Spontaneous symmetry breaking and vector-meson dominance

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This article is a continuation of our previous work on the ρ-π puzzle. It examines further consequences of the unification of partial conservation of axial-vector current (PCAC) and vector-meson dominance (VMD) in which the pion is a Goldstone state and the ρ is "dormant" Goldstone state. Our new picture of the vector mesons does not require an A_1 meson although such a state is not ruled out. The Weinberg sum rules which provide the *raison d'être* for the A_1 are reexamined. The first Weinberg sum rule can be accommodated without a narrow A_1 state although some enhancement seems required. Examining the $J^{PC}=1^{+-}$ nonet we conclude that a new state, the isoscalar octet partner of the $B(1235)$ should exist around 1.7 GeV assuming ideal mixing. Without a detailed assumption on the mixing angle, its mass should be in the range 1.4 to 1.7 GeV. We also discuss the photon-ρ interactions. In the standard VMD picture the predicted rate for $\rho \to \pi + \gamma$ fails by five standard deviations, while in our picture this undesired result is averted.

I. INTRODUCTION

In a previous paper,[1] hereafter referred to as I, we proposed a new interpretation of vector mesons and vector-meson dominance (VMD) in which VMD is seen to be a consequence of spontaneous symmetry breaking. In this way partial conservation of axial-vector current (PCAC) and (VMD) are unified. In this article we shall develop this idea further by considering how our picture deals with some aspects of vector-meson phenomenology which we did not discuss in I.

The essential feature of the interpretation of vector mesons which we proposed in I is that they transform as members of a $(3,\overline{3})\oplus(\overline{3},3)$ representation of chiral $SU(3)\otimes SU(3)$. Hence, the ρ vector meson, for example, has the same chiral representation content as the pion. So the ρ and the π can be in the same SU(6) multiplet, the $\underline{35}$, and still the π can be a Nambu-Goldstone boson without difficulty. Under chiral $SU(6)\otimes SU(6)$ the vectors and pseudoscalars transform as members of $(6,\overline{6})\oplus(\overline{6},6)$.

This solves the ρ-π puzzle. In this picture the ρ is considered to be a "dormant" Goldstone boson. A dormant Goldstone boson is a state which in a nonrelativistic theory would be a true Goldstone boson. So in the static SU(6) limit both the π and the ρ masses are degenerate and zero; they are both Goldstone bosons.

The importance of the chiral representation of the vector mesons is that in order for the vector mesons to be in a $(3,\overline{3})\oplus(\overline{3},3)$ representation, the components of the vector-meson field operator must be part of an antisymmetric tensor operator in the quark model, that is, $\overline{q}\sigma_{\mu\nu}\frac{1}{2}\lambda^a q$. Letting

$$t^a_{\mu\nu}(x)=\overline{q}(x)\sigma_{\mu\nu}\tfrac{1}{2}\lambda^a q(x)\,, \tag{1.1}$$

we then project out the phenomenological vector-meson field,

$$\rho^a_\nu(x)=\frac{\partial^\mu t^a_{\mu\nu}(x)}{m_\rho Z_\rho^{\,1/2}}\,, \tag{1.2a}$$

$$\partial^\nu \rho^a_\nu=0\,. \tag{1.2b}$$

This effective vector field ρ^a_ν is normalized so that

$$\begin{aligned}\langle 0|t^a_{\mu\nu}(0)|\rho^b(k,\epsilon)\rangle&=\frac{Z_\rho^{\,1/2}}{m_\rho}\delta^{ab}(k_\mu\epsilon_\nu-k_\nu\epsilon_\mu)\,,\\ \langle 0|\rho^a_\mu(0)|\rho^b(k,\epsilon)\rangle&=-i\delta^{ab}\epsilon_\mu\,.\end{aligned} \tag{1.3}$$

The relation (1.2a) is called partial conservation of tensor current (PCTC). It serves the purpose of defining a three-component field ρ^a_ν transforming like $(3,\overline{3})\oplus(\overline{3},3)$ under the chiral group.

The implication of putting the vector mesons in $(3,\overline{3})\oplus(\overline{3},3)$ is that vector-meson dominance is a consequence of spontaneous symmetry breaking. Just as the $\pi[(3,\overline{3})\oplus(\overline{3},3)]$ couples to the axial-vector current $[(1,8)\oplus(8,1)]$ via a σ^0 going into the vacuum [Fig. 1(a)], so also the $\rho[(3,\overline{3})\oplus(\overline{3},3)]$ couples to the vector current $[(1,8)\oplus(8,1)]$ by the same mechanism [Fig. 1(b)]. Using this picture we have obtained (I) an expression for the current–vector-meson coupling, $1/\gamma_\rho$, defined by

$$\langle 0|V^a_\mu(0)|\rho^b(k,\epsilon)\rangle=-i\epsilon_\mu\delta^{ab}\frac{m_\rho^{\,2}}{\gamma_\rho}\,. \tag{1.4a}$$

This is analogous to the expression for the pion–axial-vector-current coupling

$$\langle 0|A^a_\mu(0)|\pi^b(k)\rangle=ik_\mu f_\pi\delta^{ab}\,. \tag{1.4b}$$

The relation we obtain between $1/\gamma_\rho$ and f_π is

$$\frac{1}{\gamma_\rho}=\left(\frac{f_\pi}{m_\rho}\right)\left(\frac{Z_\pi}{Z_\rho}\right)^{1/2}, \tag{1.5}$$

which is similar to the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin (KSRF) relation as we have discussed in I.

Another important consequence of the chiral-representation assignment of $(3,\overline{3})\oplus(\overline{3},3)$ to the vector mesons is that soft vector mesons decouple, just as soft pions decouple. This decoupling can be seen from the relations

$$\partial^\mu A^a_\mu(x)=m_\pi{}^2 f_\pi \pi^a , \tag{1.6a}$$

$$\partial^\mu t^a_{\mu\nu}(x)=m_\rho Z_\rho{}^{1/2}\rho^a_\nu . \tag{1.6b}$$

Since the left side of (1.6a) must vanish between states of zero momentum transfer ($q_\mu=0$), pions must decouple as $q_\mu\to 0$. The same argument applies to (1.6b), so vector mesons must decouple as $q_\mu\to 0$. But physical vector mesons are not soft, so this decoupling theorem is hard to test experimentally. Nevertheless, it does present a different picture for the way in which the vector current couples to hadronic states in that a direct coupling is required as well as a vector-meson pole term. Otherwise the charge associated with V^a_μ would vanish. (See Fig. 2.)

It should be noted that the relation (1.6b), usually called PCTC (1.2), is *not* the analog of the PCAC relation (1.6a). Rather, one must separate the vector current into two pieces as depicted in Fig. 2(a),

$$V^a_\mu=\tilde{V}^a_\mu+\frac{f_\pi Z_\pi{}^{1/2}}{Z_\rho}\,\partial^\lambda t^a_{\lambda\mu}, \tag{1.7}$$

where $\tilde{V}^a_\mu$ has no ρ-pole terms. This corresponds to to the standard separation of the axial-vector current,

$$A^a_\mu=\tilde{A}^a_\mu-f_\pi Z_\pi{}^{-1/2}\partial_\mu \pi^a , \tag{1.8}$$

where $\tilde{A}^a_\mu$ has no pion pole [Fig. 2(b)]. Then one can derive a tensor-field identity (TFI),

$$\partial_\alpha V^a_\beta-\partial_\beta V^a_\alpha=\partial_\alpha \tilde{V}^a_\beta-\partial_\beta \tilde{V}^a_\alpha+\frac{f_\pi Z_\pi{}^{1/2}}{Z_\rho}\left(\Box t^a_{\alpha\beta}-\epsilon_{\alpha\beta\gamma\delta}\,\partial^\gamma C^{a\delta}\right), \tag{1.9a}$$

which is the analog of PCAC, written in the form

$$\partial^\mu A^a_\mu=\partial^\mu\tilde{A}^a_\mu-f_\pi Z_\pi{}^{-1/2}\Box\pi^a . \tag{1.9b}$$

In (1.9a) C^a_δ is a second-class axial-vector current.

The picture of vector mesons presented here is consistent with the observed behavior of electromagnetic form factors. For example, let us consider the pion form factor $F_\pi(q^2)$ defined by

$$\langle\pi^b(p)\,|\,V^a_\mu\,|\,\pi^c(k)\rangle=F_\pi(q^2)\epsilon^{abc}P_\mu , \tag{1.10}$$

($q=p-k$, $P=p+k$) where we have SU(2) internal

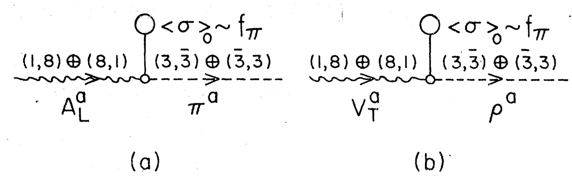

FIG. 1. Coupling of (a) axial-vector current to pion and (b) vector current to the ρ via spontaneous breaking of chiral symmetry.

symmetry for simplicity, and

$$F_\pi(0)=1 .$$

The pionic matrix element of the tensor current is

$$\langle\pi^b(p)\,|\,t^a_{\mu\nu}\,|\,\pi^c(k)\rangle=iT(q^2)\epsilon^{abc}(q_\mu P_\nu-q_\nu P_\mu) . \tag{1.11}$$

The effective ρ field is defined by (1.2) and the ρ source is defined by

$$(\Box+m_\rho{}^2)\rho_\mu{}^a={}^\rho J^a_\mu .$$

Then the $\rho\pi\pi$ matrix element,

$$\langle\pi^b(p)\,|\,{}^\rho J^a_\mu\,|\,\pi^c(k)\rangle=G_{\rho\pi\pi}(q^2)\epsilon^{abc}P_\mu , \tag{1.12}$$

is related to $T(q^2)$ in this way:

$$G_{\rho\pi\pi}(q^2)=\frac{q^2(-q^2+m_\rho{}^2)}{m_\rho Z_\rho{}^{1/2}}\;T(q^2) . \tag{1.13}$$

Since $G_{\rho\pi\pi}(m_\rho{}^2)=g_{\rho\pi\pi}$, $T(q^2)$ has a ρ pole. Furthermore, since $T(q^2)$ has no pole at $q^2=0$ (assuming no zero-mass vector-meson states), we have $G_{\rho\pi\pi}(0)$

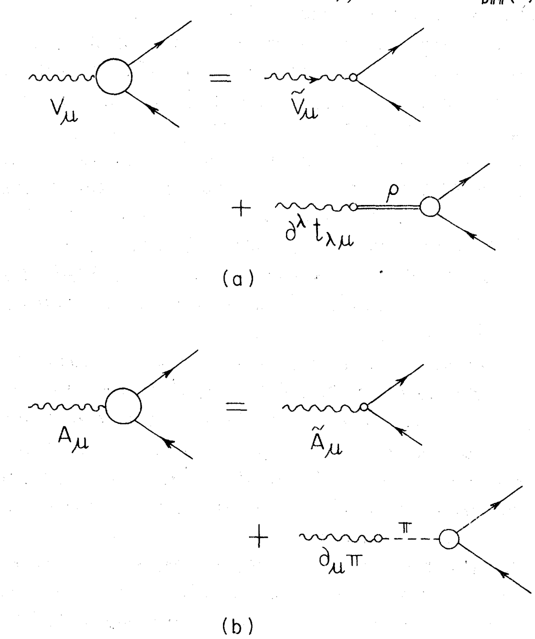

FIG. 2. (a) Separation of the vector form factor into a direct plus a ρ-pole contribution. The ρ coupling to hadrons vanishes at zero-momentum transfer. (b) Separation of the axial-vector current into a pion-pole piece and nonpole contribution.

$=0$, the decoupling theorem. So $G_{\rho\pi\pi}(q^2)$ is not a slowly varying function.

Now we assume that after removing the ρ pole from the tensor current what remains is smooth (i.e., does not vary rapidly), so that $(-q^2+m_\rho^2)T(q^2)=$ constant is slowly varying, and

$$G_{\rho\pi\pi}(q^2)=\frac{q^2}{m_\rho^2}\,g_{\rho\pi\pi} \tag{1.14}$$

from (1.13). If we calculate the contributions to the pion form factor $F_\pi(q^2)$ as depicted in Fig. 2(a), they are given by

$$F_\pi(q^2)=1+\frac{m_\rho^2}{\gamma_\rho}\,\frac{G_{\rho\pi\pi}(q^2)}{-q^2+m_\rho^2}, \tag{1.15}$$

where we have denoted the direct coupling by 1. Using (1.14) we finally obtain

$$F_\pi(q^2)=1+\frac{g_{\rho\pi\pi}}{\gamma_\rho}\,\frac{q^2}{-q^2+m_\rho^2}. \tag{1.16}$$

The no-subtraction hypothesis, $F_\pi(\infty)=0$, yields the usual vector universality relation

$$\gamma_\rho=G_{\rho\pi\pi}(m_\rho^2)=g_{\rho\pi\pi}. \tag{1.17}$$

Hence (1.16) is identical to

$$F_\pi(q^2)=\frac{m_\rho^2}{-q^2+m_\rho^2}\left(\frac{g_{\rho\pi\pi}}{\gamma_\rho}\right), \tag{1.18}$$

the usual vector-meson dominance (VMD). For alternative derivations using dispersion relations, as well as treatment of nucleonic form factors, see I.

The treatment of the form factor can be generalized to other matrix elements. In the conventional treatment of VMD, besides definitions one must invoke a smoothness hypothesis to extract interesting results. In our new treatment of the vector mesons, besides definitions, one must also introduce a smoothness (or equivalently a no-subtraction) hypothesis as we saw above. In either our new treatment or in the old treatments one requires a single smoothness assumption.

In the present instance the general smoothness assumption is that

$$[(q_\alpha-q_\beta)^2-m_V^2]\,\langle\alpha\,|t^a_{\mu\nu}(0)|\,\beta\rangle,$$

where α and β are *hadronic* states, is slowly varying for $0\le(q_\alpha-q_\beta)^2\le m_V^2$ in those invariant amplitudes corresponding to channels coupling to the vector mesons. There are also invariant amplitudes in this matrix element to which axial-vector mesons can contribute, and for these one may make another smoothness hypothesis.

Although the picture presented here may seem at first more complicated than the usual picture of VMD owing to the requirement of a direct coupling, there is no inconsistency with the experimental behavior of the vector mesons. Furthermore, there are a number of advantages and simplifications to this scheme as we have discussed in detail in I. What we might stress here is that VMD and PCAC are consequences of the same mechanism in this picture.

Another advantage of having the ρ in a $(3,\bar{3})\oplus(\bar{3},3)$ representation of chiral $SU(3)\otimes SU(3)$ is that the chiral partner of the ρ is then the $B(1235)$ meson, $J^{PC}=1^{+-}$, a well-established resonant state. This is in contrast to the case of the A_1 meson, $J^{PC}=1^{++}$, which would be the chiral partner of the ρ if the ρ were in a $(1,8)\oplus(8,1)$ representation. But the canonical A_1 does not seem to exist experimentally.[2]

The remainder of this paper is organized as follows. In Sec. II we discuss the Weinberg[3] spectral-function sum rules in the absence of an A_1 resonance. We find that saturating the vector spectral function with just the ρ pole in the first Weinberg sum rule leads to the relation

$$\frac{1}{\gamma_\rho}=\frac{f_\pi}{m_\rho}, \tag{1.19}$$

which agrees with (1.5) in the SU(6) limit for which $Z_\pi=Z_\rho$. Including finite-ρ-width corrections alters this relation by only 10%. However, the relationship which agrees with experiment is

$$\frac{1}{\gamma_\rho}=\frac{f_\pi}{m_\rho}\ (1.5)\,. \tag{1.20}$$

We discuss the possible reasons and significance of the discrepancy between (1.19) and (1.20), and point out the need for better experimental information before the sum rule can be accurately tested. What is evidently required for the sum rule to be saturated by just low-lying states is an enhancement in the axial-vector spectral function. This need not be a narrow A_1 state but could be a large continuum or broad enhancement as is suggested by recent experiments.[4] An alternate possibility is that there is no large structure in this spectral function. Instead the integral converges very slowly with the axial-vector spectral function slightly larger than the vector spectral function asymptotically.

Section III describes the situation concerning the remaining members of the axial-vector nonet of which the B meson is the $I=1$ member. We find that the mass relation among the low-lying meson octets derived in I,

$$m_\rho^2-m_\pi^2=m_B^2-m_\sigma^2, \tag{1.21}$$

is well satisfied for all the observed members of the octets. We also estimate the mass of the missing $I=0$, $J^{PC}=1^{+-}$ octet member to be approximately 1.5 GeV. The physical mass of this state then depends on the mass of the $I=0$, SU(3) singlet and the mixing between the two (which need not be ideal). If one assumes, as given by the quark model, ideal mixing and a physical mass for the singlet around the mass of the $B(1235)$, then the physical mass of the $I=0$ octet member comes out to be 1.7 GeV.

Section IV deals with the interaction of vector mesons with photons. In particular, we try to dispel some confusion that may result in considering the matrix element

$$\langle 0 \,|\, \partial^\mu t^a_{\mu\nu}(0) \,|\, \gamma\rangle ,$$

which it must be realized never appears in any physical process. We illustrate the correct approach with a discussion of photoproduction. We also investigate the process $\rho \to \pi\gamma$ from our viewpoint and find that the disagreement with the measured rate[5] which results from applying the conventional picture of VMD does not occur in our scheme.

In Sec. V we offer some concluding remarks.

II. WEINBERG SUM RULES

Using the commutation relations of current algebra, Weinberg[3] derived spectral-function sum rules for chiral $SU(2)\otimes SU(2)$. The first Weinberg sum rule is

$$\int_0^\infty [\rho_V(m^2) - \rho_A(m^2)]\, \frac{dm^2}{m^2} = f_\pi^{\,2} , \qquad (2.1)$$

where ρ_V and ρ_A are the spectral functions of the vector and axial-vector currents defined by

$$\langle 0 \,|\, V^a_\mu(x) V^b_\nu(0) \,|\, 0\rangle = (2\pi)^{-3}\delta^{ab}\int d^4p\, \theta(p^0) e^{-ip\cdot x}\rho_V(p^2)\left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2}\right) , \qquad (2.2)$$

$$\langle 0 \,|\, A^a_\mu(x) A^b_\nu(0) \,|\, 0\rangle = (2\pi)^{-3}\delta^{ab}\int d^4p\, \theta(p^0) e^{-ip\cdot x}\left[\rho_A(p^2)\left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2}\right) + f_\pi^{\,2}\delta(p^2) p_\mu p_\nu\right] . \qquad (2.3)$$

The sum rule (2.1) follows just from current algebra with the assumption that Schwinger terms are either c numbers or, if operators, contain no $\Delta I = 1$ terms. To derive the second sum rule,

$$\int_0^\infty [\rho_V(m^2) - \rho_A(m^2)]\, dm^2 = 0 , \qquad (2.4)$$

requires additional assumptions.

To examine the question of the convergence of these spectral-function sum rules, Wilson[6] pointed out the usefulness of the operator-product expansion. He showed that the first and second sum rules were valid only in the limit of exact chiral $SU(2)\otimes SU(2)$ invariance. This analysis is in the context of chiral-symmetry breaking by hadron-mass terms, assuming the scale dimension of the quark-mass term to be three or larger.

A further analysis,[7] assuming that the strong interactions are asymptotically free, concludes that the sum rules are valid even in the world of broken chiral symmetry if one takes an appropriate combination of current propagators involving strange and/or charmed quarks, as well as the up and down quarks already present in the original SU(2)-internal-symmetry spectral functions. For the first sum rule to be valid only one term, involving either the strange quark or the charmed quark, need be subtracted from the "ρ-A_1 sum rule" (2.1). However, for the second sum rule to converge in the broken-chiral-symmetry world, more terms are necessary and these require a detailed knowledge of the high-frequency behavior of the quark-mass spectrum. Hence we do not believe the second sum rule is amenable to saturation by low-energy states.

Since the single correction term to the original ρ-A_1 first sum rule is proportional to m_0/m_s or m_0/m_c, where m_0 is the nonstrange-quark mass, m_s is the strange-quark mass, and m_c is the charmed-quark mass, it is plausible that this correction term is small, as we shall assume. Then in order to test the original first Weinberg sum rule (2.1) it is necessary to know the spectral functions $\rho_V(m^2)$, ideally for all m^2. These spectral functions are measurable in principle from the cross sections for hadron production in electron-positron annihilation, for example. However, the actual experiments are rather demanding. To date we still lack adequate data on even the isovector electromagnetic spectral function [which is related to $\rho_V(m^2)$], let alone the axial-vector spectral function, which can be obtained from inclusive soft-pion production in e^+-e^- annihilation.[8]

While awaiting more satisfactory data, one may attempt to saturate the sum rule with low-lying resonances. This approach assumes that the integral converges already at low energies.[9,10] But what precisely is meant by low energy is unfortunately unknown.

In this vein Weinberg estimated $\rho_V(m^2)$ by using ρ dominance:

$$\rho_V(m^2) \simeq g_\rho^2 \delta(m^2 - m_\rho^2). \tag{2.5}$$

He also estimated $\rho_A(m^2)$ by assuming dominance by an axial-vector meson,

$$\rho_A(m^2) \simeq g_A^2 \delta(m^2 - m_A^2). \tag{2.6}$$

Then using (2.4) which implies $g_\rho^2 = g_A^2$ and the relation[11] $g_\rho^2 = 2f_\pi^2 m_\rho^2$ Weinberg obtained his relation

$$m_{A_1} \simeq m_\rho \sqrt{2} \tag{2.7}$$

for the mass of the A_1.

However, it has turned out that an A_1 meson at such a mass does not seem to exist.[2] Furthermore, in our solution to the ρ-π puzzle we do not require an A_1 as the chiral partner of the ρ or strong couplings to the A_1 if it does exist.

On this basis we have estimated the first sum rule (2.1) by using the measured[12] cross section for $e^+e^- \to \rho$ and the relation[8,13]

$$\sigma(p^2) = (16\pi^3\alpha^2/p^2)\,\frac{\rho_V(p^2)}{p^2}\;, \tag{2.8}$$

along with the estimate for the continuum contribution to $\rho_A(p^2) = \rho_A^{\text{cont}}(p^2) \simeq \rho_V^{\text{cont}}(p^2)$. This procedure yields a value for the left side of (2.1) approximately $2f_\pi^2$ instead of f_π^2. Essentially the same result is obtained by using (2.5), i.e., dropping any continuum, and setting $\rho_A(p^2) = 0$. Hence the continuum contribution to $\rho_A(p^2)$ up to about 1.2 GeV^2 was either badly estimated or is negligible. The latter alternative is the conclusion of Chen *et al.*[9] from their examination of Weinberg's second sum rule (2.4).

But one must not take the failure to obtain agreement very seriously with the crude data currently available. As we have already remarked, the procedure is merely makeshift until adequate data are available. In particular, no measurement of $\rho_A(m^2)$ exists. Nevertheless, it is surprising that $\rho_V(m^2)$ and $\rho_A(m^2)$ should behave so differently at low energies. Indeed, there are now some preliminary indications that this may not be so. Apparently a candidate for an A_1 axial-vector meson (i.e., $J^{PC} = 1^{++}$) exists,[4] but at higher mass (1.4–1.5 GeV) and broader width (~ 300 MeV) than previously expected. If such a state proves to be real, then depending on the strength of its coupling to the axial-vector current it may be what is needed to saturate the first sum rule at low energies. It is worth remarking that without the original second sum rule (2.4) there is no detailed constraint on the A_1 mass since g_ρ and g_A are not necessarily equal.

We should point out, however, that were such an A_1 state to exist, it still would not necessarily be the chiral partner of the ρ nor would it conflict with our solution to the ρ-π puzzle. To begin with the mass, it is more than $\frac{1}{3}$ too large compared with the Weinberg prediction (2.7). Furthermore, having the ρ in a $(3,\bar{3}) \oplus (\bar{3},3)$ representation of chiral SU(3) $\otimes$ SU(3) and having the $B(1235)$ as the chiral partner of the ρ is independent of the existence or not of an A_1. To be sure, there is always the possibility of representation mixing, but so far this seems an unnecessary complication.

Our general conclusion is that with just the ρ contribution to the vector spectral function the first Weinberg sum rule fails by a factor of 2. Additional contributions to the vector spectral function can only make the agreement worse. Hence there must be structure in the axial-vector spectral function but this need not be the conventional A_1 meson. Furthermore, there remains the possibility that the correction term due to broken chiral SU(2) $\otimes$ SU(2) is not negligible.

III. THE $J^{PC} = 1^{+-}$ NONET

In I we derived the following mass relation among the low-lying meson octets:

$$m_\rho^2 - m_\pi^2 = m_B^2 - m_\sigma^2. \tag{3.1}$$

This relation follows from breaking chiral U(6) $\otimes$ U(6) by an explicit spin-dependent U(6)-breaking term independent of SU(3) breaking. It was pointed out in I that the relation holds very well for the $I=1$ members of the octets, i.e., $\vec{\pi}$, $\vec{\rho}$, $\vec{\text{B}}$, and $\vec{\delta}$. One obtains

$$m_{\vec{\rho}}^2 - m_{\vec{\pi}}^2 = m_{\vec{\text{B}}}^2 - m_{\vec{\delta}}^2\,, \qquad 0.574 \text{ vs } 0.577 \text{ GeV}^2. \tag{3.2}$$

We can now test the relation further since a strange partner for the B meson seems fairly well identified. A Stanford Linear Accelerator Group[14] has recently reported evidence for the existence of two strange axial-vector mesons Q_1 and Q_2. A European collaboration[15] favors the existence of only one such state. Furthermore, a unitary-Deck-model analysis[16] of the data also concludes that there is only one axial-vector strange resonance, the Q_B between 1.3 and 1.4 GeV with a width of order 150 MeV. This is in agreement with the prediction of our mass relation (3.1), now putting in the strange members of the octets,

$$m_{K^*}^2 - m_K^2 = m_{Q_B}^2 - m_\kappa^2. \tag{3.3}$$

Again each side of the relation is approximately 0.6 GeV^2. Here we have used $m_\kappa = 1250 \pm 100$ MeV.[17]

Turning our attention to the $I=0$ octet member we can make a prediction of its mass, but as yet no $I=0$ partners of the B have been identified. Since the relation (3.1) is valid for octet members only, we make use of the Gell-Mann–Okubo formula to determine the masses of the unmixed isoscalars. We obtain then the following masses: $m_{\phi_0} = 0.93$ GeV and $m_{\epsilon_0} = 1.33$ GeV. Hence the unmixed mass of the $I = 0$ octet partner of the B should be given by

$$\begin{aligned} m_{X_0}^{\ 2} &= m_{\varphi_0}^{\ 2} - m_\eta^{\ 2} + m_{\epsilon_0}^{\ 2} \\ &= 2.33\ \mathrm{GeV}^2\,, \\ m_{X_0} &\approx 1.5\ \mathrm{GeV}. \end{aligned} \tag{3.4}$$

This value agrees with that determined from the Gell-Mann–Okubo formula

$$m_{X_0}^{\ 2} = \tfrac{1}{3}\,(4m_{Q_B}^{\ 2} - m_B^{\ 2}) \tag{3.5}$$

using $m_{Q_B}^{\ 2} \approx 2\ \mathrm{GeV}^2$. This is not surprising since (3.4) is calculated using Gell-Mann–Okubo formulas, and Eq. (3.1) is consistent with the inputs to the formulas. We might also mention that $m_{\phi_0}^{\ 2} - m_\eta^{\ 2}$ also equals $0.57\ \mathrm{GeV}^2$, in agreement with (3.2) and (3.3).

The value predicted for the unmixed mass of the isoscalar octet partner of the B, which we call here X_0, may of course be different from the physical mass. This depends on the unmixed mass (unknown) of the singlet isoscalar axial-vector meson ($C = -1$) which we call here X_0', and on the mixing between the two isoscalar mesons. Although the mixing is ideal for the vector and tensor mesons, it is not for the pseudoscalars or scalars.[18] So there seems to be no compelling reason to assume that it is ideal here. At any rate, *we expect an isoscalar partner of the B to lie roughly in the range 1.4 to 1.7 GeV.*

If we do assume ideal mixing, as would occur in the quark model, and we also assume, again from the quark model, that the physical mass of the isoscalar SU(3) singlet, the X', is approximately degenerate with the B meson mass (so say 1250 MeV), then *we can determine the physical mass of the X to be 1.7 GeV.* The unmixed mass of the isoscalar singlet, the X_0', comes out to be 1.5 GeV, degenerate with the mass of the unmixed X_0.

Although a state with the quantum numbers of the X has not yet been identified around 1.7 GeV or even in the range 1.4 to 1.7 GeV, neither has it been assiduously searched for. However, such a search is now being undertaken.[19]

IV. PHOTON-ρ INTERACTIONS

Because of the decoupling theorem for soft vector mesons and the existence of a direct coupling term as shown in Fig. 2(a), it may appear that some discrepancies might arise when one considers the interactions of real photons. On the contrary, it turns out that we retain all the good results of VMD, while avoiding those which conflict with experiment.

First we wish to clear up some confusion which may occur if one chooses to consider the matrix element

$$\langle 0 | \rho_\mu | \gamma \rangle\,. \tag{4.1}$$

In our picture the ρ is in a $(3,\bar{3}) \oplus (\bar{3},3)$ representation and the effective ρ field is defined by

$$\rho_\nu^{a(3,\bar{3})\oplus(\bar{3},3)} \propto \partial^\mu t^a_{\mu\nu} = \partial^\mu \bar{q}\sigma_{\mu\nu}\tfrac{1}{2}\lambda^a q\,. \tag{4.2}$$

The conventional picture has the ρ in a $(1,8)\oplus(8,1)$ representation, so its definition in the quark model is

$$\rho_\mu^{a(1,8)\oplus(8,1)} \propto \bar{q}\gamma_\mu \tfrac{1}{2}\lambda^a q\,. \tag{4.3}$$

If one calculates (4.1) using (4.2) one obtains

$$\langle 0 | \rho_\nu^{a(3,\bar{3})\oplus(\bar{3},3)} | \gamma \rangle \propto \langle 0 | \partial^\mu t^a_{\mu\nu} | \gamma \rangle = 0\,, \tag{4.4a}$$

since

$$\langle 0 | t^a_{\mu\nu} | \gamma(q,\epsilon) \rangle = c\,\delta^{aQ}(q_\mu \epsilon_\nu - q_\nu \epsilon_\mu) \tag{4.4b}$$

and $q^2 = 0$. On the other hand, in the old treatment, $\rho_\mu^{(1,8)\oplus(8,1)}$, one finds that (4.1) does not necessarily equal zero,

$$\langle 0 | \rho_\mu^{(1,8)\oplus(8,1)} | \gamma \rangle \neq 0\,. \tag{4.5}$$

Thus in our treatment it looks as though the ρ-γ coupling vanishes for real photons. If one were to assume that this matrix element (4.1) was the ρ-γ vertex shown in Fig. 3(a) and that this vertex was interpolated smoothly from $q^2 = 0$ to $q^2 = m_\rho^{\ 2}$, then one would reach the false conclusion that the pho-

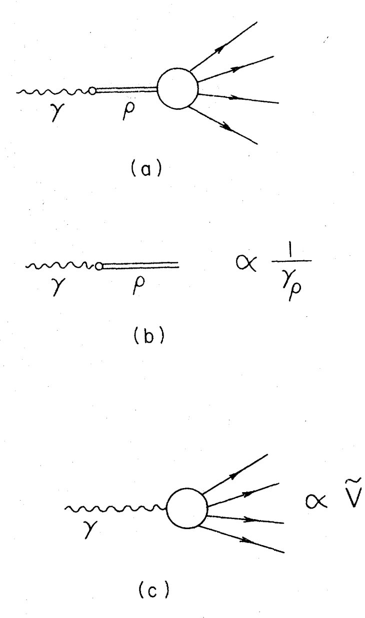

FIG. 3. Photon-hadron interaction via (a) VMD and (c) direct coupling.

tons had no couplings to vector mesons.

The reason the preceding argument is erroneous is that the matrix element (4.1) never appears in the calculation of any physical process. Only matrix elements of on-mass-shell particles enter into the computation of the S matrix. In (4.1) the ρ field is off its mass shell since $q^2=0$, the physical photon mass. But interpolating fields are not measurable. The relevant matrix element for considering the ρ-γ coupling is

$$\langle 0|J_\mu^{\text{em}}|\rho\rangle\,, \tag{4.6}$$

and this is defined on the ρ mass shell by

$$\langle 0|J_\mu^{\text{em}}|\rho(k,\epsilon)\rangle = -i\epsilon_\mu \frac{em_\rho{}^2}{\gamma_\rho}\,, \tag{4.7}$$

no matter what the chiral representation content of the ρ.

We might also point out that even though the matrix element

$$\langle A|t^a_{\alpha\beta}|B\rangle \tag{4.8}$$

is not measurable in strong and electromagnetic interactions, it could possibly be measured in weak-interaction processes if the weak currents included $t^a_{\alpha\beta}$.

In the usual picture, with $\rho\propto\rho^{(1,8)\oplus(8,1)}$, diagrams such as Fig. 3(b) can be isolated from Fig. 3(a) and still retain meaning. However, in our picture, with $\rho\propto\rho^{(3,\bar{3})\oplus(\bar{3},3)}$, the diagram Fig. 3(a) is interpreted differently. One cannot treat Fig. 3(b) in isolation; it is always part of the matrix element Fig. 3(a). Furthermore, our picture also requires Fig. 3(c), a direct photon-hadron coupling. This is because Fig. 3(a) vanishes as $q_\mu\to 0$ since it is proportional to $\langle 0|\partial^\mu t^a_{\mu\nu}|\text{hadrons}\rangle$. But one should note that it is the decoupling of soft ρ's from hadrons which causes Fig. 3(a) to vanish as $q_\mu\to 0$, and not any soft ρ-photon decoupling.

We illustrate the correct procedure for treating ρ-photon interactions by examining photoproduction of hadrons off nucleons, Fig. 4.

In the conventional treatment, Fig. 4(a),

$$A_\gamma(q^2) = \frac{e}{\gamma_\rho}\frac{m_\rho{}^2}{m_\rho{}^2-q^2}A_\rho\,. \tag{4.9}$$

So for real photons,

$$A_\gamma(0) = \frac{e}{\gamma_\rho}A_\rho\,. \tag{4.10}$$

In our treatment, Fig. 4(b),

$$A_\gamma(q^2) = C + \frac{e}{\gamma_\rho}\frac{m_\rho{}^2}{m_\rho{}^2-q^2}g_{\rho J}\frac{q^2}{m_\rho{}^2}A_J\,. \tag{4.11}$$

There are some things to notice about Eq. (4.11). The ρ-photon coupling is a constant independent of q^2 just as in the conventional treatment. In addition, a factor of $q^2/m_\rho{}^2$ appears because our ρ de-

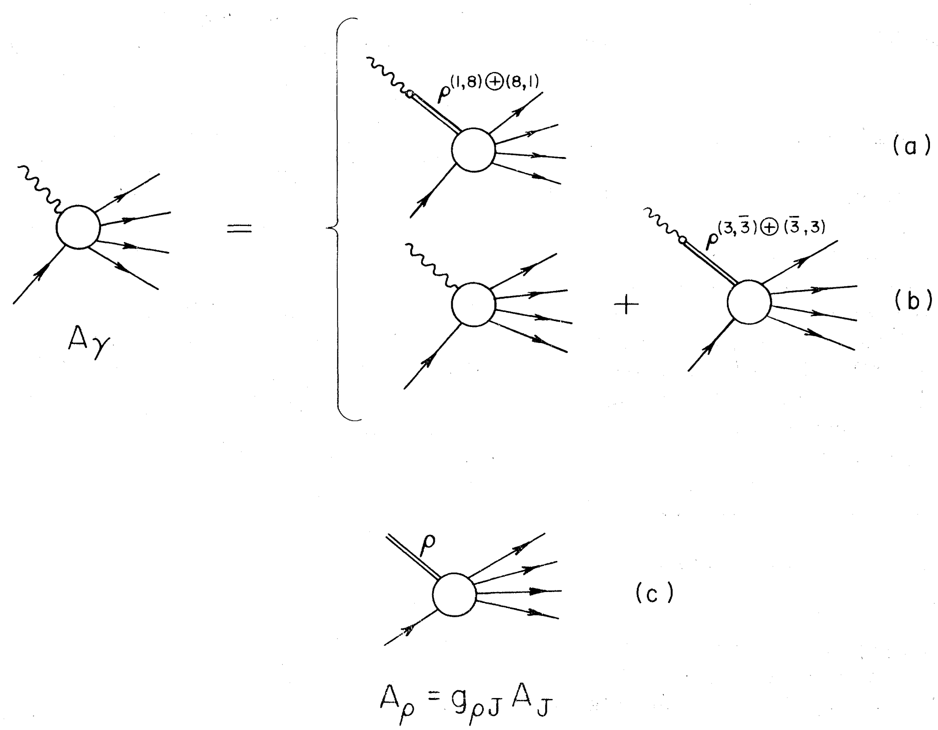

FIG. 4. Photoproduction of hadrons off nucleons in the (a) $\rho^{(1,8)\oplus(8,1)}$ picture and (b) $\rho^{(3,\bar{3})\oplus(\bar{3},3)}$ picture of VMD. (c) ρ-nucleon interaction vertex.

couples from hadrons in this process at $q^2=0$.

There are two arguments one can employ to recover the conventional and experimentally verified result, Eq. (4.10). First, as $q^2\to\infty$, $A_\gamma(q^2)$ is assumed to go to zero, that is, we assume a no-subtraction dispersion relation. Then

$$0=C-\frac{e}{\gamma_\rho}g_{\rho J}A_J\,. \tag{4.12}$$

But at $q^2=0$,

$$A_\gamma(0)=C\,. \tag{4.13}$$

Hence,

$$A_\gamma(0)=\frac{e}{\gamma_\rho}g_{\rho J}A_J=\frac{e}{\gamma_\rho}A_\rho\,. \tag{4.14}$$

One may instead use a smoothness argument to arrive at the same result. That is, we assume that after removing the ρ pole from $A_\gamma(q^2)$,

$$(m_\rho{}^2-q^2)A_\gamma(q^2)=F(q^2)\,, \tag{4.15}$$

$F(q^2)$ is slowly varying so that

$$F(0)\approx F(m_\rho{}^2)\,. \tag{4.16}$$

But from Eq. (4.11),

$$F(q^2)=(m_\rho{}^2-q^2)C+\frac{e}{\gamma_\rho}g_{\rho J}q^2A_J\,, \tag{4.17}$$

and at $q^2=m_\rho{}^2$,

$$\begin{aligned}F(m_\rho{}^2)&=\frac{e}{\gamma_\rho}g_{\rho J}m_\rho{}^2A_J \\ &\approx F(0)\,.\end{aligned} \tag{4.18}$$

Hence from (4.15) at $q^2=0$ we obtain

$$m_\rho{}^2A_\gamma(0)=\frac{e}{\gamma_\rho}g_{\rho J}m_\rho{}^2A_J\,, \tag{4.19}$$

and therefore we again recover (4.10).

The preceding example shows that our treatment of VMD is consistent with the conventional one not only for off-shell photons, as discussed in I, but for real photons as well. The general nature of the arguments leads us to believe that this agreement will hold for most cases. However, we have examined at least one process where our picture of VMD does not lead to the result of the conventional treatment, but in this case the conventional answer turns out to be at variance with present experiments.

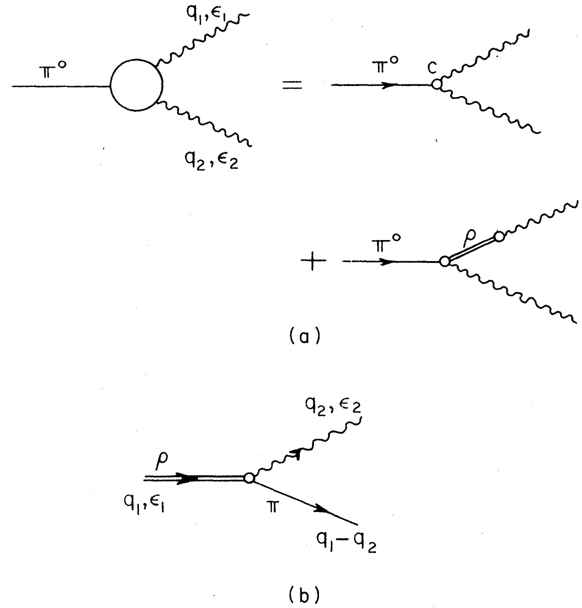

FIG. 5. (a) Separation of the π^0-$\gamma\gamma$ vertex into a direct photon coupling term and a ρ-pole contribution. (b) ρ-$\pi\gamma$ vertex.

The process is $\rho\to\pi\gamma$. The latest measurement of the $\rho^-\to\pi^-\gamma$ decay width[5] is approximately three times smaller (more than five standard deviations) than the prediction[20] from conventional VMD and the well-known $\pi^0\to\gamma\gamma$ rate, as well as predictions using quark or higher-symmetry models.[21]

The conventional VMD analysis relates the coupling constants $g_{\pi^0\gamma\gamma}$ and $g_{\rho\pi\gamma}$ by

$$g_{\pi^0\gamma\gamma}=\frac{2eg_{\rho\pi\gamma}}{\gamma_\rho}\,, \tag{4.20}$$

and it is this result that is in apparent disagreement with the observed rate. Our treatment is depicted in Fig. 5(a). We examine VMD of only one photon to simplify the analysis. This will not change the qualitative result.

The amplitude is a sum of two pieces: a direct coupling as well as the ρ-mediated term,

$$\epsilon_{\mu\nu\lambda\delta}\epsilon_1^\mu\epsilon_2^\nu q_1^\lambda q_2^\delta A_{\pi^0\gamma\gamma}(q_1{}^2,\ldots)=\epsilon_{\mu\nu\lambda\delta}\epsilon_1^\mu\epsilon_2^\nu q_1^\lambda q_2^\delta\left[A^c_{\pi^0\gamma\gamma}+\frac{em_\rho{}^2}{\gamma_\rho}\frac{1}{m_\rho{}^2-q_1{}^2}g_{\rho\pi\gamma}(q_1{}^2,\ldots)\right]. \tag{4.21}$$

This decomposition just corresponds to the decomposition of the vector current in (1.7). We again assume smoothness of the amplitude after removing the ρ pole,

$$F(q^2)=(m_\rho{}^2-q_1{}^2)A_{\pi^0\gamma\gamma}\,, \tag{4.22a}$$

$$F(m_\rho{}^2)\approx F(0)\,. \tag{4.22b}$$

At $q_1{}^2=m_\rho{}^2$ we have

$$\begin{aligned}F(m_\rho{}^2)&=\frac{em_\rho{}^2}{\gamma_\rho}g_{\rho\pi\gamma}(m_\rho{}^2,\ldots) \\ &\approx F(0)\,.\end{aligned} \tag{4.23}$$

This development reproduces the conventional result, (4.20), providing

$$F(0)=m_\rho^2 A^c_{\pi^0\gamma\gamma}\,. \tag{4.24}$$

But this assumes $g_{\rho\pi\gamma}(q_1^2=0,\ldots)=0$. That the decoupling theorem at $q_1^2=0$ does not apply to $g_{\rho\pi\gamma}(q_1^2,\ldots)$ in this case can be seen from studying the $\rho\to\pi\gamma$ decay directly as shown in Fig. 5(b). Here the interpolating amplitude is given by

$$\frac{1}{m_\rho Z_\rho^{1/2}}(q_1^2-m_\rho^2)q_1^\alpha(\epsilon_1^\beta\langle\pi^a|t^b_{\alpha\beta}|\gamma(q_2,\epsilon_2)\rangle)$$
$$=q_1^\alpha\epsilon_1^\beta\epsilon_{\alpha\beta\gamma\delta}\epsilon_2^\gamma q_2^\delta g_{\rho\pi\gamma}(q_1^2,\ldots)\,. \tag{4.25}$$

The decoupling theorem for soft ρ's, as $q_1^\alpha\to 0$, is an automatic consequence of the Lorentz structure of the matrix element and there is no condition on the invariant amplitude $g_{\rho\pi\gamma}(q_1^2,\ldots)$. Hence $g_{\rho\pi\gamma}(q_1^2,\ldots)$ is not required to have a zero at $q_1^2=0$.

The result is that we cannot make the identification (4.20), and hence we avoid its disagreement with experiment. Unfortunately we are not able to calculate independently the value of the contact amplitude $A^c_{\pi^0\gamma\gamma}$, so we cannot offer a prediction for the $\rho\to\pi\gamma$ rate.

V. CONCLUSIONS

The picture we have presented is that the ρ is in the $(3,\overline{3})\oplus(\overline{3},3)$ representation of chiral $SU(3)\otimes SU(3)$. This solves the ρ-π puzzle. In addition, it offers a picture of VMD which is unified with PCAC—they both result from spontaneous symmetry breaking.

This treatment of the vector mesons requires the chiral partner of the ρ to be the B meson rather than the A_1. The A_1 meson may or may not exist—it is not a requirement of our approach. The Weinberg sum rules when properly analyzed do not require an A_1 in the conventional way. More experimental information is needed before the first Weinberg sum rule can be adequately tested.

The remaining members of the $J^{PC}=1^{+-}$ nonet are beginning to be identified. These are the SU(3) partners of the B meson. The Q_B meson seems fairly well established, and its mass of order 1.4 GeV fits within experimental error our prediction from the mass relation $m_{K^*}^2-m_K^2=m_{Q_B}^2-m_\kappa^2$. We would like to emphasize the importance of searching for the isoscalar members of this nonet, one of which we predict to have a mass around 1.7 GeV assuming ideal mixing.

Our picture of VMD is quite successful in recovering the good results of the conventional treatment, as has been demonstrated in I and in this paper, in particular, for interactions involving real photons. In addition, our treatment does not lead to the bad result of the conventional treatment of $\rho\to\pi\gamma$ decay.

Putting all this together we believe that we have demonstrated that this picture of vector mesons is a reasonable and even a preferable one. The conventional prejudice that the ρ is in a $(1,8)\oplus(8,1)$ representation just like the vector current is only superficially simple. The ρ-π puzzle that results from this representation assignment is a fundamental complication. By putting the ρ in a $(3,\overline{3})\oplus(\overline{3},3)$ representation, not only is this complication resolved, but a simple, unified account of VMD and PCAC results.

It is difficult to imagine the construction of a future theory of the strong interactions which does not take into account these ideas about the relation of spontaneously broken chiral symmetry and VMD. The collective model of the pion can be extended to the vector mesons as well.[22] However, there is a problem. For high-mass states such as charmonium and $D^{\pm,0}$ mesons the spectroscopy is well accounted for by an atomic picture. Indeed one might say that the charm family is the "hydrogen atom" of the hadrons. How does one interpolate between this atomic picture for the high-mass systems and the collective picture for the light mesons such as the pion and the ρ? One cannot help but be reminded of a similar duality between the collective and individual-particle models of the nucleus. For the hadrons the solution of this problem is still forthcoming. It would be valuable to have some parametric control on this transition. Is there some parameter such as f_π^2/m_H^2, where m_H is a hadron mass, that describes the transition between the collective and the atomic picture? These and other questions are quite open.

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