Classical limit of the quantum theory of radiation damping. II

Richard L. Knight* and Jack E. Chatelain
Department of Physics, Utah State University, Logan, Utah 84321
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In part I of this work the classical limit of the quantum theory of radiation damping was studied for scattering processes involving photons and neutral or charge-symmetric mesons interacting with nucleons through $S(S)$, $P(A)$, $V(V)$, and $V(T)$ coupling. Here this investigation is extended to include the $A(A)$ coupling. In both papers the resulting cross sections are compared with corresponding results from the classical field and action-at-a-distance theories of Havas. The cross sections agree with those of the action-at-a-distance theory in all cases which have been considered.

I. INTRODUCTION

In a previous paper,[1] which will henceforth be designated I, scattering processes involving photons, neutral and charge-symmetric mesons interacting with nucleons through $S(S)$, $P(A)$, $V(V)$, and $V(T)$ couplings were calculated using the Heitler radiation damping theory. These calculations were made in both a neutral theory and a charge-symmetric theory. In the neutral theory, the couplings $S(S)$ and $A(A)$ brought about scattering through nucleon recoil. Since we were interested in a classical (low-energy) limit the derived matrix elements were obtained by performing a Foldy-Wouthuysen transformation of the Hamiltonian to the first order in reciprocal nucleon mass. The resulting first-order matrix elements together with the possible Feynman diagrams led to the following compound matrix elements for scalar and vector coupling:

$$H_{fi}=\frac{2\pi\hbar^4c^2g^2K^2}{M\epsilon^3}\,\vec{n}_f\cdot\vec{n},\quad S(S)$$

$$H_{LL}=\frac{2\pi\hbar^2g^2\mu^2c^4}{M\epsilon^3}\,\vec{n}_f\cdot\vec{n}_i,\quad V(V)$$

$$H_{TL}=-\frac{i2\pi\hbar^2g^2\mu c^2}{M\epsilon^2}\,\vec{j}_f\cdot\vec{n}_i,\quad V(V)$$

$$H_{TT}=\frac{2\pi\hbar^2g^2}{M\epsilon}\,\vec{j}_f\cdot\vec{j}_i,\quad V(V)$$

$$H_{LP}=\frac{2\pi\hbar^2ge\mu c^2}{M\epsilon^2}\,\vec{n}_f\cdot\vec{J}_i,\quad V(V)$$

$$H_{TP}=\frac{2\pi\hbar^2ge}{M\epsilon}\,\vec{j}_f\cdot\vec{J}_i,\quad V(V)$$

$$H_{PP}=\frac{2\pi\hbar^2e^2}{M\epsilon}\,\vec{J}_f\cdot\vec{J}_i,\quad V(V)\,.$$

For the tensor and pseudovector coupling the lowest-order scattering is due to the nucleon spin, so the nonrelativistic infinite-nucleon-mass approximation applies. The matrix elements are

$$H_{fi}=\frac{2\pi f^2\hbar^2c^2K^2}{\chi^2\epsilon^2}\times(\vec{\sigma}\cdot\vec{n}_f\vec{\sigma}\cdot\vec{n}_i-\vec{\sigma}\cdot\vec{n}_i\vec{\sigma}\cdot\vec{n}_f),\quad P(A)$$

$$H_{fi}=\frac{2\pi f^2\hbar^2c^2K^2}{\chi^2\epsilon^2}\times(\vec{\sigma}\cdot\vec{j}_f\times\vec{n}_f\vec{\sigma}\cdot\vec{j}_f\times\vec{n}_i-\vec{\sigma}\cdot\vec{j}_i\times\vec{n}_i\vec{\sigma}\cdot\vec{j}_f\times\vec{n}_f),\quad V(T)$$

$$H_{fi}=\frac{2\pi\hbar^2c^2}{\epsilon^2}\,(\vec{\sigma}\cdot\vec{A}\vec{\sigma}\cdot\vec{B}-\vec{\sigma}\cdot\vec{B}\vec{\sigma}\cdot A),\quad V(T)$$

where $\vec{A}$ or $\vec{B}=Kg\vec{j}$ or $ke\vec{J}$.

In the charge-symmetric theory the lowest-order scattering can be thought of as a consequence of the isotopic spin. The scattering matrix elements in the $V(T)$ case are as follows:

$$H_{LL}=\frac{2\pi c^2g^2\hbar^2K^2}{\epsilon^2\chi^2},$$

$$H_{TL}\text{ or }H_{LT}=\frac{2\pi c^2fgK^2\hbar^2}{\epsilon^2\chi^2}\,\vec{\sigma}\cdot\vec{j}_{i\,\mathrm{or}\,f},$$

$$H_{TT}=\frac{2\pi c^2f^2\hbar^2K^2}{\epsilon^2\chi^2}\,\vec{\sigma}\cdot\vec{j}_f\vec{\sigma}\cdot\vec{j}_i,$$

$$H_{LL_0}=H_{L_0L}=\frac{4\pi c^2gg_0\hbar^2K^2}{\epsilon^2\chi^2},$$

$$H_{LT_0}\text{ or }H_{T_0L}=\frac{4\pi c^2gf_0\hbar^2K^2}{\epsilon^2\chi^2}\,\vec{\sigma}\cdot\vec{j}_{0\,i\,\mathrm{or}\,f},$$

$$H_{TL_0}\text{ or }H_{L_0T}=\frac{4\pi c^2g_0f\hbar^2K^2}{\epsilon^2\chi^2}\,\vec{\sigma}\cdot\vec{j}_{i\,\mathrm{or}\,f},$$

$$H_{TT_0}\text{ or }H_{T_0T}=(2\pi c^2f_0f\hbar^2K^2)\times[(\vec{\sigma}\cdot\vec{j}_f\vec{\sigma}\cdot\vec{j}_{0i}+\vec{\sigma}\cdot\vec{j}_{0i}\vec{\sigma}\cdot\vec{j}_f)\text{ or }(\vec{\sigma}\cdot\vec{j}_{0f}\vec{\sigma}\cdot\vec{j}_i+\vec{\sigma}\cdot\vec{j}_i\vec{\sigma}\cdot\vec{j}_{0f})],$$

$$H_{L_0L_0}=H_{L_0T_0}=H_{T_0L_0}=0,$$

$$H_{T_0T_0}=\frac{2\pi c^2f_0^2\hbar^2K^2}{\epsilon^2\chi^2}\times(\vec{\sigma}\cdot\vec{j}_{0f}\vec{\sigma}\cdot\vec{j}_{0i}-\vec{\sigma}\cdot\vec{j}_{0i}\vec{\sigma}\cdot\vec{j}_{0f}).$$

The above matrix elements are then used in the Heitler damping equation, i.e.,

$$U_{AB} = H_{AB} - i\pi H_{AC} U_{CB} \rho \, d\Omega . \quad (1)$$

The solutions U_{AB} are related to the transition probabilities by

$$w = (2\pi/\hbar) \, |U_{AB}|^2 \rho . \quad (2)$$

As indicated in I the summations involved in solving the damping equations can either be "classical" or "quantum mechanical" depending on whether or not a classical limit is desired. The cross sections in I were compared in the classical limit with corresponding action-at-a-distance cross sections and were in agreement in all cases.

Cross sections based on this formalism have been calculated by Craft[2] for the scattering of charged vector mesons and pseudovector mesons by nucleons. The calculations were carried out for both the classical action-at-a-distance and field theories. The differences in the predictions of these two theories for pseudovector mesons were found to be much more pronounced than those already compared in I.

Because of these greater differences, we have extended the comparison study initiated in I to include processes involving $A(A)$ coupling. Our procedure is to calculate scattering cross sections from Heitler's radiation damping theory, to take the classical low-energy limit of these, and to compare the final cross sections with those of Craft. In order to compare the charged cross sections, we must set Craft's vector coupling constants equal to zero. His neutral cross sections are valid to order $1/M$ in the nucleon mass. Since we have used the infinite-mass approximation, we must let $M \to \infty$ for purposes of comparison. In these limiting cases our results are in agreement with Craft's classical action-at-a-distance cross sections.

II. SCATTERING CROSS SECTIONS FOR CHARGED MESONS

In this section cross sections will be obtained for the scattering of charged mesons by nucleons with the assumption that the total charge of the system is +2 or −1 times the electronic charge. In order to calculate the matrix elements H_{AB}, it is necessary to quantize the pseudovector field. For the quantized interaction Hamiltonian density [with $A(A)$ coupling], we find

$$H = \psi^* \left(\frac{\hbar}{2V}\right)^{1/2} \sum_{i,j} \left\{ \lambda f \left(\frac{c^2(\lambda^2 + K_i^2 \delta_{ij})}{\lambda^2 + K_i^2(1-\delta_{ij})} \right)^{1/4} \left[(A_{ij} - B_{ij}^*) \vec{\sigma} \cdot \vec{e}_{ij} \, e^{i\vec{K}_i \cdot \vec{r}} + (A_{ij}^* - B_{ij}) \vec{\delta} \cdot \vec{e}_{ij} \, e^{-i\vec{K}_i \cdot \vec{r}} \right] \right.$$
$$\left. + icf \left[\frac{\lambda^2 + K_i^2(1-\delta_{ij})}{(\lambda^2 + K_i^2 \delta_{ij}) c^2} \right]^{1/4} \left[(A_{ij} + B_{ij}^*) \gamma_5 \vec{\nabla} \cdot \vec{e}_{ij} \, e^{i\vec{K}_i \cdot \vec{r}} - (A_{ij}^* + B_{ij}) \gamma_5 \vec{\nabla} \cdot \vec{e}_{ij} \, e^{-i\vec{K}_i \cdot \vec{r}} \right] \right\} \psi . \quad (3)$$

The approximation will be made that the nucleon mass is infinite, in which case the nonrelativistic unit of the above equation is obtained by putting

$$\alpha \to 0, \quad \beta \to 1, \quad \gamma_\mu \to 0, \quad \vec{\sigma}_{(\mathrm{Dirac})} \to \vec{\sigma}_{(\mathrm{Pauli})} . \quad (4)$$

From the resulting Hamiltonian we obtain the following second-order matrix elements in the usual way:

$$\begin{aligned} H_{ll} &= -A \vec{\sigma} \cdot \vec{n}_i \, \vec{\sigma} \cdot \vec{n}_f , \\ H_{lt} &= -B \vec{\sigma} \cdot \vec{n}_i \, \vec{\sigma} \cdot \vec{j}_f , \\ H_{tl} &= -B \vec{\sigma} \cdot \vec{j}_i \, \vec{\sigma} \cdot \vec{n}_f , \\ H_{tt} &= -D \vec{\sigma} \cdot \vec{j}_i \, \vec{\sigma} \cdot \vec{j}_f . \end{aligned} \quad (5)$$

The coefficients A, B, and D are

$$\begin{aligned} A &= \frac{\lambda f^2}{2V} , \\ B &= \frac{c\hbar \lambda^2 f^2}{2\epsilon V} , \\ D &= \frac{c^2 \hbar^2 \lambda^2 f^2}{2\epsilon^2 V} . \end{aligned} \quad (6)$$

Equation (1) is conveniently solved by putting

$$U_{fi} = - X_{fi} K_i K_f - Y_{fi} K_f K_i \quad (7)$$

and properly determining the coefficients X_{fi} and Y_{fi}. Here K is the inner product of $\vec{\sigma}$ with the appropriate polarization vector. Since taking a classical limit is anticipated, we treat $\vec{\sigma}$ as a classical unit vector in the summations over intermediate states (see I). The results are

$$\begin{aligned} X_{ll} &= \frac{A}{1 - i\frac{8}{3}\pi^2 \rho(A+2D)} , \\ X_{lt} = X_{tl} &= \frac{B}{1 - i\frac{8}{3}\pi^2 e(A+2D)} , \\ X_{tt} &= \frac{D}{1 - i\frac{8}{3}\pi^2 \rho(A+2D)} , \\ Y_{ll} &= \frac{-i\frac{4}{3}\pi^2 \rho(A+2D)}{[1 - i\frac{4}{3}\pi^2 \rho(A+2D)][1 - i\frac{8}{3}\pi^2 \rho(A+2D)]} , \\ Y_{lt} = Y_{tl} &= \frac{-i\frac{4}{3}\pi^2 \rho B(A+2D)}{[1 - i\frac{4}{3}\pi^2 \rho(A+2D)][1 - i\frac{8}{3}\pi^2 \rho(A+2D)]} , \\ Y_{tt} &= \frac{-i\frac{4}{3}\pi^2 \rho D(A+2D)}{[1 - i\frac{4}{3}\pi^2 \rho(A+2D)][1 - i\frac{4}{3}\pi^2 \rho(A+2D)]} . \end{aligned} \quad (8)$$

Equations (2), (6), and (7) yield the differential cross sections. When integrating to obtain total cross sections, we again treat $\vec{\sigma}$ as a classical unit vector. In addition, we substitute $\hbar/2 \rightarrow I$. The total cross sections are then

$$\left.\begin{aligned} \phi_{ll} &= \frac{\omega^2\lambda^2 f^4 \cos^2\theta}{48\pi I^2 c^4} \\ \phi_{lt} &= \frac{\lambda^4 f^4 \cos^2\theta}{48\pi I^2 c^2} \\ \phi_{tl} &= \frac{\lambda^4 f^4 \cos^2\theta}{24\pi I^2 c^2} \\ \phi_{tt} &= \frac{\lambda^6 f^4 \cos^2\theta}{24\pi I^2 \omega^2} \end{aligned}\right\} \times \frac{1}{1+\left(\dfrac{\lambda K\omega f^2}{24\pi I c^2}\right)^2\left[1+2\left(\dfrac{c\lambda}{\omega}\right)^2\right]^2}, \tag{9}$$

where θ is the angle between the polarization vector of the incident meson. These cross sections are in agreement with the classical action-at-a-distance cross sections calculated by Craft.

III. SCATTERING CROSS SECTIONS FOR NEUTRAL MESONS

The scattering problem considered here is similar to the one of Sec. II. The major difference is that conservation of charge rules out one Feynman diagram in the charged case. In addition, the first-order matrix elements for the creation or annihilation of neutral mesons is larger than the corresponding ones for charged mesons by a factor of $\sqrt{2}$.

The compound matrix elements are

$$\begin{aligned} H_{ll} &= 2A(\vec{\sigma}\cdot\vec{n}_f\vec{\sigma}\cdot\vec{n}_i - \vec{\sigma}\cdot\vec{n}_i\vec{\sigma}\cdot\vec{n}_f), \\ H_{lt} &= 2B(\vec{\sigma}\cdot\vec{n}_f\vec{\sigma}\cdot\vec{j}_i - \vec{\sigma}\cdot\vec{j}_i\vec{\sigma}\cdot\vec{n}_f), \\ H_{tl} &= 2B(\vec{\sigma}\cdot\vec{j}_f\vec{\sigma}\cdot\vec{n}_i - \vec{\sigma}\cdot\vec{n}_i\vec{\sigma}\cdot\vec{j}_f), \\ H_{tt} &= 2D(\vec{\sigma}\cdot\vec{j}_f\vec{\sigma}\cdot\vec{j}_i - \vec{\sigma}\cdot\vec{j}_i\vec{\sigma}\cdot\vec{j}_f). \end{aligned} \tag{10}$$

In the same manner as before we obtain

$$\left.\begin{aligned} \phi_{ll} &= \frac{\lambda^2\omega^2 f^4 \sin^2\theta}{3\pi I^2 c^4} \\ \phi_{lt} &= \frac{\lambda^4 f^4 \sin^2\theta}{3\pi I^2 c^2} \\ \phi_{tl} &= \frac{2\lambda^4 f^4 \sin^2\theta}{3\pi I^2 c^2} \\ \phi_{tt} &= \frac{2\lambda^6 f^4 \sin^2\theta}{3\pi I^2 \omega^2} \end{aligned}\right\} \times \frac{1}{1+\left(\dfrac{\lambda K\omega f^2}{6\pi I c^2}\right)^2\left[1+2\left(\dfrac{\lambda c}{\omega}\right)^2\right]^2}. \tag{11}$$

These cross sections also agree with Craft's corresponding action-at-a-distance cross sections.

IV. DISCUSSION

The results of this calculation broaden the area of agreement between the classical limit of the Heitler theory of radiation damping and the action-at-a-distance theory of Wheeler and Feynman.

*Present address: Sandia Laboratories, Albuquerque, New Mexico.

[1]J. Chatelain and P. Havas, Phys. Rev. 129, 1459 (1963).

[2]Alba D. Craft and Peter Havas, Phys. Rev. 154, 1460 (1967).