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Nonperturbative approach to quantum chromodynamics*

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In this article a nonperturbative approach to quantum chromodynamics in the confinement phase is developed. The principal idea is to assume confinement, rather than attempting to prove it, and then to examine the consequences of the resulting quantum field theory. We implement confinement by assuming that the gauge field propagator becomes infrared singular like q^{-4} at small momenta. The consequences of this confinement *Ansatz* are explored for the quark propagator, ghost propagator, and quark-gluon proper vertex. Slavnov-Taylor-Ward identities are used to fix the low-momentum behavior of the Green's functions, so no approximations are made in the Dyson equations. In both the covariant Landau gauge and the noncovariant axial gauge we obtain differential equations for the quark propagator which are solved. Vacuum γ_5 invariance is dynamically broken so the solutions satisfy partial conservation of axial-vector current. We also obtain the solution for the quark propagator in the case in which the flavor symmetry is explicitly broken by a quark mass term. Some implications of this approach, which is an expansion about the infrared behavior of the amplitudes, for the bound-state problem are briefly discussed.

I. INTRODUCTION

In this article we will describe a nonperturbative approach to quantum chromodynamics (QCD). Our approach will be to assume confinement, rather than attempting to prove it, and then to examine the consistency of the resulting quantum field theory.

QCD is the $SU_c(3)$ gauge field theory of strong interactions in which a gauge group triplet of colored quarks with $N \geq 4$ flavors interacts with a color octet of gauge fields, $A_\mu^a(x)$. The flavor chiral symmetry of $SU(N)\times SU(N)$ can be explicitly broken by a quark mass term while the $SU_c(3)$ symmetry is to remain exact. This model, to which we restrict our attention, has been described in detail elsewhere[1] and is a strong candidate for a theory of the strong interaction.

QCD, if it is to describe the real world of the observed hadrons, must undergo at least two phase transitions from the free-field state of real quarks and gluons. These phase transitions are the PCAC (partial conservation of axial-vector current) phase transition of the flavor symmetry and the confinement phase transition of the color symmetry. We discuss these in turn.

If one ignores the flavor breaking due to a quark mass term the flavor symmetry of QCD is $SU(N)\times SU(N)\times U_B(1)\times U_A(1)$. The PCAC phase is one in which the chiral $SU(N)\times SU(N)$ symmetry is dynamically broken to $SU(N)$ in the vacuum. Then the states are classified by the irreducible representations of $SU(N)$, and the Goldstone theorem assures us of an (N^2-1)-plet of massless pseudoscalar bosons. These massless Goldstone bosons acquire mass if explicit flavor breaking is introduced. An undesired alternate to this scheme is to have no spontaneous dynamical breaking of the chiral group. Then all states are parity-doubled.

The PCAC phase transition is relatively well understood and has been studied in renormalizable field theories in which the Goldstone states associated with the dynamically broken chiral symmetry are fermion-antifermion bound states.[2] We assume that QCD is in the PCAC phase. This assumption is easily implemented by assuming that in the absence of explicit flavor breaking

$$[S^{-1}(p),\gamma_5]_+ \neq 0\,, \tag{1.1}$$

so that the γ_5 invariance of the quark propagator, $S(p)$, is broken. This condition is color gauge invariant if the ground state is gauge invariant. Further, (1.1) is the necessary and sufficient condition for the flavor axial-vector Ward identity

$$q^\mu\ {}^a\Gamma_\mu^5(p,q) = S^{-1}(p-q)\gamma_5\tfrac{1}{2}\lambda^a + \gamma_5\tfrac{1}{2}\lambda^a S^{-1}(p) \tag{1.2}$$

to imply Goldstone states. Using (1.1) this Ward identity implies

$${}^a\Gamma_\mu^5(p,q) \underset{q^2\to 0}{\sim} F(p^2,p\cdot q)\lambda^a q_\mu/q^2\,,\quad F(p^2,p\cdot q)\neq 0 \tag{1.3}$$

exhibiting the zero-mass boson pole. Here λ^a is an $SU(N)$ flavor matrix.

In the axial singlet channel corresponding to the $U_A(1)$ symmetry the above Ward identity is known to fail as $q_\mu \to 0$ on account of the existence of topological solitons in QCD.[3] In this channel there is an extra term in (1.2) arising from an effective $U_A(1)$-breaking interaction $\det M + \det M^\dagger$, where M is the flavor matrix $M = \bar{q}(1+\gamma_5)q$.[4] So in this channel no Goldstone state is required as is desired phenomenologically.[5]

An insistence of our approach to solving QCD is that the PCAC phase and the collective nature of the low-lying mesons be respected. Evidently the π, a Goldstone state, and the ρ, a dormant Goldstone state, are general features of a collective model.[6] The conflict between PCAC, which requires a Nambu-Goldstone pion, and the quark-model symmetry SU($2N$), which places the pion in a Wigner multiplet with the ρ, is avoided if both the π and ρ transform like members of the $(\overline{N},N)\oplus(N,\overline{N})$ representation of the flavor chiral group e.g. such as

$$\pi^a \sim \overline{q}i\gamma_5\lambda^a q\ ,\quad \rho^a_\mu \sim \partial_\lambda(\overline{q}\sigma_{\lambda\mu}\lambda^a q)\ . \tag{1.4}$$

This unusual representation content for the ground-state vector mesons then implies that PCAC and vector-meson dominance are unified consequences of a dynamically broken chiral group.

By contrast little is known about the confinement phase transition for the color gauge symmetry. Confinement means all the physical states are color singlets. In this model it means the physical states are all bound states. Equivalently the cluster property fails for matrix elements of color-nonsinglet gauge-covariant operators. There is no proof that confinement occurs in QCD.

Attempts have been made to see if perturbation theory in the gauge coupling constant g would signal confinement. The hope was that the infrared divergences of QCD would provide damping factors for the emission of colored states from color-singlet states. Soft-gluon emissions, for example, do not obey a Poisson distribution as they do in QED because of the gluon self-coupling. However, there can be no signal of confinement, if it indeed occurs, from perturbation theory for at least two reasons.

First, the Kinoshita-Lee-Nauenberg theorem[7] applies to QCD order by order in g. Hence exclusive processes are free of infrared divergences in perturbation theory. The infrared structure of QCD amplitudes in perturbation theory is not qualitatively different from QED.

Second, if confinement occurs, so that there is a mass gap, then the S matrix has an essential singularity at the origin of the coupling-constant plane (see Fig. 1). So the critical coupling is $g_c = 0$. Perturbation theory has zero radius of convergence and will never see the essential singularity.

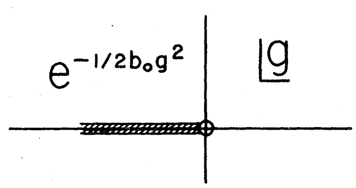

FIG. 1. Analytic properties in the coupling constant in the confinement phase.

The argument for the essential singularity given by Gross and Neveu[8] follows from the observation that confinement in 3+1 dimensions requires a mass scale M but there is none in the Lagrangian. M is a renormalization-group invariant so

$$\left(\overline{\mu}\frac{\partial}{\partial\overline{\mu}} + \beta(g)\frac{\partial}{\partial g}\right)M = 0\ , \tag{1.5}$$

where $\beta(g)$ is the Callan-Symanzik function and g is the gauge coupling defined at momenta $p_i^2 = -\overline{\mu}^2$. Since $M = \overline{\mu}f(g)$ it follows by solving (1.5) that

$$M = \overline{\mu}\exp\left(-\int^g \frac{dg'}{\beta(g')}\right) \underset{g\to 0}{\sim} \overline{\mu}e^{-1/2b_0g^2},\quad b_0 > 0\ . \tag{1.6}$$

Since all masses in QCD exhibit the solution (1.6) up to a multiplicative constant of integration, it follows that the ratio of all bound-state masses are determined independent of any parameters (for no explicit flavor breaking of course).

While this elementary argument destroys the possibility of ever seeing a signal of confinement in perturbation theory, it also points to a distinction between (3+1)-dimensional QCD on one hand and (1+1)- and (2+1)-dimensional QED on the other. (1+1)-dimensional QED has confinement for all values of the coupling as does (2+1)-dimensional QED at least for small values of the coupling in perturbation theory. Confinement does not entail an essential singularity because the coupling constant, which is not dimensionless, provides the requisite mass scale. This suggests that the confinement mechanism of QCD could be qualitatively different from what our experience in low dimensions has taught us.

Our first problem even if we assume QCD confines is to technically implement it. Consider the value of the commutator for a gauge field $A^a_\mu(x)$ satisfying the gauge condition $\partial_\mu A^a_\mu = 0$,

$$\langle 0|[gA^a_\mu(x), gA^b_\nu(0)]|0\rangle = \delta^{ab}F(x^2)\theta(x^2)\left(g_{\mu\nu} - \frac{x_\mu x_\nu}{x^2}\right)\ , \tag{1.7}$$

and the associated propagator

$$D^{ab}_{\mu\nu}(q) = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)\frac{d(q^2)}{q^2}\delta^{ab}$$
$$= \int d^4x\, e^{-iq\cdot x}\delta^{ab}F(x^2)\theta(x^2)\theta(x_0)\left(g_{\mu\nu} - \frac{x_\mu x_\nu}{x^2}\right)\ . \tag{1.8}$$

In perturbation theory one makes the perturbative *Ansatz* $F(x^2) \sim g^2/x^2$ or

$$d_p(q^2) = g^2\ . \tag{1.9}$$

This *Ansatz* is then inserted into the regulated Dyson equations and one may calculate the corrections to the ansatz.

For QCD our primary assumption is to make the alternate confining *Ansatz*[9] $F(x^2) \sim \mu^2 \ln(-x^2/\mu^2)$, $-x^2 \to \infty$ or

$$d_c(q^2) \underset{q^2 \to 0}{\sim} \mu^2/q^2\,, \tag{1.10}$$

where μ^2 is a mass characterizing the scale of confinement. To actually define the integral equations in the infrared region we will introduce an analytic regulation $\epsilon \sim 0^+$:

$$d(q^2) \underset{q^2 \to 0}{\sim} (\mu^2/q^2)^{1-\epsilon}\,. \tag{1.11}$$

If the propagator is more singular than (1.11) it is not possible to control infrared divergences in the Dyson equations. To control ultraviolet divergences one may use dimensional regularization with the prescription $d^4q \to d^{4-\gamma}q$, $\epsilon > \gamma > 0$.

Our confining *Ansatz* violates the cluster property since $F(x^2)$, $-x^2 \to \infty$, does not vanish. Strocchi[10] has shown that in a local field theory the proof of the cluster property for QED fails for QCD. Thus while the confining ansatz cannot be true for QED it may be true for QCD.

It is tempting to justify the confining ansatz (1.10) on the basis of the phenomenological success of linearly rising potential models of bound quarks. This would be so if there were a direct connection between the potential and single-gluon exchange since this corresponds to a linear potential. However, such a connection assumes the validity of perturbation theory in the number of exchanged gluons with the propagator given by the confining *Ansatz*. It is easy to convince oneself that multigluon exchanges become successively stronger in the infrared region and any connection with a potential must be with the sum of all possible exchanges.

Recent perturbative studies by Cornwall[11] and Frenkel, Meuldermans, Mohammad, and Taylor[12] have attempted to go beyond perturbation theory. It is worth remarking on the connection of their work with ours. They find that to $O(g^4)$ the perturbative *Ansatz* (1.9), $d_p(q^2) = g^2$ is replaced by

$$d(q^2) = \bar{g}^2(q^2) \tag{1.12}$$

for large q^2 where $\bar{g}(q^2)$ is the solution to

$$q^2 \frac{d}{dq^2}\, \bar{g}(q^2) = \beta(\bar{g}(q^2))\;,\quad \bar{g}(-\bar{\mu}^2) = g\,. \tag{1.13}$$

If a result such as (1.12) holds for all q^2 then we obtain our confining *Ansatz* (1.10) from (1.12) providing $\beta(g) \to -g/2$, $g \to \infty$. In perturbation theory $\beta(g) = -b_0 g^3 + \cdots$, $b_0 > 0$. Nothing is known about $\beta(g)$ in the strong-coupling regime.

Perturbation theory in the coupling provides a natural approximation scheme by truncating the number of gluon exchanges. For our ansatz this is not possible and one must consider the sum of all such exchanges. A problem is to find a nonperturbative approximation scheme appropriate for the confinement phase. What we will do is to examine only the region $|p^2/\mu^2| < \epsilon^{-1}$ so a suitable small parameter will turn out to be ϵ, the infrared regulator. The important observation that makes this approach viable is that the small-momentum region is controlled by the Ward identities. The usual criticism that the Ward identities are not respected will not apply to our approach. No such control is easily achieved for finite momenta. For large momenta we have control of the amplitudes in terms of a self-consistent application of perturbation theory and asymptotic freedom.[13] The interpolation between the infrared region, which we will study here, and the ultraviolet region specified by asymptotic freedom is the *terra incognita* of QCD.

In this article we examine the quark and ghost propagators in the Landau gauge and the quark propagator in the axial gauge for which ghost complications are absent. Our result will be to establish and solve differential equations for the quark propagator. The solution has the PCAC property (1.1) and has no quark pole. The quark self-energy is infinite as $\epsilon \to 0$. In the axial gauge the quark propagator is obtained in complete generality. While free of ghost problems, it is noncovariant. We also give a solution for the quark propagator for the case that the flavor symmetry is broken by a mass term.

What is not examined in this article are the gluon self-couplings and the Dyson equations for the gluon Green's functions. The self-consistency of the confining *Ansatz* for the gluon propagator is an important and unsolved problem in our approach.

The results given here generalize the results given in a previous paper of mine.[14] The intent of that more phenomenologically oriented work was to give a derivation of the Gorkov equations for QCD for the gap fluctuations of confined quarks and antiquarks.[15] The mesons were flavored bilocal excitations of the color-singlet condensate. A nonlinear gap equation (the nonlinear Gorkov equation) for the meson excitations was obtained. In principle, the meson spectrum is obtained from this equation. If one linearizes the integral equation the resulting eigenvalue problem yields a phenomenologically reasonable low-lying meson spectrum which respects PCAC. Further, meson scattering amplitudes, in the duality diagram approximation, were finite in the infrared $\epsilon \to 0$ limit. One found a simple dynamics for confinement in the color-singlet scattering processes in which the quarks, while fundamental for the dynamical scattering processes, never appeared as physical states. The discontinuity across quark lines in the duality diagram vanished. Further, the amplitude for meson $\to \bar{q}+q$ vanishes as $\epsilon \to 0$ as is required for the consistency of the confining *Ansatz*.

II. THE DYSON EQUATIONS

The Dyson equations are proved in perturbation theory. We assume that they are valid even if perturbation theory is not.

We will only consider the Dyson equations for unrenormalized Green's functions. QCD is renormalizable. This means that a well-defined finite set of subtractions on the unrenormalized amplitudes renders finite those amplitudes that have ultraviolet divergences *in perturbation theory*. The ultraviolet divergences can be regulated by dimensional regularization in $4-\gamma$ dimensions. The usual renormalization prescription is to specify the renormalized amplitudes at some fixed momentum so one can construct them from the subtracted unrenormalized amplitudes.

It is not known how to implement the renormalization program outside of perturbation theory. Since our *Ansatz* is inherently nonperturbative (like confinement) we do not attempt to renormalize our solutions for the amplitudes. Presumably this can be done by the usual subtractive procedure, but a nonperturbative proof that this yields ultraviolet-finite amplitudes is lacking. We suppose that the infrared behavior of QCD is effectively decoupled from the ultraviolet region for which a renormalization prescription is essential.

It should also be remarked that the usual integral representations for the Dyson equation that we will use for the quark and ghost propagators have overlapping divergences in perturbation theory. Representations free of overlapping divergences for QCD have been constructed by Baker and Lee.[16] As we ignore the ultraviolet behavior completely we assume that the usual integral representations suffice to extract the correct infrared behavior. With these caveats we proceed to the examination of some of the simpler Dyson equations.

A. The quark propagator

The Dyson equation for the quark propagator $S(p)=i/\not{p}-\Sigma(p)$ is given in Fig. 2 or

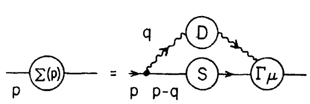

FIG. 2. Quark Dyson equation.

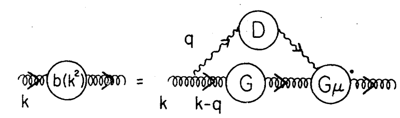

FIG. 3. Ghost Dyson equation.

$$\Sigma(p)=-\int \frac{d^4q}{(2\pi)^4}\, \Gamma^a_\mu(p,q)S(p-q)\gamma_\nu \tfrac{1}{2}\lambda^b D^{ab}_{\mu\nu}(q)\,. \tag{2.1}$$

Here the proper vertex is $\Gamma^a_\mu(p,q)=-i\Gamma_\mu(p,q)\frac{1}{2}\lambda^a$ with the color matrix $\frac{1}{2}\lambda^a$ obeying $[\frac{1}{2}\lambda^a,\frac{1}{2}\lambda^b]=iC^{abc}\times\frac{1}{2}\lambda^c$ with C^{abc} a Lie coefficient, and

$$D^{ab}_{\mu\nu}(q)=-i\delta^{ab}\left[\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)\frac{d(q^2,\alpha)}{q^2}+\alpha g_0{}^2\frac{q_\mu q_\nu}{q^4}\right] \tag{2.2}$$

is the general form of the gauge field propagator consistent with the Slavnov identity $iq^\mu q^\nu D^{ab}_{\mu\nu}(q)=\alpha g_0{}^2\delta^{ab}$. The coupling constant has been included in $d(q^2,\alpha)$. Here α is a gauge parameter corresponding to a gauge-fixing term $(\partial_\mu A^a_\mu)^2/2\alpha$.

We will assume our ansatz, for definiteness, in the Landau gauge $\alpha=0$:

$$d(q^2,0)\underset{q^2\to 0}{\sim}\left(\frac{\mu^2}{q^2}\right)^{1-\epsilon} \tag{2.3}$$

Substituting this and (2.2) into (2.1), letting $\epsilon\to 0$, and taking the limit symmetrically, $q^2\to 0$, $q_\mu q_\nu/q^2\to g_{\mu\nu}/4$, the integral diverges like ϵ^{-1} and

$$\Sigma(p)=\frac{m^2}{\epsilon}\,\Gamma_\mu(p,0)\frac{1}{\not{p}-\Sigma(p)}\gamma_\mu + \text{terms less singular in } \epsilon\,. \tag{2.4}$$

Here $m^2=C_F 3\pi^2\mu^2/4(2\pi)^4$ and $C_F=\frac{4}{3}$ is the Casimir invariant for the triplet of fermions, $(\lambda^a/2)(\lambda^a/2)=C_F$.

In what follows we denote with a bar amplitudes which are finite as $\epsilon\to 0$. We will find that $\bar{\Gamma}_\mu(p)=\epsilon\Gamma_\mu(p,0)$ and $\Sigma(p)=\bar{\Sigma}(p)/\epsilon$ so that as $\epsilon\to 0$ the quark self-energy $\Sigma(p)$ diverges relative to the kinetic energy $\not{p}$ as is expected in the confinement phase. This is the case for momenta $|p^2/m^2|\ll\epsilon^{-1}$. For momenta $|p^2/m^2|\gg\epsilon^{-1}$ the quark propagator is given by asymptotic freedom, that is, $1/\not{p}$ up to logarithms. From (2.4) one has as $\epsilon\to 0$

$$\bar{\Sigma}(p)=-m^2\bar{\Gamma}_\mu(p)\bar{\Sigma}^{-1}(p)\gamma_\mu\,,\quad |p^2/m^2|\ll\epsilon^{-1} \tag{2.5}$$

as the infrared content of the quark Dyson equation.

B. The ghost propagator

The ghost propagator also obeys an elementary Dyson equation and can be analyzed in a similar fashion (see Fig. 3). The ghost propagator is

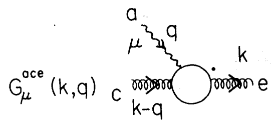

FIG. 4. Ghost-gluon vertex.

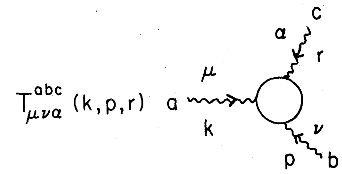

FIG. 5. Triple-gluon vertex.

$$G^{ab}(k)=\frac{i\delta^{ab}}{k^2(1+b(k^2))} \tag{2.6}$$

and the Dyson equation

$$\delta^{ef}k^2 b(k^2)=-i\int\frac{d^4q}{(2\pi)^4}\,G_\mu^{ace}(k,q)G^{cd}(k-q)\times C^{bdf}(k-q)_\nu D_{\mu\nu}^{ab}(q)\,. \tag{2.7}$$

Here

$$G_\mu^{ace}(k,q)=C^{ace}G_\mu(k,q)=C^{ace}k^\lambda G_{\lambda\mu}(k,q) \tag{2.8}$$

is the ghost-gluon vertex (Fig. 4) ($G_{\lambda\mu}=g_{\lambda\mu}$ in perturbation theory). Defining

$$G_\mu(k,0)=k_\mu A(k^2) \tag{2.9}$$

and the gauge field Casimir invariant $\sum_{c,d}C^{acd}C^{bcd}=C_2\delta^{ab}$ and using the *Ansatz* (2.3), one obtains from the integral (2.7) as $\epsilon\to 0$

$$b(k^2)=\frac{m^2A(k^2)(C_2/C_F)}{\epsilon k^2(1+b(k^2))},\quad |k^2/m^2|\ll\epsilon^{-1}. \tag{2.10}$$

This is the infrared content of the ghost Dyson equation, a result that is useful in our analysis of the Ward identities.

III. SLAVNOV-TAYLOR-WARD IDENTITIES

In order to solve the Dyson equations (2.5) and (2.10) for the quark and ghost propagators one needs to know the proper vertex functions for gauge fields of zero momentum. Such information can be provided by the Slavnov-Taylor-Ward identities.[17]

For $\Gamma_\mu^a(p,p)$ we can obtain an elementary identity provided that $b(0)$, the ghost self-energy at $k^2=0$, exists. From the ghost Dyson equation (2.10) we see that $b(0)$ exists providing that the vertex satisfies

$$A(k^2)=\epsilon k^2R(k^2)\quad |R(0)|\underset{\epsilon\to 0}{<}\infty\,, \tag{3.1}$$

where $R(0)$ exists as $\epsilon\to 0$. We assume this is so. One might hope to verify such an assumption, or at least check its consistency from the Ward identities for the ghost-gauge field vertex $G_\mu^{ace}(k,q)$ since $G^{ace}(k,0)=C^{ace}k_\mu A(k^2)$. Such formal Ward indentities for the ghost field vertices have been obtained by Joglekar and Lee.[18] However, these identities involve the matrix elements of composite operators of ghost and gauge fields and we have found that no useful information can be obtained. The ghost-gauge field vertex also makes its appearance in the Ward identity for the triple gauge field vertex $T_{\mu\nu\alpha}^{abc}(k,p,r)=C^{abc}T_{\mu\nu\alpha}(k,p,r)$ (see Fig. 5). The identity is

$$[1+b(k^2)]k^\mu T_{\mu\nu\alpha}(k,p,r)=-G^{\nu\beta}(k+p,p)d^{-1}((k+p)^2)[(p+k)^2g^{\beta\alpha}-(p+k)^\beta(p+k)^\alpha]+G^{\alpha\lambda}(k+p,p)d^{-1}(p^2)(p^2g^{\lambda\nu}-p^\lambda p^\nu)\,, \tag{3.2}$$

where $(k+p)^\nu G^{\nu\beta}(k+p,p)=G_\beta(k+p,p)$. However, unless one has independent information on $T_{\mu\nu\alpha}$ no useful result for G_β can be found. Such an assumption as (3.1) on the ghost coupling is unnecessary in the axial gauge which we subsequently discuss in Sec. VI.

The quark Slavnov-Taylor[17] identity is

$$-ik^\mu\Gamma_\mu(p,k)[1+b(k^2)]=[1-B(k,p)]S^{-1}(p+k)-S^{-1}(p)[1-B(k,p)]\,, \tag{3.3}$$

where $B(k,p)$ is defined in Fig. 6. In the Landau gauge Taylor[17] has shown that

$$B(0,p)=0\,. \tag{3.4}$$

Using this result and differentiating (3.3) with respect to k_μ, assuming regular behavior, $k_\mu\partial b(k^2)/\partial k_\lambda\to 0$, $k^\lambda\partial\Gamma_\lambda/\partial k_\beta\to 0$ as $k_\mu\to 0$ one obtains, in the Landau gauge, the identity

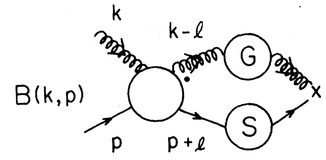

FIG. 6. Definition of $B(k,p)$ in the quark vertex Ward identity.

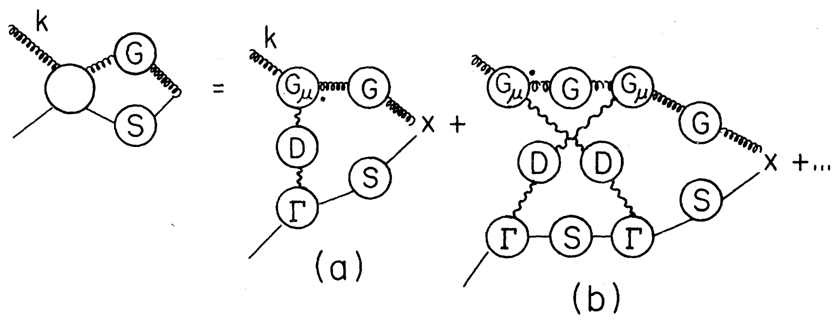

FIG. 7. Ghost-quark scattering kernel.

$$-i\Gamma_\mu(p,0)(1+b(0)) = \frac{\partial S^{-1}(p+k)}{\partial k_\mu}\bigg|_{k=0} + \left[S^{-1}(p),\ \frac{\partial B(k,p)}{\partial k_\mu}\bigg|_{k=0}\right]. \tag{3.5}$$

This is still not a useful form since the term $\partial B(k,p)/\partial k\ |k=0$ appears. However, this can be computed in terms of $\Gamma_\mu(p,0)$ and $S(p)$, as we now show.

In the integral representation for $B(k,p)$ shown in Fig. 6 the ghost-quark scattering kernel appears. This kernel has a skeleton expansion the first term of which, $B_{(1)}(k,p)$, is shown in Fig. 7. The feature that permits a computation of the derivative of $B(k,p)$ at $k=0$ is that all other contributions to the kernel contribute to $B(k,p)$ terms of $O(k^2)$ and higher (in the Landau gauge). For example, the diagram shown in Fig. 7(b) can be explicitly written down. Using our *Ansatz* for the gluon propagator only, the zero-momentum part in the gluon lines contributes and hence each ghost-gluon vertex contributes a factor k_μ. So the diagram is of $O(k^2)$. These factors of k_μ are not canceled by ghost propagator poles if we use the ghost Dyson equation (2.10) and make the assumption that $b(k^2)$ is regular. This argument is valid term by term in the skeleton expansion. Consequently the regularity of $b(k^2)$ implies

$$\frac{\partial B(k,p)}{\partial k_\mu}\bigg|_{k=0} = \frac{\partial B_{(1)}(k,p)}{\partial k_\mu}\bigg|_{k=0}. \tag{3.6}$$

We have

$$\tfrac{1}{2}\lambda^a B_{(1)}(k,p) = -i\tfrac{1}{2}\lambda^b\int\frac{d^4l}{(2\pi)^4}\, S(p+l)D^{ec}_{\mu\nu}(l)\Gamma_\nu(p+l,l)\times\lambda^c G^{ear}_\mu(k-l,-l)G^{rb}(k-l)\,, \tag{3.7}$$

which in the $\epsilon\to 0$ limit becomes, using (2.2), (2.3), (2.8), (2.9), and (2.10),

$$B_{(1)}(k,p) = -\tfrac{1}{2}iS(p)k^\nu\Gamma_\nu(p,0)\ \frac{m^2A(k^2)(C_2/C_F)}{\epsilon k^2(1+b(k^2))} = -\tfrac{1}{2}iS(p)k^\nu\Gamma_\nu(p,0)b(k^2)\,. \tag{3.8}$$

Consequently from (3.6) we obtain

$$\frac{\partial B(k,p)}{\partial k_\mu}\bigg|_{k=0} = -\tfrac{1}{2}iS(p)\Gamma_\mu(p,0)b(0) \tag{3.9}$$

and substituting this into (3.5) we have the final form of the Ward identity

$$(1+\tfrac{1}{2}b)\Gamma_\mu(p,0) = \frac{i\partial S^{-1}(p+k)}{\partial k_\mu}\bigg|_{k=0} - \tfrac{1}{2}bS(p)\Gamma_\mu(p,0)S^{-1}(p)\,,\quad b=b(0)\,. \tag{3.10}$$

The parameter b cannot be determined by this method. If $b=0$ we recover from (3.10) the standard QED-type Ward identity.

IV. DIFFERENTIAL EQUATIONS FOR THE QUARK PROPAGATOR

Denoting

$$\begin{aligned}
&-iS(p)/\epsilon = \bar S(p) = -\bar\Sigma^{-1}(p) = A(p^2)\not p + B(p^2)\,,\\
&\bar\Gamma_\mu(p) = \epsilon\,\Gamma_\mu(p,0)\,,\\
&\bar S_\mu(p) = \partial\bar S(p+k)/\partial k_\mu\,|_{k=0}\\
&\qquad = A(p^2)\gamma_\mu + (A'(p^2)\not p + B'(p^2))2p_\mu\,,\\
&\bar S_\mu^{-1}(p) = \partial\bar S^{-1}(p+k)/\partial k_\mu\,|_{k=0}\,,
\end{aligned} \tag{4.1}$$

We assume $\bar S(p)$ has an inverse so that

$$\bar S_\mu^{-1}(p)\bar S(p) + \bar S^{-1}(p)\bar S_\mu(p) = 0\,. \tag{4.2}$$

The quark Dyson equation (2.5) and Ward identity (3.10) read

$$\bar S^{-1}(p) = -m^2\bar\Gamma_\mu(p)\bar S(p)\gamma_\mu \tag{4.3}$$

and

$$(1+\tfrac{1}{2}b)\bar\Gamma_\mu(p) = \bar S_\mu^{-1}(p) - \tfrac{1}{2}b\bar S(p)\bar\Gamma_\mu(p)\bar S^{-1}(p)\,. \tag{4.4}$$

These are a system of first-order differential equations for $\bar{S}(p)$ and $\bar{\Gamma}_\mu(p)$ which are parametrized in terms of b. The algebraic problem of finding the differential equations for $\bar{S}(p)$, that is, for the functions $A(p^2)$, $B(p^2)$, is not difficult. In order to satisfy PCAC (1.1) we must have $[S^{-1}(p),\gamma_5]_+ \neq 0$ or $B(p^2)\neq 0$.

The algebraic problem is simplified by noting that $p^\mu\Gamma_\mu(p), \gamma^\mu\Gamma_\mu(p), \Gamma_\mu(p)\gamma^\mu$ all have Dirac structure $\alpha\not{p}+\beta$ so they commute with $\bar{S}(p)$ and $\bar{S}^{-1}(p)$. Using this,

$$[\gamma^\mu\bar{\Gamma}_\mu(p),\bar{S}(p)] = [\bar{\Gamma}_\mu(p)\gamma^\mu,\bar{S}(p)] = [p^\mu\bar{\Gamma}_\mu(p),\bar{S}(p)] = 0\,, \tag{4.5}$$

it follows from (4.4) that

$$(1+b)\,p^\mu\bar{\Gamma}_\mu(p) = p^\mu\bar{S}^{-1}{}_\mu(p)\,. \tag{4.6}$$

A special case is if $1+b=0$. Then (4.6) implies

$$p^\mu\bar{S}_\mu{}^{-1}(p)=0 \quad (1+b=0) \tag{4.7}$$

or using (4.2) $p^\mu\bar{S}_\mu(p)=0$ or

$$\begin{aligned} &2p^2A'(p^2)+A(p^2)=0\,,\\ &B'(p^2)=0 \quad (1+b=0)\,. \end{aligned} \tag{4.8}$$

These equations have the general solution

$$\begin{aligned} &A(p^2)=\frac{a_1}{(p^2)^{1/2}}\,,\quad B(p^2)=c_1\,,\\ &\bar{S}(p)=\frac{a_1}{(p^2)^{1/2}}\,\not{p}+c_1 \quad (1+b=0) \end{aligned} \tag{4.9}$$

witht a_1 and c_1 constants. So for $b+1=0$ the problem is solved and $\bar{\Gamma}_\mu(p)$ can be obtained from the other equations.

In what follows we assume that $b+1\neq 0$. Another elementary case that can be solved exactly is if $b=0$ so the Ward identity is the same as for QED,

$$\bar{\Gamma}_\mu(p)=\bar{S}^{-1}{}_\mu(p) \quad (b=0)\,. \tag{4.10}$$

Using this in (4.3) and using (4.2)

$$\begin{aligned} \bar{S}^{-1}(p) &= -m^2\bar{S}^{-1}{}_\mu(p)\bar{S}(p)\gamma_\mu\\ &= m^2\bar{S}^{-1}(p)\bar{S}_\mu(p)\gamma_\mu \end{aligned} \tag{4.11}$$

or

$$\begin{aligned} &\frac{1}{2m^2}=2A(p^2)+p^2A'(p^2)\,,\\ &B'(p^2)=0 \quad (b=0)\,, \end{aligned} \tag{4.12}$$

which has the general solution[19]

$$\begin{aligned} &A(p^2)=\frac{a_2}{p^4}+\frac{1}{4m^2}\,,\quad B(p^2)=c_2\,,\\ &\bar{S}(p)=\left(\frac{a_2}{p^4}+\frac{1}{4m^2}\right)\not{p}+c_2 \quad (b=0)\,. \end{aligned} \tag{4.13}$$

with a_2 and c_2 constants. Equations (4.12) are a more transparent version of the same equations in Ref. 14. The vertex function $\bar{\Gamma}_\mu(p)$ is easily obtained from (4.13) and (4.10).

In the general case $1+b\neq 0$ a computation given in the appendix gives the differential equations

$$\begin{aligned} \frac{A(p^2)}{m^2} &= \left(\frac{1-b}{1+b}\right)p^2(A^2(p^2))' - (B^2(p^2))' + \frac{4+3b}{1+b}\,A^2(p^2)\,,\\ (1+b)\,\frac{B^2(p^2)}{m^2} &= \left(\frac{b-1}{b+1}\right)p^2A(p^2)(B^2(p^2))' + B^2(p^2)(2A'(p^2)p^2+4A(p^2))\,. \end{aligned} \tag{4.14}$$

We have not succeeded in finding an analytic solution to these equations in the general case. A few special cases can be noted. If $B=0$, which violates the PCAC condition (1.1), then (4.14) is a linear system for A with the general solution

$$A(p^2)=a_3(p^2)^\gamma+\left(\frac{1+b}{4+3b}\right)\frac{1}{m^2}\,,\quad \gamma=\frac{4+3b}{2(b-1)} \tag{4.15}$$

with a_3 a constant. A specific solution to (4.14) for which $B\neq 0$ valid if $b\neq 0$ (so the propagator has an inverse) is

$$\begin{aligned} &A(p^2)=\frac{(1+b)^2}{(3+5b)}\,\frac{1}{m^2}=\text{constant}\,,\\ &B(p^2)=c_3p^2\,,\\ &m^4c_3=\left(\frac{1+b}{3+5b}\right)(1+2b+3b^2)=\text{constant}\,. \end{aligned} \tag{4.16}$$

We see that for covariant gauges the complications due to ghosts can be considerable. Not only are we required to make an additional assumption for the ghost vertex (3.1), but granting this assumption the differential equations are rather complicated in the general case. These problems are averted in the axial gauge.

V. SOLUTION FOR BROKEN FLAVOR SYMMETRY

So far we have assumed the absence of flavor SU(N) symmetry breaking. We now suppose that flavor symmetry is explicitly broken in the Lagrangian by a quark mass term. Then the pseudoscalar Goldstone bosons acquire mass. The Dyson equation (2.1) has an inhomogeneous term m_0 which, without loss of generality, can be assumed diagonal in flavor space. This inhomogeneous term also modifies the infrared Dyson equation (2.5) according to

$$\Sigma(p)=\bar{m}_0-m^2\bar{\Gamma}_\mu(p)\Sigma^{-1}(p)\gamma_\mu\,, \tag{5.1}$$

where $\bar{m}_0=m_0\epsilon^{-1}$.

The solution for the quark propagator in this explicitly broken symmetry can also be given in terms of elementary functions in the case that $b=0$ so the Ward identity is simply

$$\overline{\Gamma}_\mu(p)=\overline{S}^{-1}{}_\mu(p) \quad (b=0). \tag{5.2}$$

The Ward identity for the color symmetry is unaffected by flavor breaking. With $-\overline{\Sigma}^{-1}(p)=\overline{S}(p^2)=A(p^2)\not{p}+B(p^2)$, (5.2) and (5.1) imply

$$1=m^2(4A(p^2)+2p^2A'(p^2))-\overline{m}_0B(p^2), \tag{5.3a}$$

$$0=2m^2B'(p^2)-\overline{m}_0A(p^2), \tag{5.3b}$$

a system of first-order equations which are soluble.

Differentiating (5.3a) and substituting (5.3b) into the result, one obtains a second-order equation for A. This equation is of a modified type of Bessel's equation and has a solution given by modified Bessel functions. By integrating (5.3b) one can obtain B from A. One can check that a specific linear combination of the modified Bessel functions actually solves the first order equations.

We simply state the results for the general solution to (5.3).

$$\begin{aligned} A(p^2)&=\frac{1}{\overline{m}_0{}^2p^2}\left[\left(\frac{2}{m^2}+\gamma\overline{m}_0\right)I_2(z)+\beta\overline{m}_0{}^4K_2(z)\right],\\ B(p^2)&=-\frac{1}{\overline{m}_0}+\frac{m^2}{\overline{m}_0z}\left[\left(\frac{2}{m^2}+\gamma\overline{m}_0\right)I_1(z)\right.\\ &\qquad\left.-\beta\overline{m}_0{}^4K_1(z)\right],\\ z&=(\overline{m}_0{}^2p^2/m^4)^{1/2}, \end{aligned} \tag{5.4}$$

where γ and β are arbitrary constants of integration and I_n and K_n are modified Bessel functions of the first and third kind. In the limit of symmetry restoration $\overline{m}_0\to 0$, if $\gamma\neq 0$ and $|\beta|<\infty$ we recover the symmetric solution (4.13) given previously.

VI. THE QUARK PROPAGATOR IN THE AXIAL GAUGE

Attempts to study QCD in a manifestly covariant gauge are necessarily plagued by ghosts. Following Kummer and others[20] we will adopt the axial gauge with a gauge-fixing term $C^aA_\mu^an_\mu$ with n_μ a constant non-null vector and C^a a trivial field. While the resulting Ward identities are simple because the ghosts are absent, one loses manifest covariance.

The general structure of the gauge field propagator in the axial gauge $n_\mu A_\mu^a=0$ is

$$\begin{aligned} iD_{\mu\nu}^{ab}(q)&=\delta^{ab}\left[a(q^2,q\cdot n)P_{\mu\nu}+b(q^2,q\cdot n)g_{\mu\nu}^T\right],\\ P_{\mu\nu}&=g_{\mu\nu}-\frac{q_\mu n_\nu+q_\nu n_\mu}{n\cdot q}+\frac{n^2\ q_\mu q_\nu}{(n\cdot q)^2},\\ g_{\mu\nu}^T&=g_{\mu\nu}-n_\mu n_\nu/n^2. \end{aligned} \tag{6.1}$$

This satisfies the gauge condition $n_\mu D_{\mu\nu}^{ab}(q)=0$. The confinement *Ansatz* which respects this gauge condition is

$$iD_{\mu\nu}^{ab}(q)\underset{q^2\to 0}{\sim}\delta^{ab}g_{\mu\nu}^T\ \frac{1}{q^2}\left(\frac{\mu^2}{q^2}\right)^{1-\epsilon}. \tag{6.2}$$

Using this ansatz in the quark Dyson equation (2.1) one finds using (6.2) the infrared content for the propagator $\Sigma^{-1}(p,n)=-\overline{S}(p,n)$ which is n_μ dependent,

$$\overline{S}^{-1}(p,n)=-m^2\overline{\Gamma}_\mu(p,n)\overline{S}(p,n)\gamma_\nu g_{\mu\nu}^T. \tag{6.3}$$

In this gauge the Ward identity is simply

$$\overline{\Gamma}_\mu(p,n)=\overline{S}^{-1}{}_\mu(p,n), \tag{6.4}$$

where the quark propagator has the Dirac decomposition

$$\overline{S}(p,n)=A(x,y)\not{p}+B(x,y)+C(x,y)\not{n}+D(x,y)\not{n}\not{p},\qquad x=p^2,\quad y=p\cdot n. \tag{6.5}$$

We set $n^2=-1$ without loss of generality.

Substituting (6.4) into (6.3) and using (4.2) one finds

$$\frac{1}{m^2}=\overline{S}_\mu(p,n)\gamma_\nu g_{\mu\nu}^T, \tag{6.6}$$

where

$$\begin{aligned}\overline{S}_\mu(p,n)&=A(x,y)\gamma_\mu+D(x,y)\not{n}\gamma_\mu\\ &\quad+(A_x(x,y)\not{p}+B_x(x,y)+C_x(x,y)\not{n}\\ &\quad+D_x(x,y)\not{n}\not{p})\,2p_\mu+G(p,n)n_\mu,\end{aligned} \tag{6.7}$$

where $A_x=dA/dx$ and $G(p,n)n_\mu$ will not contribute to (6.6). Substituting (6.7) into (6.6) implies

$$\begin{aligned}\frac{1}{m^2}&=2xA_x(x,y)+3A(x,y)-2C_x(x,y)y\\ &\quad-4y^2A_x(x,y),\\ 0&=B_x(x,y)+yD_x(x,y),\\ 0&=2xD_x(x,y)+3D(x,y)-2yB_x(x,y)\\ &\quad-4y^2D_x(x,y),\\ 0&=C_x(x,y)+yA_x(x,y).\end{aligned} \tag{6.8}$$

The general solution to these equations is not difficult to find. It is

$$\begin{aligned}A(x,y)&=\frac{\beta_1(y)}{(x-y^2)^{3/2}}+\frac{1}{3m^2},\\ D(x,y)&=\frac{\beta_2(y)}{(x-y^2)^{3/2}},\\ B(x,y)&=-yD(x,y)+\beta_3(y),\\ C(x,y)&=-yA(x,y)+\beta_4(y),\end{aligned} \tag{6.9}$$

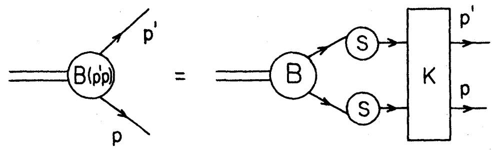

FIG. 8. Bethe-Salpeter equation for flavored mesons.

with $\beta_i(y)$ arbitrary functions of $y=p\cdot n$ alone.

No physical amplitude can depend on $y=p\cdot n$ since it is gauge dependent. So there should be no loss of generality for gauge-invariant physical amplitudes if we specify $\beta_i(y)$. Choosing $\beta_1(y)=\beta_2(y)=0$, $\beta_4(y)=y/3M^2$, $\beta_3(y)=\beta_3=$ constant the quark propagator takes the simple form

$$\overline{S}(p)=\frac{\not{p}}{3m^2}+\beta_3, \tag{6.10}$$

which is an entire function. Up to unimportant constants this is the same propagator used in Ref. 14.

The axial gauge, we conclude, leads to simple results upon application. This indicates that the ghost field complications encountered previously may be only superficial manifestations of an inconvenient gauge choice.

VII. REMARKS ON THE BOUND-STATE PROBLEM

Our purpose in constructing the quark propagator and the proper vertex is that these amplitudes enter the bound-state problem. A major problem is gauge field theory is to find a gauge-invariant approximation to the bound-state problem since one holds no hope for an exact solution. A gauge-invariant approximation to the bound-state problem is not known.

In our previous work[14] we examine the bound states as gap fluctuations in analogy with a similar development in superconductivity theory. The Dyson equation for the quark propagator in the presence of sources was considered in the approximation of ignoring the influence of sources on the gauge field propagator and the proper vertex function as they entered the Dyson equation. This approximation, since it treats the propagator and vertex differently, violates the Ward identities for Green's functions in the presence of sources. Then using the confining ansatz a nonlinear integral equation for the gap fluctuations resulted. This is the nonlinear Gorkov equation. The integral equation respects PCAC and in a linear approximation yields a reasonable spectrum of low-lying mesons. This development suggests that as far as meson phenomenology is concerned the nonlinear Gorkov equation can offer a viable approach.

If one wishes to go beyond this phenomenological approach one is led to consider the exact Bethe-Salpeter (BS) equation for the bound-state meson amplitude $B(p',p)$. For color-singlet, flavor-nonsinglet amplitudes $B(p',p)$ the exact BS equation is shown in Fig. 8. Flavor-singlet mesons require special treatment since pairs, etc. of gluons in color-singlet states can contribute to the direct-channel process. The skeleton expansion of the two-particle irreducible kernel K is indicated in Fig. 9.

Now consider the kernel if we make the confining *Ansatz* for the gluon propagator. From our previous work we note that the combination $\Gamma_\mu(p)S(p)=\overline{\Gamma}_\mu(p)\overline{S}(p)$, which is ubiquitous in the BS equation, is finite as $\epsilon\to 0$. Considering only the infrared singular part of the integrations in the BS equation and the kernel the first two terms of the kernel shown in Fig. 9 give the BS equation as $\epsilon\to 0$:

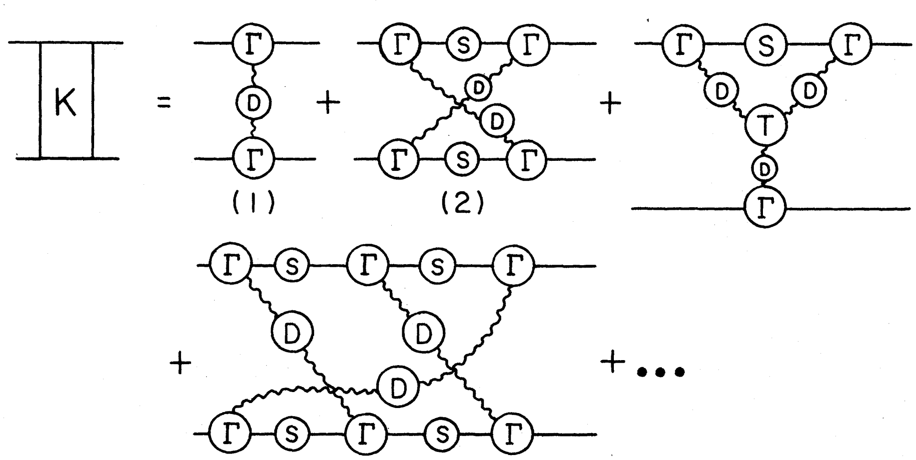

FIG. 9. Skeleton expansion for the kernel.

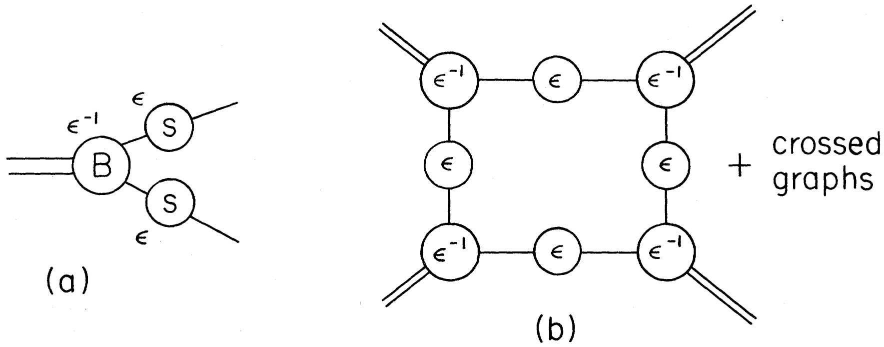

FIG. 10. (a) Meson-quark decay. (b) Color-singlet scattering in the duality diagram approximation.

$$B(p',p)=\frac{m^2}{\epsilon}\,\overline{\Gamma}_\mu(p')\overline{S}(p')B(p',p)\overline{S}(p)\overline{\Gamma}_\mu(p)$$
$$+c\left(\frac{m^2}{\epsilon}\right)^2\overline{\Gamma}_\mu(p')\overline{S}(p')\overline{\Gamma}_\nu(p')\overline{S}(p')B(p',p)\overline{S}(p)\overline{\Gamma}_\mu(p)\overline{S}(p)\overline{\Gamma}_\nu(p)+\cdots\ , \tag{7.1}$$

where c is a numerical constant.

It is clear that the truncation of the skeleton expansion is an inconsistent procedure since the powers of ϵ on each side of the equation cannot match. Evidently one must first sum the most infrared singular parts in the skeleton expansion and then take $\epsilon\to 0$. The actual summation of the series may be tractable if one considers only the most singular terms. This computation will not be undertaken here. We suggest that the result of this procedure will be to effectively replace the gluon propagator of the lowest-order kernel $d(q^2)/q^2$ with $\epsilon d(q^2)/q^2$. Then the powers of ϵ match and one obtains an eigenvalue problem for $B(p',p)$.

What this exercise shows is that truncation in gluon exchanges is not a consistent approximation if the gluon propagator is as singular as our *Ansatz*. This lesson is also made clear if one considers the Dyson equation for the proper vertex $\Gamma_\mu(p',p)$. Truncation of the skeleton expansion for the kernel in this Dyson equation also gives a result inconsistent (in powers of ϵ^{-1}) with our result obtained from an analysis of the exact Ward identity.

There are a few additional remarks on the bound-state problem which can be made that indicate the consistency of the confinement *Ansatz*. In Ref. 14 we noted that the bound-state amplitude $B(p',p)\sim\Sigma^*(p',p)$, defined in (14), was of $O(\epsilon^{-1})$. Consequently, since the quark propagator is of $O(\epsilon)$ the amplitude for meson $\to\overline{q}+q$ decay given by $S(p')B(p',p)S(p)\sim O(\epsilon)$ vanishes in the infrared limit $\epsilon\to 0$, consistent with confinement [see Fig. 10(a)]. However, if one considers any meson (color singlet) scattering amplitude in the duality diagram approximation as shown in Fig. 10(b) all the factors of ϵ^{-1} in the wave function cancel exactly against factors of ϵ in the quark propagators to produce a nontrivial scattering amplitude. This is an elementary intimation of the confinement mechanism in the bound-state sector.

A further result regarding gluon emissions is also easily obtained. Suppose color-singlet amplitudes are finite as $\epsilon\to 0$. This is true in the duality diagram approximation; here we assume it is generally true. Then we consider a representation of the color-singlet amplitude $T(p_i)$ by explicitly exhibiting a gluon-loop integration as shown in Fig. 11. Hence

$$T(p_i)=\int\frac{d^4q}{(2\pi)^4}\;D^{ab}_{\mu\nu}(q)T^{ab}_{\mu\nu}(q,p_i)\ ,$$

where $T^{ab}_{\mu\nu}(q,p_i)=\delta^{ab}T_{\mu\nu}(q,p_i)$ is the amplitude for emitting a pair of virtual gluons in a color-singlet state. Using our ansatz one obtains as $\epsilon\to 0$

$$T(p_i)=c\left(\frac{m^2}{\epsilon}\right)T_{\mu\mu}(0,p_i)\ ,$$

with c a numerical constant. If the color-singlet amplitude $T(p_i)$ is finite $T_{\mu\mu}(0,p_i)$ must be of $O(\epsilon)$

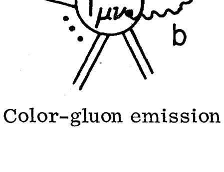

FIG. 11. Color-gluon emission amplitude.

as $\epsilon \to 0$ if it does not vanish identically. Hence the amplitude for soft-gluon pair emissions vanishes in the infrared limit. This is also the case for any finite number of gluon emissions from a color-singlet system. This result again supports the consistency of our approach.

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APPENDIX

Here we give a derivation of the differential equations (4.14). Substituting (4.4) into (4.3) and using (4.2) we have

$$(1+\tfrac{1}{2}b)/m^2 = \bar{S}_\mu\gamma_\mu + \tfrac{1}{2}b\bar{S}^2\bar{\Gamma}_\mu\gamma_\mu . \tag{A1}$$

Writing $\bar{S}\bar{\Gamma}_\mu\gamma_\mu = -\bar{\Gamma}_\mu\bar{S}_\mu + [\bar{S},\bar{\Gamma}_\mu]_+\gamma_\mu$ and using (4.3) and $[\bar{S},\bar{\Gamma}_\mu]_+\gamma_\mu = \bar{\Gamma}_\mu[\bar{S},\gamma_\mu]_+$ (A1) becomes

$$\begin{aligned} m^{-2} &= \bar{S}_\mu\gamma_\mu + \tfrac{1}{2}b\bar{S}\bar{\Gamma}_\mu[\bar{S},\gamma_\mu]_+ \\ &= \bar{S}_\mu\gamma_\mu + b\bar{S}(A\bar{\Gamma}_\mu p_\mu + B\bar{\Gamma}_\mu\gamma_\mu) \end{aligned} \tag{A2}$$

with $\bar{S} = A\not{p} + B$. For $\bar{\Gamma}_\mu p_\mu$ we use (4.6) or

$$p^\mu\bar{\Gamma}_\mu = p^\mu\bar{S}_\mu^{-1}/(1+b) . \tag{A3}$$

If we multiply (4.4) on the right by $\bar{S}\gamma_\mu$ then

$$(1+\tfrac{1}{2}b)\bar{\Gamma}_\mu\bar{S}\gamma_\mu = \bar{S}_\mu^{-1}\bar{S}\gamma_\mu - \tfrac{1}{2}b\,\bar{S}\bar{\Gamma}_\mu\gamma_\mu , \tag{A4}$$

and using $\bar{S}\gamma_\mu = -\gamma_\mu\bar{S} + [\bar{S},\gamma_\mu]_+ = -\gamma_\mu\bar{S} + 2(Ap_\mu + B\gamma_\mu)$ (A4) implies

$$-\bar{\Gamma}_\mu\gamma_\mu\bar{S} + (1+\tfrac{1}{2}b)2\bar{\Gamma}_\mu(Ap_\mu + B\gamma_\mu) = -\bar{S}^{-1}\bar{S}_\mu\gamma_\mu . \tag{A5}$$

Upon using (A3) we find from (A5)

$$(\bar{S} - (2+b)B)\bar{\Gamma}_\mu\gamma_\mu = \bar{S}^{-1}\bar{S}_\mu\gamma_\mu + \frac{2+b}{1+b}Ap^\mu\bar{S}_\mu^{-1} . \tag{A6}$$

Multiplying (A2) by $\bar{S} - (2+b)B$ and using (A6) and (A3) implies

$$(A\not{p} - (1+b)B)m^{-2} - (A\not{p} - B)\bar{S}_\mu\gamma_\mu - \frac{bA}{1+b}\,p_\mu\bar{S}_\mu , \tag{A7}$$

where $\bar{S}_\mu = A\gamma_\mu + 2p_\mu(A'\not{p} + B')$. Equation (A7) directly gives the differential equations (4.14).

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