tilde{\rho}_0=22$ mb allows us to calculate ρ'; the results together with $\Delta\rho\equiv\rho_\pi^{\text{expt}}-\tilde{\rho}_0$ are also listed in Table II, where ρ_π^{expt} is from Ref. 10. We see that $\Delta\rho$, the increase over the bare multiperipheral inclusive cross section, is comparable to ρ'. We emphasize that because of various uncertainties (both experimental and theoretical), we should not put too much emphasis on the specific numbers. Our purpose is only to point out that once the $N\bar{N}$ cluster threshold is surpassed, there is an additional contribution ρ' to ρ_π, and under certain reasonable assumptions the magnitude of this contribution can account for the observed increase of ρ_π.

TABLE II. Determination of ρ'.

$\sqrt{s}$ (GeV)	$\langle n_{\bar{p}}\rangle$	B	$1+\gamma$	ρ' (mb)	$\Delta\rho$ (mb)
23	0.06	1.4	1.07	1.5	1.4
31	0.11	1.8	1.18	4.0	4.1
45	0.15	2.0	1.29	6.4	5.8
53	0.16	2.2	1.35	7.7	8.3

In this paper we only tried to calculate the rise of ρ_π in the ISR energy range. The rise of ρ_π from Brookhaven energies to Fermilab energies may be attributed to the previously mentioned threshold effects in meson production and a negative secondary internal Mueller-Regge coupling; there was no attempt here to calculate the rate of increase due to these effects.

If $\eta>1$, then the detected π^- could result from the final-state interaction of the $N\bar{N}$ cluster. Therefore, there will be an additional contribution ρ'' to ρ_π. For completeness, we mention that it is given by

$$\rho''(y_\pi)=2(\eta-1)\int_{-\infty}^{\infty}dy_N\,\frac{d\sigma^{\bar{p}}}{dy_N}\,g(y_\pi,y_N)\,, \tag{13}$$

where $g(y_\pi,y_N)$ is the inclusive differential multiplicity of π^- from the $N\bar{N}$ cluster at y_N.

The method we have presented can also be applied to ρ_K, the K^- inclusive cross section. With $\eta=1$, the only change is that in the final result, Eqs. (11a) and (12), $\tilde{\rho}_0$ is replaced by $\tilde{\rho}_{K_0}$. If $\tilde{\rho}_{K_0}$ may effectively be replaced by a constant over ISR energy range, then the present mechanism can account for about 30% rise.[25]

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[22]For estimating Δ', we use $\sigma_{in} = 32$ mb because the multiplicity data of Ref. 7 was normalized to $\sigma_{in} = 32$ mb.
[23]We numerically integrated their p_T distributions and obtained numbers for $\rho_{\bar{p}}$ slightly different from theirs.
[24]Ref. 10 does not give $B^{\bar{p}}$ because of the limited y_N range measured.
[25]Unfortunately, the kaon data in Ref. 10 have large fluctuations preventing us from making a meaningful comparison.