SO(4)-invariant extended supergravity*

Ashok Das

Institute for Theoretical Physics, State University of New York at Stony Brook, Stony Brook, New York 11794

(Received 26 January 1977)

We present results on an extended supergravity theory which has an internal SO(4) symmetry. All terms in the Lagrangian and transformation rules are determined through order κ^2, and some κ^3 terms in $\mathcal{L}$ are given. A complicated feature of this theory, not present in previous extended supergravity models, is a strongly indicated nonpolynomial structure in the scalar fields. This theory contains as a special case SO(2) extended supergravity coupled to its Abelian vector multiplet.

I. INTRODUCTION

Locally supersymmetric theories corresponding to various irreducible representations of the global supersymmetry algebra have recently been constructed. The theory corresponding to the simplest representation is pure supergravity,[1] whose spin content is $(2, \frac{3}{2})$. Theories with a doublet[2] and a triplet[3] of Majorana spin-$\frac{3}{2}$ fields have also been constructed. In the former case, the theory has an SO(2) global symmetry with particle content $(2, \frac{3}{2}, \frac{3}{2}, 1)$, whereas the latter with a particle content $(2, \frac{3}{2}, \frac{3}{2}, \frac{3}{2}, 1, 1, 1, \frac{1}{2})$ is invariant under a global SO(3) rotation. Such theories are important because it has been argued that in such extended supergravity theories many one-loop[4] and leading two-loop[5] divergences cancel.

The next supersymmetric theory in this line of development has a global SO(4) symmetry and a local supersymmetry involving a quartet of Majorana spinor parameters $\epsilon^i(x)$, $i=1,2,3,4$. The theory describes massless particles with multiplicity and spin as follows: one spin-2, four spin-$\frac{3}{2}$, six spin-1, four spin-$\frac{1}{2}$, and one spin-0 parity doublet. It is the first extended supergravity construction in which scalar fields enter.

The method of construction is similar to Ref. 3 and uses symmetry arguments. We have determined the Lagrangian up to all terms of order κ^3 in the Lagrangian which are necessary to make the theory locally supersymmetric up to order κ^2. This means that all κ^3 terms in the Lagrangian involving the spin-$\frac{3}{2}$ fields have been determined. There may, of course, be other terms of order κ^3 which would only lead to variations of order κ^3 or higher. The transformation laws are determined up to order κ^2. We are presenting in this paper partial results on SO(4) theory because this theory exhibits qualitatively new features compared to previous extended supergravity constructions, namely, a strongly indicated nonpolynomial structure in the scalar fields. This feature makes a complete derivation quite difficult. However, it must be noted that such nonpolynomial structure was also observed in the case of the scalar multiplet[6] coupled to supergravity when the scalar field was given a mass.

The SO(4) extended supergravity theory contains as a special case the locally supersymmetric theory of the SO(2) extended supergravity multiplet $(2, \frac{3}{2}, \frac{3}{2}, 1)$ coupled to its Abelian vector multiplet of spin content $(1, \frac{1}{2}, \frac{1}{2}, 0^{\pm})$. This has not been treated previously. We find that the indications of nonpolynomial structure persist in this special case.

A calculation of the one-loop correction to the photon-photon scattering has already been done[7] for the SO(4) theory and the results turn out to be finite.

In Sec. II we present the Lagrangian and the transformation rules which leave the theory supersymmetric up to κ^2 variations. In Sec. III we discuss the proof of supersymmetry and finally discuss various new features in Sec. IV.

II. LAGRANGIAN

The Lagrangian contains a vierbein field $V_{a\mu}$, quartets of spin-$\frac{3}{2}$ Majorana fields ψ^i_μ and spin-$\frac{1}{2}$ Majorana fields χ^i, six vector fields A^{ij}_μ, a scalar field A, and a pseudoscalar field B. Here i and j run from 1 to 4 and the vector field is antisymmetrical in its internal indices. The Lagrangian in second-order form is

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 \, , \qquad (1)$$

where

$$\mathcal{L}_0 = -\frac{1}{4\kappa^2} VR - \tfrac{1}{2}\epsilon^{\lambda\rho\mu\nu}\bar{\psi}^i_\lambda \gamma_5 \gamma_\mu D_\nu \psi^i_\rho - \tfrac{1}{8} V F^{ij}_{\mu\nu} F^{\mu\nu ij} + \tfrac{1}{2} i V \bar{\chi}^i \not{D} \chi^i + \tfrac{1}{2} V(\partial_\mu A)^2 + \tfrac{1}{2} V(\partial_\mu B)^2 \, , \qquad (2)$$

$$\mathcal{L}_1 = -\frac{\kappa}{2}\bar{\psi}^i_\mu\left(VF^{\mu\nu ij} - \frac{i}{2}\gamma_5\tilde{F}^{\mu\nu ij}\right)\psi^j_\nu - \frac{i\kappa}{2\sqrt{2}}V\epsilon^{ijkl}\bar{\psi}^i_\lambda\sigma^{\mu\nu}F^{kl}_{\mu\nu}\gamma^\lambda\chi^j - \frac{\kappa}{\sqrt{2}}V\bar{\psi}^i_\mu\partial_\nu(A + i\gamma_5 B)\gamma^\nu\gamma^\mu\chi^i\,, \tag{3}$$

$$\begin{aligned}\mathcal{L}_2 = &-\frac{V\kappa}{8}\epsilon^{ijkl}AF^{ij}_{\mu\nu}F^{\mu\nu kl} + \frac{V\kappa}{16}\epsilon^{ijkl}B\tilde{F}^{ij}_{\mu\nu}F^{\mu\nu kl} - \frac{i}{\sqrt{2}}V\kappa^2 A\bar{\psi}^i_\lambda\sigma^{\mu\nu}F^{ij}_{\mu\nu}\gamma^\lambda\chi^j + \frac{i}{2\sqrt{2}}V\kappa^2 B\bar{\psi}^i_\lambda\sigma^{\mu\nu}\tilde{F}^{ij}_{\mu\nu}\gamma^\lambda\chi^j\\
&-\frac{\kappa^2}{4}\epsilon^{ijkl}A\bar{\psi}^i_\mu\left(VF^{\mu\nu kl} - \frac{i}{2}\gamma_5\tilde{F}^{\mu\nu kl}\right)\psi^j_\nu + \frac{i\kappa^2}{4}\epsilon^{ijkl}B\bar{\psi}^i_\mu\left(VF^{\mu\nu kl} - \frac{i}{2}\gamma_5\tilde{F}^{\mu\nu kl}\right)\psi^j_\nu + \frac{i\kappa^2}{4}\epsilon^{\mu\nu\rho\lambda}(\bar{\psi}^i_\mu\gamma_\rho\psi^i_\lambda)A\overleftrightarrow{\partial}_\nu B\\
&-\frac{\kappa^2}{8}\epsilon^{ijkl}\epsilon^{klpq}(\bar{\psi}^p_\mu\psi^q_\nu)\left[V(\bar{\psi}^{\mu i}\psi^{\nu j}) - \frac{i}{2}\epsilon^{\mu\nu\alpha\beta}\bar{\psi}^i_\alpha\gamma_5\psi^j_\beta\right] - \frac{i\kappa^2}{\sqrt{2}}V\epsilon^{ijkl}(\bar{\psi}^k_\mu\psi^l_\nu)(\bar{\psi}^i_\lambda\sigma^{\mu\nu}\gamma^\lambda\chi^j)\\
&-\frac{\kappa^2}{4}V\epsilon^{ijkl}\epsilon^{klpq}(\bar{\psi}^p_\mu\gamma_\nu\chi^q)(\bar{\psi}^i_\lambda\sigma^{\mu\nu}\gamma^\lambda\chi^j) + \frac{\kappa^2}{8}V(\bar{\psi}^i_\mu\gamma^a\psi^{\mu j})(\bar{\chi}^i\gamma_a\chi^j) + \frac{\kappa^2}{8}V(\bar{\psi}^i_\mu\gamma_5\gamma^a\psi^{\mu j})(\bar{\chi}^i\gamma_5\gamma_a\chi^j)\\
&+\frac{i\kappa^2}{8}\epsilon^{\mu\nu\rho\sigma}(\bar{\psi}^i_\mu\gamma_\sigma\psi^j_\nu)(\bar{\chi}^i\gamma_5\gamma_\rho\chi^j) + \frac{i\kappa^2}{8}\epsilon^{\mu\nu\rho\sigma}(\bar{\psi}^i_\mu\gamma_5\gamma_\sigma\psi^j_\nu)(\bar{\chi}^i\gamma_\rho\chi^j) - \frac{i\kappa^2}{16}\epsilon^{\mu\nu\rho\sigma}(\bar{\psi}^i_\mu\gamma_\sigma\psi^i_\nu)(\bar{\chi}^j\gamma_5\gamma_\rho\chi^j)\\
&-\tfrac{1}{16}\kappa^2 V[(\bar{\psi}^{\lambda i}\gamma^\mu\psi^{\rho i})(\bar{\psi}^j_\lambda\gamma_\mu\psi^j_\rho) + 2(\bar{\psi}^{\lambda i}\gamma^\mu\psi^{\rho i})(\bar{\psi}^j_\mu\gamma_\lambda\psi^j_\rho) - 4(\bar{\psi}^{i}\cdot\gamma\psi^{\rho i})(\bar{\psi}^{j}\cdot\gamma\psi^j_\rho)]\\
&+\frac{3\kappa^2}{4}V(\bar{\chi}^i\gamma_5\gamma^a\chi^i)[\tfrac{1}{8}(\bar{\chi}^j\gamma_5\gamma_a\chi^j) - A\overleftrightarrow{\partial}_a B] + \kappa^2 V(A^2+B^2)[(\partial_\mu A)^2 + (\partial_\mu B)^2] - \frac{\kappa^3 V}{\sqrt{2}}(A^2+B^2)\bar{\psi}^i_\mu\partial_\nu(A + i\gamma_5 B)\gamma^\nu\gamma^\mu\chi^i\,,\end{aligned} \tag{4}$$

$$\begin{aligned}\mathcal{L}_3 = &-\frac{V\kappa^2}{4}(A^2 - B^2)F^{ij}_{\mu\nu}F^{\mu\nu ij} + \frac{\kappa^2}{4}AB\tilde{F}^{ij}_{\mu\nu}F^{\mu\nu ij} - \frac{iV\kappa^3}{4\sqrt{2}}A^2\epsilon^{ijkl}\bar{\psi}^i_\lambda\sigma^{\mu\nu}F^{kl}_{\mu\nu}\gamma^\lambda\chi^j + \frac{3iV\kappa^3}{4\sqrt{2}}B^2\epsilon^{ijkl}\bar{\psi}^i_\lambda\sigma^{\mu\nu}F^{kl}_{\mu\nu}\gamma^\lambda\chi^j\\
&+\frac{1}{\sqrt{2}}V\kappa^3 AB\epsilon^{ijkl}\bar{\psi}^i_\lambda\gamma_5\sigma^{\mu\nu}F^{kl}_{\mu\nu}\gamma^\lambda\chi^j - \frac{\kappa^3}{4}A^2\bar{\psi}^i_\mu\left(VF^{\mu\nu ij} - \frac{i}{2}\gamma_5\tilde{F}^{\mu\nu ij}\right)\psi^j_\nu + \frac{3\kappa^3}{4}B^2\bar{\psi}^i_\mu\left(VF^{\mu\nu ij} - \frac{i}{2}\gamma_5\tilde{F}^{\mu\nu ij}\right)\psi^j_\nu\\
&+i\kappa^3 AB\bar{\psi}^i_\mu\gamma_5\left(VF^{\mu\nu ij} - \frac{i}{2}\gamma_5\tilde{F}^{\mu\nu ij}\right)\psi^j_\nu + \frac{3i\kappa^3}{4}VB\epsilon^{ijkl}(\bar{\psi}^k_\mu\psi^l_\nu)(\bar{\chi}^i\gamma_5\sigma^{\mu\nu}\chi^j) - \frac{3i\kappa^3}{4}VB\epsilon^{ijkl}(\bar{\psi}^k_\mu\gamma_5\psi^l_\nu)(\bar{\chi}^i\sigma^{\mu\nu}\chi^j)\\
&-\frac{i\kappa^3}{4}VB\epsilon^{ijkl}(\bar{\psi}^k_\mu\sigma^{ab}\psi^l_\nu)(\bar{\chi}^i\gamma_5\gamma^\mu\sigma_{ab}\gamma^\nu\chi^j) + \frac{i\kappa^3}{4}A\epsilon^{ijkl}\epsilon^{\mu\nu\alpha\beta}(\bar{\psi}^i_\mu\psi^j_\nu)(\bar{\psi}^k_\alpha\gamma_5\psi^l_\beta)\\
&+\frac{i\kappa^3}{2}VB\epsilon^{ijkl}(\bar{\psi}^i_\mu\psi^j_\nu)(\bar{\psi}^{\mu k}\gamma_5\psi^{\nu l}) - \frac{\kappa^3}{2\sqrt{2}}\epsilon^{ijkl}\epsilon^{klpq}B(\bar{\psi}^i_\lambda\sigma^{\mu\nu}\gamma^\lambda\chi^j)\left(V\bar{\psi}^p_\mu\gamma_5\psi^q_\nu - \frac{i}{2}\epsilon_{\mu\nu}{}^{\alpha\beta}\bar{\psi}^p_\alpha\psi^q_\beta\right)\end{aligned} \tag{5}$$

where

$$\begin{aligned}&D_\nu\psi^i_\rho = (\partial_\nu + \tfrac{1}{2}\omega_{\nu ab}\sigma^{ab})\psi^i_\rho\,,\\
&D_\nu\chi^i = (\partial_\nu + \tfrac{1}{2}\omega_{\nu ab}\sigma^{ab})\chi^i\,,\\
&F^{ij}_{\mu\nu} = \partial_\mu A^{ij}_\nu - \partial_\nu A^{ij}_\mu\,,\\
&\tilde{F}^{ij}_{\mu\nu} = \epsilon_{\mu\nu}{}^{\alpha\beta}F^{ij}_{\alpha\beta}\,.\end{aligned} \tag{6}$$

We work in the metric $(+---)$. In our convention $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$, $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, and $\epsilon^{0123} = +1$. There is a summation over the internal indices wherever they are repeated.

This Lagrangian is invariant under a global rotation in the internal SO(4) space. It is also invariant under the following local supersymmetry transformations:

$$\begin{aligned}&\delta A = \frac{1}{\sqrt{2}}\bar{\epsilon}^i\chi^i[1 - \kappa^2(A^2+B^2)]\,,\\
&\delta B = \frac{i}{\sqrt{2}}\bar{\epsilon}^i\gamma_5\chi^i[1 - \kappa^2(A^2+B^2)]\,,\\
&\delta V_{a\mu} = -i\kappa(\bar{\epsilon}^i\gamma_a\psi^i_\mu)\,,\end{aligned}$$

$$
\begin{aligned}
\delta\bar{\chi}^i = {} & \frac{i}{\sqrt{2}}\bar{\epsilon}^i(A+i\gamma_5 B)\overleftarrow{\not{\partial}}[1+\kappa^2(A^2+B^2)]+\frac{1}{2\sqrt{2}}\epsilon^{ijkl}\bar{\epsilon}^j\sigma^{\alpha\beta}F^{kl}_{\alpha\beta}+\frac{\kappa}{\sqrt{2}}A\bar{\epsilon}^j\sigma^{\alpha\beta}F^{ij}_{\alpha\beta}-\frac{\kappa}{2\sqrt{2}}B\bar{\epsilon}^j\sigma^{\alpha\beta}\tilde{F}^{ij}_{\alpha\beta} \\
& +\frac{\kappa}{\sqrt{2}}\epsilon^{ijkl}\bar{\epsilon}^j\sigma^{\alpha\beta}(\bar{\psi}^k_\alpha\psi^l_\beta)-\frac{i\kappa}{2}(\bar{\psi}^j_\rho\chi^j)\bar{\epsilon}^i\gamma^\rho-\frac{i\kappa}{2}(\bar{\psi}^j_\rho\gamma_5\chi^j)\bar{\epsilon}^i\gamma^\rho\gamma_5-i\kappa(\bar{\psi}^i_\mu\gamma_\nu\chi^j-\bar{\psi}^j_\mu\gamma_\nu\chi^i)\bar{\epsilon}^j\sigma^{\mu\nu} \\
& +\frac{\kappa^2}{4\sqrt{2}}\epsilon^{ijkl}A^2\bar{\epsilon}^j\sigma^{\alpha\beta}F^{kl}_{\alpha\beta}-\frac{3\kappa^2}{4}\epsilon^{ijkl}B^2\bar{\epsilon}^j\sigma^{\alpha\beta}F^{kl}_{\alpha\beta}+\frac{i\kappa^2}{\sqrt{2}}\epsilon^{ijkl}AB\bar{\epsilon}^j\gamma_5\sigma^{\alpha\beta}F^{kl}_{\alpha\beta} \\
& -\frac{i\kappa^2}{\sqrt{2}}B\left[\left(\bar{\psi}^i_\mu\gamma_5\psi^j_\nu-\frac{i}{2}\epsilon_{\mu\nu}{}^{\alpha\beta}\bar{\psi}^i_\alpha\psi^j_\beta\right)-(i\leftrightarrow j)\right]\bar{\epsilon}^j\sigma^{\mu\nu}-\frac{3\kappa^2}{2\sqrt{2}}[\bar{\epsilon}^j\gamma_5(A+i\gamma_5B)\chi^j]\bar{\chi}^i\gamma_5 \\
& -\tfrac{1}{4}\kappa^2B\epsilon^{ijkl}[-3(\bar{\epsilon}^k\psi^l_\lambda)\bar{\chi}^j\gamma_5\gamma^\lambda+3(\bar{\epsilon}^k\gamma_5\psi^l_\lambda)\bar{\chi}^j\gamma^\lambda-2(\bar{\epsilon}^k\sigma^{ab}\psi^l_\lambda)\bar{\chi}^j\gamma_5\gamma^\lambda\sigma_{ab}],
\end{aligned}
$$

$$
\begin{aligned}
\delta A^{ij}_\mu = {} & \frac{i}{\sqrt{2}}\epsilon^{ijkl}\bar{\epsilon}^k\gamma_\mu\chi^l-(\bar{\epsilon}^i\psi^j_\mu-\bar{\epsilon}^j\psi^i_\mu)-\frac{i\kappa}{\sqrt{2}}A(\bar{\epsilon}^i\gamma_\mu\chi^j-\bar{\epsilon}^j\gamma_\mu\chi^i)-\frac{\kappa}{\sqrt{2}}B(\bar{\epsilon}^i\gamma_5\gamma_\mu\chi^j-\bar{\epsilon}^j\gamma_5\gamma_\mu\chi^i) \\
& +\kappa A\epsilon^{ijkl}\bar{\epsilon}^k\psi^l_\mu+i\kappa B\epsilon^{ijkl}\bar{\epsilon}^k\gamma_5\psi^l_\mu+\frac{i\kappa^2}{2\sqrt{2}}A^2\epsilon^{ijkl}\bar{\epsilon}^k\gamma_\mu\chi^l+\frac{i\kappa^2}{2\sqrt{2}}B^2\epsilon^{ijkl}\bar{\epsilon}^k\gamma_\mu\chi^l \\
& -\tfrac{1}{2}\kappa^2A^2(\bar{\epsilon}^i\psi^j_\mu-\bar{\epsilon}^j\psi^i_\mu)-\tfrac{1}{2}\kappa^2B^2(\bar{\epsilon}^i\psi^j_\mu-\bar{\epsilon}^j\psi^i_\mu),
\end{aligned}
\tag{7}
$$

$$
\begin{aligned}
\delta\bar{\psi}^i_\lambda = {} & \kappa^{-1}\bar{\epsilon}^i\overleftarrow{D}_\lambda-\frac{i}{2}\bar{\epsilon}^j\gamma_\lambda\sigma^{\alpha\beta}F^{ij}_{\alpha\beta}-\frac{i\kappa}{4}A\epsilon^{ijkl}\bar{\epsilon}^j\gamma_\lambda\sigma^{\alpha\beta}F^{kl}_{\alpha\beta}-\frac{\kappa}{4}B\epsilon^{ijkl}\bar{\epsilon}^j\gamma_5\gamma_\lambda\sigma^{\alpha\beta}F^{kl}_{\alpha\beta}-\frac{i\kappa}{2}\bar{\epsilon}^i\gamma_5(A\overleftrightarrow{\partial}_\lambda B) \\
& +i\kappa\bar{\epsilon}^j\gamma_\lambda\sigma^{\alpha\beta}(\bar{\psi}^j_\alpha\psi^i_\beta)-\frac{\kappa}{\sqrt{2}}\epsilon^{ijkl}(\bar{\psi}^k_\mu\gamma_\nu\chi^l)\bar{\epsilon}^j\gamma_\lambda\sigma^{\mu\nu}+\frac{\kappa}{2\sqrt{2}}\epsilon^{ijkl}[(\bar{\psi}^k_\lambda\gamma^a\chi^l)\bar{\epsilon}^j\gamma_a+(\bar{\psi}^k_\lambda\gamma_5\gamma^a\chi^l)\bar{\epsilon}^j\gamma_5\gamma_a] \\
& +\frac{i\kappa}{8}(\bar{\chi}^j\gamma_5\gamma^a\chi^j)\bar{\epsilon}^i\gamma_5(\gamma_\lambda\gamma_a+g_{\lambda a})+\frac{i\kappa}{4}(2\bar{\psi}^j_\lambda\gamma_a\psi^j_b+\bar{\psi}^j_a\gamma_\lambda\psi^j_b)\bar{\epsilon}^i\sigma^{ab}+\frac{i\kappa}{4}\bar{\epsilon}^j\gamma_\lambda[(\bar{\chi}^i\gamma^a\chi^j)+(\bar{\chi}^i\gamma_5\gamma^a\chi^j)\gamma_5]\gamma_a \\
& -\frac{i\kappa^2}{4}A^2\bar{\epsilon}^j\gamma_\lambda\sigma^{\alpha\beta}F^{ij}_{\alpha\beta}+\frac{3i\kappa^2}{4}B^2\bar{\epsilon}^j\gamma_\lambda\sigma^{\alpha\beta}F^{ij}_{\alpha\beta}-\kappa^2AB\bar{\epsilon}^j\gamma_5\gamma_\lambda\sigma^{\alpha\beta}F^{ij}_{\alpha\beta}-\kappa^2B\epsilon^{ijkl}[(\bar{\psi}^l_\mu\psi^j_\nu)\bar{\epsilon}^k\gamma_5+(\bar{\psi}^l_\mu\gamma_5\psi^j_\nu)\bar{\epsilon}^k]\sigma^{\mu\nu} \\
& +\frac{\kappa^2}{2}B\epsilon^{ijkl}(\bar{\chi}^l\sigma^{\alpha\beta}\chi^j)\bar{\epsilon}^k(\gamma_5\gamma_\lambda\sigma_{\alpha\beta}+\tfrac{1}{2}\gamma_\beta\gamma_\lambda\gamma_\alpha-\tfrac{1}{2}\gamma_5\gamma_\beta\gamma_\lambda\gamma_\alpha) \\
& -\frac{\kappa^2}{2\sqrt{2}}A(\bar{\epsilon}^j\gamma_5\chi^j)\bar{\psi}^i_\lambda\gamma_5-\frac{i\kappa^2}{2\sqrt{2}}B(\bar{\epsilon}^j\chi^j)\bar{\psi}^i_\lambda\gamma_5 \\
& -\frac{i\kappa^2}{\sqrt{2}}B[(\bar{\psi}^i_\mu\sigma^{\alpha\beta}\gamma^\mu\chi^j)-(i\leftrightarrow j)]\bar{\epsilon}^j\gamma_5\gamma_\lambda\sigma_{\alpha\beta}.
\end{aligned}
$$

We emphasize here again that this Lagrangian and the transformation laws do not constitute the complete theory. Rather, they represent the theory up to κ^2 variations, which means that they contain all κ^3 terms in the Lagrangian which involve ψ fields. But one expects other terms, for example of the form $\kappa^3A^3F^2$ and $\kappa^3B^3F^2$, which do not involve ψ.

It is of interest to note that this theory reduces to SO(3) and SO(2) supergravity coupled to its vector multiplet as special cases. First of all, if we set $\psi^4_\mu=0$, $A=B=0$, $\chi^4=\chi\neq 0$, $F^{i4}_{\mu\nu}=0$ and if we write $F^{ij}_{\mu\nu}=\epsilon^{ijk}F^k_{\mu\nu}$, where i, j, and k now run over 1 to 3, then we get the familiar SO(3) theory. On the other hand, if we restrict ourselves to the case where $\psi^3_\mu=\psi^4_\mu=0$, $\chi^3=\chi^4=0$, $F^{12}_{\mu\nu}=G_{\mu\nu}$, and $F^{34}_{\mu\nu}=F_{\mu\nu}$, then we get back SO(2) supergravity coupled to its Abelian vector multiplet. Both special cases are compatible truncations of the SO(4) theory. This means that the variation of fields which vanish also vanish.

III. LOCAL SUPERSYMMETRY

The philosophy behind this construction is the same as in all other supergravity models, namely, at each order one introduces new terms to the Lagrangian and to the transformation laws to maintain local supersymmetry. However, the present construction is technically much more complicated because of the extra group indices various fields carry and because of the distinct ways they can be coupled.

In the notation that we are working with, the ψ and χ Noether currents can, of course, have both direct coupling or dual coupling in the internal space or both. There is *a priori* no reason for ruling out any coupling. Hence one starts out with a Lagrangian whose Noether currents have the most general group couplings and a set of transformation laws which reflect the same feature. For example, we started out with a Lagrangian

$$\mathcal{L}=\mathcal{L}_0+\mathcal{L}_1' , \tag{8}$$

where

$$\begin{aligned}\mathcal{L}_1' = \kappa\bar{\psi}^i_\mu[(aVF^{\mu\nu ij}+b\epsilon^{ijkl}F^{\mu\nu kl}) \\ -\tfrac{1}{2}i\gamma_5(a\tilde{F}^{\mu\nu ij}+b\epsilon^{ijkl}\tilde{F}^{\mu\nu kl})]\psi^j_\nu \\ -\frac{i\kappa}{\sqrt{2}}V\bar{\psi}^i_\lambda\sigma^{\mu\nu}(cF^{ij}_{\mu\nu}+d\epsilon^{ijkl}F^{kl}_{\mu\nu})\gamma^\lambda\chi^j \\ -\frac{\kappa}{\sqrt{2}}V\bar{\psi}^i_\mu\partial_\nu(A+i\gamma_5B)\gamma^\nu\gamma^\mu\chi^i ,\end{aligned} \tag{9}$$

and a set of transformation laws

$$\begin{aligned}\delta A &= \frac{1}{\sqrt{2}}\bar{\epsilon}^i\chi^i ,\\ \delta B &= \frac{i}{\sqrt{2}}\bar{\epsilon}^i\gamma_5\chi^i ,\\ \delta\bar{\chi}^i &= \frac{i}{\sqrt{2}}\bar{\epsilon}^i(A+i\gamma_5B)\overleftarrow{\not\partial} \\ &\quad +\frac{1}{\sqrt{2}}\bar{\epsilon}^j\sigma^{\mu\nu}(eF^{ij}_{\mu\nu}+f\epsilon^{ijkl}F^{kl}_{\mu\nu}),\\ \delta A^{ij}_\mu &= \frac{i}{\sqrt{2}}[g(\bar{\epsilon}^i\gamma_\mu\chi^j-\bar{\epsilon}^j\gamma_\mu\chi^i)+h\epsilon^{ijkl}\bar{\epsilon}^k\gamma_\mu\chi^l] \\ &\quad +[m(\bar{\epsilon}^i\psi^j_\mu-\bar{\epsilon}^j\psi^i_\mu)+n\epsilon^{ijkl}\bar{\epsilon}^k\psi^l_\mu],\\ \delta\bar{\psi}^i_\lambda &= \kappa^{-1}\bar{\epsilon}^i\overleftarrow{D}_\lambda+\bar{\epsilon}^j\gamma_\lambda\sigma^{\mu\nu}(pF^{ij}_{\mu\nu}+q\epsilon^{ijkl}F^{kl}_{\mu\nu}),\\ \delta V_{a\mu} &= -i\kappa(\bar{\epsilon}^i\gamma_a\psi^i_\mu) .\end{aligned} \tag{10}$$

Then the requirement that the lower-order variations like $\bar{\epsilon}F\psi, \bar{\epsilon}F\chi$ vanish gives relations between various parameters of the theory. They become uniquely determined when one demands that variations of the type $\kappa\bar{\epsilon}F^2\psi$ vanish. Interestingly enough, local supersymmetry allows only two distinct couplings:

(1) ψ Noether current has a direct coupling and the χ Noether current has dual coupling.

(2) ψ Noether current has dual coupling and the χ Noether current has a direct coupling.

One observes here that these two couplings are trivially mapped into each other under a duality transformation in the internal space, viz., $F^{ij}_{\mu\nu}\rightarrow\frac{1}{2}\epsilon^{ijkl}F^{kl}_{\mu\nu}$.

The starting theory is now uniquely determined and one goes to check various other types of variations. Even at the order κ we have new types of variations that did not occur previously. For example, one has

(a) $\kappa\bar{\epsilon}F^2\chi$, (b) $\kappa\bar{\epsilon}AF\chi$, (c) $\kappa\bar{\epsilon}BF\chi$,

(d) $\kappa\bar{\epsilon}AF\psi$, (e) $\kappa\bar{\epsilon}BF\psi$.

In addition, one also has the old familiar type of variations

(f) $\kappa\bar{\epsilon}A^2\psi$, (g) $\kappa\bar{\epsilon}B^2\psi$, (h) $\kappa\bar{\epsilon}\psi^2\chi$,

(i) $\kappa\bar{\epsilon}\chi^2\psi$, (j) $\kappa\bar{\epsilon}\psi^3$, (k) $\kappa\bar{\epsilon}AB\psi$.

The $\kappa\bar{\epsilon}F^2\chi$ terms have only one source in the Lagrangian, namely, the χ Noether current with $\delta\bar{\psi}\sim\bar{\epsilon}F$ and they do not vanish. Hence one needs to add new terms to the Lagrangian. There are three types of terms that can be added:

$$\kappa\bar{\chi}^i\sigma^{\mu\nu}F^{ij}_{\mu\nu}\chi^j ,\quad \kappa A\epsilon^{ijkl}F^{ij}_{\mu\nu}F^{\mu\nu kl} ,$$
$$\kappa B\epsilon^{ijkl}\tilde{F}^{ij}_{\mu\nu}F^{\mu\nu kl} ,$$

and the unique combination whose change would cancel the previous variation is the particular combination of AFF and $B\tilde{F}F$ in our Lagrangian. The $\kappa\bar{\epsilon}AF\chi$ and $\kappa\bar{\epsilon}BF\chi$ have two sources in the Lagrangian, viz., the scalar Noether current with $\delta\bar{\psi}\sim\bar{\epsilon}F$ and the new terms added to the Lagrangian with $\delta F\sim\bar{\epsilon}\chi$. These variations do not cancel and one needs terms of the type $\kappa^2A\psi F\chi$ and $\kappa^2B\psi F\chi$ terms in the Lagrangian. These also lead to the change in transformation laws like

$$\delta\bar{\chi}\sim A\bar{\epsilon}F ,\quad B\bar{\epsilon}F ,$$
$$\delta A_\mu\sim A\bar{\epsilon}\gamma_\mu\chi ,\quad B\bar{\epsilon}\gamma_\mu\chi .$$

$\kappa\bar{\epsilon}AF\psi$ and $\kappa\bar{\epsilon}BF\psi$ have three sources in the Lagrangian, namely, the χ Noether current with $\delta\bar{\chi}\sim\bar{\epsilon}A,\bar{\epsilon}B$, the scalar Noether current with $\delta\bar{\chi}\sim\bar{\epsilon}F$, and the $AFF, B\tilde{F}F$ terms with $\delta F\sim\bar{\epsilon}\psi$. To cancel them, one needs terms of the type $\kappa^2A\bar{\psi}F\psi$ and $\kappa^2B\bar{\psi}F\psi$ in the Lagrangian, and one picks up transformation changes of the type

$$\delta A_\mu\sim A\bar{\epsilon}\psi_\mu ,\quad B\bar{\epsilon}\psi_\mu ,$$
$$\delta\bar{\psi}\sim A\bar{\epsilon}F ,\quad B\bar{\epsilon}F .$$

The rest of the terms were present in various other multiplets and cancel by themselves or by adding known terms to the Lagrangian with an obvious SO(4) coupling. It may be of some interest to point out here that, although all the $\psi^2\chi^2$ contact terms do not look familiar, they can be absorbed into supercovariant derivatives (up to a factor of $\frac{1}{2}$) if one remembers that both ψ and χ now carry internal indices. For example,

$$-\frac{\kappa^2}{4}V\epsilon^{ijkl}\epsilon^{klpq}(\bar{\psi}^p_\mu\gamma_\nu\chi^q)(\bar{\psi}^i_\lambda\sigma^{\mu\nu}\gamma^\lambda\chi^j)$$

is the supercovariant completion of $F^{ij}_{\mu\nu}$, whereas the rest of the $\psi^2\chi^2$ contact terms can be absorbed if one defines

$$D_\nu A = \partial_\nu A + \frac{1}{\sqrt{2}} (\bar{\chi}^j \psi_\nu^j) ,$$
$$D_\nu B = \partial_\nu B + \frac{i}{\sqrt{2}} (\bar{\chi}^j \gamma_5 \psi_\nu^j) . \quad (11)$$

Rest of the terms in the Lagrangian and the transformation laws are determined in a similar manner by demanding that all variations in the Lagrangian of order κ^2 vanish.

IV. DISCUSSION

The SO(4)-invariant theory has a number of unusual features which need some discussion. As we have pointed out before the theory seems to have a nonpolynomial structure. The terms in the Lagrangian and in transformation laws seem to be building up in powers of A and B fields. Direct and dual couplings seem to alternate at every higher order. The nonpolynomial feature has been noted before in the case of the scalar multiplet coupled to supergravity, and qualitatively seems to be unique to scalar fields. Although the naive dimensional arguments[8] and on-shell gauge invariance of S-matrix elements restrict severely the maximum number of fermion fields and photon fields that can be present in the contact terms of any supergravity theory, they allow an infinite series in powers of the A and B fields. One can, of course, try to find a function of the scalar fields which would sum the series and would allow the Lagrangian to be written in a more compact way. However, we have not studied this problem.

The other feature that one may feel uncomfortable about is the occurrence of terms like AFF, $B\tilde{F}F$, A^2FF, B^2FF, and so on. They would seem to change the Maxwell kinetic action. However, such a feature is not new either. This is because if one looks at the covariantized Maxwell kinetic action and expands $g^{\mu\nu}$ in terms of gravitational fields, then one gets back a similar power series except that the gravitational fields now replace the scalar fields.

It is worth pointing out that this theory does not reduce to the case of the scalar multiplet coupled to pure supergravity. This is again not surprising because at a higher-order reduction of a theory simultaneously to SO(3) and scalar multiplet coupled to supergravity is incompatible. This can be seen from the fact that although a term of the type $\kappa^2(\bar{\chi}\gamma_5\gamma^a\chi)^2$ occurs both in SO(3) and scalar-multiplet theory, the coefficients in front are different. In case of the former, the value of the coefficient is $\frac{3}{32}$, whereas it is $-\frac{1}{32}$ for the scalar multiplet. So a theory can only reduce to one case and our theory selects SO(3), which is logical.

The other interesting feature of this theory is a strong asymmetry in A and B fields. For example, in order κ^3 in the Lagrangian and order κ^2 in the transformation laws we have terms with only B fields multiplied, whereas the A-field counterpart does not occur. Since our theory does not possess a chiral invariance ($A \to -B$, $B \to A$) one, of course, does not expect a complete symmetry. But whether such an asymmetry reflects a termination of the theory at a higher level is something we do not know.

ACKNOWLEDGMENTS

I have learned much about this subject from Professor D. Z. Freedman and would like to thank him for his guidance and encouragement and for reading this manuscript. It is also a pleasure to thank Professor P. van Nieuwenhuizen for many useful conversations. I would like to thank M. Roček for pointing out an error in an earlier version of the manuscript.

*Work supported in part by the National Science Foundation under Grant No. PHY-76-15328.

[1]D. Z. Freedman, P. van Nieuwenhuizen, and S. Ferrara, Phys. Rev. D 13, 3214 (1976); S. Deser and B. Zumino, Phys. Lett. 62B, 335 (1976).

[2]S. Ferrara and P. van Nieuwenhuizen, Phys. Rev. Lett. 37, 1669 (1976).

[3]D. Z. Freedman, Phys. Rev. Lett. 38, 105 (1977); S. Ferrara, J. Scherk, and B. Zumino, Phys. Lett. 66B, 35 (1977).

[4]M. T. Grisaru, P. van Nieuwenhuizen, and J. A. M. Vermaseren, Phys. Rev. Lett. 37, 1662 (1976).

[5]M. T. Grisaru, Phys. Lett. 66B, 75 (1977).

[6]S. Ferrara, D. Z. Freedman, P. van Nieuwenhuizen, P. Breitenlohner, F. Gliozzi, and J. Scherk, Phys. Rev. D 15, 1013 (1977).

[7]P. van Nieuwenhuizen and J. A. M. Vermaseren, Phys. Rev. D (to be published).

[8]S. Ferrara, F. Gliozzi, J. Scherk, and P. van Nieuwenhuizen, Nucl. Phys. B117, 333 (1976).