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Asymptotic chiral invariance*

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We discuss the conditions under which chiral symmetries become exact at asymptotic momenta, utilizing the homogeneous Callan-Symanzik equations so that momenta are not restricted to the deep Euclidean domain as others have done previously. In particular, we consider as our prototype the chiral SU(2)⊗SU(2) σ model of Gell-Mann and Levy without fermions, where we allow for both spontaneous symmetry breaking and explicit breaking of the symmetry via a term in the Lagrangian linear in the σ field. To check our general results, we calculate to two-loop order in perturbation theory the coefficients of the homogeneous Callan-Symanzik equations along with the interesting amplitudes of the theory and finally show that under suitable conditions the symmetry-broken theory and the symmetry-conserved theory both approach the same massless, symmetric theory as the momenta become asymptotic.

I. INTRODUCTION

There have been numerous attempts to explore the implications of exact chiral invariance at asymptotic momenta. Hara[1] shows that if one assumes exact SU(2)⊗SU(2) symmetry at high energy and large momentum transfer, then for nucleon-nucleon scattering the Goldberger-Grisaru-MacDowell-Wong (GGMW)[2] helicity amplitudes are related as follows:

$$\phi_2/\phi_1 = O(s^{-1}),$$
$$t\phi_3 \approx \left(\frac{s\sqrt{-t}}{m_N}\right)\phi_5 + (s-2m_N{}^2)\phi_4 . \qquad (1.1)$$

Weinberg[3] assumes exact SU(2)⊗SU(2) to derive sum rules for the spectral functions of the propagators of the vector and axial-vector currents:

$$\int_0^\infty [\rho_V(\mu^2)-\rho_A(\mu^2)]\mu^{-2}d\mu^2 = F_\pi{}^2 , \qquad (1.2)$$

where F_π is the pion decay amplitude. And, under the additional assumption that the matrix elements of the currents act at high momenta as if the currents were free $1^\pm$ fields, he further derives a second sum rule:

$$\int_0^\infty [\rho_V(\mu^2)-\rho_A(\mu^2)]d\mu^2 = 0 . \qquad (1.3)$$

Others who have looked at such problems are Sakurai,[4] Akiba and Kang,[5] Hsu,[6] and Nambu.[7]

To provide a field-theoretic basis for such investigations, Lee and Weisberger[8] have employed the renormalization group in the more recent form of the inhomogeneous Callan-Symanzik equations[9] to show that at large spacelike momenta (the deep Euclidean domain) spontaneous symmetry breaking and explicit symmetry breaking due to a term in the Lagrangian linear in the scalar field(s) both disappear under suitable conditions.

In our investigations we use the homogeneous form of the Callan-Symanzik equations which were first applied to fermion field theories by Weinberg[10] and subsequently to scalar field theories by Callan.[11] This formulation of the renormalization-group equations has the advantage of giving simple solutions without restricting momenta to the deep Euclidean domain and of explicitly showing how mass parameters behave asymptotically; thus, we are able to study a wider variety of processes asymptotically.

As a prototype for the various chiral groups, we study the chiral SU(2)⊗SU(2) σ model of Gell-Mann and Levy[12] (excluding fermions for simplicity) and allow for both spontaneous symmetry breaking and explicit breaking of the symmetry via a term in the Lagrangian linear in the σ field. Using the homogeneous Callan-Symanzik equations with their treatment of masses as simply additional coupling constants, we both prove in general and demonstrate explicitly to two-loop order that, under suitable conditions for certain anomalous dimensions of the theory, in the asymptotic limit the symmetry-broken and symmetry-conserved theories both approach the same massless, symmetric theory.

In the next section we present the essential features of the SU(2)⊗SU(2) σ model and the renormalization program and then discuss our general results for asymptotic symmetries. In Sec. III we demonstrate our results to two-loop order by first calculating both the Callan-Symanzik coefficients and the relevant one-particle-irreducible (1PI) amplitudes and then by discussing the asymptotic limit of these amplitudes. Finally, in Sec. IV we offer some concluding remarks.

II. THE σ MODEL IN THE ASYMPTOTIC LIMIT

In this section we apply the machinery which the homogeneous Callan-Symanzik equations afford to our prototype theory, the chiral SU(2)⊗SU(2) σ model of Gell-Mann and Levy,[12] where we omit fermions in order to simplify later calculations.

A. σ-model formalism

We take as our Lagrangian

$$\mathcal{L}=\tfrac{1}{2}\partial_\mu\sigma\partial^\mu\sigma+\tfrac{1}{2}\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi}+\tfrac{1}{2}\phi(\sigma^2+\vec{\pi}^2)-\frac{g}{4!}(\sigma^2+\vec{\pi}^2)^2+c\sigma\,, \tag{2.1}$$

where σ is a scalar meson, π^i are three pseudoscalar mesons, $-\phi$ is the mass squared of the meson fields leading to spontaneous symmetry breakdown, and the term $c\sigma$, linear in the scalar field, provides for explicit breaking of SU(2) ⊗SU(2) by $\mathcal{L}$.

Rotations in isotopic-spin space are given by

$$\sigma\to\sigma\,,\qquad \vec{\pi}\to\vec{\pi}+\vec{\alpha}\times\vec{\pi}\,, \tag{2.2}$$

with the following conserved vector currents:

$$\vec{V}_\mu=\vec{\pi}\times\partial_\mu\vec{\pi}\,,\qquad \partial^\mu\vec{V}_\mu=0\,. \tag{2.3}$$

$\mathcal{L}$ is partially invariant under the following transformations:

$$\sigma\to\sigma+\vec{\beta}\cdot\vec{\pi}\,,\qquad \vec{\pi}\to\vec{\pi}-\vec{\beta}\sigma\,, \tag{2.4}$$

with the following partially conserved axial-vector currents (PCAC):

$$\vec{A}_\mu=\vec{\pi}\overleftrightarrow{\partial}_\mu\sigma\,,\qquad \partial^\mu\vec{A}_\mu=c\vec{\pi}\quad(\text{PCAC})\,. \tag{2.5}$$

The linear symmetry-breaking term and the imaginary mass cause σ to develop a vacuum expectation value, and hence we can write down a shifted Lagrangian in the form

$$\mathcal{L}=\tfrac{1}{2}\partial_\mu\sigma\partial^\mu\sigma+\tfrac{1}{2}\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi}-\tfrac{1}{2}\phi_\sigma\sigma^2-\tfrac{1}{2}\phi_\pi\vec{\pi}^2-\frac{gF\sigma}{3!}(\sigma^2+\vec{\pi}^2)-\frac{g}{4!}(\sigma^2+\vec{\pi}^2)^2\,, \tag{2.6a}$$

where we have slightly altered the notation in this equation so that the σ from Eq. (2.1) we now call σ' with

$$\sigma'=\sigma+F\,,\qquad \langle\sigma'\rangle_0\equiv\langle 0|\sigma'|0\rangle=F\,,\qquad \langle\sigma\rangle_0=0\,. \tag{2.7}$$

Whenever we use Eq. (2.1) it is understood that $\langle\sigma\rangle_0=F$ and when using Eq. (2.6a) it is understood that $\langle\sigma\rangle_0=0$. We refer to these two equivalent representations of the Lagrangian as the u (unshifted) and s (shifted) representations, respectively. Along with Eq. (2.6a) we add the following auxiliary relations:

$$\text{(i) } c+F\phi-\frac{g}{3!}F^3=0\,,\qquad \text{(ii) } \phi_\sigma=\frac{gF^2}{2}-\phi\,,\qquad \text{(iii) } \phi_\pi=\frac{gF^2}{6}-\phi\,, \tag{2.6b}$$

with Eq. (i) being valid only to tree order in perturbation theory. If $c=0$ we get the usual Goldstone result, namely $\phi_\pi=0$, $\phi_\sigma>0$.

In studying the amplitudes of the theory, we limit ourselves to the one-particle-irreducible (1PI) amplitudes, that is, amplitudes which include only those Feynman diagrams which are connected and cannot be separated into two parts by cutting a single propagator. It is customary to multiply an amplitude by each external particle's full (as opposed to free) inverse propagator. Also, except for two-particle amplitudes, all momenta are headed into the diagrams so that $\sum p_i=0$.

B. Callan-Symanzik equations for symmetric theory

Lee has shown how to renormalize the chiral SU(2)⊗SU(2) σ model in the presence of both spontaneous symmetry breaking and explicit breaking of the symmetry by a term in the Lagrangian linear in the σ field.[13] He has shown that the only counterterms needed to render the symmetry-broken theory finite are just those which renormalize the symmetry-conserved theory. The symmetric theory from which we calculate the counterterms is defined by the following Lagrangian yielding finite amplitudes:

$$\mathcal{L}+\delta\mathcal{L}=\frac{1+A}{2}\partial_\mu\sigma\partial^\mu\sigma+\frac{1+A}{2}\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi}-\frac{1+C}{2}\phi(\sigma^2+\vec{\pi}^2)-\frac{D}{2}(\sigma^2+\vec{\pi}^2)-\frac{g+B}{4!}(\sigma^2+\vec{\pi}^2)^2\,. \tag{2.8}$$

In the manner of Callan,[11] we consider ϕ in Eqs. (2.1), (2.6), and (2.8) to be an external, space-time-independent field providing mass for the mesons. To renormalize the symmetric theory we have the freedom to use any of the two- and four-particle amplitudes. We use the σ propagator:

$$\Gamma_s^{(2,0,0,0)}=i[(1+A)p^2-\Sigma_\sigma] \quad (2.9)$$

(where $\Gamma^{(n,m_1,m_2,m_3)}$ denotes the 1PI amplitude for n external σ's and m_i external π^i's and the subscript s refers to the symmetric theory) and the σ^4 vertex. To avoid infrared divergences and to establish the connection with the zero-mass theory, the counterterms A, B, C, and D are determined by the following renormalization conditions:

$$\left.\frac{\partial\Gamma_s^{(2,0,0,0)}}{\partial p^2}\right|_{\substack{p^2=0\\ \phi=M^2}}=i\,,$$

$$\left.\frac{\partial\Gamma_s^{(2,0,0,0)}}{\partial \phi}\right|_{\substack{p^2=0\\ \phi=M^2}}=-i\,, \quad (2.10)$$

$$\Gamma_s^{(2,0,0,0)}(p^2=0,\phi=0)=0\,,$$

$$\Gamma_s^{(4,0,0,0)}(p_i=0,\phi=M^2)=-ig\,.$$

In the manner of Callan we alternatively write the Lagrangian yielding finite amplitudes in the following form:

$$\mathcal{L}+\delta\mathcal{L}=\frac{Z}{2}\,\partial_\mu\sigma\partial^\mu\sigma+\frac{Z}{2}\,\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi}-\frac{Z}{2}\,\phi_0(\sigma^2+\vec{\pi}^2)$$
$$-\frac{Z}{2}\,\delta\mu^2(\sigma^2+\vec{\pi}^2)-\frac{Z^2}{4!}\,g_0(\sigma^2+\vec{\pi}^2)^2\,, \quad (2.11)$$

where the zero subscripts denote unrenormalized quantities and

$$g=F_1\left(g_0,\frac{\Lambda}{M}\right),$$
$$Z=F_2\left(g_0,\frac{\Lambda}{M}\right), \quad (2.12)$$
$$\phi=\phi_0F_3\left(g_0,\frac{\Lambda}{M}\right),$$
$$\delta\mu^2=\Lambda^2F_4(g_0)\,,$$

with Λ being the cutoff used in calculating the unrenormalized amplitudes. Then upon noting that the renormalized amplitudes $\Gamma_s^{(n,m_i)}$ can be written in terms of the unrenormalized ones $\Gamma_{s0}^{(n,m_i)}$ via

$$\Gamma^{(n,m_i)}_{s(p_j,g,\phi,M)}=Z^{(n+\Sigma_{i=1}^3 m_i)/2}\,\Gamma^{(n,m_i)}_{s0(p_j,g_0,\phi_0,\delta\mu^2,\Lambda)}\,, \quad (2.13)$$

we can immediately write

$$0=M\frac{\partial}{\partial M}\,\Gamma_{s0}^{(n,m_i)}$$
$$=M\frac{\partial}{\partial M}\left[Z\left(g_0,\frac{\Lambda}{M}\right)^{-(n+\Sigma_{i=1}^3 m_i)/2}\times\Gamma^{(n,m_i)}_{s(p_j,g,\phi,M)}\right]_{g_0,\phi_0,\Lambda\text{ fixed}}\,, \quad (2.14)$$

which implies the homogeneous form of the Callan-Symanzik equations:

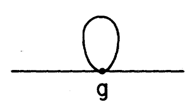
$$\left[M\frac{\partial}{\partial M}+\beta(g)\frac{\partial}{\partial g}+\hat{\gamma}_\phi(g)\phi\frac{\partial}{\partial\phi}-\left(n+\sum_{i=1}^3 m_i\right)\gamma(g)\right]\times\Gamma^{(n,m_i)}_{s(p_j,g,\phi,M)}=0\,, \quad (2.15)$$

where

$$\beta=M\frac{\partial}{\partial M}\,g=M\frac{\partial}{\partial M}F_1\left(g_0,\frac{\Lambda}{M}\right),$$
$$\gamma=\frac{1}{2}M\frac{\partial}{\partial M}\,\ln Z=\frac{1}{2}M\frac{\partial}{\partial M}\,\ln F_2\left(g_0,\frac{\Lambda}{M}\right), \quad (2.16)$$
$$\hat{\gamma}_\phi=M\frac{\partial}{\partial M}\,\ln\frac{\phi}{\phi_0}=M\frac{\partial}{\partial M}\,\ln F_3\left(g_0,\frac{\Lambda}{M}\right),$$

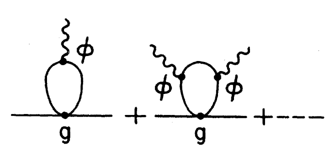
and all depend only upon the renormalized coupling constant g.

The key to the above procedure is being able to multiplicatively renormalize the bare mass ϕ_0 (by a factor F_3 which does not depend upon either ϕ or ϕ_0). Ordinarily one would not be able to include diagrams for the self-energy such as



$$\sim\int\frac{d^4l}{(2\pi)^4}\,\frac{1}{l^2-\phi+i\epsilon}\,, \quad (2.17)$$

because in perturbation expansions one is not normally allowed to contract fields inside normal-ordering signs, a restriction which leads to a non-multiplicative renormalization of the mass. However, if we adopt Callan's approach of treating ϕ as an external field, we can "legally" include diagrams of the form



$$\sim\int\frac{d^4l}{(2\pi)^4}\left(\frac{1}{l^2-\phi+i\epsilon}-\frac{1}{l^2+i\epsilon}\right), \quad (2.18)$$

where the wavy-line insertions of the ϕ field onto zero-mass propagators sum up to give an integral involving one massive propagator minus an integral involving one massless propagator. Further, the counterterm D allows us to forget altogether the normal ordering and hence to forget about subtracting the second term in Eq. (2.18) because such terms can be automatically subtracted by properly defining D; thus, summing ϕ insertions on a zero-mass propagator from zero to infinity gives a single propagator of mass ϕ, and we are led to a theory with a multiplicative mass renormalization. In Lee's work on the renormalization of the σ model, he found that normal ordering leads to an inconsistent renormalization procedure.[13] So, Callan has in a sense anticipated Lee's problem by allowing us to throw out normal ordering altogether in the renormalization-group formalism.

In applying the homogeneous Callan-Symanzik equation [Eq. (2.15)] to the study of the high-energy behavior of the theory, it is useful to write down its general solution:

$$\Gamma_s^{(n,m_i)}(\lambda p_j, g, \phi, M) = \lambda^{4-(n+\Sigma_{i=1}^3 m_i)}\Gamma_s^{(n,m_i)}(p_j, g(\lambda), \phi(\lambda), M) \times \exp\left[-\left(n+\sum_{i=1}^{3} m_i\right)\int_1^\lambda \frac{d\lambda'}{\lambda'}\gamma(g(\lambda'))\right], \tag{2.19}$$

with

$$\lambda\frac{\partial}{\partial\lambda}g(\lambda)=\beta(g(\lambda))g, \quad g(1)=g$$
$$\lambda\frac{\partial}{\partial\lambda}\phi(\lambda)=-[2-\hat{\gamma}_\phi(g(\lambda))]\phi(\lambda), \quad \phi(1)=\phi. \tag{2.20}$$

So that as $\lambda\to\infty$, g gets driven to an ultraviolet fixed point g_0 for $\beta(g_0)=0$, and $\phi(\lambda)\to 0$ as long as $\gamma(g_0)<2$. And since the theory with $\phi=0$ is the massless theory, we conclude that as long as $2>\gamma(g_0)$, the asymptotic behavior of the massive, symmetric theory is without approximation that of the massless, symmetric theory. We thus require that this zero-mass limit of the theory exists, such as in the study of high-energy fixed-angle hadronic scattering processes.

C. Callan-Symanzik equations for broken-symmetry theory

What we have done in first renormalizing the symmetric theory is what Lee refers to as an intermediate renormalization of the broken-symmetry theory,[13] because when we pass to the broken-symmetry situation, the theory is already made finite by the symmetric theory's counterterms; however, to complete the renormalization process, we have to fix propagator poles at the physical σ and π masses, and we get the conventional field normalizations (which we do not worry about in our discussion) by multiplying the fields, coupling constant, and amplitudes by factors of finite $\sqrt{z_\pi}$ and $\sqrt{z_\sigma}$, as in the following:

$$\langle 0|\sqrt{z_\pi}\ \Pi(x)|\pi(p)\rangle=\frac{e^{-ip\cdot x}}{(2\pi)^{3/2}\sqrt{2p_0}}, \tag{2.21}$$

where $\Pi(x)$ is the pion field.

When we pass to the broken-symmetry theory defined by Eqs. (2.1) and (2.6), we must use $\delta\mathcal{L}=\text{terms}+\frac{1}{2}C\phi(\sigma^2+\vec{\pi}^2)$ instead of $\delta\mathcal{L}=\text{terms}-\frac{1}{2}C\phi(\sigma^2+\vec{\pi}^2)$ because our theory now involves an imaginary mass. Also, it is simpler to calculate the amplitudes $\Gamma^{(n,m_i)}$ in the s representation, and the Lagrangian with its counterterms leading to finite amplitudes is

$$\mathcal{L}+\delta\mathcal{L}=\frac{1+A}{2}\partial_\mu\sigma\partial^\mu\sigma+\frac{1+A}{2}\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi}-\tfrac{1}{2}\phi_\sigma'\sigma^2-\tfrac{1}{2}\phi_\pi'\vec{\pi}^2-\frac{gF\sigma}{3!}(\sigma^2+\vec{\pi}^2)$$
$$-\frac{g}{4!}(\sigma^2+\vec{\pi}^2)^2-\frac{D}{2}(\sigma^2+\vec{\pi}^2)+\frac{C}{2}\phi(\sigma^2+\vec{\pi}^2)-\frac{BF\sigma}{3!}(\sigma^2+\vec{\pi}^2)-\frac{B}{4!}(\sigma^2+\vec{\pi}^2)^2, \tag{2.22}$$

where

$$\phi_\sigma'=\phi_\sigma+\frac{BF^2}{2}, \quad \phi_\pi'=\phi_\pi+\frac{BF^2}{6}, \tag{2.23}$$

owing to coupling-constant renormalizations inside ϕ_σ and ϕ_π which are defined by Eq. (2.6b).

The Callan-Symanzik equation for the broken-symmetry theory is easily derived by paralleling the derivation for the symmetric theory. We get

$$\left[M\frac{\partial}{\partial M}+\beta(g)\frac{\partial}{\partial g}+\hat{\gamma}_\phi(g)\phi\frac{\partial}{\partial\phi}+\hat{\gamma}_c(g)c\frac{\partial}{\partial c}-\left(n+\sum_{i=1}^{3}m_i\right)\gamma(g)\right]\Gamma^{(n,m_i)}(p_j,g,\phi_\sigma,\phi_\pi,M,F)=0, \tag{2.24}$$

where ϕ_σ, ϕ_π, and F are to be viewed as functions of g, ϕ, c, and M^2 as defined by Eq. (2.6b) [with higher-order radiative corrections to relation (i)].

The only difference between the Callan-Symanzik equations for the symmetry-broken and symmetry-conserved theories is that the symmetry-broken theory has an extra term $\hat{\gamma}_c c\partial\Gamma^{(n,m_i)}/\partial c$. But upon closer scrutiny one realizes that $\hat{\gamma}_c(g)$ is calculated from a symmetric-theory counterterm. Lee has pointed out that the relationship between the renormalized c and the unrenormalized c_0 is[13]

$$c_0=\frac{1}{\sqrt{Z}}c, \tag{2.25}$$

whereas for the fields we have

$$\sigma_0=\sqrt{Z}\,\sigma. \tag{2.26}$$

Hence extending the program leading up to the symmetric theory's Callan-Symanzik equation, we have

$$\hat{\gamma}_c(g) = M\frac{\partial}{\partial M}\ln\frac{c}{c_0} = \frac{1}{2}M\frac{\partial}{\partial M}\ln Z = \gamma(g), \qquad (2.27)$$

so that the broken-symmetry theory does not have a new coefficient after all. The other coefficients, $\beta(g)$, $\hat{\gamma}_\phi(g)$, and $\gamma(g)$, have the same values as for the symmetric theory.

$\Gamma^{(1,0,0,0)} \equiv F = \langle\sigma\rangle_0$ (in the u representation for the theory) satisfies a homogeneous Callan-Symanzik equation which is not quite the same as those for the other amplitudes $\Gamma^{(n,m_i)}$. To see this, note that Eq. (2.24) for the other $\Gamma^{(n,m_i)}$ is derived from the following relationship between the renormalized and unrenormalized amplitudes:

$$\Gamma^{(n,m_i)}(p_j,g,\phi,M,c) = Z^{(n+\Sigma_{i=1}^3 m_i)/2}\Gamma_0^{(n,m_i)}(p_j,g_0,\phi_0,\delta\mu^2,\Lambda,c_0). \qquad (2.28)$$

However, σ and σ_0 are related by Eq (2.26) so that $F=\langle\sigma\rangle_0$ and $F_0=\langle\sigma_0\rangle_0$ are related by

$$F_0 = \sqrt{Z}\,F. \qquad (2.29)$$

Consequently, the same line of reasoning leading to Eq. (2.24) when applied to F gives

$$\left[M\frac{\partial}{\partial M} + \beta(g)\frac{\partial}{\partial g} + \hat{\gamma}_\phi(g)\phi\frac{\partial}{\partial\phi} + \hat{\gamma}_c(g)c\frac{\partial}{\partial c} + \gamma(g)\right]F = 0, \qquad (2.30)$$

so that the last terms on the left-hand side of Eqs. (2.24) and (2.30) differ by a minus sign. We can further exploit Eq. (2.30). Remembering that in the expressions for $\Gamma^{(n,m_i)}$ in the s representation, F is not considered an independent parameter but depends upon g, ϕ, c, and M^2 via its implicit equation [Eq. (2.6b), relation (i)] plus radiative corrections, we can rewrite the Callan-Symanzik equations for $\Gamma^{(n,m_i)}$ as follows:

$$\left[\frac{\partial}{\partial F}\Gamma^{(n,m_i)}\right]\left[M\frac{\partial}{\partial M} + \beta(g)\frac{\partial}{\partial g} + \hat{\gamma}_\phi(g)\phi\frac{\partial}{\partial\phi} + \hat{\gamma}_c(g)c\frac{\partial}{\partial c}\right]F + \left[M\frac{\partial}{\partial M} + \beta(g)\frac{\partial}{\partial g} + \hat{\gamma}_\phi\phi\frac{\partial}{\partial\phi} + \hat{\gamma}_c(g)c\frac{\partial}{\partial c} - \left(n+\sum_{i=1}^3 m_i\right)\gamma(g)\right]\Gamma^{(n,m_i)}\Big|_{F\ \text{fixed}} = 0. \qquad (2.31)$$

Then, upon defining

$$\hat{\gamma}_F \equiv \frac{1}{F}\left[M\frac{\partial}{\partial M} + \beta(g)\frac{\partial}{\partial g} + \hat{\gamma}_\phi(g)\phi\frac{\partial}{\partial\phi} + \hat{\gamma}_c(g)c\frac{\partial}{\partial c}\right]F, \qquad (2.32)$$

Eq. (2.30) tells us

$$\hat{\gamma}_F(g) = -\gamma(g) = -\hat{\gamma}_c(g). \qquad (2.33)$$

Hence, as long as $\Gamma^{(n,m_i)}$ depends upon c only through F, we can consider F an independent parameter and rewrite Eq. (2.24) as follows:

$$\left[M\frac{\partial}{\partial M} + \beta(g)\frac{\partial}{\partial g} + \hat{\gamma}_\phi(g)\phi\frac{\partial}{\partial\phi} + \hat{\gamma}_F(g)F\frac{\partial}{\partial F} - \left(n+\sum_{i=1}^3 m_i\right)\gamma(g)\right]\Gamma^{(n,m_i)}(p_j,g,\phi,M,F) = 0, \qquad (2.34)$$

which is analogous to what Lee and Weisberger found for the inhomogeneous Callan-Symanzik equations.[8] Since more often than not it is more convenient to use the s representation in calculating amplitudes, we find that Eq. (2.34) is the more palatable form of the homogeneous Callan-Symanzik equations. Thus F enters the Callan-Symanzik solutions as a momentum-dependent effective mass parameter.

D. Asymptotic solutions

To study the asymptotic behavior of the broken-symmetry theory, we write down the general solution to Eq. (2.34):

$$\Gamma^{(n,m_i)}(\lambda p_j,g,\phi,F,M) = \lambda^{4-(n+\Sigma_{i=1}^3 m_i)}\Gamma^{(n,m_i)}(p_j,g(\lambda),\phi(\lambda),F(\lambda),M) \times\exp\left[-\left(n+\sum_{i=1}^3 m_i\right)\int_1^{\lambda}\frac{d\lambda'}{\lambda'}\gamma(g(\lambda'))\right], \qquad (2.35)$$

with

$$\lambda\frac{\partial}{\partial\lambda}g(\lambda) = \beta(g(\lambda)),\quad g(1)=g$$

$$\lambda\frac{\partial}{\partial\lambda}\phi(\lambda) = -[2-\hat{\gamma}_\phi(g(\lambda))]\phi(\lambda),\quad \phi(1)=\phi \qquad (2.36)$$

$$\lambda\frac{\partial}{\partial\lambda}F(\lambda) = -[1-\hat{\gamma}_F(g(\lambda))]F(\lambda),\quad F(1)=F.$$

Had we been interested in how c behaves asymptotically, we would have solved Eq. (2.24) giving

$$\lambda \frac{\partial}{\partial \lambda} c(\lambda) = -[3 - \hat{\gamma}_c(g(\lambda))]c(\lambda), \quad c(1) = c. \tag{2.37}$$

From the above we see that, without approximation, the conditions for the broken-symmetry theory to approach the massless, symmetric theory at asymptotic momenta, or $\lambda \to \infty$, are as follows:

(i) β has a zero at g_0 so that $g(\lambda) \to g_0$,

(ii) $\hat{\gamma}_\phi(g_0) < 2$ so that $\phi(\lambda) \to 0$, and

(iii) $\hat{\gamma}_F(g_0) < 1$ so that $F(\lambda) \to 0$.

In addition, conditions (ii) and (iii) in conjunction with the implicit equation for F in terms of g, ϕ, c and M^2 [cf. Eq. (2.6b), relation (i) plus radiative corrections] are sufficient to ensure that order by order $c(\lambda) \to 0$ as $\lambda \to \infty$; however, we also see from Eq. (2.37) that $c(\lambda) \to 0$ is ensured immediately to all orders if $\hat{\gamma}_c(g_0) < 3$.

To conclude this section, we compare the two quite different broken-symmetry theories $\mathcal{L}^+$ and $\mathcal{L}^-$ defined by

$$\mathcal{L}^\pm + \delta\mathcal{L}^\pm = \frac{1+A}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1+A}{2}\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi} \pm \frac{1+C}{2}\phi(\sigma^2+\vec{\pi}^2) - \frac{D}{2}(\sigma^2+\vec{\pi}^2) - \frac{g+B}{4!}(\sigma^2+\vec{\pi}^2)^2 + c\sigma, \tag{2.38}$$

where the counterterms are still those from the symmetric theory and $\mathcal{L}^-$ has no spontaneous symmetry breaking (but only explicit breaking of the symmetry) since the mass is real and positive. In light of the above discussion, we note that even though for finite momenta the worlds described by $\mathcal{L}^+$ and $\mathcal{L}^-$ look quite different, they converge to the same massless, symmetric theory in the asymptotic limit.

III. VERIFICATION OF RESULTS TO TWO-LOOP ORDER

In this section we verify the results obtained in Sec. II to two-loop order in perturbation theory. We work to this order since for such scalar theories as we are dealing with, some underlying features (such as anomalous field dimensions) do not arise until two-loop order. Owing to limitations in space in writing down the expressions and diagrams of selected amplitudes, we give only the one-loop-order terms. To see the calculations in their full, rather lengthy two-loop-order detail, we refer the reader to Ref. 14. Further, we have checked in detail that all the appropriate one-, two-, three-, and four-particle amplitudes of the massive symmetric, massless symmetric, and broken-symmetry theories indeed do satisfy their respective Callan-Symanzik equations.

To regulate diagrams with no overlapping divergences, we simply cut the momentum integrations off at large Λ; however, for diagrams with overlapping divergences, we have to be a bit more careful, so we use Pauli-Villars regularization on each propagator via[15]

$$\frac{1}{p^2-\phi+i\epsilon} - \frac{1}{p^2-\Lambda^2+i\epsilon} = -\Lambda^2\int_0^1 \frac{dx}{[p^2 - x\phi - (1-x)\Lambda^2 + i\epsilon]^2}\,. \tag{3.1}$$

As mentioned earlier, to renormalize the symmetric theory we can use any of the two- and four-particle amplitudes. To calculate the wave-function renormalization and the mass counterterms we use the σ propagator $\Gamma_s^{(2,0,0,0)}$ shown in Eq. (2.9). The diagrams contributing to Σ_σ are shown in Fig. 1. To calculate the coupling-constant counterterm we use $\Gamma_s^{(4,0,0,0)}(p_i)$, where the contributing diagrams are shown in Fig. 2. Subject to the renormalization conditions shown in Eq. (2.10), the counterterms to two-loop order are calculated to be the following:

$$\begin{aligned}
Z &= 1 + A = 1 - \frac{2g^2}{3(32\pi^2)^2}\ln\frac{\Lambda^2}{M^2}\,,\\
B &= \frac{g^2}{8\pi^2}\left(\ln\frac{\Lambda^2}{M^2} - 1\right)\\
&\quad + \frac{8g^3}{(32\pi^2)^2}\left(2\ln^2\frac{\Lambda^2}{M^2} - \frac{19}{3}\ln\frac{\Lambda^2}{M^2} + \frac{25}{6}\right),\\
C &= \frac{g}{16\pi^2}\left(\ln\frac{\Lambda^2}{M^2} - 1\right)\\
&\quad + \frac{2g^2}{(32\pi^2)^2}\left(2\ln^2\frac{\Lambda^2}{M^2} - 6\ln\frac{\Lambda^2}{M^2} + 6\right),\\
D &= -\frac{g\Lambda^2}{16\pi^2} + D_2\,,
\end{aligned} \tag{3.2}$$

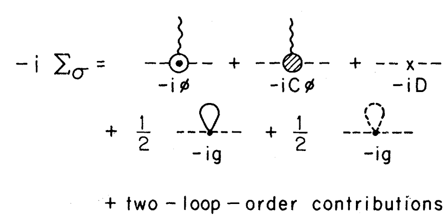


FIG. 1. Diagrams contributing to Σ_σ. σ lines are dashed and pion lines are solid.

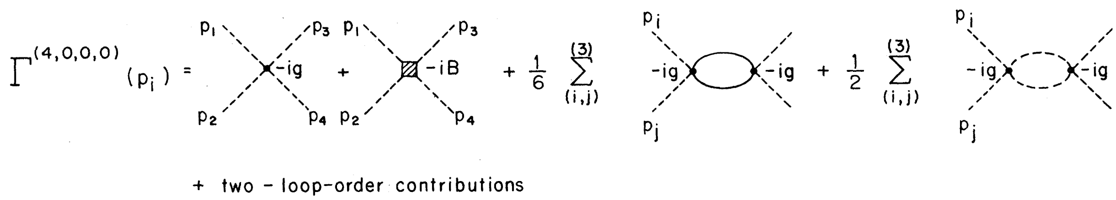


FIG. 2. Diagrams contributing to $\Gamma^{(4,0,0,0)}$. $\sum_{(i,j)}^{(3)}$ means to sum over the three pairs (p_1, p_2), (p_1, p_3), and (p_1, p_4).

where it is not necessary to know $D_2 \sim g^2$ to this order; D is defined so that we simply drop all terms in $\Gamma_s^{(2,0,0,0)}$ which vary as Λ^2. Substituting these counterterms into Eq. (2.16) yields the following coefficients of the Callan-Symanzik equations to two-loop order:

$$\beta(g) = \frac{g^2}{4\pi^2} - \frac{104g^3}{3(32\pi^2)^2},$$

$$\gamma(g) = \hat{\gamma}_c(g) = -\hat{\gamma}_F(g) = \frac{2g^2}{3(32\pi^2)^2}, \tag{3.3}$$

$$\hat{\gamma}_\phi(g) = \frac{g}{8\pi^2} - \frac{20g^2}{3(32\pi^2)^2}.$$

The finite σ propagator for the symmetric theory is

$$-i\Gamma_s^{(2,0,0,0)} = p^2 - \phi + \frac{g\phi}{16\pi^2}\left(\ln\frac{M^2}{\phi} + 1\right) + \text{two-loop-order contributions}, \tag{3.4}$$

and the σ^4 vertex at $p_i = 0$ is

$$\Gamma_s^{(4,0,0,0)}(p_i = 0) = -ig + \frac{ig^2}{8\pi^2}\left(\ln\frac{M^2}{\phi}\right) + \text{two-loop-order contributions}. \tag{3.5}$$

We define the massive, symmetric theory for the purpose of calculating counterterms and Callan-Symanzik coefficients. And before investigating whether the broken-symmetry amplitudes reduce to the massless, symmetric amplitudes, we first define the massless, symmetric theory as that corresponding to the following Lagrangian plus counterterms:

$$\hat{\mathcal{L}} + \delta\hat{\mathcal{L}} = \frac{1+\hat{A}}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1+\hat{A}}{2}\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi} - \frac{\hat{D}}{2}(\sigma^2 + \vec{\pi}^2) - \frac{g+\hat{B}}{4!}(\sigma^2+\vec{\pi}^2)^2, \tag{3.6}$$

where the carets refer to the massless theory. This time, since $\phi = 0$, we do not need the counterterm C nor the anomalous mass dimension $\hat{\gamma}_\phi(g)$ [since it is the product $\phi\hat{\gamma}_\phi(g)$ which occurs in the Callan-Symanzik equations]. Further, we only need the following three renormalization conditions:

$$\left.\frac{\partial\hat{\Gamma}^{(2,0,0,0)}}{\partial p^2}\right|_{p^2=0} = i,$$

$$\hat{\Gamma}^{(2,0,0,0)}(p^2 = 0) = 0, \tag{3.7}$$

$$\hat{\Gamma}^{(4,0,0,0)}(p_{ij}^2 = -M^2) = -ig,$$

where $p_{ij} \equiv p_i + p_j$. The counterterms $\hat{A}$, $\hat{B}$, and $\hat{D}$ are just equal to A, B, and D, respectively, as shown in Eq. (3.2). The finite amplitudes of greatest interest are

$$\hat{\Gamma}^{(2,0,0,0)}(p^2) = \hat{\Gamma}^{(0,2,0,0)}(p^2) = ip^2 + \text{two-loop-order contributions},$$

$$\hat{\Gamma}^{(4,0,0,0)}(p_i) = -ig - \frac{4ig^2}{3(32\pi^2)}\sum_{(i,j)}^{(3)}\ln\left(\frac{-p_{ij}^2}{M^2}\right) + \text{two-loop-order contributions}, \tag{3.8}$$

$$\hat{\Gamma}^{(1,2,0,0)}(p_i) = 0,$$

where $\sum_{(i,j)}^{(3)}$ means to sum over p_{12}, p_{13}, and p_{14}. Equation (3.8) defines what we mean by amplitudes of the massless, symmetric theory. The Callan-Symanzik coefficients $\beta(g)$ and $\gamma(g)$ are the same as given in Eq. (3.3).

Having obtained the necessary counterterms to two-loop order, we now pass to the broken-symmetry theory and first calculate the implicit equation for $\Gamma^{(1,0,0,0)} \equiv F = \langle\sigma\rangle_0$. We prefer to work in the u representation:

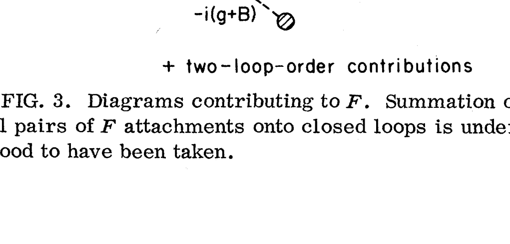


FIG. 3. Diagrams contributing to F. Summation over all pairs of F attachments onto closed loops is understood to have been taken.

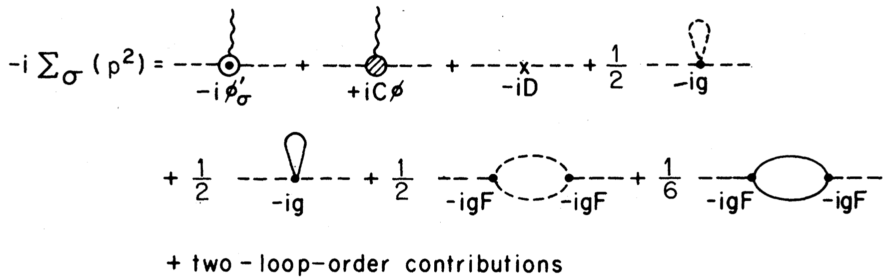


FIG. 4. Diagrams for $-i\,\Sigma_\sigma(p^2)$.

$$\mathcal{L}+\delta\mathcal{L}=\frac{1+A}{2}\partial_\mu\sigma\partial^\mu\sigma+\frac{1+A}{2}\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi}+\tfrac{1}{2}[(1+C)\phi-D](\sigma^2+\vec{\pi}^2)-\frac{g+B}{4!}(\sigma^2+\vec{\pi}^2)^2+c\sigma. \quad (3.9)$$

The diagrams contributing to F are shown in Fig. 3, where it is understood that we have already summed over all pairs of F attachments onto the closed loops. Equation (2.6b), relation (i) gets modified to

$$c+F\phi\left\{1-\frac{g}{16\pi^2}\left[1+\frac{1}{2}\left(\ln\frac{M^2}{\phi_\pi}+\ln\frac{M^2}{\phi_\sigma}\right)\right]+\text{two-loop-order contributions}\right\}$$
$$-\frac{gF^3}{3!}\left\{1-\frac{g}{8\pi^2}\left[1+\frac{1}{4}\left(\ln\frac{M^2}{\phi_\pi}+3\ln\frac{M^2}{\phi_\sigma}\right)\right]+\text{two-loop-order contributions}\right\}=0. \quad (3.10)$$

Next we calculate several of the most interesting amplitudes for the broken-symmetry theory. As mentioned earlier, it is easier to use the s representation of the Lagrangian which is shown in Eq. (2.22), with the counterterms given in Eq. (3.2) and ϕ'_σ and ϕ'_π defined by Eq. (2.23).

The diagrams for $-i\Sigma_\sigma(p^2)$, $-i\Sigma_\pi(p^2)$, $\Gamma^{(4,0,0,0)}(p_i)$, $\Gamma^{(0,4,0,0)}(p_i)$, and $\Gamma^{(1,2,0,0)}(p_i)$ are given in Figs. 4–8, respectively. The summations over all topologically inequivalent momentum labelings are indicated, with the numbers above the summation signs indicating the number of such inequivalent labelings.

The finite amplitudes calculated from Figs. 4–8 are the following (where as before $p_{ij}=p_i+p_j$):

$$\text{(i)}\quad -i\,\Gamma^{(2,0,0,0)}(p^2)=p^2-\phi_\sigma+\frac{g\phi_\sigma}{32\pi^2}\left(\ln\frac{M^2}{\phi_\sigma}+1\right)+\frac{g\phi_\pi}{32\pi^2}\left(\ln\frac{M^2}{\phi_\pi}+1\right)+\frac{g^2F^2}{32\pi^2}\left(\ln\frac{M^2}{\phi_\sigma}+\frac{1}{3}\ln\frac{M^2}{\phi_\pi}\right)$$
$$-\frac{g^2F^2}{32\pi^2}\int_0^1 dz\,\ln\left\{\left[1-\frac{p^2}{\phi_\sigma}z(1-z)\right]\left[1-\frac{p^2}{\phi_\pi}z(1-z)\right]^{1/3}\right\}+\text{two-loop-order contributions;} \quad (3.11)$$

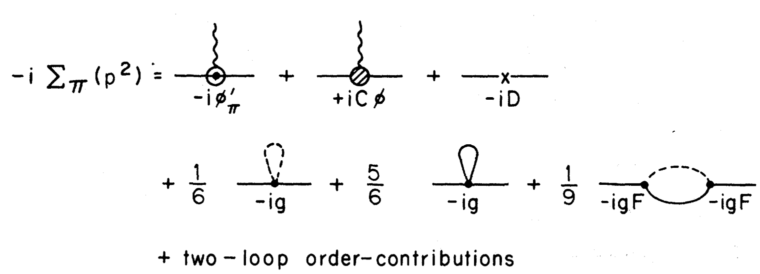


FIG. 5. Diagrams for $-i\,\Sigma_\pi(p^2)$.

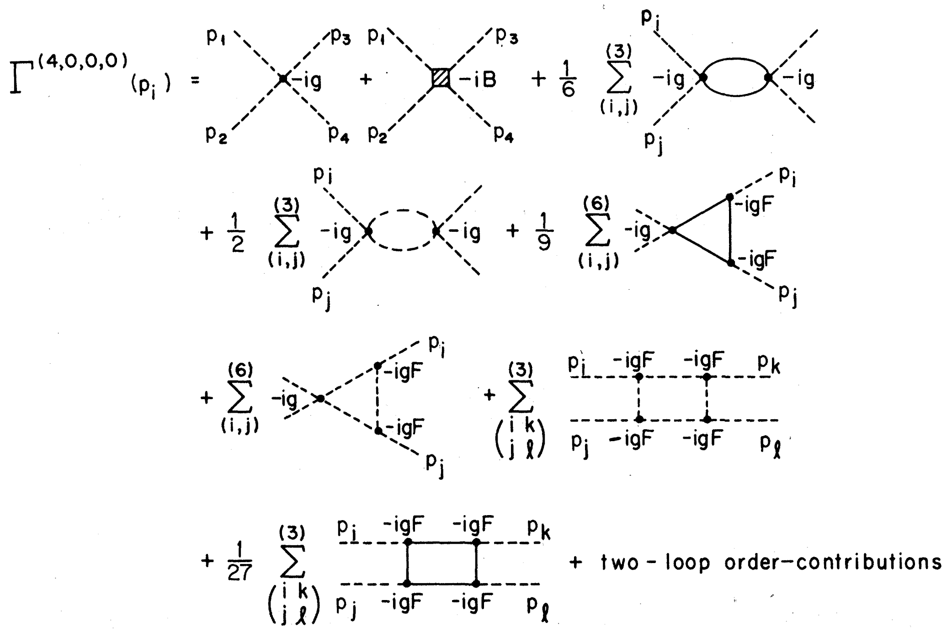


FIG. 6. Diagrams for $\Gamma^{(4,0,0,0)}(p_i)$. Numbers in parentheses indicate the number of inequivalent momentum labelings.

(ii)
$$-i\Gamma^{(0,2,0,0)}(p^2)=p^2-\phi_\pi+\frac{g\phi_\sigma}{3(32\pi^2)}\left(\ln\frac{M^2}{\phi_\sigma}+1\right)+\frac{5g\phi_\pi}{3(32\pi^2)}\left(\ln\frac{M^2}{\phi_\pi}+1\right)+\frac{2g^2F^2}{9(32\pi^2)}\ln\frac{M^2}{\phi_\pi}$$
$$-\frac{2g^2F^2}{9(32\pi^2)}\int_0^1 dz\ln\left[1+\frac{zgF}{3\phi_\pi}-\frac{p^2z(1-z)}{\phi_\pi}\right]+\text{two-loop-order contributions;} \tag{3.12}$$

(iii)
$$\Gamma^{(4,0,0,0)}(p_i)=-ig-\frac{ig^2}{32\pi^2}\left\{\ln\frac{\phi_\pi}{M^2}+3\ln\frac{\phi_\sigma}{M^2}+\sum_{(i,j)}^{(3)}\int_0^1 dz\ln\left[\left(1-\frac{p_{ij}^{\,2}}{\phi_\sigma}z(1-z)\right)\left(1-\frac{p_{ij}^{\,2}}{\phi_\pi}z(1-z)\right)^{1/3}\right]\right\}$$
$$-\frac{2ig^3F^2}{32\pi^2}\sum_{(i,j)}^{(6)}\left[C_\sigma^\sigma(p_i,p_j)+\tfrac{1}{9}C_\pi^\sigma(p_i,p_j)\right]-\frac{2ig^4F^4}{32\pi^2}\sum_{\binom{i\,k}{j\,l}}^{(3)}\left[D_\sigma^\sigma\begin{pmatrix}i&k\\j&l\end{pmatrix}+\frac{1}{27}D_\pi^\sigma\begin{pmatrix}i&k\\j&l\end{pmatrix}\right]$$
$$+\text{two-loop-order contributions}, \tag{3.13}$$

where

$$C_\pi^\sigma(p_i,p_j)=i\int\frac{d^4l}{\pi^2}\,\frac{1}{[(l+p_i)^2-\phi_\pi+i\epsilon][(l-p_j)^2-\phi_\pi+i\epsilon](l^2-\phi_\pi+i\epsilon)} \tag{3.14}$$

arising from the diagram shown in Fig. 9(a), and

$$D_\pi^\sigma\begin{pmatrix}i&k\\j&l\end{pmatrix}=i\int\frac{d^4l}{\pi^2}\,\frac{1}{(l^2-\phi_\pi+i\epsilon)[(l+p_i)^2-\phi_\pi+i\epsilon][(l+p_i+p_j)^2-\phi_\pi+i\epsilon][(l-p_k)^2-\phi_\pi+i\epsilon]} \tag{3.15}$$

arising from the diagram shown in Fig. 9(b), with the superscripts denoting external lines, the subscripts denoting internal lines, and both the C's and D's being ultraviolet convergent;

(iv)
$$\Gamma^{(0,4,0,0)}(p_i)=-ig-\frac{ig^2}{3(32\pi^2)}\left\{\ln\frac{\phi_\sigma}{M^2}+11\ln\frac{\phi_\pi}{M^2}+\tfrac{1}{3}\sum_{(i,j)}^{(3)}\int_0^1 dz\ln\left[\left(1-\frac{p_{ij}^{\,2}}{\phi_\sigma}z(1-z)\right)\left(1-\frac{p_{ij}^{\,2}}{\phi_\pi}z(1-z)\right)^{11}\right]\right\}$$
$$-\frac{2ig^3F^2}{9(32\pi^2)}\sum_{(i,j)}^{(6)}\left[C_\sigma^\pi(p_i,p_j)+\tfrac{1}{3}C_\pi^\pi(p_i,p_j)\right]-\frac{2ig^4F^4}{81(32\pi^2)}\sum_{\binom{ik}{jl}}^{(6)}D^\pi\begin{pmatrix}i&k\\j&l\end{pmatrix}$$
$$+\text{two-loop-order contributions}, \tag{3.16}$$

where

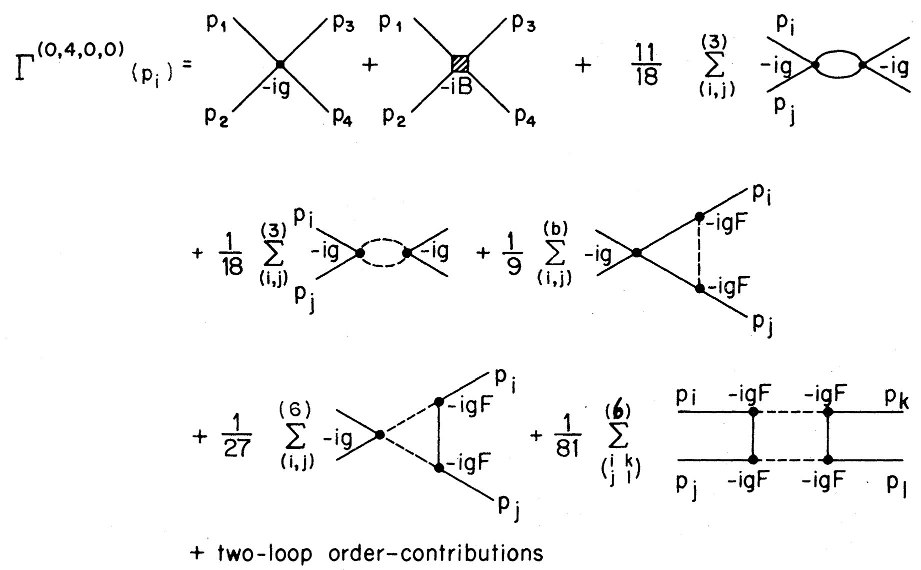


FIG. 7. Diagrams for $\Gamma^{(0,4,0,0)}(p_i)$.

$$C_\sigma^\pi(p_i,p_j)=i\int\frac{d^4l}{\pi^2}\frac{1}{(l^2-\phi_\sigma+i\epsilon)[(l+p_i)^2-\phi_\pi+i\epsilon][(l-p_j)^2-\phi_\pi+i\epsilon]} \tag{3.17}$$

arising from the diagram shown in Fig. 10(a), interchanging σ and π on the internal lines gives C_π^π, and

$$D^\pi\begin{pmatrix} i & k \\ j & l\end{pmatrix}=i\int\frac{d^4l}{\pi^2}\frac{1}{(l^2-\phi_\sigma+i\epsilon)[(l+p_i)^2-\phi_\pi+i\epsilon][(l+p_i+p_j)^2-\phi_\sigma+i\epsilon][(l-p_k)^2-\phi_\pi+i\epsilon]} \tag{3.18}$$

arising from the diagram shown in Fig. 10(b);

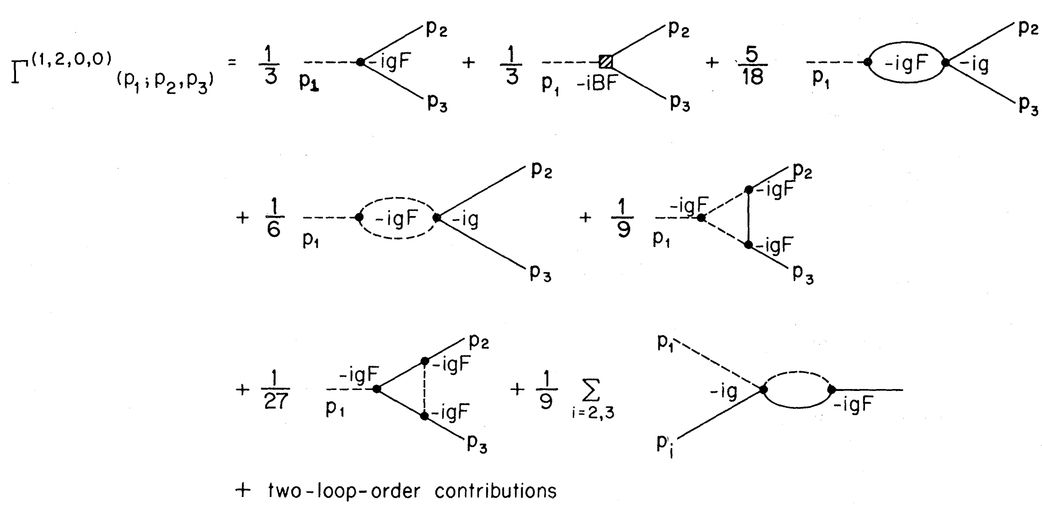


FIG. 8. Diagrams for $\Gamma^{(1,2,0,0)}(p_1;p_2,p_3)$.

$$\text{(v)}\quad \Gamma^{(1,2,0,0)}(p_1;p_2,p_3) = -\frac{igF}{3} - \frac{ig^2F}{3(32\pi^2)}\Bigg\{\frac{7}{3}\ln\frac{\phi_\sigma}{M^2} + \frac{5}{3}\ln\frac{\phi_\pi}{M^2} + \int_0^1 dz \ln\Bigg[\left(1-\frac{p_1^{\,2}}{\phi_\sigma}z(1-z)\right)\left(1-\frac{p_1^{\,2}}{\phi_\pi}z(1-z)\right)^{5/3} \times \prod_{i=1,2}\left(1-\frac{zgF^2}{3\phi_\sigma}-\frac{p_i^{\,2}}{\phi_\sigma}z(1-z)\right)^{2/3}\Bigg]\Bigg\}$$
$$-\frac{2ig^3F^3}{9(32\pi^2)}C_\pi^\sigma(p_2,p_3) - \frac{2ig^3F^3}{27(32\pi^2)}C_\sigma^\pi(p_2,p_3) + \text{two-loop-order contributions}\,, \qquad (3.19)$$

where the C's are defined above although here the superscripts do not refer to external lines.

We have checked explicitly that all the above amplitudes satisfy to two-loop order their respective Callan-Symanzik equations shown in Eq. (2.34) and hence must scale as shown in Eqs. (2.35) and (2.36) for asymptotic momenta. Thus, we anticipate the summation to all orders in perturbation theory and assume that $\beta(g)$ has a zero at g_0 so that $g(\lambda)\to g_0$ as $\lambda\to\infty$. Also we assume that $\hat{\gamma}_\phi(g_0)<2$ and $\hat{\gamma}_F(g_0)<1$ so that $\phi(\lambda)\to 0$ and $F(\lambda)\to 0$ as $\lambda\to\infty$. The result is that for asymptotic momenta, the amplitudes shown in Eqs. (3.11)–(3.19) simplify to the following:

$$\text{(i), (ii)}\quad \Gamma^{(2,0,0,0)}(\lambda^2p^2) \underset{\lambda\to\infty}{\sim} \Gamma^{(0,2,0,0)}(\lambda^2p^2) \underset{\lambda\to\infty}{\sim} \left\{\lambda^2\exp\left[-2\int_1^\lambda \frac{d\lambda'}{\lambda'}\gamma(g(\lambda'))\right]\right\} \times (ip^2 + \text{two-loop-order contributions})\,, \qquad (3.20)$$

$$\text{(iii), (iv)}\quad \Gamma^{(4,0,0,0)}(\lambda p_i) \underset{\lambda\to\infty}{\sim} \Gamma^{(0,4,0,0)}(\lambda p_i) \underset{\lambda\to\infty}{\sim} \left\{\exp\left[-4\int_1^\lambda \frac{d\lambda'}{\lambda'}\gamma(g(\lambda'))\right]\right\} \times\left[-ig_0 - \frac{4ig_0^{\,2}}{3(32\pi^2)}\sum_{(i,j)}^{(3)}\ln\left(-\frac{p_{ij}^{\,2}}{M^2}\right) + \text{two-loop-order contributions}\right], \qquad (3.21)$$

$$\text{(v)}\quad \Gamma^{(1,2,0,0)}(\lambda p_i) = 0\,, \qquad (3.22)$$

so that apart from overall scale factors we are led to the massless, symmetric theory defined by Eq. (3.8).

IV. CONCLUSION

We have made a field-theoretic application of the homogeneous Callan-Symanzik equations to theories with chiral symmetry breaking, using the SU(2) ⊗ SU(2) σ model as a prototype. General conditions were derived under which such symmetries become exact for asymptotic momenta, and these results were verified to two-loop order in perturbation theory by (a) calculating counterterms and Callan-Symanzik coefficients from the

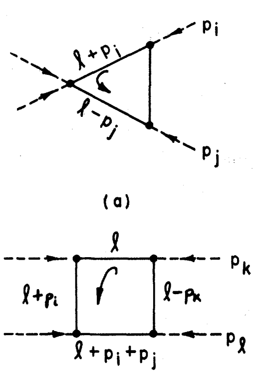


FIG. 9. Diagrams corresponding to (a) $C_\pi^\sigma(p_i,p_j)$ and (b) $D_\pi^\sigma\binom{i\,k}{j\,l}$.

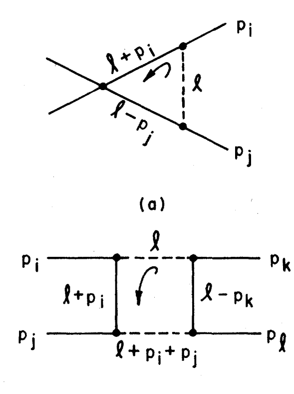


FIG. 10. Diagrams corresponding to (a) $C_\sigma^\pi(p_i,p_j)$ and (b) $D^\pi\binom{i\,k}{j\,l}$.

symmetric theory, (b) demonstrating that these are the only counterterms and independent coefficients necessary for the broken-symmetry theory, (c) checking that the one-, two-, three-, and four-particle amplitudes satisfy their respective homogeneous Callan-Symanzik equations, and finally (d) showing explicitly that the asymptotic energy realm looks like the massless, symmetric theory.

The moral to be learned is that if the strongly interacting particles are viewed as fundamental fields, then at asymptotic momenta, all mass parameters scale to zero; consequently, all semblances of chiral symmetry breaking vanish. However, what happens to chiral symmetry breaking for hadrons viewed as bound states is quite a different story, and we shall discuss this problem in a subsequent paper.

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