PHYSICAL REVIEW D VOLUME 15, NUMBER 9 1 MAY 1977

$\Delta I = 1/2$ rule and right-handed currents: Heavy-quark expansion and limitation on Zweig's rule*

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(Received 13 August 1976; revised manuscript received 24 March 1977)

We reexamine and extend the analysis of the effects of right-handed currents on the $\Delta I = 1/2$ rule in nonleptonic decays. The Wilson coefficients and anomalous dimensions of new operators generated by right-handed currents are computed. We apply the recently developed techniques of heavy-quark expansion to the resulting effective weak Hamiltonian. We point out that the short-distance-enhanced operator $(\bar{\mathfrak{N}}_L \gamma_\mu c_L \bar{c}_R \gamma^\mu \lambda_R)$ is expected to have large matrix elements, contrary to the naive implication of Zweig's rule. This may well account for the $\Delta I = 1/2$ rule, if right-handed currents exist.

I. INTRODUCTION AND SUMMARY

An understanding of the $\Delta I = \frac{1}{2}$ rule in nonleptonic decay is of importance in helping us unravel both the strong and the weak interactions. In this paper, we would like to make some more remarks on the subject, especially on the effect of introducing right-handed currents[1-4] into the weak interaction. This aspect of the subject has received some attention in the recent literature.[5] We will rely particularly on Ref. 3, summarizing, amplifying, and amending some of the remarks made there.

We will restrict ourselves to the standard view[6] that the nonleptonic weak interaction is formed out of a product of the hadronic weak currents occurring in semileptonic weak interaction. (We will assume the standard framework of gauge theories for both weak and strong interactions.) Thus the effective nonleptonic interaction density is of the form

$$\mathcal{H} \sim \int d^4x\, \Delta^{\mu\nu}(x, M_W)\, T(J_\mu(x) J_\nu^\dagger(0)) , \qquad (1.1)$$

where $\Delta^{\mu\nu}$ denotes the intermediate-boson propagator. Since $\Delta^{\mu\nu}$ is largest for $x \lesssim M_W^{-1}$ the suitable framework for analyzing (1.1) is the short-distance operator-product expansion:

$$J_\mu(x) J_\nu^\dagger(0) \underset{x\approx 0}{\approx} \sum_i C_i(x) \mathcal{O}_i(0) . \qquad (1.2)$$

We suppress Lorentz indices on the operators $\mathcal{O}_i$. The operators $\mathcal{O}_i$ relevant for the nonleptonic decays of strange hyperons can then be classified according to whether they carry isospin $\frac{1}{2}$ or isospin $\frac{3}{2}$. We denote these operators and their associated Wilson coefficient functions C_i by the generic notation

$$\mathcal{O}_{1/2},\ \mathcal{O}_{3/2},\ C_{1/2},\ \text{and}\ C_{3/2} .$$

The $\Delta I = \frac{1}{2}$ rule could then arise because of one or the other, or both, of the following possibilities:

(A) The matrix elements $\langle \alpha | \mathcal{O}_{1/2} | \beta \rangle$ for a given decay process $\alpha \to \beta$ are larger than the matrix elements $\langle \alpha | \mathcal{O}_{3/2} | \beta \rangle$.

(B) The coefficient functions $C_{1/2}(x)$ are larger than $C_{3/2}(x)$ as $x \to 0$.

It is most likely that in the real world (A) and (B) are both operative. Let us examine each of these in turn.

There are a number of dynamical arguments[7] suggesting that (A) is true. Of these, the most reliable is the well-known one based on soft-pion theorems,[8] which, however, does not apply to all nonleptonic weak decays. This argument also provides an important constraint on the handedness[9] of the nonleptonic weak interaction, namely that right-handed $\mathcal{P}$ and $\mathfrak{N}$ quark fields cannot play a significant role in ordinary nonleptonic decays. The other arguments are somewhat weaker than the soft-pion argument. However, taken together they add significant plausibility to (A). Indeed, the $\Delta I = \frac{1}{2}$ rule is undoubtedly partly due to the same mechanism that renders the particle spectrum into low-lying multiplets.[10]

The possibility (B), first suggested by Wilson,[11] has been examined[12] in the gauge theory of strong interaction and found to be valid. However, the evaluated enhancement of $C_{1/2}(x)$ over $C_{3/2}(x)$ appears to be somewhat too small to account for the observed accuracy of the $\Delta I = \frac{1}{2}$ rule.

In summary, we deem it fair to say that while the $\Delta I = \frac{1}{2}$ rule is far from completely understood, it is not a total mystery either.

How does the possible existence of right-handed hadronic currents affect the situation? This question was first raised in Ref. 1 (see Ref. 13) and was subsequently discussed by the authors of Ref. 3 and by others.[5] It is the purpose of this paper to add to this discussion.

Right-handed currents can affect the $\Delta I = \frac{1}{2}$ rule in two ways:

(α) They can generate new operators, of the type $\mathcal{O}_{1/2}$ (and in general also of the type $\mathcal{O}_{3/2}$).

(β) They can modify substantially the behavior of the Wilson coefficient $C(x)$.

Let us take up both of these possibilities in turn, but first, some preliminary background remarks are necessary. Right-handed currents (which are relevant to $\Delta S = 1$ nonleptonic decays) proposed in the literature may be broadly divided into two classes, those in which $\mathfrak{N}_R$ is coupled[1] to a heavy quark (typically the charmed quark) and those in which λ_R is coupled.[2,3] We will not enter into the dispute of which (if either) will prove to be ultimately correct. It will be seen that the following discussion will apply to both types of theories. (Differing types of theories differ in various overall proportionality constants which will not particularly concern us here.) Right-handed currents involving $\mathcal{P}_R$ and a heavy (negatively) charged quark will not contribute to $\Delta S \neq 0$ decays and will not be relevant to our discussion. It should be straightforward to extend our discussion to cover $\Delta S = 0$ nuclear processes and charm-changing decays. We may perhaps also stress that our discussion, since it is not specific to any given model, is *a fortiori* not committed to the so-called vector theories.

Having made these remarks, we will now consider (α) and (β).

(α) $\Delta S = 1$ operators coupling $\mathfrak{N}_L$ to λ_R (or equivalently $\mathfrak{N}_R$ to λ_L) can now participate in nonleptonic interactions. However, one must take care to note that they are in fact operative. For example, the operator $\overline{\mathfrak{N}}_L\lambda_R$ may be absorbed in the mass term.[14] Other operators may be reduced by the equation of motion to operators of lower dimension. A list of these operators up to canonical dimension 6 was given in Ref. 3 and will not be repeated here. In this paper we would particularly like to focus much of our attention on the canonical-dimension-5 operators $\mathcal{O}_1 \equiv \overline{\mathfrak{N}}_L D_\mu D^\mu \lambda_R$ and $\mathcal{O}_2 \equiv \overline{\mathfrak{N}}_L \sigma_{\mu\nu} F^{\mu\nu}\lambda_R$. (Here D_μ denotes the covariant derivative and $F^{\mu\nu}$ denotes the gluon field strength.) Another operator that requires attention is the canonical-dimension-6 four-quark operator $\mathcal{O}_c \equiv \overline{\mathfrak{N}}_L \gamma_\mu c_L \overline{c}_R \gamma^\mu \lambda_R$.

(β) The anomalous dimension of the operator $\mathcal{O}_c$ happens to be twice as large[15] as the corresponding four-quark operators of "pure-handedness" $\overline{\mathfrak{N}}_L\gamma_\mu c_L \overline{c}_L \gamma^\mu \lambda_L$ and $\overline{\mathfrak{N}}_R\gamma_\mu c_R \overline{c}_R \gamma^\mu \lambda_R$. Recalling that the anomalous dimension appears effectively in the exponent of a $(\ln M_W)$ factor, one sees that this "arithmetical" fact makes a substantial difference.[3] To the extent that theories involving right-handed currents often involve more quarks, this also has an effect on $C(x)$ as follows: The addition of more quarks (up to a point) increases the value of the β function of the renormalization-group equation and thus enhances the exponent of the $(\ln M_W)$ factor. See Ref. 3 for a discussion on this point.

Before presenting the reader with our discussion we will for the sake of clarity summarize and highlight the chief assertions of this paper.

(1) In Ref. 3 it was asserted that the operator $\mathcal{O}_2$ does not appear in the operator-product expansion of the currents. It was then claimed, erroneously, that the operator $\mathcal{O}_1$ can then be neglected. In fact, $\mathcal{O}_1$ does appear. This was pointed out recently by R. K. Ellis.[16] The numerical values of these anomalous dimensions will be given below. It turns out that they are such as to suppress the appearance of these operators in the operator-product expansion.

(2) While there is no reliable way in general of evaluating the matrix element of a given operator, it was thought[3,17] that the operator $\mathcal{O}_c \equiv \overline{\mathfrak{N}}_L\gamma_\mu c_L \overline{c}_R \gamma^\mu \lambda_R$ would have a rather small matrix element between ordinary noncharmed hadrons on the basis that it might be valid to apply some version of the so-called Zwig rule[18] to this case. Through the recent work of Witten,[19] however, it has become clear that there are fundamental limitations to the validity of this rule, and that blind and indiscriminate invocation of this rule could in some cases be totally misleading. It may be proved[19] that, in the limit in which the charmed-quark mass is much larger than the relevant strong-interaction mass scale, the matrix elements of $\mathcal{O}_c$ are equal to the matrix elements of $\mathcal{O}_1$ and $\mathcal{O}_2$ times a universal factor calculable in terms of the effective strong-interaction coupling constant $\overline{g}$ and proportional to the charmed-quark mass. The charmed-quark mass enters because charmed quark fields of opposite handedness enter into $\mathcal{O}_c$. This situation is to be contrasted with the apparently successful application of Zweig's rule to the decay of $J/\psi(3100)$.

Note that the proportionality factor relating the dimension-6 operator $\mathcal{O}_c$ to the dimension-5 operators $\mathcal{O}_1$ and $\mathcal{O}_2$ is not set by the ordinary hadronic mass scale. To be sure, in the real world, the relevant mass of the charmed quark may well not be large enough for this theorem to be of quantitative relevance. It is important to note, however, that the remark made here applies to any quark x which couples to both $\mathcal{N}_R$ and λ_L generating the operator $\mathcal{O}_x \equiv \overline{\mathcal{N}}_R \gamma_\mu x_R \bar{x}_L \gamma^\mu \lambda_L$. If there exist several such heavy quarks in the world their cumulative effect could be quite substantial despite the fact that these operators $\mathcal{O}_x$ are associated with small "Cabibbo-type" angles.

These two effects are both favorable to the enhancement of $\Delta I = \frac{1}{2}$ amplitudes. We are thus encouraged to think that the $\Delta I = \frac{1}{2}$ rule is quite understandable within the present framework of gauge theories for weak and strong interactions, if the appropriate right-handed currents exist. These remarks will be explained and elaborated on in Secs. II and III. Technical details will be relegated to the Appendixes.

II. DIMENSION-FIVE OPERATORS FROM OPERATOR-PRODUCT EXPANSION

There are two operators of canonical dimension five and transforming like $I = \frac{1}{2}$ which may appear in the product expansion of the currents $J_\mu = \overline{\mathcal{N}}_R \gamma^\mu c_R$ and $J'_\mu = \bar{c}_L \gamma^\mu \lambda_L$. These operators are

$$\mathcal{O}_1 \equiv \overline{\mathcal{N}}_R D^\mu D_\mu \lambda_L \,, \tag{2.1}$$

$$\mathcal{O}_2 \equiv g\, \overline{\mathcal{N}}_R \sigma_{\mu\nu} F_a^{\mu\nu} \tfrac{1}{2}\lambda^a \lambda_L \,. \tag{2.2}$$

These two operators are related to another operator

$$\mathcal{O}_3 \equiv \overline{\mathcal{N}}_R \not{D}\not{D} \lambda_L \tag{2.3}$$

by the identity

$$\mathcal{O}_3 = \mathcal{O}_1 - \tfrac{1}{2}\mathcal{O}_2 \,. \tag{2.4}$$

It must be emphasized that this relation is purely arithmetical and follows from the identity $\gamma_\mu \gamma_\nu = g_{\mu\nu} - \frac{1}{2} i \sigma_{\mu\nu}$. However, using the equation of motion, one finds that the operator $\mathcal{O}_3$ is actually equal to $m_\lambda^2 \overline{\mathcal{N}}_R \lambda_L$, an operator of lower (scaling) dimension. (One should keep in mind, however, that the equation of motion applies only in matrix elements between on-shell states.)

To determine whether these two operators in fact appear we calculate the (off-shell) matrix element of the effective weak interaction $\int d^4x\, \Delta(x)\, T(J^\mu(x) J'_\mu(0))$ [$\Delta(x)$ = intermediate-boson propagator] between two fermions, between two fermions and a gluon, and between two fermions and two gluons. The relevant graphs are displayed in Fig. 1.

By chirality, these graphs are all proportional to m_c. We wish to compare the value of these graphs in the limit of large M_W with the matrix elements of $\mathcal{O}_1$ and $\mathcal{O}_2$ between two fermions, between two fermions and a gluon, and between two fermions and two gluons. (These matrix elements are given in Fig. 2.)

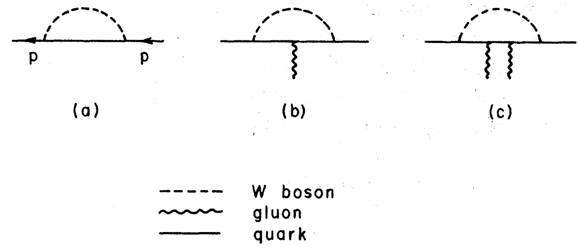


FIG. 1. Calculation of the Wilson coefficients of the operators $\mathcal{O}_1$ and $\mathcal{O}_2$ (see Sec. II).

To begin with let us extract the coefficient of p^2 in the graph of Fig. 1(a) by differentiating the graph with respect to p^2. A simple calculation then yields in the limit of large M^2

$$\frac{\partial}{\partial p^2}[\text{Fig. 1(a)}] \to \frac{1}{M_W^2}(-2m_c)\left[-\frac{i}{16\pi^2}\left(\frac{1+\gamma_5}{2}\right)\right] . \tag{2.5}$$

This result informs us that the operator $\mathcal{O}_1$ does appear in the operator-product expansion of J_μ with J'_μ. To determine whether or not $\mathcal{O}_2$ also appears we have to calculate the graph in Fig. 1(b). We find

$$[\text{Fig. 1(b)}] \to \frac{1}{M_W^2}(2m_c)\left(-\frac{g\lambda^a}{2}\right)(2p+q)_\mu \times \left[-\frac{i}{16\pi^2}\left(\frac{1+\gamma_5}{2}\right)\right] . \tag{2.6}$$

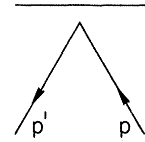
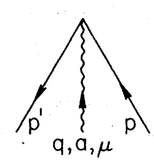
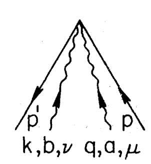
	$\mathcal{O}_1$	$\mathcal{O}_2$
	$-p^2$	0
	$-g\frac{\lambda^a}{2}(2p+q)_\mu$	$2ig\frac{\lambda^a}{2}\sigma_{\mu\nu}q^\nu$
	$-g^2 g_{\mu\nu}\left\{\frac{\lambda^a}{2},\frac{\lambda^b}{2}\right\}$	$2g^2\frac{\lambda^a}{2}f^{abc}\sigma_{\mu\nu}$

FIG. 2. The off-shell matrix element of $\mathcal{O}_1$ and $\mathcal{O}_2$ between two fermions, two fermions and a gluon, and two fermions and two gluons, respectively.

We thus conclude that $\mathcal{O}_2$ does not appear in the operator-product expansion.

It is not necessary to calculate Fig. 1(c) except as a check on our calculations of Figs. 1(a) and 1(b). Indeed, some arithmetic leads to

$$[\text{Fig. } 1(\text{c})] \sim \frac{1}{M_W{}^2}(2m_c)(-g^2 g_{\mu\nu})\left\{\frac{\lambda^b}{2},\frac{\lambda^a}{2}\right\} \times\left[\frac{-i}{16\pi^2}\left(\frac{1+\gamma_5}{2}\right)\right], \tag{2.7}$$

which is a result consistent with gauge invariance as it should be.

In conclusion, we have found that

$$J'_\mu(x)J^\mu(0) \sim c(x)\mathcal{O}_1 + \text{other terms}. \tag{2.8}$$

In Ref. 3 it was asserted correctly that $\mathcal{O}_2$ does not appear in the operator-product expansion. However, it would be wrong to conclude that $\mathcal{O}_1$ also does not appear.

We next turn to the evaluation of the anomalous dimensions of these operators. The two operators $\mathcal{O}_1$ and $\mathcal{O}_2$ mix under renormalization. Using the subscripts R and 0 to denote renormalized and bare quantities respectively we can write

$$\vec{\mathcal{O}}_R = \underline{Z}^{-1}\vec{\mathcal{O}}_0. \tag{2.9}$$

Here $\vec{\mathcal{O}}$ is a two-dimensional vector which equals $(\mathcal{O}_1, \mathcal{O}_2)$, and the cutoff-dependent renormalization factor $\underline{Z}$ is a two-by-two matrix. The multiplicatively renormalizable operators are then

$$\vec{e}\cdot\vec{\mathcal{O}}_R = \vec{e}\underline{Z}^{-1}\vec{\mathcal{O}}_0 = z^{-1}\vec{e}\cdot\vec{\mathcal{O}}_0, \tag{2.10}$$

where $\vec{e}$ are the two left eigenvectors of $\underline{Z}^{-1}$:

$$\vec{e}\underline{Z}^{-1} = z^{-1}\vec{e}. \tag{2.11}$$

The operators $\vec{e}\cdot\vec{\mathcal{O}}_R$ have anomalous dimension

$$\gamma = -\Lambda\frac{\partial}{\partial\Lambda}\ln z, \tag{2.12}$$

where Λ is the ultraviolet cutoff. The determination of $\underline{Z}$ involves the evaluation of all the graphs listed in Fig. 3. We may perhaps remark that this is the most tedious anomalous-dimension calculation which either of the two authors has ever encountered. The details are relegated to the Appendix and the values of the graphs are listed in Table I. Suffice it here to state that the result is

$$\underline{Z}^{-1} = 1 - \begin{pmatrix} -\frac{37}{24} & +\frac{7}{16} \\ -\frac{23}{4} & \frac{53}{24} \end{pmatrix} \frac{g^2}{16\pi^2}\ln\frac{\Lambda^2}{\mu^2}. \tag{2.13}$$

(We should note that the relation between $\mathcal{O}_1$, $\mathcal{O}_2$, and $\mathcal{O}_3$ [Eq. (2.4)] is manifested here in the fact that $(Z_{11}-\frac{1}{2}Z_{21}) = -2(Z_{12}-\frac{1}{2}Z_{22})$.)

The two left eigenvectors of $\underline{Z}^{-1}$ are

$$\vec{e}^{(1)} = (1, -\tfrac{1}{2}) \tag{2.14}$$

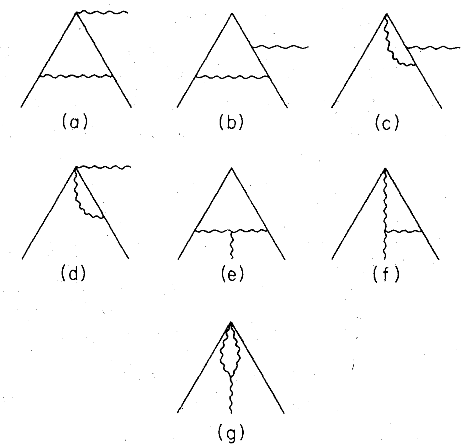


FIG. 3. The relevant graphs for computing the anomalous dimensions of $\mathcal{O}_1$ and $\mathcal{O}_2$. (It is understood that graphs obtained by crossing these shown are not displayed separately.)

and

$$\vec{e}^{(2)} = (1, -\tfrac{7}{46}). \tag{2.15}$$

The corresponding operators $\mathcal{O}^{(i)} = \vec{e}^{(i)}\cdot\vec{\mathcal{O}}$ have anomalous dimension

$$\gamma^{(1)} = -\frac{8}{3}\left(\frac{g^2}{16\pi^2}\right), \tag{2.16}$$

$$\gamma^{(2)} = +\frac{4}{3}\left(\frac{g^2}{16\pi^2}\right). \tag{2.17}$$

It is not surprising that the operator $\mathcal{O}^{(1)}$ is merely the operator $\mathcal{O}_3 = \bar{\mathfrak{N}}_R \not{D}\not{D}\lambda_L$ in another notation. In a regime where light-quark masses can be set equal to zero, this operator vanishes by virtue

TABLE I. The matrix elements of $\mathcal{O}_1$ and $\mathcal{O}_2$ corresponding to each group of graphs in Fig. 3 are given in terms of the two invariants I_1 and I_2 defined in the Appendix. The entries are to be multiplied by $g((g^2/16\pi^2)\ln\Lambda^2)$. Thus, for example, the graph in Fig. 3(a) with $\mathcal{O}_1$ inserted at the apex has the value $-\frac{1}{3}[(g^2/16\pi^2)\ln\Lambda^2](-g\frac{1}{2}\lambda^a)(2p+q)_\mu$.

Figure	Matrix elements of $\mathcal{O}_1$	$\mathcal{O}_2$
3(a)	$-\frac{1}{3}I_1$	0
3(b)	$+\frac{1}{3}I_1$	0
3(c)	$+\frac{1}{6}I_2$	$I_1+\frac{1}{6}I_2$
3(d)	$-\frac{7}{12}(I_1+\frac{1}{2}I_2)$	$+\frac{3}{2}(-3I_1+\frac{1}{2}I_2)$
3(e)	$+\frac{9}{4}I_2$	0
3(f)	$\frac{27}{8}(I_1-\frac{1}{2}I_2)$	$\frac{3}{4}(-3I_1+\frac{27}{2}I_2)$
3(g)	0	$-\frac{9}{2}I_2$

of the equation of motion in the following sense: The matrix element of $\mathcal{O}_3$ vanishes between on-shell states of quarks and gluons. If gauge theory indeed confines quarks and gluons the matrix elements of $\mathcal{O}_3$ between hadronic states will also vanish. Alternatively, if one prefers, one may say that the matrix elements of $\mathcal{O}_3$ between hadronic states is related, by virtue of the equation of motion, to the matrix elements of the mass operator $\overline{\mathfrak{N}}_R\lambda_L$ and so can be absorbed.

Putting together the usual renormalization-group machinery we can now conclude that the effects of strong interaction is to associate with the operator $\mathcal{O}_1$ appearing in the operator-product expansion the suppression factor

$$(\ln M_W^2/\mu^2)^{-2/(33-2n)}. \tag{2.18}$$

Here n is the number of flavors. On top of this, we have an additional suppression factor coming from the explicit mass factor m_c which appears associated with $\mathcal{O}_2$ in the operator-product expansion. This suppression factor is

$$(\ln M_W^2/\mu^2)^{-12/25}. \tag{2.19}$$

We thus conclude that the appearance of this operator $\mathcal{O}_1$ with the introduction of a right-handed current does not convincingly account for the $\Delta I = \frac{1}{2}$ rule.

III. DIMENSION-FIVE OPERATORS FROM HEAVY-QUARK EXPANSION

As was mentioned in the Introduction, the incorporation of a right-handed current leads to the appearance of the four-quark operator

$$\mathcal{O}_c \equiv \overline{\mathfrak{N}}_R \gamma_\mu c_R \bar{c}_L \gamma^\mu \lambda_L, \tag{3.1}$$

which obviously transforms with isospin $\frac{1}{2}$. Furthermore, the anomalous dimension of this operator is twice as large as the corresponding four-quark operators of pure handedness. Since the anomalous dimension appears in the exponent of $(\ln M_W)$, the contribution of this operator is substantially enhanced. This observation might have offered a promising expansion of the $\Delta I = \frac{1}{2}$ rule were it not for the suspicion that the matrix elements of $\mathcal{O}_c$ between "ordinary" noncharmed hadrons are likely to be small because of some sort of Zweig rule.

We will argue in this section that this suspicion is perhaps not well founded; in any case, one should be cautions in applying the so-called Zweig rule too freely.

The actual evaluation of strong-interaction matrix elements is, of course, beyond the capabilities of high-energy theory at present. However, it turns out that it is, in fact, possible to say something fairly rigorously about matrix elements of operators involving charmed quarks between ordinary hadrons in the limit in which m_c, the "mass" of the charmed quark, goes to infinity. (We have already taken the limit of large M_W; thus what is meant here is the situation in which $M_W \gg m_c \gg$ typical noncharmed-hadronic mass scale.) The statements that can be made have been discussed recently in detail by Witten and applied by him to other processes in weak interactions and in deep-inelastic production.[19] These statements will be referred to as the heavy-quark expansion. In the present context, it is true that in the limit of large m_c

$$\langle\beta|\mathcal{O}_c|\alpha\rangle \approx F(m_c)\langle\beta|\mathcal{O}_2|\alpha\rangle. \tag{3.2}$$

Here α and β are any two states consisting of ordinary hadrons. $F(m_c)$ is a function of m_c and is universal in the sense of not depending on α and β. The heavy-quark expansion clearly bears a close resemblance to Wilson's operator-product expansion. Indeed, the proof given by Witten[19] follows closely one proof of the operator-product expansion. In particular, one applies much the same reasoning used in Sec. II, namely that for a suitably convergent integral one may extract the $1/M_W^2$ dependence of a lowest-order weak amplitude.

For instance, consider the off-shell element of $\mathcal{O}_c$ between two fermions and a gluon. A contributing graph is displayed in Fig. 4(a). Chirality im-

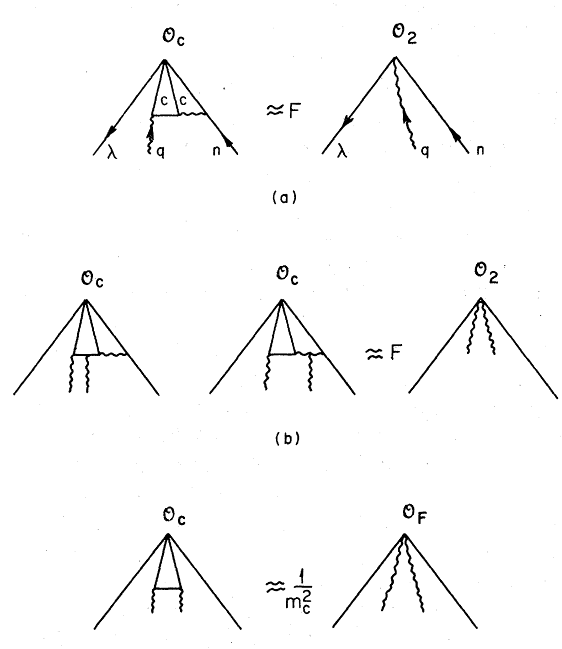


FIG. 4. Illustrative examples of the heavy-quark expansion (see Sec. III).

plies that the graph is proportional to m_c. Noting that the matrix element of $\mathcal{O}_2$ is proportional to the external momentum q we differentiate with respect to q, thus rendering the graph sufficiently convergent to conclude that

$$F(m_c) \sim \text{const} \times m_c(\ln m_c)^p \frac{\bar{g}^2(m_c)}{4\pi}\,. \tag{3.3}$$

The foregoing is certainly not meant to be anything more than suggestive. There are a number of technical questions one must answer. Firstly, the powers of $(\ln m_c)$ indicated in Eq. (3.3) can, in fact, be controlled by using standard renormalization-group arguments. Secondly, the quantity m_c must be eliminated in favor of a more physical measure of the charmed-quark mass, such as some specified fraction of the ψ/J mass or better, of the location of the new structure observed in the electron-positron annihilation hadronic cross section. It turns out that with reasonable assumptions, these points can be dealt with satisfactorily. It will be beyond the scope and province of this paper to review these points which Witten discussed in detail. The reader is referred to Ref. 19 for further explanation.

We should perhaps remark that the present situation is to be sharply contrasted with the applications of Zweig's rule to deep-inelastic production. In these applications the relevant operators are such that charmed-quark fields of the same handedness enter and their matrix elements between ordinary noncharmed-hadronic states are then suppressed by factors of $\bar{g}(m_c)$.

We could also determine F by considering the matrix element of $\mathcal{O}_c$ between two fermions and two gluons. The relevant graphs are given in Fig. 4(b).

As another example, the graph in Fig. 4(c) leads to the dimension-7 operator $\mathcal{O}_F \equiv \mathfrak{N}_R \lambda_L F_{\mu\nu} F^{\mu\nu}$ and can be disregarded.

We presume that the operator $\mathcal{O}_1$ does not appear because all relevant graphs either vanish or are of higher order in $\bar{g}^2(m_c)$. For instance, the graph in Fig. 5(a) vanishes. The reason is that by a Fierz transformation $\mathcal{O}_c$ may be rewritten as $\mathfrak{N}_R \lambda_L \bar{c}_L c_R$ and the graph in Fig. 5(e) vanishes by gauge invariance. As another example, the graph in Fig. 5(c) is of higher order in $\bar{g}^2(m_c)$.

We have not undertaken the task of determining $F(m_c)$ more precisely in view of the fact that the matrix element of $\mathcal{O}_2$ is not reliably calculable. Indeed $F(m_c)$ depends on the precise subtraction prescription used to define the various operators $\mathcal{O}_c$ and $\mathcal{O}_2$. We do not insist on the technical details here. Rather, we would like to suggest that the moral of our discussion is that one should not apply Zweig's rule indiscriminately. The matrix element of $\mathcal{O}_c$ between ordinary hadrons may be a good deal larger than many people had suspected. If our suggestion here is correct and if right-handed currents of the appropriate type exist, then a substantial portion of the $\Delta I=\frac{1}{2}$ rule may be explained.

IV. $\Delta I=\frac{1}{2}$ RULE EXPLAINED BY RIGHT-HANDED CURRENT

In conclusion, let us summarize the main points of our discussions.

(1) In the presence of right-handed currents new operators $\mathcal{O}_1$, $\mathcal{O}_2$, and $\mathcal{O}_c$, all transforming as $\Delta I=\frac{1}{2}$, appear in the operator-product expansion of currents.

(2) The operators $\mathcal{O}_1$ and $\mathcal{O}_2$ are found to be suppressed for an asymptotically free theory of strong interaction.

(3) In contrast, the dimension-6 operator $\mathcal{O}_c$ is highly enhanced.

(4) In defiance of a naive interpretation of Zweig's rule the operator $\mathcal{O}_c$ may well have *large* matrix elements between ordinary hadrons.

(5) More speculatively, we note that if there are other heavy quarks participating in right-handed currents of the appropriate type the sort of effect mentioned under (4) may accumulate.

(6) If point (4), and more uncertainly point (5), are correct, then the $\Delta I=\frac{1}{2}$ rule becomes less mysterious than ever before.

The important point here is that the charmed quark provides a mass scale larger than ordinary hadronic mass scale.

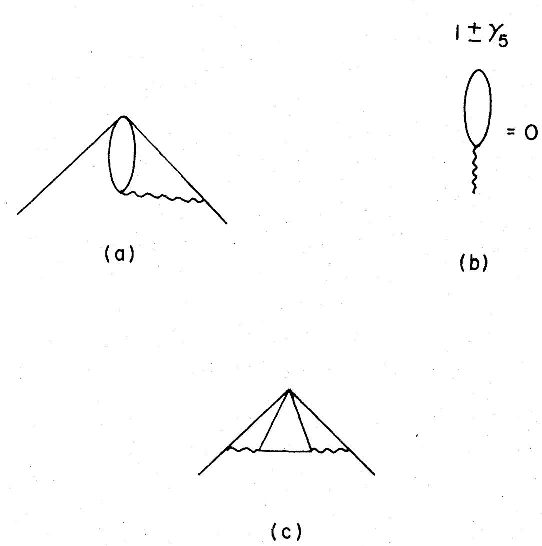


FIG. 5. Graphs illustrating the nonappearance of the operator $\mathcal{O}_1$ in the heavy-quark expansion of $\mathcal{O}_c$ (see Sec. III for explanations).

In closing, we should remind the reader that there is at present no clear-cut experimental[1-3] evidence for right-handed currents. Our entire discussion is clearly predicated on the assumption that these currents do exist.

Note added. There is another contribution to $|\Delta S|=1$ decays in the models considered here which, although certainly negligible in the mathematical limit $M_W \to \infty$, may be comparable to the contribution discussed in Sec. IV for physical values of M_W. This comes about in the following way. Our operator $\mathcal{O}_c$ mixes with $\mathcal{O}_2$ with an anomalous-dimension matrix[20] of the form

$$\vec{\gamma}(\bar{g}^2)=\begin{pmatrix} a\bar{g}^2 & c\bar{m}\bar{g}^2 \\ 0 & b\bar{g}^2 \end{pmatrix}.$$

Notice that under a change of normalization point the dimension-6 operator $\mathcal{O}_c$ transforms into a combination of itself and $\bar{m}$ (the effective mass) times the dimension-5 operator $\mathcal{O}_2$. This mixing does not change the anomalous dimensions we have computed, since the eigenvalues of a triangular matrix are just the diagonal entries. It does, however, affect which operators are multiplicatively renormalized. In particular, not simply $\mathcal{O}_c$ but some linear combination of $\mathcal{O}_c$ and $\mathcal{O}_2$ is the operator which receives the large enhancement factor. Thus there is an enhanced piece of the effective Hamiltonian directly from $\mathcal{O}_2$ as well as the indirect piece we discussed in Sec. IV. Now $\bar{m}$ approaches zero at large momenta, but only slowly, so that although the mixing vanishes asymptotically (as a fractional power of $\ln M_W^2$), it may not be negligible for the physical value of M_W. We have made a crude estimate which indicates that this contribution is probably smaller than the one we emphasized. In any case, the qualitative conclusion that a $\bar{c}_R\gamma_\mu\lambda_R$ weak current could help explain the $\Delta I=\frac{1}{2}$ rule remains unchanged.

After this work was completed we learned of a related work by M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Zh. Eksp. Teor. Fiz. Pis'ma Red. 23, 656 (1976) [JETP Lett. 23, 602 (1976)]. The emphasis and the viewpoint here are somewhat different.

ACKNOWLEDGMENT

It is a pleasure to thank S. Joglekar, N. Vasanti, and E. Witten for instructive and enlightening comments. We are especially indebted to S. Treiman for many stimulating discussions.

We are grateful to R. K. Ellis and M. Shifman, A. Vainshtein, and V. I. Zakharov for correspondence and for detecting an error in an earlier version of our calculation. One of us (A.Z.) thanks M. Shifman, A. Vainshtein, and V. Zakharov for interesting discussions.

APPENDIX A

To calculate the anomalous dimensions of $\mathcal{O}_1$ and $\mathcal{O}_2$ we must evaluate their off-shell matrix elements to the one-loop level. There are some technical problems concerning the mixing of these gauge-invariant operators with gauge-variant operators. We refer the reader to the literature[21] for a discussion of these problems. Suffice it to say that if we evaluate the matrix elements between two fermions and one gluon these problems can be ignored.[22] The relevant graphs are given in Fig. 3. We use the schematic notation $\langle \mathcal{O}_i \psi A \bar{\psi}\rangle$ for the one-particle-irreducible matrix elements of $\mathcal{O}_i$ between two fermions and a gluon and the subscripts R and 0 to denote renormalized and bare quantities, respectively. The renormalized operators $\mathcal{O}_R$ are given by

$$\mathcal{O}_R = Z^{-1}\mathcal{O}_0,$$

where the cutoff-dependent renormalization factor Z is a two-by-two matrix. The anomalous-dimension matrix γ is given by $\gamma=-Z^{-1}\Lambda(\partial/\partial\Lambda)Z$. The matrix elements $\langle \mathcal{O}_i \psi A\bar{\psi}\rangle_0$ may be written in terms of two invariants of the form $I_1=(-\frac{1}{2}\lambda^a)(2p+q)_\mu$ and $I_2=2i\frac{1}{2}\lambda^a\sigma_{\mu\nu}q^\nu$ (see Fig. 2), thus

$$\langle \mathcal{O}_i\psi A\bar{\psi}\rangle_0 = g_0\sum_j v_{ij}I_j \cdots . \tag{A1}$$

Notice the appearance of the bare coupling constant g_0. The two-by-two matrix v is to be evaluated by calculating all the graphs in Fig. 3. To determine Z one writes down the relation between renormalized and bare matrix elements:

$$\begin{aligned}\langle \mathcal{O}_i\psi A\bar{\psi}\rangle_R &= \sum_j S_{ij}{}^{-1}A_2 Z_3{}^{1/2}\langle \mathcal{O}_j\psi A\bar{\psi}\rangle_0 \\ &= \sum_{jk} Z_{ij}{}^{-1}Z_1 g_R v_{jk}. \end{aligned} \tag{A2}$$

Here Z_2 and Z_3 are the usual wave-function renormalization factor for the fermion field ψ and the gluon field A. The renormalized coupling g_R is given by $g_R=Z_2Z_1{}^{-1}Z_3{}^{1/2}g_0$. The value of each of the graphs of Fig. 3 is tabulated in Table I. From this table we obtain the matrix Z quoted in the text by requiring that $\langle \mathcal{O}_i\psi A\bar{\psi}\rangle_R$ be cutoff-independent.

*Research supported in part by the National Science Foundation under Grant No. MPS75-22514 and ERDA under Contract No. E(11-1)3072.

† Work supported in part by the A. P. Sloan Foundation.

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