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ν- and $\bar{\nu}$-induced neutral-current processes and SU(2) × U(1) gauge models of quarks

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By examining both ν- and $\bar{\nu}$-induced neutral-current inclusive processes on an isoscalar target and elastic scattering processes on a proton target, we derive two constraints on the fermionic representation in SU(2) × U(1) gauge models. There are five classes of models that are consistent with these constraints. For each class the upper bound of $\sin^2\theta_W$ is determined. We also list models for each class with a minimum number of quarks. For these models we analyze the data of νp and $\bar{\nu}p$ elastic scattering processes. We also examine both ν- and $\bar{\nu}$-induced charged- and neutral-current inclusive processes on a proton target.

I. INTRODUCTION

Recently several authors[1,2] have compared SU(2) × U(1) gauge models[3] of quarks with the experimental data for both ν- and $\bar{\nu}$-induced neutral-current processes.[4-6] The models they have examined, however, are limited to those in which there are quarks of charge $\frac{2}{3}$ and $-\frac{1}{3}$ which form either doublets or singlets. In their analysis there are ambiguities about estimating isoscalar contributions.[7] For the inclusive processes, they had to assume a detailed model for quark-parton distributions.

In our previous paper[8] we discussed SU(2) × U(1) gauge models with more general quark representations which allow heavy quarks of exotic charges other than $\frac{2}{3}$ or $-\frac{1}{3}$ and quarks in larger representations. We derived a constraint on the quark representations in SU(2) × U(1) gauge models from the experimental observation[5] that $\sigma_{\rm nc}^{\nu N_0} > \sigma_{\rm nc}^{\bar{\nu} N_0}$, where $\sigma_{\rm nc}^{\nu(\bar{\nu})N_0}$ is the ν- ($\bar{\nu}$-) induced neutral-current inclusive cross section on an isoscalar, $N_0 = \frac{1}{2}(p+n)$, nucleon target. The constraint is energy-independent and is free from ambituities of quark-parton distributions. There are five classes of gauge models that satisfy the constraint. For each class, the upper bound of $\sin^2\theta_W$ was determined.

In this paper we examine ν- and $\bar{\nu}$-induced neutral-current inclusive processes on a proton target and νp and $\bar{\nu}p$ neutral-current elastic scattering processes in those five classes of gauge models. We derive another constraint on quark representations from the experimental observation[6] that $\sigma_{\rm el}^{\nu p} > \sigma_{\rm el}^{\bar{\nu} p}$, where $\sigma_{\rm el}^{\nu p(\bar{\nu}p)}$ is a $\nu p(\bar{\nu}p)$ neutral-current elastic scattering cross section. All of the five classes of models respect the constraint. The improved bound for $\sin^2\theta_W$ and the bound for $\kappa = F_A^0(Q^2)/F_A^1(Q^2)$ are obtained for each class of models, where $F_A^{0(1)}$ is an isoscalar (isovector) axial-vector form factor of the nucleonic neutral current. We discuss constraints on quark representations which will be given by future experiments of neutral-current inclusive processes on a proton and a neutron target.

In order to obtain the bound for $\sin^2\theta_W$, we examine, in Sec. II, the neutrino-lepton elastic neutral-current processes for a general class of SU(2) × U(1) gauge models of leptons, where a left-handed neutrino and a left-handed lepton belong to either a doublet or a triplet while the multiplet to which a right-handed lepton belongs is not specified. From the experimental data, we obtain a broad range of $\sin^2\theta_W$ values. In Sec. III, we discuss the general SU(2) × U(1) gauge models and in Sec. IV we derive constraints on quark representations from both ν- and $\bar{\nu}$-induced neutral-current inclusive processes on a nucleon target and an isoscalar target. The five classes of models discussed in our previous paper[8] are reconsidered. In Sec. V, we derive another constraint from νp- and $\bar{\nu}p$-elastic neutral-current processes. Models for each class are listed in Sec. VI, with a minimum number of quarks. For these models we analyze in Sec. VII both ν- and $\bar{\nu}$-induced quasi-elastic and elastic scattering processes on a nucleon target. We also examine both ν- and $\bar{\nu}$-induced charged- and neutral-current inclusive processes on a proton target. The results of our investigation are summarized in Sec. VIII.

II. LEPTONS IN SU(2) × U(1) GAUGE MODELS

We discuss SU(2) × U(1) gauge models of leptons. We assume that the leading effective weak-interaction Hamiltonian in SU(2) × U(1) models is given by

$$\mathcal{H}_W \sim \frac{G_F}{\sqrt{2}} (J_\mu^{cc\dagger} J_\mu^{cc} + \sqrt{\eta}\, J_\mu^{nc} J_\mu^{nc}) \,, \tag{2.1}$$

where $J_\mu^{cc\,(nc)}$ is the charged (neutral) fermionic current of vector and axial-vector type. Interactions mediated by Higgs scalar bosons are ignored. The parameter η in Eq. (2.1) stands for the relative strength of neutral-current weak interactions to charged-current interactions (see Appendix A for the definition of η).

Let

$$\begin{pmatrix} \cdot \\ \cdot \\ \cdot \\ \nu_l \\ l^- \\ \cdot \\ \cdot \\ \cdot \end{pmatrix}_L \text{ and } \begin{pmatrix} \cdot \\ \cdot \\ \cdot \\ l^- \\ \cdot \\ \cdot \\ \cdot \end{pmatrix}_R$$

be the multiplets that contain the left-handed neutrino and lepton and the right-handed lepton, respectively. The dots stand for still unspecified leptons. Let $I_{L(R)}$ be the magnitude of the weak isospin for the left- (right-) handed multiplet. We denote the third component of weak isospin for l^- ($l = e$ or μ) as

$$(I_3)_{l_L} = \tfrac{1}{2}\beta_L \,, \quad (I_3)_{l_R} = \tfrac{1}{2}\beta_R \,, \tag{2.2}$$

where $\beta_{L(R)}$ takes an integer value. As both the neutrino and left-handed lepton belong to the same left-handed I_L multiplet, the third component of weak isospin for ν_l is given by

$$(I_3)_\nu = \tfrac{1}{2}\beta_\nu = \tfrac{1}{2}(\beta_L + 2). \tag{2.3}$$

For example,

$$\begin{pmatrix} \nu, L^0 \\ l^- \end{pmatrix}_{L,R}$$

belongs to a class ($I_{L,R} = \frac{1}{2}$, $\beta_{L,R} = -1$) of models while

$$\begin{pmatrix} L^+ \\ \nu, L^0 \\ l^- \end{pmatrix}_{L,R}$$

belongs to a class ($I_{L,R} = 1$, $\beta_{L,R} = -2$). The standard Weinberg-Salam model is in the ($I_L = \frac{1}{2}$, $\beta_L = 1$, $I_R = 0$, $\beta_R = 0$) class

$$\begin{pmatrix} \nu_l \\ l^- \end{pmatrix}_L , \quad l_R^- . \tag{2.4}$$

The weak neutral current of leptons is given by

$$J_\mu^{nc} = \beta_\nu \bar{\nu}_L \gamma_\mu \nu_L + \beta_L \bar{l}_L \gamma_\mu l_L + \beta_R \bar{l}_R \gamma_\mu l_R - 2Q_l \sin^2\theta_W \bar{l}\gamma_\mu l + \cdots \,, \tag{2.5}$$

where $Q_l = -1$ is the electric charge of the lepton e or μ, and $\sin^2\theta_W$ is a parameter of the model. The dots stand for the contributions of the still unspecified leptons. Neutral-current coupling constants $g_{V,A}$ are defined as

$$J_\mu^{nc} = \tfrac{1}{2}\beta_\nu \bar{\nu}\gamma_\mu (1-\gamma_5)\nu + \bar{l}\gamma_\mu (g_V - g_A\gamma_5) l + \cdots \,. \tag{2.6}$$

For a (β_L, β_R) class of models, they are given by

$$g_V = \tfrac{1}{2}(\beta_L + \beta_R + 4\sin^2\theta_W), \quad g_A = \tfrac{1}{2}(\beta_L - \beta_R). \tag{2.7}$$

The restrictions on $\beta_{L,R}$ and $\sin^2\theta_W$ are then deduced by examining neutrino-lepton elastic scattering processes.[9] The effective weak-interaction Hamiltonian for $\nu_\mu(\bar{\nu}_\mu)e$-elastic scattering processes is given by

$$\mathcal{H}_W = \frac{G_F}{\sqrt{2}} \bar{\nu}\gamma_\mu (1-\gamma_5)\nu \bar{e}\gamma_\mu (G_V - G_A\gamma_5) e, \tag{2.8}$$

where

$$G_{V,A} = \sqrt{\eta}\,\beta_\nu g_{V,A}.$$

For $\nu_e(\bar{\nu}_e)e$ scattering, the charged-current interaction contributes to the process, and we observe that

$$G_{V,A}^{\nu_e e} = 1 + G_{V,A} = 1 + \sqrt{\eta}\,\beta_\nu g_{V,A}. \tag{2.9}$$

The total cross sections for $\nu_l(\bar{\nu}_l)e$ elastic scattering processes are

$$\sigma\begin{pmatrix} \nu_l e \to \nu_l e \\ \bar{\nu}_l e \to \bar{\nu}_l e \end{pmatrix} = \frac{2G_F^2 m_e E}{3\pi}(G_V^2 + G_A^2 \pm G_V G_A). \tag{2.10}$$

Recent experimental data[4] provide a strong restriction on $\beta_{L,R}$, while $\sin^2\theta_W$ is hardly determined [see Figs. 1(a) and 1(b)]. We consider cases where the left-handed multiplet of ν_l and l_l^- is either a doublet ($I_L = \frac{1}{2}$) or a triplet ($I_L = 1$). For the ($I_L = \frac{1}{2}$, $\beta_L = -1$) class, as in the standard Weinberg-Salam model, $\beta_R = -2$ is not allowed while models with $\beta_R = 0$ and -1 are permitted only if, for instance,

$$\begin{aligned} &\text{for } \beta_R = 0: \ 0.2 \lesssim \sin^2\theta_W \lesssim 0.3, \text{ with } \eta \sim 1 \\ &\qquad\qquad 0.3 \lesssim \sin^2\theta_W \lesssim 0.4, \text{ with } \eta \sim \tfrac{2}{5} \\ &\text{for } \beta_R = -1: \ 0.2 \lesssim \sin^2\theta_W \lesssim 0.4, \text{ with } \eta \sim 1 \\ &\qquad\qquad 0.05 \lesssim \sin^2\theta_W \lesssim 0.2, \text{ with } \eta \sim \tfrac{2}{5} \end{aligned} \tag{2.11}$$

(see Fig. 1). Thus the leptonic sector of the gauge models must be either

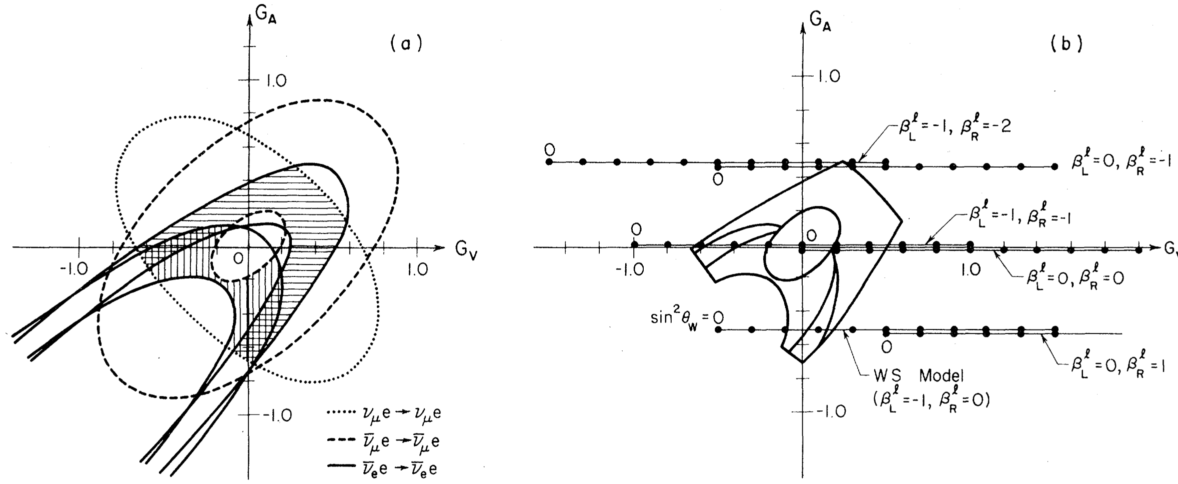

FIG. 1. (a) The experimental restrictions for the leptonic neutral-current coupling constants, G_V and G_A. The areas with vertical and horizontal lines are taken from the experimental data (Ref. 4) of the $\bar{\nu}_e e \to \bar{\nu}_e e$ elastic process for the recoiled electron energies at 1.5 MeV $< E_e <$ 3.0 MeV and 3.0 MeV $< E_e <$ 4.5 MeV, respectively. (b) The theoretical predictions of G_V and G_A for the various β_L and β_R with $\eta = 1$. For $\eta < 1$, the G_V and G_A are scaled down by a factor of $\sqrt{\eta}$.

$$L0 \text{ model: } \begin{pmatrix} \nu_l \\ l^- \end{pmatrix}_L , \quad l_R^-$$

or (2.12)

$$L1 \text{ model: } \begin{pmatrix} \nu_l \\ l^- \end{pmatrix}_L , \quad \begin{pmatrix} L^0 \\ l^- \end{pmatrix}_R$$

with a minimum number of leptons, without doubly charged leptons. In the latter model, $\beta_R = -1$, there may exist a nondiagonal neutral current involving ν_l and a left-handed L^0. An electronic neutral current is of vector type,[10] therefore, the model predicts $\sigma_{\nu_\mu e} = \sigma_{\bar{\nu}_\mu e}$ (Table I).

For a ($I_L = 1$, $\beta_L = -2$, $\beta_\nu = 0$) class, there is no ν-induced neutral-current weak interaction to

TABLE I. Leptonic multiplets in the SU(2)×U(1) model. A left-handed multiplet of lepton $l^-(=\mu, e)$ and its neutrino is assumed to be either a doublet ($I_L = \frac{1}{2}$, $\beta_L = -1$) or a triplet ($I_L = 1$, $\beta_L = 0$, or -2). The simplest right-handed multiplet is given for each value of β_R. The data of the νe-elastic scattering processes (in Ref. 4) restrict the range of $\sin^2\theta_W$ for each class of models [see Fig. 1(b)].

I_L	β_L	β_R	Left-handed multiplet	Right-handed multiplet	η	$\sin^2\theta_W$	Remarks
$\frac{1}{2}$ doublet	-1	0	$\begin{pmatrix} \nu \\ l^- \end{pmatrix}_L$	l_R^-	1	$0.2 \lesssim \sin^2\theta_W \lesssim 0.3$	$L0$ model
					$\frac{2}{5}$	$0.3 \lesssim \sin^2\theta_W \lesssim 0.4$	Not compatible with inclusive data (Ref. 8)
		-1		$\begin{pmatrix} L^0 \\ l^- \end{pmatrix}_R$	1	$0.2 \lesssim \sin^2\theta_W \lesssim 0.4$	$L1$ model; electronic nc is vector
					$\frac{2}{5}$	$0.05 \lesssim \sin^2\theta_W \lesssim 0.2$	
		-2		$\begin{pmatrix} L^+ \\ L^0 \\ l^- \end{pmatrix}_R$	Any	Not compatible with the data of $\bar{\nu}_e e$ elastic scattering (Ref. 4) for any value of η and $\sin^2\theta_W$	
1 triplet	0	Any	$\begin{pmatrix} \nu \\ l^- \\ L^{--} \end{pmatrix}_L$		Any	Not compatible with the data of $\bar{\nu}_e e$ elastic scattering (Ref. 4) for any value of η and $\sin^2\theta_W$	
	-2	Any	$\begin{pmatrix} L^+ \\ \nu \\ l^- \end{pmatrix}_L$		Any	No ν- ($\bar{\nu}$-) induced nc processes to order G_F	

order G_F. As is shown in Fig. 1, models of the class ($I_L=1$, $\beta_L=0$, $\beta_\nu=2$) are not compatible with the experimental data for any values of η, $\sin^2\theta_W$, and β_R. Thus models with a left-handed triplet

$$\begin{pmatrix} \nu_l \\ l^- \\ L^{--} \end{pmatrix}_L$$

must be rejected. In this paper we do not consider models with a larger left-handed multiplet.

III. NEUTRAL CURRENT OF QUARKS IN SU(2) × U(1) GAUGE MODELS

For the quarks, the left-handed sector of gauge models in this paper is restricted to that of the standard Weinberg-Salam model with the Glashow-Iliopoulos-Maiani (GIM) mechanism,[11] with respect to light leptons and SU(4) quarks:

$$\begin{pmatrix} \nu_l \\ l^- \end{pmatrix}_L, \quad \begin{pmatrix} u \\ d_C \end{pmatrix}_L, \quad \begin{pmatrix} c \\ s_C \end{pmatrix}_L, \qquad (3.1)$$

where d_C and s_C are the Cabibbo rotated d and s quarks. If the left-handed neutrino and lepton form a doublet, as in Eq. (2.12), then the left-handed u quark and d_C quark must form a doublet, too, because of the μ-β Cabibbo universality in the low-energy weak interaction.

Let

$$\begin{pmatrix} \cdot \\ \cdot \\ \cdot \\ u \\ \cdot \\ \cdot \\ \cdot \end{pmatrix}_R \text{ and } \begin{pmatrix} \cdot \\ \cdot \\ \cdot \\ d \\ \cdot \\ \cdot \\ \cdot \end{pmatrix}_R$$

be quark mulitplets containing a right-handed u and d quark, respectively. The dots stands for unspecified quarks.

Let us denote the third component of weak isospin for a right-handed u and d quark by

$$(I_3)_{u_R}=\tfrac{1}{2}\alpha, \quad (I_3)_{d_R}=\tfrac{1}{2}\beta, \qquad (3.2)$$

where α and β take integer values. The neutral current in a (α,β) class of models is then given by

$$J_\mu^{\rm nc}=\bar{u}_L\gamma_\mu u_L-\bar{d}_L\gamma_\mu d_L+\alpha\bar{u}_R\gamma_\mu u_R+\beta\bar{d}_R\gamma_\mu d_R$$
$$-2\sin^2\theta_W(Q_u\bar{u}\gamma_\mu u+Q_d\bar{d}\gamma_\mu d)+\cdots, \qquad (3.3)$$

where $Q_u=+\frac{2}{3}$ and $Q_d=-\frac{1}{3}$ are electric charges of a u quark and a d quark, respectively, and the dots stand for the contribution of the unspecified quarks.

Neutral-current coupling constants are introduced as

$$J_\mu^{\rm nc}=\sum_{i=u,d,\ldots}\bar{q}^i\gamma_\mu(C_V^i+C_A^i\gamma_5)q^i, \qquad (3.4)$$

where we have assumed that there is no flavor-changing neutral current at least for parts involving u and d quarks. The C's are given in terms of the set of integers (α,β) and $\sin^2\theta_W$ as

$$\begin{aligned} C_V^u&=\tfrac{1}{2}(1+\alpha)-2Q_u\sin^2\theta_W,\\ C_A^u&=\tfrac{1}{2}(-1+\alpha),\\ C_V^d&=\tfrac{1}{2}(-1+\beta)-2Q_d\sin^2\theta_W,\\ C_A^d&=\tfrac{1}{2}(1+\beta). \end{aligned} \qquad (3.5)$$

IV. ν- (ν̄-) INDUCED NEUTRAL-CURRENT INCLUSIVE PROCESSES

In a quark-parton model[12] the differential cross section for the neutral-current inclusive process on a proton target is given, in units of $\eta G_F{}^2ME/\pi$ (see Appendix B),[13,14] by

$$\frac{d^2\sigma^{\nu(\bar\nu)}}{dxdy}=\sum_{i=u,d,\ldots}[(1-y+\tfrac{1}{2}y^2)G_2^i(x)$$
$$\mp y(1-\tfrac{1}{2}y)xG_3^i(x)]\theta(W-W_i). \qquad (4.1)$$

The θ function describes the threshold of q^i-quark production. Structure functions are given in terms of neutral-current coupling constants C's defined in Eq. (3.4) by

$$\begin{aligned} G_2^{\nu p}&=G_2^{\bar\nu p}\\ &=\sum_{i=u,d,\ldots}[(C_V^i)^2+(C_A^i)^2]x[q^i(x)+\bar{q}^i(x)],\\ G_3^{\nu p}&=G_3^{\bar\nu p}\\ &=\sum_{i=u,d,\ldots}[2C_V^iC_A^i][q^i(x)-\bar{q}^i(x)], \end{aligned} \qquad (4.2)$$

where q^i ($\bar{q}^i$) is a quark distribution of a q^i quark (antiquark) in a proton. With the assumption

$$q^i(x)=\bar{q}^i(x) \qquad (4.3)$$

for all but the u and d quarks, we recognize that $G_3^{\nu p}(x)$ is determined by the valence-quark distributions alone:

$$G_3^{\nu p}=2[C_V^uC_A^uu_V(x)+C_V^dC_A^dd_V(x)], \qquad (4.4)$$

where

$$q_V^i(x)=q^i(x)-\bar{q}^i(x) \text{ for } q^i=u,d.$$

Thus we observe

$$\delta\sigma_p=\sigma_{\rm nc}^{\nu p}-\sigma_{\rm nc}^{\bar\nu p}=\tfrac{4}{3}\langle u_V\rangle\Delta_p, \qquad (4.5)$$

where

$$\Delta_p = -(C_V^u C_A^u + \rho C_V^d C_A^d)$$

and

$$\rho = \langle d_V \rangle / \langle u_V \rangle \text{ with } \langle q_V \rangle = \int_0^1 x q_V(x) dx$$

is free from ambiguous sea-quark contributions. Similarly on a neutron target

$$\delta\sigma_n = \sigma_{\rm nc}^{\nu n} - \sigma_{\rm nc}^{\bar\nu n} = \tfrac{4}{3}\langle u_V \rangle \Delta_n, \tag{4.6}$$

where

$$\Delta_n = -(\rho C_V^u C_A^u + C_V^d C_A^d).$$

ρ dependence is factored out in

$$\delta\sigma_0 = \sigma_{\rm nc}^{\nu N_0} - \sigma_{\rm nc}^{\bar\nu N_0} = \tfrac{2}{3}\langle u_V \rangle (1+\rho)\Delta_0, \tag{4.7}$$

$$\delta\sigma_1 = \sigma_{\rm nc}^{\nu N_1} - \sigma_{\rm nc}^{\bar\nu N_1} = \tfrac{2}{3}\langle u_V \rangle (1-\rho)\Delta_1, \tag{4.8}$$

where $N_{0(1)}$ stands for an isoscalar (isovector) nucleon target $N_0 = \frac{1}{2}(p+n)$ and $N_1 = \frac{1}{2}(p-n)$. Δ_0 and Δ_1 are given by

$$\begin{aligned} \Delta_0 &= -\sum_{i=u,d} C_V^i C_A^i, \\ \Delta_1 &= -(C_V^u C_A^u - C_V^d C_A^d). \end{aligned} \tag{4.9}$$

From Eq. (3.5), Δ's are given for a (α, β) class of models by

$$\begin{aligned} \Delta_0 &= \tfrac{1}{4}[2 - \alpha^2 - \beta^2 - \tfrac{4}{3}\sin^2\theta_W(3 - 2\alpha + \beta)], \\ \Delta_1 &= \tfrac{1}{4}[-\alpha^2 + \beta^2 - \tfrac{4}{3}\sin^2\theta_W(1 - 2\alpha - \beta)], \\ \Delta_p &= \tfrac{1}{4}\{1 - \alpha^2 + \rho(1-\beta^2) \\ &\quad - \tfrac{4}{3}\sin^2\theta_W[2(1-\alpha) + \rho(1+\beta)]\}, \\ \Delta_n &= \rho\Delta_p(\rho \to 1/\rho) . \end{aligned} \tag{4.10}$$

The significance of Eqs. (4.5)–(4.9) is that if it is established experimentally that $\delta\sigma \gtrless 0$, then we can use this information to impose constraints $\Delta \gtrless 0$ on quark representations in gauge models. These constraints are independent of energy and η and are free from ambiguities about estimating sea-quark contributions. In particular, the constraints for $N_{0,1}$ are independent of ρ.

Experimentally the data on ν- and $\bar\nu$-induced neutral-current processes are observed as neutral-current inclusive cross sections on an isoscalar target normalized by the charged-current inclusive cross sections,

$$R_0^{\nu(\bar\nu)} = \sigma_{\rm nc}^{\nu(\bar\nu)N_0} / \sigma_{\rm cc}^{\nu(\bar\nu)N_0} . \tag{4.11}$$

Therefore the signature of $\delta\sigma_0$ is obtained simply by comparison of $R_0^{\bar\nu}/R_0^{\nu}$ with $\sigma_{\rm cc}^{\bar\nu N_0}/\sigma_{\rm cc}^{\nu N_0}$.[15] Indeed the experimental data published so far[5] are consistent with $\delta\sigma_0 > 0$:

Gargamelle (at $E_{\nu,\bar\nu} \simeq 2$ GeV):

$$R_0^{\bar\nu}/R_0^{\nu} = 1.95 \pm 0.33 ,$$

$$\sigma_{\rm cc}^{\nu N_0}/\sigma_{\rm cc}^{\bar\nu N_0} = 2.78 \pm 0.06 ,$$

$$\sigma_{\rm nc}^{\nu N_0}/\sigma_{\rm nc}^{\bar\nu N_0} = 1.43 \pm 0.27 ;$$

HPWF (Harvard-Pennsylvania-Wisconsin-Fermilab) collaboration (at $E_\nu \simeq 41$ GeV, $E_{\bar\nu} \simeq 53$ GeV):

$$R_0^{\bar\nu}/R_0^{\nu} = 1.34 \pm 0.32 ,$$

$$\sigma_{\rm cc}^{\nu N_0}/\sigma_{\rm cc}^{\bar\nu N_0} \approx 2.0 ,$$

$$\sigma_{\rm nc}^{\nu N_0}/\sigma_{\rm nc}^{\bar\nu N_0} \approx 1.49 ;$$

CITF (Caltech-Fermilab) collaboration (at $E_\nu \simeq 50$ GeV):

$$R_0^{\bar\nu}/R_0^{\nu} = 1.45 \pm 0.38 .$$

From Fig. 2 we observe that there are only five classes of gauge models that satisfy $\Delta_0 > 0$.[8] They are labeled by $(\alpha, \beta) = (0,0)$, $(1,0)$, $(0,1)$, $(-1,0)$, and $(0,-1)$. The constraint $\Delta_0 > 0$ also restricts the range of $\sin^2\theta_W$ for each (α, β) class of models.

As there are no data of the inclusive processes on a proton or on a neutron target, the signatures of $\delta\sigma_1$, $\delta\sigma_p$, or $\delta\sigma_n$ are so far unknown. In Table II, we predicted these signatures for the range of $\sin^2\theta_W$ obtained from $\delta\sigma_0 > 0$. We also evaluated the ratio $r_{1,0} = \delta\sigma_1/\delta\sigma_0$ (Table II and Fig. 3), which will be useful in helping us find a correct class among five, once precise measurements of $\delta\sigma_0$ and $\delta\sigma_1$ are performed.

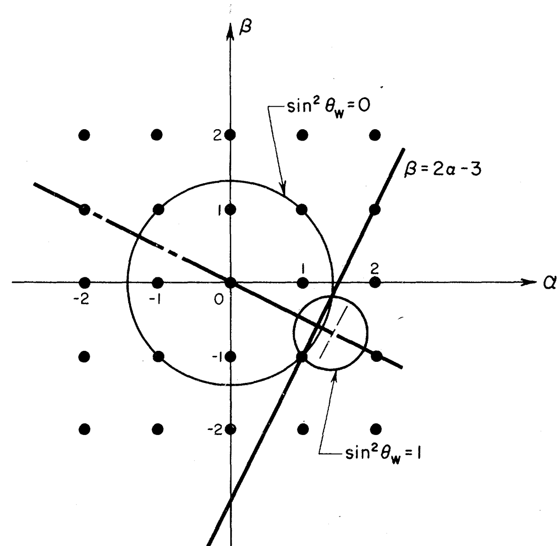

FIG. 2. The constraint on α and β by $\Delta_0(\alpha, \beta) > 0$. Two circles satisfy $\Delta_0(\alpha, \beta) = 0$ for $\sin^2\theta_W$ and $\sin^2\theta_W = 1$. Five classes, $(\alpha, \beta) = (0,0)$, $(1,0)$, $(0,1)$, $(-1,0)$, and $(0,-1)$, respect the constraint $\Delta_0 > 0$, which is implied by an experimental observation $\delta\sigma_0 = \sigma_{\rm nc}^{\nu N_0} - \sigma_{\rm nc}^{\bar\nu N_0} > 0$.

TABLE II. The predicted signatures of $\delta\sigma$ on an isovector, a proton, and a neutron target. The upper bound of $\sin^2\theta_W$ is given in the second column for each class of models with $\delta\sigma_0>0$. The signatures of $\delta\sigma_1$, $\delta\sigma_p$, and $\delta\sigma_n$ are predicted for $\sin^2\theta_W$ with $\delta\sigma_0>0$. In general, the signature of $\delta\sigma_p$ and $\delta\sigma_n$ cannot be predicted unless the value of ρ is known, where $\rho=\langle d_V\rangle/\langle u_V\rangle$ [see Eq. (4.5)]. In the last column, we computed the energy-independent ratio $[(\rho+1)/(\rho-1)]r_{1,0}=[(\rho+1)/(\rho-1)](\delta\sigma_1/\delta\sigma_0)$, as a function of $\sin^2\theta_W$ for each class (see Fig. 3).

Models (α,β)	Upper bound of $\sin^2\theta_W$	$\delta\sigma_1$	$\delta\sigma_p$	$\delta\sigma_n$	$\frac{\rho+1}{\rho-1}r_{1,0}$
(0, 0)	$\frac{1}{2}$	−	$\delta\sigma_p \gtrless 0$ for $\sin^2\theta_W \lessgtr \frac{3}{4}\frac{1+\rho}{2+\rho}$	+	$\frac{2\sin^2\theta_W}{3(1-2\sin^2\theta_W)}$
(1, 0)	$\frac{3}{4}$	−	+	+	1
(0, 1)	$\frac{3}{16}$	+	+	$\delta\sigma_n \gtrless 0$ for $\sin^2\theta_W \lessgtr \frac{3}{8}\frac{\rho}{1+\rho}$	$\frac{-3}{3-16\sin^2\theta_W}$
(−1, 0)	$\frac{3}{20}$	−	$\delta\sigma_p \gtrless 0$ for $\sin^2\theta_W \lessgtr \frac{3}{4}\frac{\rho}{4+\rho}$	+	$\frac{3(1+4\sin^2\theta_W)}{3-20\sin^2\theta_W}$
(0, −1)	$\frac{3}{8}$	+	+	+	−1
(1, −1)	No bound	0	0	0	...

V. A CONSTRAINT FROM νp AND $\bar\nu p$ ELASTIC SCATTERING

For νp and $\bar\nu p$ elastic scattering processes, we find (see Appendix C)[14]

$$\delta\sigma_{\rm el}=\sigma_{\rm el}^{\nu p}-\sigma_{\rm el}^{\bar\nu p}$$
$$=-\int dQ^2\eta\,\frac{G_F{}^2Q^2}{4\pi M^2E^2}\,(4ME-Q^2)F_{A(Q^2)}^{(1)}\times G_E^{(1)}(Q^2)\Delta_{\rm el}\,, \quad (5.1)$$

where $F_A^{(1)}$ and $G_E^{(1)}$ are the isovector axial-vector form factor and the electric Sachs form factor,[16] respectively. $\Delta_{\rm el}$ is an energy-independent quantity and is given by

$$\Delta_{\rm el}=[(C_V^u-C_A^d)+3\kappa(C_V^u+C_A^d)]\times[(1+\xi)(C_V^u-C_V^d)+3(1+\zeta)(C_V^u+C_V^d)]\,, \quad (5.2)$$

where $\xi=\mu_p-\mu_n$, $\zeta=\mu_p+\mu_n$, and $\mu_p(\mu_n)$ is an anomalous magnetic moment of a proton (neutron). κ is a parameter, $\kappa=F_A^{(0)}/F_A^{(1)}$, where $F_A^{(0)}$ is an isoscalar axial-vector form factor, where we have assumed that only u and d quarks contribute dominantly to the isoscalar form factors. In the naive quark model, $\kappa=\frac{1}{5}$.[17] With Eq. (3.5), $\Delta_{\rm el}$ should read, for an (α,β) class of gauge models, as

$$\Delta_{\rm el}=\tfrac{1}{2}[2-\alpha(1+3\kappa)+\beta(1-3\kappa)]\times[1+\mu_p-\mu_u-4\sin^2\theta_W(1+\mu_p)+\alpha(2+2\mu_p+\mu_n)+\beta(1+\mu_p+2\mu_n)]\,. \quad (5.3)$$

For $0<Q^2<4ME$, the experimental observation whether $\delta\sigma_{\rm el}\gtrless 0$, would provide another constraint to classify gauge models (α,β), according to $\Delta_{\rm el}\gtrless 0$. The published data are consistent with $\delta\sigma_{\rm el}$

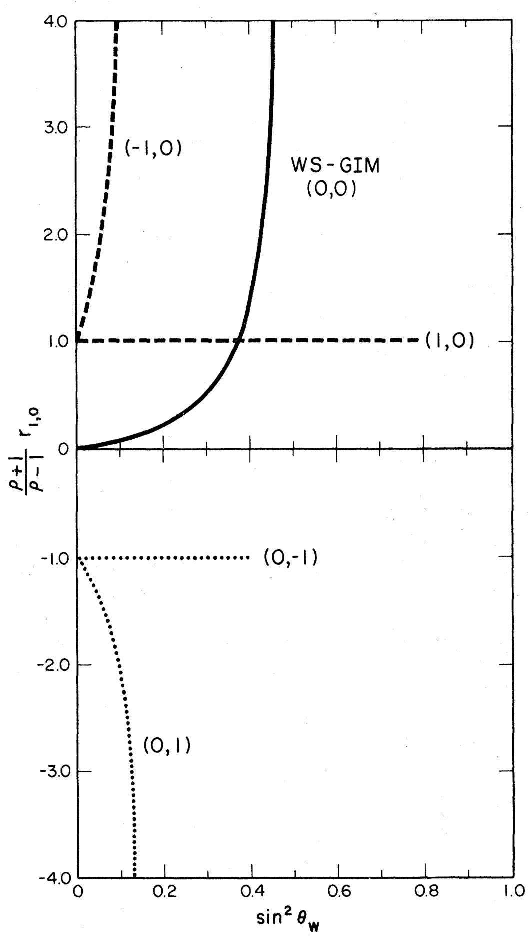

FIG. 3. $\sin^2\theta_W$ dependence of $(\rho+1/\rho-1)r_{1,0}$, given in Table II, for five classes of models. The range of $\sin^2\theta_W$ is restricted to respect $\Delta_0>0$. The (0, 0), (1, 0), and (0, 1) classes of models can be differentiated by observing this quantity unless $\sin^2\theta_W\approx\frac{3}{8}$.

>0 and $\Delta_{el}>0$. Indeed, the Harvard-Pennsylvania-Wisconsin collaboration (HPW)[6] has recently reported an experimental ratio of $\bar{\nu}p$ and νp neutral-current elastic cross sections: $\sigma_{el}^{\bar{\nu}p}/\sigma_{el}^{\nu p}=0.4\pm0.2$ for $0.3<Q^2<0.9$ $(\mathrm{Gev}/c)^2$ with $\nu(\bar{\nu})$ flux peaked at 1 GeV.

We have found that all of the five classes of gauge models that respect the "$\delta\sigma_0>0$" constraint satisfy this "$\delta\sigma_{el}>0$" constraint. The range of $\sin^2\theta_W$ and κ for each class of models is given in Table III.

VI. GAUGE MODELS OF QUARKS WITH $\delta\sigma_0>0$ AND $\delta\sigma_{el}>0$

In order to compare the experimental data with predictions of gauge models, various models are specified. In this section we will list the models to be compared with the data. A set of restrictions is imposed for the construction of the models.

(i) The absence of nondiagonal terms is assumed in the neutral current for parts involving at least a u and a d quark.

(ii) As is suggested in the low-energy phenomenology, there is no right-handed charged current among the SU(3) triplet u, d, and s quarks.

(iii) The right-handed current of $\bar{c}_R\gamma_\mu d_R$ type is not admitted.[18]

(iv) In order to explain the recent experimental data on $\bar{\nu}$-induced charged-current inclusive processes on an isoscalar target,[15] i.e., the high-y anomaly and the rise in $R_c=\sigma_{cc}^{\bar{\nu}N_0}/\sigma_{cc}^{\nu N_0}$, we assume that there exists a new quark b of charge $-\frac{1}{3}$ (or q of charge $-\frac{4}{3}$) with a right-handed current of $\bar{u}_R\gamma_\mu b_R$ (see Ref. 19) (or $\bar{d}_R\gamma_\mu q_R$; see Ref. 20).

In Table IV, we present the models we have examined for each class, assuming a minimum number of quark flavors. We show only right-handed multiplets containing a u quark or a d quark as a member. The common left-handed doublets are already specified in Eq. (3.1).

The gauge model should be anomaly-free so that it is renormalizable.[21] It is known (see Appendix D) that the triangle anomalies vanish in an SU(2) × U(1) gauge model if there exists the following relation among multiplets of the model:

$$\sum_{\substack{\text{left-handed}\\ \text{multiplet } i}} I_L^i(I_L^i+1)(2I_L^i+1)Y_L^i = \sum_{\substack{\text{right-handed}\\ \text{multiplet } j}} I_R^j(I_R^j+1)(2I_R^j+1)Y_R^j, \quad (6.1)$$

where $Y_{L(R)}^i$ is a weak hypercharge for a multiplet i with a weak isospin $I_{L(R)}^i$. However, since we are mostly interested in finding the phenomenological structure of weak currents involving a u and a d quark in the SU(2) × U(1) gauge model, the internal cancellation of the anomalies, while assumed, is not considered in detail. In the same spirit, the existence of a heavy lepton[22] is ignored.

VII. THE EXAMINATION OF THE EXPERIMENTAL DATA IN GAUGE MODELS

In our previous paper[8] we made a numerical analysis[23] of both ν- and $\bar{\nu}$-induced charged-current and neutral-current inclusive processes on an isoscalar target for the gauge models with $\delta\sigma_0>0$. Table V shows the estimated values of $\sin^2\theta_W$, η, and masses of intermediate vector bosons for each model we compared with the data.[5]

In this paper, we present predictions of both ν- and $\bar{\nu}$-induced inclusive processes on a proton target for each model with the parameters of the model given in Table V. See Figs. 4–8. We observe that there are three distinct groups of models: the standard Weinberg-Salam model, a group of $(0,0)_{II}$, $(1,0)$, and $(0,-1)$ models that contain a b quark of charge $-\frac{1}{3}$ to explain the high-y anomaly and the rise in R_c, and another group of $(0,0)_I$, $(0,1)$, and $(-1,0)$ models in which a q quark of charge $-\frac{4}{3}$ causes these new effects.

We also present predictions for νp and $\bar{\nu}p$ elastic scattering for those models, assuming the standard dipole form factors.[1,16,17] In order to make comparisons possible, we computed the elastic cross sections using the BNL neutrino-beam spectrum.[6] The experimental cuts for HPW experiments[6] are taken into consideration. The results are given in Figs. 9–11 and in Table V.

TABLE III. An upper bound of $\sin^2\theta_W$ obtained from the constraint $\delta\sigma_{el}>0$ for elastic scattering processes. An experimental observation that $\delta\sigma_{el}>0$ provides an upper bound of $\sin^2\theta_W$ as a function of μ_p and μ_n, the anomalous magnetic moment of a proton and of a neutron. $\kappa=F_A^{(0)}/F_A^{(1)}$ and $\kappa=\frac{1}{5}$ in the simple quark model.

Models (α,β)	Upper bound of $\sin^2\theta_W$	Bound of κ
$(0,0)$	$\dfrac{1+\mu_p-\mu_n}{4(1+\mu_p)}\sim0.42$	No bound
$(1,0)$	$\frac{3}{4}$	$\kappa<\frac{1}{3}$
$(0,1)$	$\dfrac{2(1+\mu_p)+\mu_n}{4(1+\mu_p)}\sim0.33$	$\kappa<1$
$(-1,0)$	$\dfrac{-(1+\mu_p+2\mu_n)}{4(1+\mu_p)}\sim0.09$	$\kappa>-1$
$(0,-1)$	$\dfrac{-3\mu_n}{4(1+\mu_p)}\sim0.51$	$\kappa>-\frac{1}{3}$
$(1,-1)$	$\delta\sigma_{el}=0$ for any $\sin^2\theta_W$ and κ	

TABLE IV. The SU(2)×U(1) gauge models with $\delta\sigma_0>0$ and $\delta\sigma_{el}>0$. For each class (α,β) we present the minimum model in the second column. The models are extended to contain a new heavy quark, a b quark or a q quark, in the other two columns. An asterisk indicates that the minimum model already contains a b (or a q) quark. The models with a dagger will be analyzed in Sec. VII.

Class of models (α,β)	Minimum model	Model with $\bar{u}_R\gamma_\mu b_R$ current	Model with $\bar{d}_R\gamma_\mu q_R$ current
(0, 0)	u_R, d_R The standard Weinberg-Salam model†	$\begin{pmatrix} g \\ u \\ b \end{pmatrix}_R, d_R$ $(0,0)_{\text{II}}$ model†	$u_R, \begin{pmatrix} t \\ d \\ q \end{pmatrix}_R$ $(0,0)_{\text{I}}$ model†
(1, 0)	$\begin{pmatrix} u \\ b \end{pmatrix}_R, d_R$	* (1, 0) model†	...
(0, 1)	$u_R, \begin{pmatrix} d \\ q \end{pmatrix}_R$	...	* (0, 1) model†
(−1, 0)	$\begin{pmatrix} g \\ u \end{pmatrix}_R, d_R$	...	$\begin{pmatrix} g \\ u \end{pmatrix}_R, \begin{pmatrix} t \\ d \\ q \end{pmatrix}_R$ (−1, 0) model†
(0, −1)	$u_R, \begin{pmatrix} t \\ d \end{pmatrix}_R$	$\begin{pmatrix} g \\ u \\ b \end{pmatrix}_R, \begin{pmatrix} t \\ d \end{pmatrix}_R$ (0, −1) model†	...
(1, −1)	$\begin{pmatrix} u \\ b \end{pmatrix}_R, \begin{pmatrix} t \\ d \end{pmatrix}_R$ Vectorlike model	*	...

TABLE V. Results of numerical analysis. The upper bounds for $\sin^2\theta_W$ implied by $\delta\sigma_0>0$ and $\delta\sigma_{el}>0$ are given in the second column. The results of the numerical analysis of ν- ($\bar{\nu}$-) induced inclusive processes on an isoscalar target are shown in the third column. In the fourth column, it is remarked that the (0, 0), (1, 0), and (0, 1) models show good agreement with the data of νp and $\bar{\nu} p$ elastic scattering processes. For each model the structure of the leptonic sector is suggested in the last column. (See Table I for identification of $L0$ and $L1$.)

Model (α,β)	Upper bound of $\sin^2\theta_W$	Inclusive processes on an isoscalar target					Elastic scattering	Lepton model
		$\sin^2\theta_W$	η	Quark masses (GeV)	Masses of gauge bosons (GeV)	Remarks		
$(0,0)_{\text{I, II}}$	0.42	~0.3	1	$m_{b,q}\approx 6$, $m_t\gtrsim 10$	$M_W\approx 68$, $M_Z\approx 81$	Good	Good, κ-independent	$L0, L1$
(1, 0)	$\frac{3}{4}$	~0.4	1	$m_b\approx 5$	$M_W\approx 59$, $M_Z\approx 76$	Good	Good for $\kappa=0$, poor for $\kappa=\frac{1}{5}$	$L0, L1$
(0, 1)	$\frac{3}{16}$	$\lesssim 0.1$	$\frac{2}{5}$	$m_q\approx 5$	$M_W\gtrsim 118$, $M_Z\gtrsim 156$	Fair	Good for $\kappa=\frac{1}{5}$	$L1$
(−1, 0)	0.09	$\lesssim 0.1$	$\frac{2}{5}$	$m_q\approx 6$, $m_{g,t}\gtrsim 10$	$M_W\gtrsim 118$, $M_Z\gtrsim 156$	Fair	Poor	$L1$
(0, −1)	$\frac{3}{8}$	$\lesssim 0.1$	$\frac{2}{5}$	$m_b\approx 6$, $m_{g,t}\lesssim 10$	$M_W\gtrsim 118$, $M_Z\gtrsim 156$	Fair	Poor	$L1$

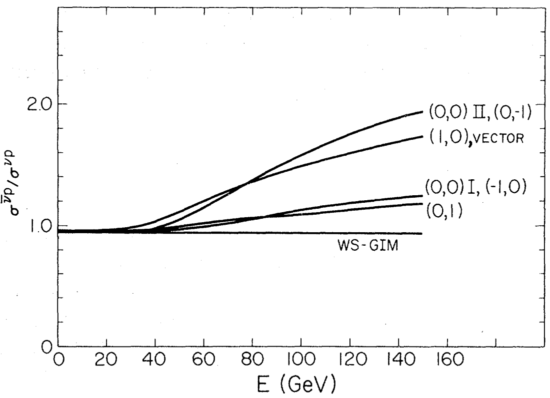

FIG. 4. The energy dependence of $R_c^p = \sigma_{cc}^{\bar{\nu}p}/\sigma_{cc}^{\nu p}$ for five classes of models. The value, heavy-quark masses, $\sin^2\theta_W$, and η given in Table V, are used for Figs. 4–8.

VIII. SUMMARY

(i) The neutrino-lepton elastic scattering data provide the restriction on $\sin^2\theta_W$:

$$0.2 \lesssim \sin^2\theta_W \lesssim 0.4, \quad \text{for } \eta \simeq 1$$

$$0.05 \lesssim \sin^2\theta_W \lesssim 0.2, \quad \text{for } \eta \simeq \tfrac{2}{5}.$$

(ii) There are only five classes of gauge models which are consistent with the experimental constraints, $\delta\sigma_0 > 0$ and $\delta\sigma_{el} > 0$. Those classes are $(\alpha, \beta) = (0,0)$, $(1,0)$, $(0,1)$, $(-1,0)$, and $(0,-1)$. An upper bound for $\sin^2\theta_W$ is given for each class of models.

(iii) From the detailed analysis of νp and $\bar{\nu}p$ elastic scattering data, we observe that the $(0,0)$ model is consistent with the data with $\sin^2\theta_W \sim 0.3$ and $\kappa \sim \frac{1}{5}$, while the $(1,0)$ model shows a reasonable fit to the data only at $\kappa \sim 0$. The $(0,1)$ model may not be rejected if $\sin^2\theta_W \sim 0$. The rest, i.e., the $(-1,0)$ and the $(0,-1)$ models, disagree with the data.

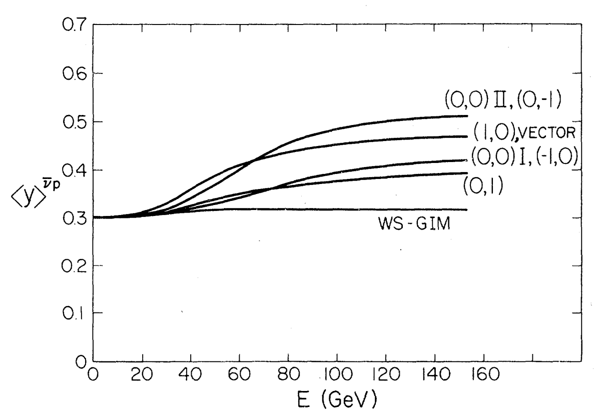

FIG. 5. The energy dependence of $\langle y \rangle^{\bar{\nu}p}$ for five classes of models.

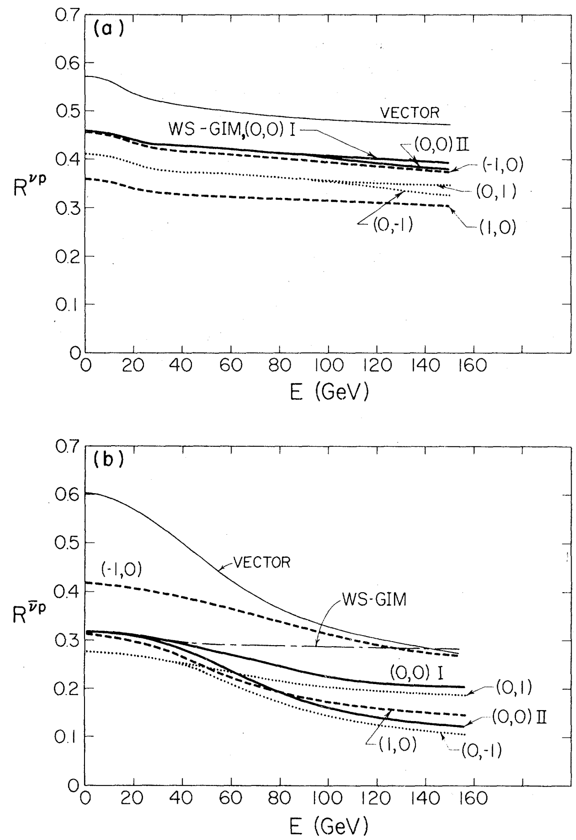

FIG. 6. The energy dependence of (a) $R^{\nu p}$ and (b) $R^{\bar{\nu}p}$ for five classes of models.

(iv) With both the ν- and $\bar{\nu}$-induced charged-current and neutral-current inclusive processes on a proton target, the group of $(0,0)_{\rm II}$, $(1,0)$, and $(0,-1)$ models show distinctively different predictions from the group of $(0,0)_{\rm I}$, $(0,1)$, and $(-1,0)$ models for R_c^p, $\langle y \rangle^{\bar{\nu}p}$, and $R^{\nu(\bar{\nu})p}$. For the former group, a new heavy b quark of charge $-\frac{1}{3}$ is present to cause the high-y anomaly and the rise in R_c, while for the latter the new heavy q quark of charge $-\frac{4}{3}$ is behind such new phenomena.

(v) The interesting question whether a b quark or a q quark is responsible for these new phenomena could be answered in the future simply by measuring $\bar{\nu}$-induced charged-current inclusive processes on a proton target.

(vi) If a b quark is assumed to exist, then the difference between the $(0,0)_{\rm II}$ and the $(1,0)$ models may be observed by taking the energy-independent ratios such as $r_{1,0}$ unless $\sin^2\theta_W \simeq \frac{3}{8}$. The precise measurement of $R^{\bar{\nu}p}/R^{\nu p}$ is also useful (see Fig. 7).

(vii) If a q quark is assumed to exist in the $(0,0)_{\rm I}$ model with $\sin^2\theta_W \simeq 0.3$ or in the $(0,1)$ model with $\sin^2\theta_W \simeq 0$, then the exotic charge $-\frac{4}{3}$ of the q quark would cause many drastic effects in future experi-

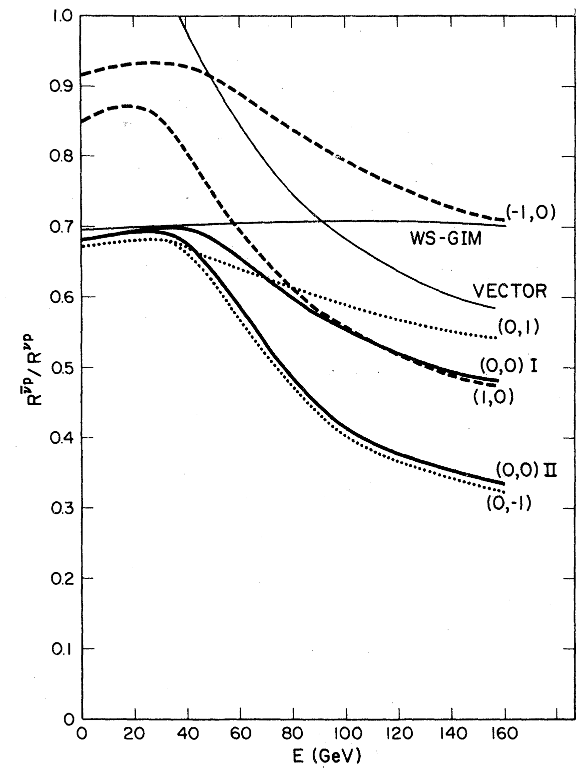

FIG. 7. The energy dependence of $R^{\bar{\nu}p}/R^{\nu p}$, which is independent of η and depends only on $\sin^2\theta_W$.

ments.[8] For instance, in e^+e^- colliding-beam experiments, the R value, $R=\sigma(e^+e^-\to \text{hadrons})/\sigma(e^+e^-\to\mu^+\mu^-)$, increases with $\Delta R=\frac{16}{3}$ at energies over $q\bar{q}$ threshold. There would be an extremely narrow ψ-like bound state whose partial decay widths to lepton pairs would be considerably large, in particular, to heavy-lepton[22] pairs. Doubly charged bosons should be observed in $\bar{\nu}$-induced inclusive experiments. For some of the models there even exists a heavier g quark of exotic charge $+\frac{5}{3}$. The effects over $g\bar{g}$ threshold would also be conspicuous.

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APPENDIX A: AN EFFECTIVE WEAK-INTERACTION HAMILTONIAN IN AN SU(2) × U(1) GAUGE MODEL

Let

$$\psi_L(I_L)=\begin{pmatrix} \circ \\ \cdot \\ \cdot \\ \nu \\ l^- \\ \cdot \\ \cdot \\ \cdot \end{pmatrix}_L \quad \text{and} \quad \psi_R(I_R)=\begin{pmatrix} \cdot \\ \circ \\ \cdot \\ l^- \\ \cdot \\ \cdot \\ \cdot \end{pmatrix}_R$$

be the multiplets containing a left-handed neutrino

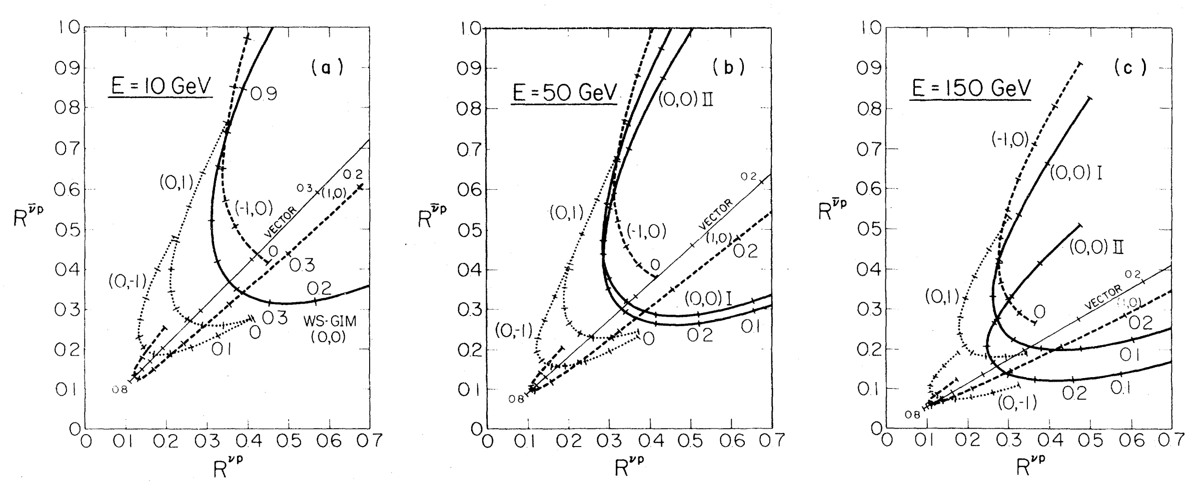

FIG. 8. The $R^{\bar{\nu}p}$-$R^{\nu p}$ correlation curves at (a) $E=10$ GeV, (b) $E=50$ GeV, and (c) $E=150$ GeV.

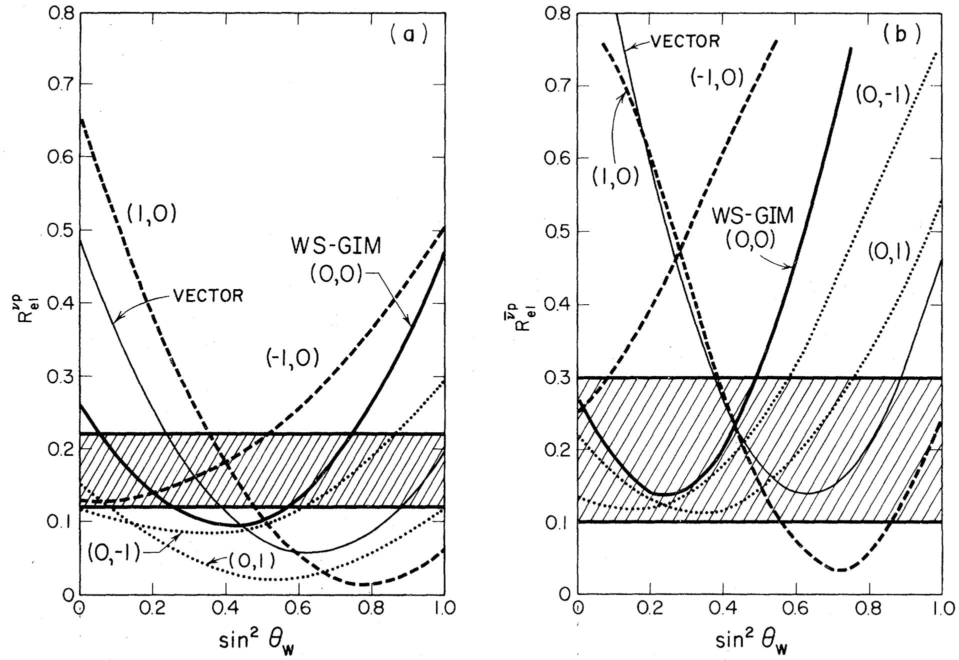

FIG. 9. $\sin^2\theta_W$ dependence of (a) $R_{el}^{\bar\nu p}$ and (b) $R_{el}^{\nu p}$ for five classes of models. The shaded area is the allowed region consistent with the data. $\kappa=0$ for the (1, 0) model and $\kappa=\frac{1}{5}$ for others.

and lepton, and a right-handed lepton, respectively. The dots denote still unspecified leptons. $I_{L(R)}$ is the magnitude of a multiplet $\psi_{L(R)}$. Let

$$(I_3)_{l_{L(R)}}$$

be the third component of weak isospin for a left- (right-) handed lepton. The Lagrangian density of our interest is

$$\mathcal{L}_{\rm int}\sim -\bar\psi_{L,R}\gamma_\mu[gA_\mu^a I_{L,R}^a + g'B_\mu \tfrac{1}{2}Y_{L,R}]\psi_{L,R}\,, \quad \text{(A1)}$$

where A_μ^a $(a=1,2,3)$ and B_μ are SU(2) and U(1) gauge bosons, with coupling constants g and g', respectively. By the Higgs mechanism, the charged vector bosons $W_\mu^\pm=(1/\sqrt{2})(A^1\mp iA^2)_\mu$, and a neutral vector boson $Z_\mu=\cos\theta_W A_\mu - \sin\theta_W B_\mu$, become massive with a mass of M_W and M_Z, respectively, where $\sin\theta_W=g'/(g^2+g'^2)^{1/2}$. A photon field, $A_\mu^{\rm em}=\sin\theta_W A_\mu^3+\cos\theta_W B_\mu$, couples to a conserved electromagnetic current with a coupling constant, $e=gg'/(g^2+g'^2)^{1/2}$. Hence a Gell-Mann–Nishijima formula holds:

$$Q^i=I_3^i+\tfrac{1}{2}Y^i\,, \quad \text{(A2)}$$

where Q^i is an electric charge of the i fermion.

The Lagrangian (A1) is rewritten for ν_l and l^- as

$$\mathcal{L}_{\rm int}\sim -\frac{g}{\sqrt{2}}\,(t_{l_L}{}^{-1/2}\bar\nu_L\gamma_\mu l_L + t_{l_R}{}^{-1/2}\bar L_R^0\gamma_\mu l_R)W_\mu^\dagger$$
$$-(g^2+g'^2)^{1/2}[(I_3)_\nu\bar\nu_L\gamma_\mu\nu_L+(I_3)_{l_L}\bar l_L\gamma_\mu l_L+(I_3)_{l_R}\bar l_R\gamma_\mu l_R - Q_l\sin^2\theta_W\bar l\gamma_\mu l]+\cdots\,, \quad \text{(A3)}$$

where $(I_3)_\nu=(I_3)_{l_L}+1$ and the dots denote a Lagrangian involving other leptons:

$$t_{l_{L,R}}=[(I_{L,R}-(I_3)_{l_{L,R}})(I_{L,R}+(I_3)_{l_{L,R}}+1)]^{-1}\,.$$

Identifying the leptonic charged current for ν and l_L^- as

$$\tfrac{1}{2}j_\mu^{\rm cc}=\bar\nu_L\gamma_\mu l_L^-\,, \quad \text{(A4)}$$

we relate the Fermi coupling constant G_F to parameters of the gauge model by

$$\frac{G_F}{\sqrt{2}}=\frac{g^2}{8M_W^2}\,t_{l_L}{}^{-1} \quad \text{(A5)}$$

to obtain an effective weak-interaction Hamiltonian of the charged currents:

$$\mathcal{H}_W=\frac{G_F}{\sqrt{2}}\,j_\mu^{\dagger\rm cc}j_\mu^{\rm cc}\,. \quad \text{(A6)}$$

Identifying the leptonic neutral current for ν and l_L^- as

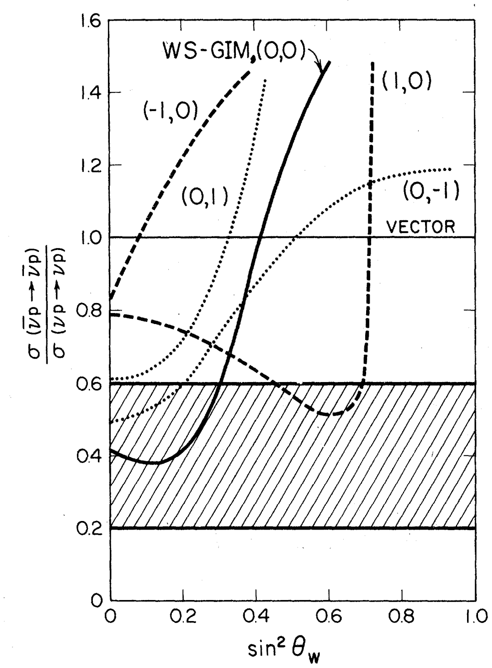

FIG. 10. $\sin^2\theta_W$ dependence of $\sigma(\bar\nu p \to \bar\nu p)/\sigma(\nu p \to \nu p)$ for five classes of models. See caption of Fig. 9.

$$\tfrac{1}{2}j_\mu^{\rm nc} = (I_3)_\nu \bar\nu_L \gamma_\mu \nu_L + (I_3)_{l_L} \bar l_L \gamma_\mu l_L + (I_3)_{l_R} \bar l_R \gamma_\mu l_R - Q_l \sin^2\theta_W \bar l \gamma_\mu l \,, \qquad (A7)$$

we find an effective weak-interaction Hamiltonian of the neutral current given by

$$\mathcal{H}_W = \frac{G_F}{\sqrt{2}} \sqrt{\eta}\, j_\mu^{\rm nc} j_\mu^{\rm nc} \,, \qquad (A8)$$

where

$$\eta = \eta_0 t_{l_L}{}^2 \,, \quad \eta_0 = (M_W/M_Z \cos\theta_W)^4 \,. \qquad (A9)$$

A factor $t_{l_L}{}^2$ is present in the definition of η as a result of the normalization of the charged-current interaction Hamiltonian [see Eq. (A5)]. For a $[I_L = \frac{1}{2},\ (I_3)_l = -\frac{1}{2}]$ class of models, $t_{l_L} = 1$, $\eta = \eta_0$. For the standard Weinberg-Salam model, $\eta = \eta_0 = 1$.

APPENDIX B: ν- (ν̄-) INDUCED NEUTRAL-CURRENT INCLUSIVE CROSS SECTIONS $\sigma_{\rm nc}^{\nu(\bar\nu)p}$ ON A PROTON TARGET IN A QUARK-PARTON MODEL

These inclusive cross sections, $\sigma_{\rm nc}^{\nu(\bar\nu)p}$, are given by

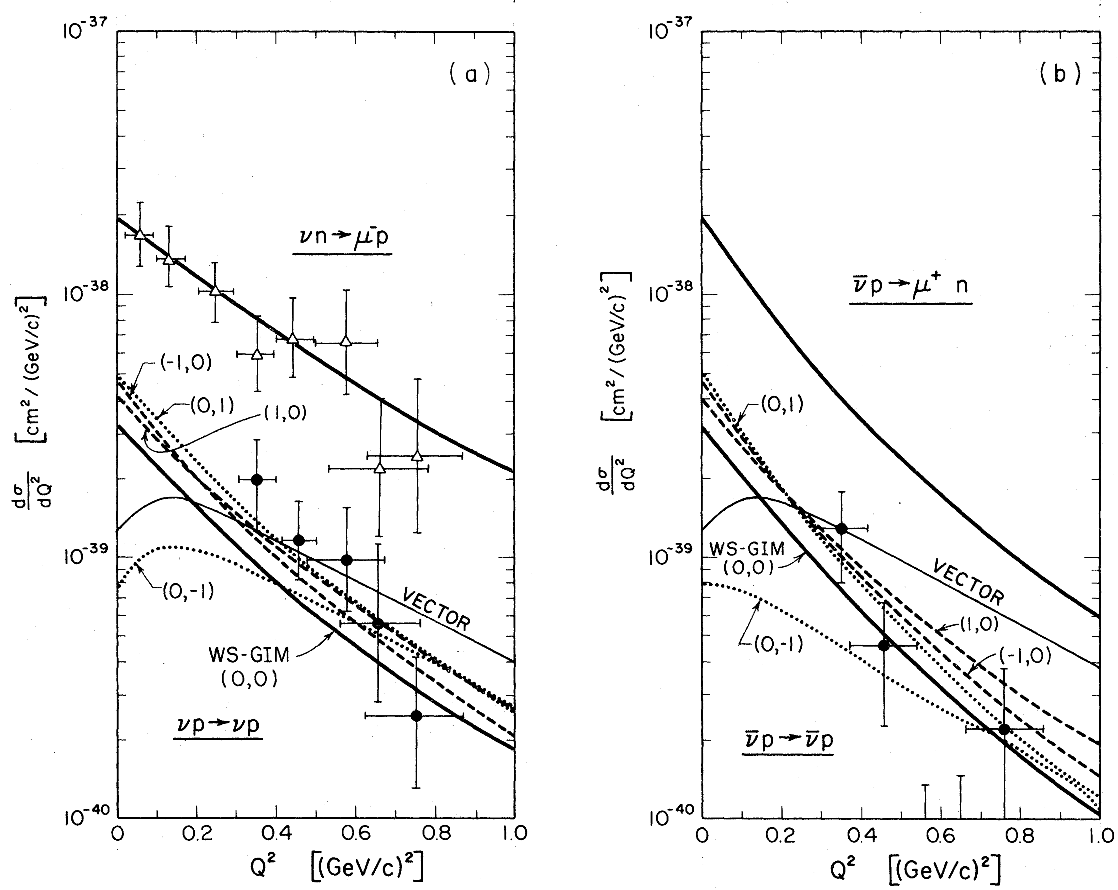

FIG. 11. The differential cross section $d\sigma/dQ^2$ for (a) $\nu p \to \nu p$ and (b) $\bar\nu p \to \bar\nu p$ elastic scattering processes for five classes of models.

$$\frac{d^2\sigma}{dE'd\Omega_{E'}} = \frac{G_F^{\,2}}{(2\pi)^2}\frac{E'}{E} l_{\mu\nu}W^{\mu\nu} ,$$

$$l_{\mu\nu} = k'_\mu k_\nu + k'_\nu k_\mu - (k'\cdot k) g_{\mu\nu} \mp i\epsilon_{\mu\nu\rho\sigma}k^\rho k'^\sigma ,$$

$$W^{\mu\nu} = \tfrac{1}{2}\sum_{p'_n} \langle P|J^\nu_{\rm nc}|p'_n\rangle\langle p'_n|J^\mu_{\rm nc}|P\rangle \times \delta^{(4)}(P+q-p'_n) , \quad \text{(B1)}$$

where $k(E=k_0)$, $k'(E'=k'_0)$, and P are a four-momentum of an incoming neutrino, an outgoing neutrino, and a target proton, respectively. The − (+) sign in $l_{\mu\nu}$ corresponds to ν- ($\bar{\nu}$-) induced reactions. $W^{\mu\nu}(P, q=k-k')$ can be written in terms of scaling structure functions as

$$W^{\mu\nu}(P,q) \to -\frac{g^{\mu\nu}}{M} G_1(x) + \frac{P^\mu P^\nu}{M^2\nu} G_2(x) + \frac{i\epsilon^{\mu\nu\rho\sigma}}{2M^2\nu} P_\rho q_\sigma G_3(x) ,$$

$$G_l = \sum_i G_l^i(x) , \quad l=1,2,3 \quad \text{(B2)}$$

where $x=-q^2/2P\cdot q$, $\nu = Ey = E-E'$. Then we observe

$$\frac{d^2\sigma}{dxdy} = \frac{G_F^{\,2}ME}{\pi}\left[y^2xG_1 + (1-y)G_2 \mp y(1-\tfrac{1}{2}y)xG_3\right] . \quad \text{(B3)}$$

In the quark-parton model, the G's are calculable from

$$W^{\mu\nu} = \tfrac{1}{2}\sum_{i,p'_i} [q^i(x)\langle q^i(xP)|J^\nu|p'_i\rangle\langle p'_i|J^\mu|q^i(xP)\rangle + \bar{q}^i(x)\langle \bar{q}^i(xP)|J^\nu|p'_i\rangle\langle p'_i|J^\mu|\bar{q}^i(xP)\rangle] \times \delta^{(4)}(xP+q-p'_i)\theta(W-W_i) , \quad \text{(B4)}$$

where $q^i(x)$, $[\bar{q}^i(x)]$ is an i-quark [antiquark] distribution in a proton target. A function θ describes the threshold effect. If the neutral current of the i quark is written by

$$J_\mu^{{\rm nc},i} = \bar{q}^i\gamma_\mu(C_V^i + C_A^i\gamma_5)q^i , \quad \text{(B5)}$$

then $G_l^i(x)$ is easily calculated to be

$$G_2^i(x) = 2xG_1^i(x) = [(C_V^i)^2 + (C_A^i)^2] [q^i(x) + \bar{q}^i(x)] x ,$$

$$G_3^i(x) = 2C_V^iC_A^i[q^i(x) - \bar{q}^i(x)] . \quad \text{(B6)}$$

For a heavy-i-quark excitation in neutral currents, the slow-rescaling variable must be assumed (see Ref. 13).

APPENDIX C: νp ($\nu\bar{p}$) NEUTRAL-CURRENT ELASTIC CROSS SECTIONS

Let us define nucleonic neutral-current form factors f_1, f_2, f_A, and f_P, by

$$\langle N(p')|J_\mu^{\rm nc}|N(p)\rangle = \frac{1}{\sqrt{2}}\bar{u}(p')\Gamma_\mu(p',p)u(p) ,$$

$$\Gamma_\mu(p',p) = \gamma_\mu f_1(q^2) + \frac{i}{2M}\sigma_{\mu\nu}q^\nu f_2(q^2) + \gamma_\mu\gamma_5 f_A(q^2) + \frac{q_\mu}{M}\gamma_5 f_p(q^2) , \quad \text{(C1)}$$

where $q=p'-p$ and we have assumed Hermiticity and time-reversal invariance of the nucleonic neutral current. It is straightforward to obtain differential cross sections for elastic processes:

$$\frac{d^2\sigma_{\rm el}^{\nu(\bar{\nu})p}}{dQ^2} = \eta\frac{G_F^{\,2}M^2}{8\pi E^2}\left[A(Q^2) \mp \left(\frac{s-u}{M^2}\right)Q^2B(Q^2) + \left(\frac{s-u}{M^2}\right)^2 C(Q^2)\right] , \quad \text{(C2)}$$

where $Q^2 = -q^2 = |\vec{Q}|^2 - Q_0^{\,2}$, $s-u = 4ME - Q^2$, and (±) for ($^\nu_{\bar{\nu}}$) p scattering. A, B, and C are given in terms of form factors. In particular,

$$B = \frac{Q^2}{M^2} f_A(Q^2)[f_1(Q^2) + f_2(Q^2)] . \quad \text{(C3)}$$

We observe

$$\frac{d\delta\sigma_{\rm el}}{dQ^2} = \frac{d\sigma_{\rm el}^{\nu p}}{dQ^2} - \frac{d\sigma_{\rm el}^{\bar{\nu}p}}{dQ^2} = -\eta\frac{G_F^{\,2}Q^2}{4\pi M^2E^2}(4ME - Q^2)f_A(Q^2) \times [f_1(Q^2) + f_2(Q^2)] . \quad \text{(C4)}$$

The nucleonic neutral current is given by

$$J_\mu^{\rm nc} = \bar{N}\tfrac{1}{2}\gamma_\mu[(C_V + C_A\gamma_5)\tau_3 + (D_V + D_A\gamma_5)1]N , \quad \text{(C5)}$$

where N is a nucleon isospin doublet, $N = ({}^p_n)$, and $C_{V(A)}$ and $D_{V(A)}$ are an isovector and an isoscalar vector (axial-vector) coupling constant, respectively. The nucleonic form factors in Eq. (C1) are given by

$$f_1 + f_2 = C_V(F_1^{(1)} + \xi F_2^{(1)}) + D_V(F_1^{(0)} + \zeta F_2^{(0)}) ,$$

$$f_A = C_AF_A^{(1)} + D_AF_A^{(0)} , \quad \text{(C6)}$$

where superscripts (1) and (0) stand for an isovector and an isoscalar contribution, respectively, and $\xi = \mu_p - \mu_n$ and $\zeta = \mu_p + \mu_n$. Introducing Sachs's magnetic and electric form factors,

$$G_M^{(1)} = F_1^{(1)} + \xi F_2^{(1)} = (1+\xi) G_E^{(1)} ,\qquad G_M^{(0)} = F_1^{(0)} + \zeta F_2^{(0)} = (1+\zeta) G_E^{(0)} , \tag{C7}$$

we obtain

$$f_1 + f_2 = [C_V(1+\xi) + D_V(1+\zeta)] G_E^{(1)} . \tag{C8}$$

Little is known about the isoscalar axial-vector form factor $F_A^{(0)}$. With a parameter $\kappa = F_A^{(0)}/F_A^{(1)}$, $f_A = (C_A + D_A\kappa) F_A^{(1)}$. It follows immediately that

$$\delta\sigma_{el} = -\int_{4ME - Q^2 > 0} dQ^2 \eta \frac{G_F{}^2 Q^2}{4\pi M^2 E^2} (4ME - Q^2) \times F_A^{(1)}(Q^2) G_E^{(1)}(Q^2) \Delta_{el} , \tag{C9}$$

$$\Delta_{el} = (C_A + x D_A)[C_V(1+\xi) + D_V(1+\zeta)] . \tag{C10}$$

The coupling constants $C_{V,A}$ and $D_{V,A}$ may be specified by those of the quark neutral current as

$$C_{V(A)} = C^u_{V(A)} - C^d_{V(A)} ,\qquad D_{V(A)} = 3[C^u_{V(A)} + C^d_{V(A)}] , \tag{C11}$$

where $\binom{u}{d}$ quarks are assumed to behave as a doublet under the isospin of a strong interaction. We assumed that the isospin structure of nucleonic form factors is governed dominantly by nucleonic u and d quarks. The effect of nonnucleonic quarks, such as s, $\bar{s}$, c, and $\bar{c}$, in the isoscalar axial-vector form factor is included in a parameter κ, which is $\frac{1}{5}$ in the naive quark model.

APPENDIX D: TRIANGLE ANOMALY IN THE SU(2) × U(1) GAUGE MODEL

In order for a gauge model to be renormalizable, it is necessary that the model be anomaly-free. For an SU(2) × U(1) gauge model, the condition to be anomaly-free is given in terms of fermionic electric charges $\{Q_i\}$, weak isospin, and its third components

$$\{I^{i_{L(R)}}, I_3^{i_{L(R)}}\}$$

for left- (right-) handed fermions $\{i_{L(R)}\}$ by

$$\text{(a)}\ \sum_{i_L} Q_{i_L} (I^{i_L})^2 = \sum_{i_R} Q_{i_R} (I^{i_R})^2 ,$$
$$\text{(b)}\ \sum_{i_L} Q_{i_L} (I_3^{i_L})^2 = \sum_{i_R} Q_{i_R} (I_3^{i_R})^2 , \tag{D1}$$
$$\text{(c)}\ \sum_{i_L} Q_{i_L}{}^2 I_3^{i_L} = \sum_{i_R} Q_{i_R}{}^2 I_3^{i_R} .$$

$\sum_{i_{L(R)}}$ stands for the summation over all the left- (right-) handed fermions in the model. These three conditions are easily obtained by requiring the absence of anomalies in $A^\pm A^\mp B$, $A^0 A^0 B$, and BBB triangle diagrams with all the possible fermionic loops, where A and B stand for an SU(2) and a U(1) vector gauge boson, respectively. Recall that a U(1) gauge boson couples to a weak hypercharge

$$Y^{i_{L(R)}}/2 = Q_{i_{L(R)}} - I_3^{i_{L(R)}}$$

(Gell-Mann–Nishijima formula).

For each multiplet with weak isospin I, the following relations hold:

$$\sum_{i\in I} Q_i{}^2 I_3^i = 2\sum_{i\in I} Q_i (I_3^i)^2 = \tfrac{2}{3} I(I+1) \sum_{i\in I} Q_i = \tfrac{2}{3} I(I+1)(2I+1) Y_I , \tag{D2}$$

where $\sum_{i\in I}$ runs over all the fermions in a multiplet and Y_I is a weak hypercharge of the multiplet. Then the three conditions in Eq. (D1) are equivalent to each other, and we find the anomaly-free condition to be

$$\sum_{I_L} I_L(I_L+1)(2I_L+1) Y_{I_L} = \sum_{I_R} I_R(I_R+1)(2I_R+1) Y_{I_R} . \tag{D3}$$

The summation $\sum_{I_{L(R)}}$ is performed over all the left- (right-) handed multiplets. In terms of charges, Eq. (D3) is identical to

$$\sum_{I_L} I_L(I_L+1) \sum_{i_L \in I_L} Q_{i_L} = \sum_{I_R} I_R(I_R+1) \sum_{i_R \in I_R} Q_{i_R} . \tag{D4}$$

For a model with only singlets and $(2I+1)$-plets, the condition Eq. (D4) reduces to

$$\sum_{i_L} Q_{i_L} = \sum_{i_R} Q_{i_R} . \tag{D5}$$

The summation is taken over all the fermions in the $(2I+1)$-plets. It is straightforward to see that the standard Weinberg-Salam model with the GIM mechanism, where fermions form doublets and singlets, is anomaly-free.

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