Cosmological constant in supergravity

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We construct an extension of pure supergravity which contains a cosmological term and a masslike term for the spin-3/2 field. Unlike another recent model which incorporates these features, that presented here is constructed from the usual spin-2, spin-3/2 fields alone.

I. INTRODUCTION

Since the advent of the new supergravity theory[1,2] much effort has been directed towards the coupling of supergravity to matter.[3,4,5] But little has been written about extensions of pure supergravity to include R^2 terms, cosmological constants, etc. Freedman and Das[6] have recently shown that the gauging of SO(2) and SO(3) internal symmetries in extended supergravity requires the presence of cosmological and spin-$\frac{3}{2}$ masslike terms for local supersymmetry. The cosmological constant in these models is proportional to the internal-symmetry gauge coupling constant and is therefore not arbitrary. One might suppose that the appearance of a cosmological term in supergravity is contingent on the couplings to matter. That this is not the case is demonstrated by the model presented below, which contains only spin-2 and spin-$\frac{3}{2}$ fields and differs from the usual supergravity theory by the addition of cosmological and spin-$\frac{3}{2}$ masslike terms. In fact, this model can be obtained from the O(2) gauge model of Freedman and Das if one eliminates from the Lagrangian of Ref. 6 the spin-$(\frac{3}{2},1)$ multiplet while keeping a nonzero gauge coupling constant.[7] However, the present author was guided by other considerations, described in Sec. III. In addition, we use here the first-order formulation of supergravity,[2] by which the verification of local supersymmetry is considerably simplified.

II. THE MODEL

The action

$$I=\int\left(-\frac{1}{4\kappa^2}\, e e^{a\mu}e^{b\nu}R_{\mu\nu ab}-\tfrac{1}{2}\,\epsilon^{\lambda\rho\mu\nu}\bar{\psi}_\lambda\gamma_5\gamma_\mu D_\nu\psi_\rho + \frac{3\lambda^2 e}{2}-\lambda\kappa e\bar{\psi}_\lambda\sigma^{\lambda\rho}\psi_\rho\right)d^4x \qquad (2.1)$$

is invariant under the following transformations:

$$\delta e_{a\mu}=-i\kappa\bar{\varepsilon}\gamma_a\psi_\mu\ , \qquad (2.2)$$

$$\delta\psi_\mu=\kappa^{-1}D_\mu\varepsilon+\tfrac{1}{2}i\lambda\gamma_\mu\varepsilon\ , \qquad (2.3)$$

$$\delta\omega_{\mu ab}=B_{\mu ab}-\tfrac{1}{2}e_{b\mu}B_{\lambda a}{}^{\lambda}+\tfrac{1}{2}e_{a\mu}B_{\lambda b}{}^{\lambda}\ , \qquad (2.4)$$

where

$$B_a{}^{\mu\nu}=\kappa e^{-1}\epsilon^{\lambda\rho\mu\nu}\bar{\varepsilon}\gamma_5\gamma_a D_\lambda\psi_\rho + i\lambda\kappa^2\epsilon^{\lambda\rho\mu\nu}\bar{\varepsilon}\gamma_5(-\tfrac{3}{2}\gamma_\lambda\gamma_a+2e_{a\lambda})\psi_\rho\ . \qquad (2.5)$$

The last two terms of (2.1) are an addition to the usual supergravity action and the last terms of (2.3) and (2.5) are additions to the usual transformation laws. The curvature tensor and covariant derivative are

$$R_{\mu\nu ab}=\omega_{\nu ab,\mu}-\omega_{\mu ab,\nu}+\omega_{\mu a}{}^{c}\omega_{\nu cb}-\omega_{\mu b}{}^{c}\omega_{\nu ca}\ , \qquad (2.6)$$

$$D_\mu=\partial_\mu+\tfrac{1}{2}\omega_{\mu ab}\sigma^{ab}\ , \qquad (2.7)$$

in which $\omega_{\mu ab}$ is an independent field.[2] $e_{a\mu}$ is the vierbein field and e is $\det(e_{a\mu})$. The metric signature is $(+---)$ and other conventions are as follows:

$$\begin{aligned}&\gamma_5=i\gamma^0\gamma^1\gamma^2\gamma^3\ ,\\ &\gamma_0{}^2=-\gamma_1{}^2=-\gamma_2{}^2=-\gamma_3{}^2=1\ ,\\ &\sigma_{ab}=\tfrac{1}{4}[\gamma_a,\gamma_b]\ ,\\ &\epsilon^{0123}=1\ ,\\ &R_{\mu a}=R_{\mu\nu ab}e^{b\nu},\quad R=R_{\mu a}e^{a\mu}\ .\end{aligned} \qquad (2.8)$$

We shall first discuss the invariance of the action of (2.1) and then comment on its construction. Only those variations containing λ need be checked here since those variations of (2.1) not containing λ are already known to vanish. We must first check that the variation of (2.1) containing terms linear in ψ_μ vanishes. Firstly, note that the variation of the cosmological term cancels the $\delta\psi_\mu=\frac{1}{2}i\lambda\gamma_\mu\epsilon$ variation of the spin-$\frac{3}{2}$ mass term. Secondly, it is easily shown that the $\delta\psi_\mu=\frac{1}{2}i\lambda\gamma_\mu\varepsilon$ variation of the spin-$\frac{3}{2}$ kinetic Lagrangian is

$$-i\lambda\epsilon^{\lambda\rho\mu\nu}\bar{\varepsilon}\gamma_5\sigma_{\lambda\mu}D_\nu\psi_\rho+\tfrac{1}{4}i\lambda\epsilon^{\lambda\rho\mu\nu}\bar{\varepsilon}\gamma_5\gamma_\lambda\gamma_\tau\psi_\rho S_{\mu\nu}{}^{\tau}\ , \qquad (2.9)$$

where the torsion is

$$2S_{\mu\nu}{}^{a}=e^{a}{}_{\nu,\mu}-e^{a}{}_{\mu,\nu}+\omega_{\mu}{}^{a}{}_{c}e^{c}{}_{\nu}-\omega_{\nu}{}^{a}{}_{c}e^{c}{}_{\mu}\ . \tag{2.10}$$

Thirdly, we rewrite the spin-$\frac{3}{2}$ mass term as

$$-\lambda\kappa e\bar{\psi}_{\lambda}\sigma^{\lambda\rho}\psi_{\rho}=-\tfrac{1}{2}i\lambda\kappa\epsilon^{\lambda\rho\mu\nu}\bar{\psi}_{\lambda}\gamma_{5}\sigma_{\mu\nu}\psi_{\rho} \tag{2.11}$$

and take the $\delta\psi_{\mu}=(1/\kappa)D_{\mu}\varepsilon$ variation to obtain

$$i\lambda\epsilon^{\lambda\rho\mu\nu}\bar{\varepsilon}\gamma_{5}\sigma_{\lambda\mu}D_{\nu}\psi_{\rho}+2i\lambda\epsilon^{\lambda\rho\mu\nu}\bar{\varepsilon}\gamma_{5}\sigma_{\tau\lambda}\psi_{\rho}S_{\mu\nu}{}^{\tau}\ . \tag{2.12}$$

Adding (2.9) to (2.12) we get

$$\tfrac{1}{2}i\lambda\epsilon^{\lambda\rho\mu\nu}\bar{\varepsilon}\gamma_{5}(-\tfrac{3}{2}\gamma_{\lambda}\gamma_{\tau}+2g_{\tau\lambda})\psi_{\rho}S_{\mu\nu}{}^{\tau}\ . \tag{2.13}$$

Now, for $\delta\omega_{\mu ab}$ of the form (2.4) the contribution to $\delta\mathcal{L}$ from $\delta\omega_{\mu ab}$ is

$$\delta\mathcal{L}=-\frac{e}{2\kappa^{2}}B_{a}{}^{\mu\nu}S_{\mu\nu}{}^{a}\ , \tag{2.14}$$

so that (2.13) is canceled by the choice of the last term in $B_{a}{}^{\mu\nu}$ given by (2.5). This completes the verification of invariance up to terms linear in ψ_{μ}.

To check that the cubic terms in δI also cancel is simple. The additional terms that we must consider come from the $\delta e_{a\mu}$ variation in the spin-$\frac{3}{2}$ mass term and the new $\delta\omega_{\mu ab}$ variation in the spin $\frac{3}{2}$ kinetic Lagrangian. The latter is easily shown to be

$$\tfrac{1}{2}\lambda\kappa^{2}\epsilon^{\lambda\rho\mu\nu}(\bar{\psi}_{\lambda}\gamma_{\nu}\psi_{\mu})(\bar{\varepsilon}\gamma_{5}\psi_{\rho}) \tag{2.15}$$

while the former is

$$-\lambda\kappa^{2}\epsilon^{\lambda\rho\mu\nu}(\bar{\psi}_{\lambda}\gamma_{5}\sigma_{\tau\nu}\psi_{\rho})(\bar{\varepsilon}\gamma^{\tau}\psi_{\mu})\ . \tag{2.16}$$

A Fierz rearrangement of (2.16) leads to a term which cancels (2.15) and a term which vanishes after a small amount of Dirac algebra. The action (2.1) is therefore fully invariant.

In obtaining Eqs. (2.15) and (2.16) we made use of the identity

$$\epsilon^{\lambda\rho\mu\nu}Q\psi_{\lambda}\bar{\psi}_{\rho}Q\psi_{\mu}\equiv 0\ , \tag{2.17}$$

where Q is any of the 16 Dirac matrices.

III. CONSTRUCTION

A nonzero cosmological constant implies a constant curvature of spacetime independent of matter. The symmetry properties of such spaces are those of the de Sitter group rather than the Poincaré group. The covariant derivative, D_{μ}, of (2.7) is appropriate to the Poincaré group. When considering the possibility of a cosmological term it is therefore natural to form the modified covariant derivative

$$\mathfrak{D}_{\mu}=D_{\mu}+\tfrac{1}{2}i\lambda\kappa\gamma_{\mu}\ , \tag{3.1}$$

which is appropriate to the de Sitter group.[8] The covariant derivative has the commutator

$$[\mathfrak{D}_{\mu},\mathfrak{D}_{\nu}]=\tfrac{1}{2}\mathcal{R}_{\mu\nu ab}\sigma^{ab}+i\lambda\kappa S_{\mu\nu a}\gamma^{a}\ , \tag{3.2}$$

where

$$\mathcal{R}_{\mu\nu ab}=R_{\mu\nu ab}-\lambda^{2}\kappa^{2}(e_{a\mu}e_{b\nu}-e_{a\nu}e_{b\mu})\ . \tag{3.3}$$

Let us now replace D_{μ} in the usual supergravity action and transformation laws by $\mathfrak{D}_{\mu}$. This gives precisely the spin-$\frac{3}{2}$ mass term of (2.1) and the new transformation law, (2.3), for ψ_{μ}. The spin-$\frac{3}{2}$ field equation is now that of conventional supergravity but with $\mathfrak{D}_{\mu}$ replacing D_{μ}. Secondly, let us construct a modified Einstein tensor from $\mathcal{R}_{\mu\nu ab}$,

$$\mathcal{G}_{\mu a}=\mathcal{R}_{\mu\nu ab}e^{b\nu}-\tfrac{1}{2}\mathcal{R}_{\mu\nu ab}e^{b\nu}e^{a\mu}=G_{\mu a}+3\lambda^{2}\kappa^{2}e_{a\mu}\ , \tag{3.4}$$

where $G_{\mu a}$ is the usual Einstein tensor constructed from $R_{\mu\nu ab}$. We will now require that the new spin-2 field equation be obtained from the usual one by replacing $\mathcal{G}_{\mu a}$ for $G_{\mu a}$ and $\mathfrak{D}_{\mu}$ for D_{μ}. This equation is

$$\mathcal{G}_{\tau}{}^{\mu}=e^{-1}\kappa^{2}\epsilon^{\lambda\rho\mu\nu}\bar{\psi}_{\lambda}\gamma_{5}\gamma_{\tau}\mathfrak{D}_{\nu}\psi_{\rho}\ . \tag{3.5}$$

This requirement is met by adding, in addition to the spin-$\frac{3}{2}$ mass term, just the new cosmological term of (2.1) to the usual supergravity action. This completes the construction of (2.1). It is seen that the new action is determined by the prescription that the quantities D_{μ} and $R_{\mu\nu ab}$ which are characteristic of the Poincaré group, be replaced *in the field equations*[9] by the quantities $\mathfrak{D}_{\mu}$ and $\mathcal{R}_{\mu\nu ab}$ which are characteristic of the de Sitter group.

IV. COMMENTS

We have shown that pure supergravity may be generalized to include an arbitrary cosmological constant. As in previous models,[6] this requires the introduction of a masslike term for the spin-$\frac{3}{2}$ field. The presence of a nonzero cosmological constant implies a constant curvature of spacetime. Consequently the spin-$\frac{3}{2}$ mass term cannot be interpreted as a physical mass.[10]

Further, it is hoped that the use of de Sitter covariant quantities will be useful in the construction of more complicated models.[11]

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[11]Since the original version of this paper, S. W. MacDowell and F. Mansouri have obtained the model presented here as the gauge theory of the graded de Sitter group OSp(4,1) [Phys. Rev. Lett. 38, 739 (1977)]. This has also been extended to SO(2) extended supergravity [see P. K. Townsend and P. van Nieuwenhuizen, Phys. Lett. (to be published)].