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# Failure of the U(12) algebra in quantum chromodynamics*

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We show that the full U(12) algebra does not exist in quantum chromodynamics.

It is well known that certain algebras, derived from free field theory, are anomalous in the renormalizable field theories. The $U(6)\times U(6)$ algebra of Feynman, Gell-Mann, and Zweig[1] is one such example. The commutation rules involving the space components of vector and axial-vector currents contain anomalies.[2] But in quantum chromodynamics (QCD), the color gauge theory of strong interactions, Bég[3] has pointed out that owing to asymptotic freedom the anomalies vanish, and the Feynman–Gell-Mann–Zweig (FGZ) $U(6)\times U(6)$ algebra exists in QCD. One then naturally asks the question: Does the full U(12) algebra also exist in QCD? The purpose of this note is to demonstrate that the answer is *no*.

For QCD with $f$ flavors the Lagrangian is

$$\mathcal{L} = -\tfrac{1}{4}(F^i_{\mu\nu})^2 + \bar\psi i(\partial_\mu - igt^iA^i_\mu)\gamma^\mu\psi - \bar\psi\mathfrak{M}\psi. \qquad (1)$$

We can construct color-singlet currents

$$\begin{aligned} V^a_\mu &= \bar\psi\tfrac{1}{2}\lambda^a\gamma_\mu\psi,\\ A^a_\mu &= \bar\psi\tfrac{1}{2}\lambda^a\gamma_\mu\gamma_5\psi,\\ S^a &= \bar\psi\tfrac{1}{2}\lambda^a\psi,\\ P^a &= \psi\tfrac{1}{2}\lambda^a\gamma_5\psi,\\ T^a_{\mu\nu} &= \bar\psi\tfrac{1}{2}\lambda^a\sigma_{\mu\nu}\psi. \end{aligned} \qquad (2)$$

They belong to the adjoint representation of SU($f$). For the free field case the $V,A$ currents form a $U(2f)\times U(2f)$ algebra, $V$, $A$, $S$, $P$, and $T$ currents from a $U(4f)$ algebra. In what follows we will restrict ourselves to the case $f=3$. One key element in Bég's proof of the closure of the $U(6)\times U(6)$ algebra is that the anomalous dimensions of the $V$ and $A$ currents vanish. Let us study the anomalous dimensions of $V$, $A$, $S$, $P$, and $T$ currents in QCD. In leading order of the coupling constant the relevant Feynman diagrams are depicted in Fig. 1. We have, in a general covariant gauge with gauge parameter $\alpha$, the quark wave-function renormalizations

$$Z_F = 1 - G(1-\alpha)\ln\Lambda, \qquad (3)$$

and proper vertex renormalization of currents,

$$\begin{aligned} Z_{V,A} &= 1 - G(1-\alpha)\ln\Lambda,\\ Z_{S,P} &= 1 - G(4-\alpha)\ln\Lambda,\\ Z_T &= 1 + G\alpha\ln\Lambda, \end{aligned} \qquad (4)$$

where $G=(g^2/8\pi^2)\mathrm{Tr}\,t^it^i$, and $\Lambda$ is the cutoff.

The anomalous dimension of a current $\theta$ is

$$\gamma_\theta \equiv -\frac{\partial}{\partial\ln\Lambda}\left(\frac{Z_F}{Z_\theta}\right). \qquad (5)$$

We find that $\gamma_{V,A}=0$,

$$\begin{aligned} \gamma_{S,P} &= -6G,\\ \gamma_T &= 2G. \end{aligned} \qquad (6)$$

They are independent of the gauge parameter $\alpha$ as was to be expected. Since the anomalous dimensions of the $S$, $P$, and $T$ currents are not equal, we conclude that most of the commutation rules in the U(12) algebra do not exist in QCD. In particular, the computation rules of $U(6)\times U(6)$ generators

$$\begin{aligned} Q^a_i &= \int d^3x\, V^a_i,\\ {}^5Q^a_i &= \int d^3x\, A^a_i \end{aligned} \qquad (7)$$

with $S$, $P$, and $T$ currents cannot be realized. Similar conclusions in perturbation theory have been made by Adler and Tung,[2] Jackiw and Preparata.[2]

Recently in a very interesting paper Caldi and Pagels[4] propose to use one consequence of the

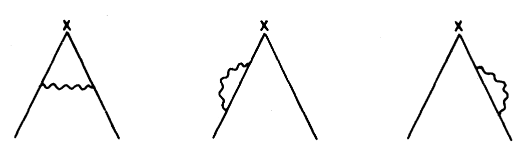

FIG. 1. Feynman diagrams for calculation of the anomalous dimensions of currents.

U(12) algebra. They treat the $S^a$, $P^a$, and $T^a_{\mu\nu}$ as members of a $(6,\overline{6})+(\overline{6},6)$ representation under the $U(6)\times U(6)$ algebra of FGZ. Our result shows that this cannot in general be realized in QCD. It must be made as an additional assumption.

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[1]R. P. Feynman, M. Gell-Mann, and G. Zweig, Phys. Rev. Lett. 13, 678 (1964).

[2]S. L. Adler and Wu-ki Tung, Phys. Rev. D 1, 2846 (1970); R. Jackiw and G. Preparata, Phys. Rev. Lett. 22, 975 (1969); 22, 1162(E) (1969).

[3]M. A. B. Bêg, Phys. Rev. D 11, 1165 (1975).

[4]D. G. Caldi and H. Pagels, Phys. Rev. D 14, 809 (1976).