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High-energy total cross sections on nuclei

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We discuss some features of the description of hadron-nucleus total and inelastic cross sections at present and foreseeable energies within the framework of Reggeon calculus. In the first (theoretical) part we discuss the Amati-Stanghellini-Fubini cancellation, emphasizing the analytic features involved, especially the consistency with unitarity in the Reggeon-particle channel. In the second (phenomenological) part we show that the observed triple-Pomeron coupling is so small that effects of Reggeon interactions can probably be ignored both at Fermilab energies and for observations of extensive air showers. It follows that the Glauber formula should describe all current experiments with only small corrections for inelastic intermediate states.

I. INTRODUCTION

The object of this paper is to investigate some aspects of the description of hadron-nucleus total cross sections within the framework of Reggeon calculus at present and foreseeable energies.

We can illustrate the main problems by considering the double-scattering term for a nucleon incident on a heavy nucleus of radius R (Fig. 1). The forward scattering amplitude can be reduced in the lab frame to the form[1]

$$F=\frac{1}{m(2\pi)^3}\binom{A}{2}\int d^3k\,\rho(4\vec{k}^2)f(s,s_1,\vec{k}^2)\ , \tag{1.1}$$

where $s=(p_1+p_2)^2$, $s_1=(p_1-k)^2$, ρ is the nuclear form factor $[\rho(0)=1]$, and f is the six-point function in the top half of Fig. 1. At fixed s_1, as $s\to\infty$ it approaches the form shown in Fig. 2. (We neglect rescattering on individual nucleons.)

Since negligible energy can be transferred to a nucleon without knocking it out of the nucleus we have

$$k_{11}\simeq\frac{s_1-m^2}{2E_L}\ , \tag{1.2}$$

where $k_{11}=k\cdot p_1/|\vec{p}_1|$ and E_L is the lab energy of the projectile. The essential features of this situation can be seen by supposing that the nuclear form factor is Gaussian:

$$\rho(4k^2)=e^{-\gamma k^2}\ . \tag{1.3}$$

We then obtain

$$F=\frac{1}{8\pi^2 m\gamma}\binom{A}{2}\int\frac{ds_1}{2E_L}\,f(s,s_1,\vec{k}^2=0)e^{-\gamma k_{11}{}^2}\ . \tag{1.4}$$

The integration is over a causal contour. The role of the nuclear form factor is to confine the s_1 integration to

$$|s_1-m^2|\lesssim 2E_L/\sqrt{\gamma}\ . \tag{1.5}$$

If E_L and A are fixed, and $\gamma\to\infty$ so that the target is very rarified, scattering takes place entirely through one-particle intermediate states of the projectile (the Glauber result). If $E_L\to\infty$ at fixed γ, A then the endpoints of the s_1 contour approach infinity, and propagation through heavy intermediate states can be important.

In this paper we discuss two questions. In Sec. II we summarize our view of the transition from pure elastic to allowed inelastic intermediate states. We try to give a fairly complete description of one aspect of this transition—the Amati-Stanghellini-Fubini (ASF) cancellation.[2]

The second part of this paper concerns the phenomenology of the Reggeon interactions that occur inside the six-point function (Fig. 2). These interactions correspond to heavy inelastic intermediate states ($s_1\gg m^2$). In Sec. III we discuss the interpretation of data at energies ranging from those at Fermilab to those of extensive air showers (EAS). On the basis of the value of the triple-Pomeron coupling deduced from $pp\to pX$ experiments we conclude that Reggeon interactions are only a small effect at Fermilab and are also not very important in the interpretation of inelastic cross sections on air as deduced from EAS. The Glauber formula is not an unreliable way to analyze those data, at least with present-day errors on the data.

II. ASF PROBLEM

We have already noted that for lab energies such that only elastic rescattering is coherent on the nucleus, the s_1 integration is dominated by the pole at $s_1=m^2$. Picking up this pole we obtain the Glauber approximation. However, as first noted in Ref. 3 in a field-theory model, the pole is contained in completely planar graphs. Therefore, when E_L increases so that inelastic intermediate

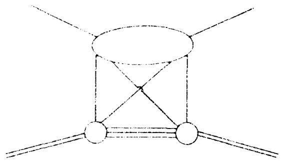

FIG. 1. Double scattering on a heavy nucleus.

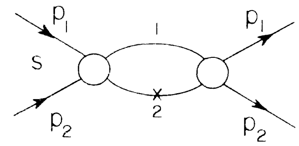

FIG. 3. ASF diagram.

states are coherent on the nucleus, the pole term is canceled,[1] and the Glauber approximation should cease to be valid. This point has recently been emphasized and discussed from a configuration-space point of view in Ref. 5.

On the other hand, it has become clear from experiments at Fermilab[6] that the Glauber approximation is quite good for all nuclei even at 300 GeV/c. This observation involves no real contradiction with soft field theory.[7,8] Firstly, unitarity in the Pomeron-particle channel guarantees positivity of the s_1 discontinuity. Secondly, it appears from experiment that $\alpha(0)>1$. This means that no subtraction is necessary in the Gribov-Migdal sum rule. The pole at $s_1=m^2$ does indeed give a lower bound to the cut strength.

Our objective in this section is merely to clarify the discussion of the ASF cancellation, and so is of only theoretical interest. In particular we want to show that the ASF cancellation can be understood for any sum of graphs in the soft field theory (with one technical assumption explained below) and for any Regge intercept.

We first examine arbitrary planar graphs to see if they can ever build a Regge cut. Recall the situation for the simplest planar graph (Fig. 3) where the blobs have high-energy behavior determined by a Regge pole. We set the denominator 2 on shell. We only need to examine the cancellation in the upper P-p amplitude. We could as well couple the Reggeons to external sources (nucleons). Choose Sudakov variables

$$k=\alpha p_2'+\beta p_1'+K\ , \tag{2.1}$$

$$p_{1,2}'=p_{1,2}-\frac{m^2}{s}p_{2,1}\ . \tag{2.2}$$

In the limit $s\to\infty$, α,β are light-cone variables when they take finite values. The Reggeon-particle subenergies are

$$s_1\equiv(p_1-k)^2=\alpha\beta s-K^2-\alpha s-\beta m^2+m^2\ , \tag{2.3}$$

$$s_2\equiv(p_2+k)^2=\alpha\beta s\ -K^2+\alpha m^2+\beta s+m^2\ . \tag{2.4}$$

Let $t_1=\alpha\beta s-K^2$. We assume the whole diagram is only large when t_1 is finite and $s\to\infty$ (as in Fig. 4). Setting $s_2=m^2$ we have

$$t_1=-\alpha(\alpha m^2+t_1)-K^2\ . \tag{2.5}$$

If t_1 is finite $\alpha\sim1$ and $K^2\sim m^2$, so $\beta\sim m^2/s$. Also

$$t_1=-\frac{K^2+\alpha^2m^2}{\alpha+1} \tag{2.6}$$

so t_1 finite implies

$$\alpha\geq-1+\eta\quad\text{for }\eta>0\ . \tag{2.7}$$

In the $(-\alpha)$ plane there is a pole and a right-hand cut due to s_1 thresholds. There is another right-hand cut at $\alpha=-1+i\epsilon$ due to singularities in t_1, but this is irrelevant because of Eq. (2.7). It is clear that the α contour can be freely distorted into the region $|\alpha|\sim1$ where $s_1\sim s$ and the denominator 1 in Fig. 3 is far off shell. Therefore in soft field theory the diagram is negligible and does not contain a Regge cut.

On the other hand, if we deform the α contour to just pick up the pole at $s_1=m^2$, then the result is not small. It contains an "ASF cut." It also contains such a cut if we include only the contribution of the s_1 cut. The sum of the pole and the branch cut cancel in the s_1 integration.

Since the α contour can be distorted so that $s_1\sim s$, the cancellation must occur for any $s\gtrsim s_0$, where s_0 is the threshold for Regge asymptotic behavior.

We next consider the high-energy limit of Figs.

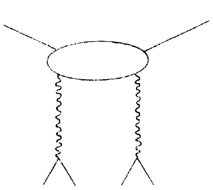

FIG. 2. High-energy form of the six-point function $f(s,s_1,\vec{K}^2)$.

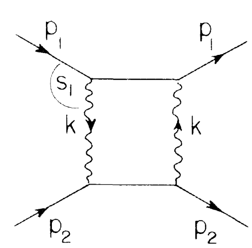

FIG. 4. High-energy form of the ASF diagram.

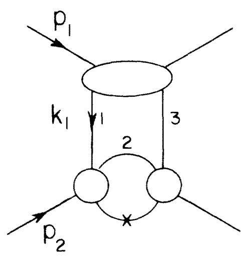

FIG. 5. A more general planar graph.

5 and 6. In each case the blobs are assumed planar but otherwise have arbitrary structure. Consider first Fig. 5. Let

$$k_1 = \alpha_1 p_2' + \beta_1 p_1' + K \,. \tag{2.8}$$

Then

$$s_{12} \equiv (p_2 + k_1)^2 = (1+\alpha_1)(m^2/s + \beta_1)s - K_1^2 \,, \tag{2.9}$$

$$\bar{s}_1 \equiv (p_1 - k_1)^2 = (1-\beta_1)(m^2/s - \alpha_1)s - K_1^2 \,. \tag{2.10}$$

As before, $\beta \sim m^2/s$ and $s_1 \equiv (p_1 - k)^2 \sim -\alpha s$. The singularities in the α_1 plane come from (2.9), (2.10), and the denominators

$$d_1 = \alpha_1\beta_1 s - K_1^2 - m^2 + i\epsilon \,, \tag{2.11}$$

$$d_2 = (\alpha_1 - \alpha)(\beta_1 - \beta)s - (K_1 - K)^2 - m^2 + i\epsilon \,, \tag{2.12}$$

$$d_3 = d_1 \,. \tag{2.13}$$

The integral over α_1 vanishes unless there are singularities on both sides of the contour. Hence $0 < \beta_1 < 1$.

Inclusion of the normal thresholds in k_1^2 does not alter this argument as they are on the same side of the α_1 contour as the poles in d_1 and d_2. The α_1 contour can be deformed entirely around the right-hand cut in $\bar{s}_1$ and hence lies in $\alpha_1 < 0$. Since the field theory is soft d_1 is finite (say $|d_1| \lesssim M_0^2$), and we have

$$K_1^2 \sim M_0^2, \quad \alpha_1\beta_1 s \sim M_0^2 \,. \tag{2.14}$$

Now $d_2 \sim M_0^2$. We conclude that the graph can only give a finite contribution if

$$\alpha\beta_1 s \simeq M_0^2 \,. \tag{2.15}$$

Therefore

$$|s_1 s_{12}| \lesssim M_0^2 s \,. \tag{2.16}$$

Now, for exactly the same reason as in discussing the ASF diagram, the α contour can be deformed into $|\alpha| \sim 1$ (so that $s_1 \sim s$). We conclude that the graph of Fig. 5 gives only a small contribution when $s_{12} \sim s$. The graph itself is not necessarily negligible; for example, the blob can contain single-Pomeron exchange.

Next we turn to Fig. 6. The discussion is the same as for Fig. 5 except that in order to determine the dominant region of β_1 integration as $0 < \beta_1 < 1$ we found it necessary to assume that the only singularity in the variable $(p_1 - k_1)^2$ is a right-hand cut. As we are discussing the analyticity of a production amplitude it is not certain this will be so. If it is not so, then it is much harder to determine the asymptotic form of Fig. 6 as $s \to \infty$ and our conclusion may or may not be altered.

We next show that the cancellation effect is consistent with unitarity in the Reggeon-particle channel. The general discontinuity equation in this channel[9] is

$$\operatorname{disc} A_{l_1 l_2} = 2i \sum_{n,j,k} \int A_{l_1}^{(j)}(s_1; n)\, A_{l_2}^{(j)*}(s_1; n)\, d\rho(n) \,. \tag{2.17}$$

This equation applies either for the full $Pp \to Pp$ amplitude or if we consider only planar graphs on each side of the equation. As noted in Ref. 10, the equation can be derived for the planar case by the methods of Ref. 11.

In the original ASF diagram, the cancellation occurs between "off-diagonal" terms in $Pp \to Pp$ discontinuity and the original pole term. This does not happen for the full planar $Pp \to Pp$ amplitude since the discontinuity of the full amplitude is necessarily positive. In other words the canceling terms are recanceled by further diagonal terms in the discontinuity equation. (This is clear from an application of Schwarz's inequality.) How then does the ASF cancellation occur?

The point is that planar graphs can only build a Reggeon with intercept $\leqslant 1$. If $\alpha(0) > 1$, then the Pomeron cannot be built consistent with direct-channel unitarity entirely from planar graphs.[10] In the unitarity equation the full two-particle amplitude is proportional to s^{α}. The two-particle discontinuity is proportional to $s^{2\alpha - 1}$ and all the other multiparticle discontinuities are positive. This situation is only self-consistent if $\alpha(0) \leqslant 1$. (This argument would fail in the presence of nonplanar graphs because they contain Regge cuts

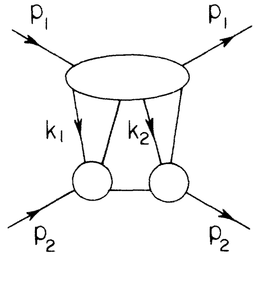

FIG. 6. Another general planar graph.

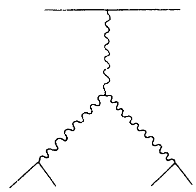

FIG. 7. Large-s_1 form of Fig. 2.

which can enforce the Froissart bound.) It follows that there is no sensible ASF problem for $\alpha(0)>1$.

On the other hand if $\alpha(0)\leq 1$, then the s_1 integration projecting the fixed pole in the Pp elastic amplitude necessarily diverges. Let the $Pp\rightarrow Pp$ amplitude be dominated by P exchange for $s_1\geq N$. Then the integral Eq. (1.4) is proportional to

$$\beta^2+\int_{(m+m_\pi)^2}^{N} ds_1\,\mathrm{Im}f(s_1)+\int_N^{s/m\sqrt{\gamma}} ds_1\,\beta r_0 s_1^{-\alpha(0)} \quad . \tag{2.18}$$

We know because of the previous argument for ASF cancellation using the Sudakov variables that the contribution from the lower limit of the final integration must cancel the previous two terms. The remaining term corresponds to single P exchange coupling to the whole nucleus.[8]

We conclude that the ASF problem is only sensible if $\alpha(0)\leq 1$, and in that case the cancellation effect is perfectly consistent with the positive discontinuity in the Reggeon-particle amplitude.

Notice also that the piece of the s_1 integration responsible for the ASF cancellation comes entirely from $s_1\leq N$. Now, once E_L is large enough that production of a state of (mass)$^2=N$ is coherent on a nucleus, the first integral of Eq. (2.18) does not differ essentially from the fixed-pole projection in hadron-hadron scattering. It follows that the ASF cancellation occurs in just the same way for hadron-nucleus scattering as for hadron-hadron scattering once $E_L\gg NR$, where R is the nuclear radius.

We can also see this cancellation in configuration space. Production of a state of (mass)$^2=s_1$ takes a longitudinal distance $Z\sim E_L/s_1$. For a planar Feynman graph the scattering process on two nucleons is sequential.[5] Hence it can only occur on a nucleus if $s_1\sim E_L/2R$. However, since a planar pP amplitude contains no fixed pole in the t channel, it is small as $s_1\rightarrow\infty$. Hence planar diagrams cannot contribute to double scattering on a finite-size nucleus as $E_L\rightarrow\infty$.

III. EFFECT OF REGGEON INTERACTIONS

In this section we investigate the importance of graphs containing the triple-Pomeron coupling assuming the validity of perturbation theory in the triple-Pomeron coupling. We first determine the size of the simplest such graph (Fig. 7) compared with the Glauber double-scattering term.

The ratio of these terms is simply the ratio of the fixed poles in the respective Pomeron-particle amplitudes. For the Glauber term the fixed pole is

$$F_1=2\pi\beta^2 \ , \tag{3.1}$$

where β is the NN coupling constant, normalized so that $\sigma_T(NN)=\beta^2$.

For Fig. 7 the fixed pole is

$$F_2=2\pi\int_{M_0^2}^{2E_L/R}\frac{ds_1}{s_1}\beta r_0 =2\pi\beta r_0\sqrt{2}\ln\frac{2E_L}{RM_0^2} \quad , \tag{3.2}$$

where M_0^2 is the lowest energy at which $pP\rightarrow pP$ is dominated by P exchange and r_0 is the triple-Pomeron coupling normalized such that

$$\frac{d\sigma}{dtdM^2}=\frac{\beta^3 r_0\sqrt{2}}{16\pi M^2} \ . \tag{3.3}$$

At these energies it is irrelevant if $\alpha(0)$ is slightly above 1. With this normalization r_0 coincides with that defined in Ref. 12.

At a level of approximation sufficient for our purpose we can assume $pp\rightarrow pp$ is dominated by one-Pomeron exchange so that $\beta\simeq 10\ \mathrm{GeV}^{-1}$. If we assume the Fermilab data on $pp\rightarrow pX$ are dominated by only the triple-Pomeron coupling we obtain $r_0\simeq 0.5\ \mathrm{GeV}^{-1}$. However, it appears[13] that about one half the observed inclusive cross section is due to non-triple-Pomeron terms. Thus we should take $r_0\simeq 0.25\ \mathrm{GeV}^{-1}$. We take $M_0=2$ GeV. Then

$$\frac{F_2}{F_1}=\frac{r_0\sqrt{2}}{\beta}\ln\frac{2E_L}{RM_0^2} \ . \tag{3.4}$$

At $E_L=200$ GeV, we have $F_2/F_1\simeq\frac{1}{30}\ln(20/R)$ if R is in fermis. Clearly this is negligible. It is equally clear that small changes in our assumptions (different model of the nuclear shape, inclusion of Regge cuts, etc.) should not change this result essentially.

We now consider the relative importance of higher graphs. A simple model incorporating these in a systematic way is that given by Schwim-

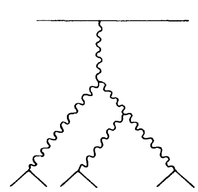

FIG. 8. Example of a "fan diagram."

mer.[14] He sums all graphs corresponding to the classical approximation; the eikonal of this model is the sum of connected "fan diagrams" [one Pomeron emitted by the projectile undergoes successive splittings ending up with n Pomerons which are absorbed on separate nucleons (see Fig. 8)]. Self-energy insertions in the Reggeon propagator are ignored because at present energies double diffraction dissociation into heavy states is on the order of only 1–2% of the total cross section and in Reggeon calculus increases as $\ln s$.

We start with a uniform-sphere nucleus and replace it by a cylindrical nucleus of the same radius whose optical thickness is the average optical thickness of the sphere. Let R be the radius:

$$U=\frac{\beta^2 A e^{\Delta y}/\pi R^2}{1+(\beta r_0 A/2\pi R^2)[(e^{\Delta y}-1)/\Delta]}\,\theta(R-|b|)\ , \tag{3.5}$$

where

$$y_0=\ln\frac{RM_0^{\,2}}{m}\ \text{and}\ \Delta=\alpha(0)-1\ .$$

Since the effect of light ($m\leqslant s_1\lesssim 2.5$ GeV) intermediate states in propagation is comparatively small[15] as long as effects of graphs renormalizing the bare Pomeron propagator are small, this model should be qualitatively correct up to very high energies (at least those in extensive air showers).

It is immediately apparent from formula (3.5) plus our original estimate of r_0 that Reggeon interactions are negligible at Fermilab energy. (We are of course assuming that $2\to 2$ couplings and all higher-order Pomeron couplings are not large.)

We now turn to a different problem: the interpretation of total cross sections extracted from observation of extensive air showers. Workers in this field usually use essentially the Glauber formula

$$\sigma_{\rm incl}(p\text{-Air})=\int d^2b\,(1-e^{-\sigma_{NN}T(b)})\ , \tag{3.6}$$

where $T(b)$ is the optical thickness of a typical air nucleus. In principle this method could result in a serious underestimate of σ_{NN} because it omits the effect of diffraction dissociation into heavy intermediate states. However, we will see below that at a typical energy of observation (10^{15} eV) this is probably not the case. The error involved in applying the Glauber formula is not large compared with other possible errors in the method. This happens partly because nitrogen is quite a small nucleus and partly because r_0/β is so small. It is also significant that it is the inelastic cross section not the total cross section that is observed.

To examine this question we use the result

$$\sigma_{\rm incl}(p\text{-Air})=\int d^2b\,(1-e^{-U(s,b)})\ , \tag{3.7}$$

where U is as above [Eq. (3.5)]. Only a crude calculation is necessary for our purpose, so we will treat the typical air nucleus as a uniform sphere of radius 3.2 F. We take $\Delta=0.1$. Then a change of r_0 from its current maximum phenomenological value (0.5 GeV^{-1}) to 0 only changes the nuclear opacity by 6%. If $r_0=0.25$ GeV^{-1} the change is only 3%. At $\Delta=0.05$ the corrections are somewhat larger, but still less than 10%. Such corrections can probably be ignored compared with, for example, the statistical errors, uncertainty of primary flux, composition, etc.

Present observations[16] see a rising $\sigma(NN)$ in the 10^{15} eV energy range, using only formula (3.6). On the basis of the above estimate we think these cross sections should be revised upwards but only by a relatively unimportant amount.

Finally consider the hypothetical world of unlimited-energy and unlimited-size nucleus. Then $U\sim 2\beta\Delta/r_0\sim 8$. Even in this case the nucleus is essentially black. This could be altered by renormalization effects in the Reggeon propagator but because of the smallness of r_0 these should not be relevant for practical observations.

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