PHYSICAL REVIEW D VOLUME 15, NUMBER 1 1 JANUARY 1977

Nature of dynamical suppressions in the generalized Veneziano model*

R. Odorico
High Energy Physics Division, Argonne National Laboratory, Argonne, Illinois 60439
and Istituto Nazionale di Fisica Nucleare, Sezione di Trieste, Trieste, Italy
(Received 24 May 1976)

It is shown by explicit numerical calculations that of a class of coupling suppressions existing in the generalized Veneziano model and associated with decay multiplicities, only a part can be attributed to the exponential growth with energy of the number of levels. The remaining suppressions have a more direct dual origin.

In a recent paper[1] we called attention to the phenomenological relevance of a suppression mechanism which affects low-multiplicity decay modes and is present in dual resonance models. The presence of suppression effects in dual resonance models is not surprising. In fact, in any duality bootstrap in which factorization is enforced, the (duality) constraints, which are as numerous as the amplitudes, systematically outnumber the available vertex couplings. Consistency can be achieved by having a sizable fraction of couplings vanish, which allows the evasion of a much larger number of duality constraints.[1] It is a well-known fact that duality bootstraps for two-body amplitudes require some resonance couplings to vanish (see Ref. 2 and other references therein). The existence in the generalized Veneziano model (GVM) of similar effects for decay models of low multiplicity was remarked some time ago by Gliozzi.[3] The resonances concerned are daughters of relatively low angular momentum with respect to that of the parent.[1]

In Ref. 3 there was expressed the view that the GVM decouplings in question are intimately connected with the exponential growth with energy of the GVM number of levels. For large enough s, in fact, the latter systematically outnumber the GVM states of any finite system of decay particles, the number of which can increase no more than polynomially with s.

The purpose of this note is to show that, as a matter of fact, only a part of the decouplings can be explained in this way, and that the remaining ones have to be regarded as a more direct consequence of the duality constraints. For this sake, the numbers of GVM levels and of the possible GVM states with k (lightest scalar) particles are explicitly calculated, for each angular momentum J, as functions of s. The excess of one over the other (when it occurs) is then compared with the actual number of decouplings existing in the model, according to the rules of Ref. 3.

First of all, some rectifications of the results of Ref. 3 are in order. The rule for the decoupling of levels, as given in Eq. (6) of Ref. 3, actually combines resonance levels with different masses, and therefore it is not fully correct. One can construct resonance levels with a well-defined $\alpha(s)=N$ and decoupling from states with k or fewer particles by considering the formal determinant of the matrix

$$(\beta_k)_{jl}=b_{\mu_l}^{(i_j+h_l)} \tag{1a}$$

with $j,l=1,2,\ldots,k$ (not $k-1$), and

$$\sum_{j=1}^{k}(i_j+h_j)=N \tag{1b}$$

(when not otherwise specified, notations are as in Ref. 3). In fact, one can verify that, substituting $b_\mu^{(i)}\to P_\mu^{(i)}$, one gets

$$\det(\beta_k)\to\sum_{l=1}^{k}p_{0\mu_l}M_l+D, \tag{2}$$

where the M_l's and D are polynomial determinants of the same type as those constructed by Gliozzi,[3] and which therefore vanish. As a consequence, following the arguments leading to Eq. (6) in Ref. 3, the resonance state $\det(\beta_k)|0\rangle$, having a well-defined mass M determined by $\alpha(M^2)=N$, cannot decay into k or fewer $|0\rangle$ "particles".

In the resonance rest frame, we disregard the time components of the $b_\mu^{(i)}$'s. They should be taken care of in the GVM by appropriate Ward-type identities. Therefore, the expression for $n_0(k)$, the minimal N at which there appear resonances decoupled from k-particle states, becomes

$$n_0(k)=6p^2+4pq+\tfrac{1}{2}q(q+1)\simeq\tfrac{2}{3}k^2, \tag{3}$$
$$k=3p+q,\quad q<3$$

As to the number of k-particle states, in order to avoid multiple counting one has to take into account (i) that all configurations differing only by a permutation of the $p_{i\mu}$'s (together with the associated variables x_i), for $i=1,\ldots,k-2$, should be counted as one, because of the symmetry of the $P_\mu^{(i)}$'s with respect to these variables, and (ii) that in the angular momentum analysis of the states contractions of $p_{i\mu}$'s having the same index i should be disregarded because they lead to configurations identical to those already counted for lower m_i's.[3] As a result of these restrictions, and because of the neglect of time components, the growth with s of the total number of k-particle states (including the total magnetic-quantum-number dependence) is then $\sim s^{3k-4}$.

With the above criteria in mind, the countings of GVM levels with given values of J and N and of the corresponding k-particle states have been performed. GVM levels have been counted exactly only up to $N=50$.[4] Beyond this value, the approximate formulas of Huang and Weinberg[5] for the total number of levels and that of Chiu, Heimann, and Schwimmer[6] for the angular momentum distribution have been used. For $J=1$ and $N=50$ the resulting approximation is found to differ from the exact value by only 25%. For simplicity, numerical results are presented only for $J=1$; this value, of course, was chosen because of its relevance to e^+e^- annihilation. The results shown, however, are typical also of the other J's. As it appears from Fig. 1, the values of N at which GVM levels begin to outnumber the available GVM k-particle states happen to be considerably higher than those marking the appearance of levels decoupled from these states. That is, the number of decouplings implied by Eq. (1) is actually much higher than what is simply required by the "excess" argument advanced in Ref. 3.

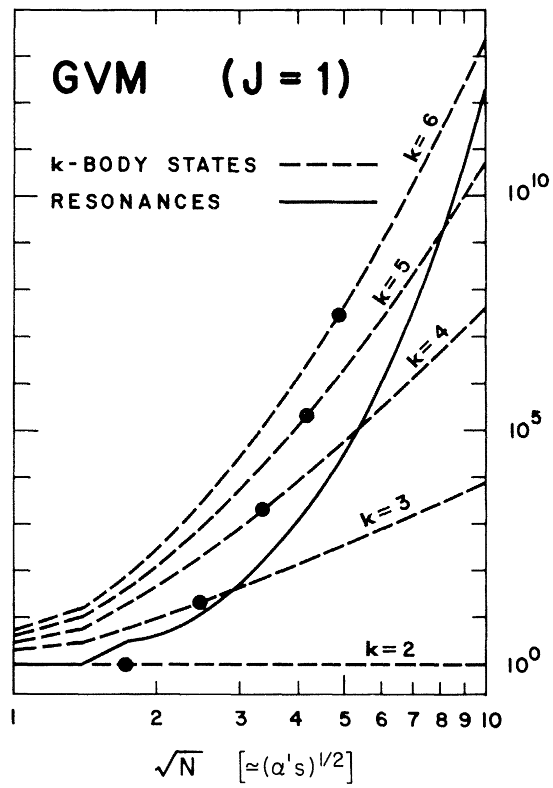

FIG. 1. Numbers of GVM levels and of GVM k-particle states with angular momentum $J=1$, as functions of $\sqrt{N}$ $(=[\alpha(s)]^{1/2}\simeq(\alpha's)^{1/2})$. Curves interpolate integer N values. The black dot on each k curve marks the $\sqrt{N}$ threshold for the appearance of levels decoupled from states of k or fewer particles, according to the determinant rule of Eq. (1).

Before ending, I would like to stress that there are no reasons to regard the decouplings described by the determinant rule of Eq. (1) as the only ones present in GVM. A comprehensive study of these effects could be done in principle by using the known form of the triple-Reggeon vertex.[7] Such an analysis, though, is hindered by the ambiguities which exist in the choice of a vector basis in a space of states having all the same quantum numbers and distinguishable among themselves only because of their different couplings. Suppose, e.g., that two such states happen to have finite but equal couplings to a given coherent state. Their difference, which from all points of view can be equally interpreted as a resonance state, does of course decouple from the state in question. Thus, a simple change of basis in this family of GVM states can bring rather drastic modifications in the physical interpretation.

---

*Work performed under the auspices of the U. S. Energy Research and Development Administration.

[1]R. Odorico, Phys. Lett. 61B, 263 (1976).

[2]R. Odorico, Phys. Rev. D 8, 3952 (1973).

[3]F. Gliozzi, Lett. Nuovo Cimento 4, 1160 (1970).

[4]This has required about half an hour of CDC7600 computer time. Going up to $N=60$ would have required about ten times as much.

[5]K. Huang and S. Weinberg, Phys. Rev. Lett. 25, 895 (1970).

[6]C. B. Chiu, R. L. Heimann, and A. Schwimmer, Phys. Rev. D 4, 3177 (1971).

[7]See, e.g., the review by S. Mandelstam, Phys. Rep. 13C, 259 (1974).