PHYSICAL REVIEW D VOLUME 15, NUMBER 9 1 MAY 1977

Scaling laws for inclusive production of hadrons in high-energy particle-nucleus collisions*

Y. Afek,† G. Berlad, and G. Eilam
Department of Physics, Technion-Israel Institute of Technology, Haifa, Israel

A. Dar‡
Laboratoire de Physique Théorique et Particules Élementaires, Université de Paris-Sud, 91405 Orsay, France
(Received 18 February 1976)

Simple scaling laws for inclusive production of hadrons in high-energy particle-nucleus collisions are derived from a model that has reproduced multiplicity distributions in high-energy particle-nucleus reactions. The success of the model, applied here to large-transverse-momentum reactions, suggests the possible use of nuclear targets and nuclear beams to investigate future energy domains of particle physics with present accelerators.

INTRODUCTION

A recent experiment at Fermilab[1] on production of hadrons at large transverse momentum p_T, with 200-, 300-, and 400-GeV protons incident on nuclear targets, showed a strong dependence of the invariant cross section for the inclusive production of $\pi^\pm, K^\pm, \bar{p}, p$ on the atomic number A of the target nucleus. While for low p_T the dependence is close to $A^{0.7}$, at high p_T the power rises, reaching numbers larger than 1.[2]

In this paper we show that a simple model[3] that was shown to successfully reproduce the average multiplicity[4] and the Koba-Nielsen-Olesen[5] scaling function in high-energy particle-nucleus collisions,[6] leads to sum rules relating inclusive production in particle-nucleus collisions to inclusive production in particle-particle collisions. These sum rules lead to an approximate scaling law which is compared with experimental data.[1] Good agreement between theory and experiment is obtained.

THE MODEL

Let us briefly summarize the model for high-energy particle-nucleus interactions that has been presented in Ref. 3. The interaction of a high-energy incident particle with the nucleus is assumed to result from its simultaneous collision with the array of nucleons that lie within a cylinder of cross section σ (the inelastic particle-nucleon cross section) along its path.

The center-of-mass energy squared in the particle-array collision is then given by $s_i \simeq 2imp_{\text{lab}}$ (neglecting nuclear binding), where p_{lab} is the laboratory momentum of the projectile, m is the nucleon mass, and i is the number of nucleons in the array. Following the observation that various quantities that characterize multiparticle production in high-energy particle-particle collisions are independent of the quantum numbers of the colliding particles, it is assumed[3] that in the center-of-mass systems the particle-array collision resembles a particle-nucleon collision at the same center-of-mass energy. Consequently particle-nucleus collisions can be predicted from particle-particle interactions, as demonstrated in Ref. 3. Here we derive additional expressions for inclusive cross sections.

DERIVATION OF RESULTS

Let $P(i,A;b)$ denote the probability that the projectile incident on a nucleus at impact parameter b will encounter i nucleons, and let $P(i,A)$ denote the probability that the projectile at any impact parameter will encounter exactly i nucleons. Then[7]

$$P(i,A;b) = \binom{A}{i}\left(\frac{\sigma T}{A}\right)^i \left(1 - \frac{\sigma T}{A}\right)^{A-i} \tag{1}$$

and

$$P(i,A) = \frac{\int d^2b\, P(i,A;b)}{\sigma_{\text{in}}^{pA}}\,, \tag{2}$$

where the inelastic projectile-nucleus cross section is

$$\sigma_{\text{in}}^{pA} = \int d^2b \left[1 - \left(1 - \frac{\sigma T}{A}\right)^A\right]. \tag{3}$$

The thickness $T(b)$ is given by

$$T(b) = \int_{-\infty}^{\infty} dz\, \rho(b,z)\,, \tag{4}$$

where ρ is the nuclear density $[\int \rho(r)\, d^3r = A]$.

According to our model the inclusive cross section for the reaction $p + i \to c +$ anything, where (without loss of generality) the projectile is called a proton, and i is the number of nucleons in any array, is given by

$$E\frac{d^3\sigma^{pi}}{dp^3}(s,E+p_\parallel,p_T)=E\frac{d^3\sigma^{pp}}{dp^3}(is,i^{1/2}(E+p_\parallel),p_T).\tag{5}$$

E, $p_\parallel$, and p_T are the energy, longitudinal, and transverse momentum, respectively, of the measured particle c; $s\simeq 2mp_{\text{lab}}$, and pp denotes proton-nucleon collision. For proton-nucleus collision $p+A\to c+\text{anything}$ we average Eq. (5) over the probabilities to find i nucleons at impact parameter b in a cylinder of cross section σ along the beam direction, and integrate the contributions from all cylinders along different impact parameters, i.e.,

$$E\frac{d^3\sigma^{pA}}{dp^3}(s,E+p_\parallel,p_T)=\int\frac{d^2b}{\sigma_{\text{in}}^{pp}}\sum_{i=1}^{A}P(i,A;b)\times E\frac{d^3\sigma^{pp}}{dp^3}(is,i^{1/2}(E+p_\parallel),p_T),\tag{6}$$

where pA denotes proton-nucleus collision. Using Eq. (2) we obtain the sum rule

$$E\frac{d^3\sigma^{pA}}{dp^3}(s,E+p_\parallel,p_T)=\frac{\sigma_{\text{in}}^{pA}}{\sigma_{\text{in}}^{pp}}\sum_{i=1}^{A}P(i,A)\,E\frac{d^3\sigma^{pp}}{dp^3}(is,i^{1/2}(E+p_\parallel),p_T).\tag{7}$$

For $p_T^2+m_c^2\ll p_\parallel^2$, where m_c is the mass of particle c, Eq. (7) reduces to

$$E\frac{d^3\sigma^{pA}}{dp^3}(s,x_\parallel,p_T)=\frac{\sigma_{\text{in}}^{pA}}{\sigma_{\text{in}}^{pp}}\sum_{i=1}^{A}P(i,A)E\frac{d^3\sigma^{pp}}{dp^3}(is,x_\parallel,p_T),\tag{8}$$

where $x_\parallel=2p_\parallel/\sqrt{s}$. Since $E(d^3\sigma^{pp}/dp^3)(s,x_\parallel,p_T)$, for large s and small p_T, is a function of $x_\parallel$ and p_T only, then

$$\frac{Ed^3\sigma^{pA}/dp^3}{Ed^3\sigma^{pp}/dp^3}\xrightarrow[p_T\text{ small}]{s\text{ large}}\frac{\sigma_{\text{in}}^{pA}}{\sigma_{\text{in}}^{pp}},\tag{9}$$

as was observed in the Fermilab experiment.[1] Application of Eq. (7) to rapidity distributions will be discussed elsewhere.

For $p_T\gtrsim 1$ GeV the data indicates that $Ed^3\sigma^{pp}/dp^3$ is independent of $p_\parallel$.[8] Using this observation and Eq. (7) we obtain

$$E\frac{d^3\sigma^{pA}}{dp^3}(s,p_T)=\frac{\sigma_{\text{in}}^{pA}}{\sigma_{\text{in}}^{pp}}\sum_{i=1}^{A}P(i,A)E\frac{d^3\sigma^{pp}}{dp^3}(is,p_T).\tag{10}$$

APPROXIMATE SCALING RULES

Comparison of Eq. (10) with experiment requires both knowledge of low-energy nuclear properties to calculate $P(i,A)$, and data for inclusive pp cross sections at momenta up to Ap_{lab}, which in the cases considered here lie far beyond the energy range of present accelerators. In order to avoid an *ad hoc* parametrization of $Ed^3\sigma^{pp}/dp^3$ at energies where no data are available, we approximate Eq. (10) in the following way: We have found that for $A\gtrsim 10$ averaging over $P(i,A)$ can be approximated by an average array of $A^{1/3}$ nucleons. Equation (10) reduces then to the simple scaling law

$$\frac{\sigma_{\text{in}}^{pp}}{\sigma_{\text{in}}^{pA}}E\frac{d^3\sigma^{pA}}{dp^3}(s,p_T)\simeq E\frac{d^3\sigma^{pp}}{dp^3}(A^{1/3}s,p_T),\tag{11}$$

i.e., the function

$$\frac{\sigma_{\text{in}}^{pp}}{\sigma_{\text{in}}^{pA}}E\frac{d^3\sigma^{pA}}{dp^3}(A^{-1/3}s,p_T)\tag{12}$$

is independent of A.

One can easily check that the appropriate integration of expression (8) leads to an average multiplicity as given in Ref. 3.

COMPARISON WITH EXPERIMENT

In Figs. 1–5 we compare the approximate sum rule [Eq. (11)] with the data as given in Ref. 1. We have plotted $(\sigma_{\text{in}}^{pp}/\sigma_{\text{in}}^{pA})E(d^3\sigma^{pA}/dp^3)$ from Ref. 1 (see Ref. 9) for W and Ti for $\sqrt{s}=19.4$, 23.8 GeV, respectively, as a function of p_T; defining $s_{\text{eff}}=A^{1/3}s$ we find that its values are almost equal to each other [$(s_{\text{eff}})^{1/2}=46.2$ and 45.2 GeV, respectively]. Data for $Ed^3\sigma^{pp}/dp^3$ were plotted at $\sqrt{s}=44.6$ GeV.[8] According to Eq. (11) $(\sigma_{\text{in}}^{pp}/\sigma_{\text{in}}^{pA})E(d^3\sigma^{pA}/dp^3)$ for W, Ti, p at $\sqrt{s}=19.4$, 23.8, 44.6 GeV, respectively, should be approximately equal.[10] Good agreement between experiment and theory is found for $c=\pi^\pm,K^\pm,\bar{p}$, as shown in Figs. 1–5. Deviations from our predictions are found only at the highest p_T values, where the replacement of the sum over $P(i,A)$ with the average $A^{1/3}$ leads to the largest error.[11,12] We exclude the comparison to the p spectra, since we have not taken yet into account the dissociation of the target arrays.

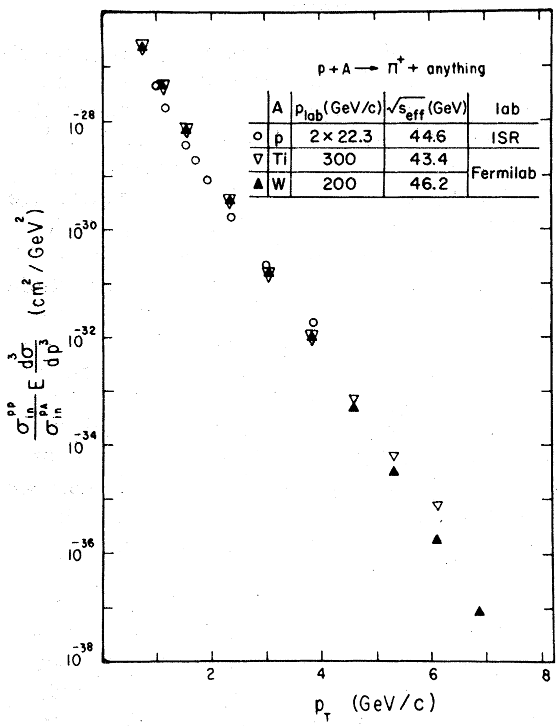

FIG. 1. Comparison between data on inclusive production of π^+ in pA collisions for $A=184$, 48, 1 at $\sqrt{s}=19.4$, 23.8, 44.6 GeV, respectively, from Refs. 1 and 8. According to the approximate scaling law [Eq. (11)], all these data should lie on the same line since they all have the same $s_{\mathrm{eff}}=A^{1/3}s$.

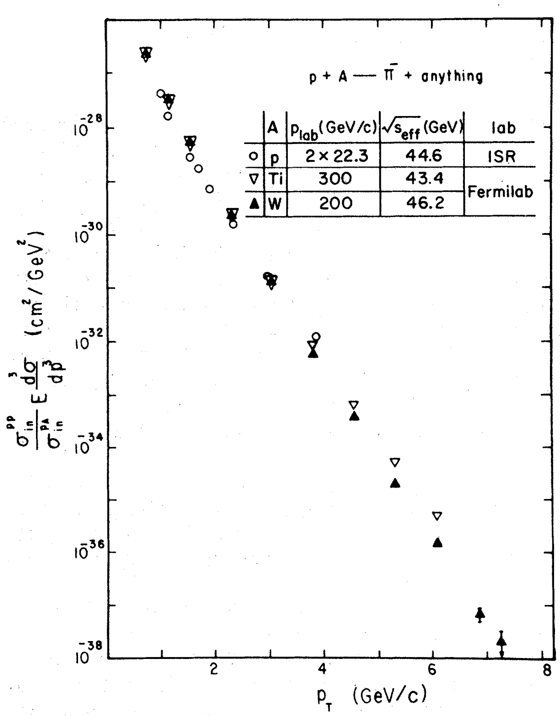

FIG. 2. The same as in Fig. 1, for π^-.

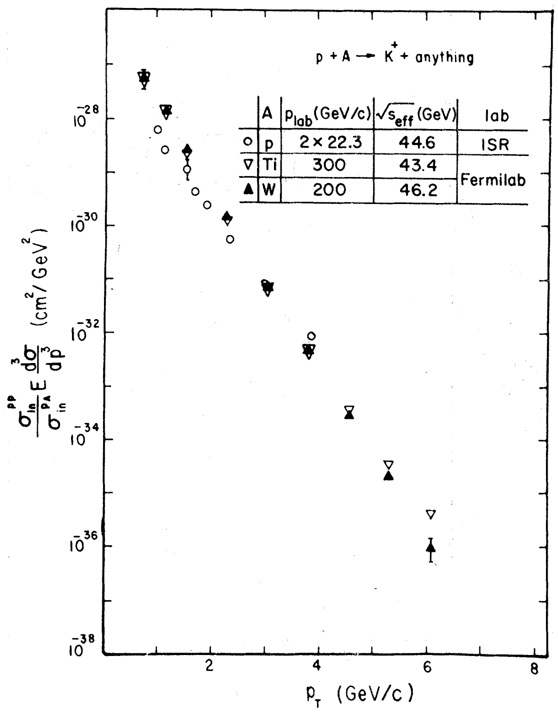

FIG. 3. The same as in Fig. 1, for K^+.

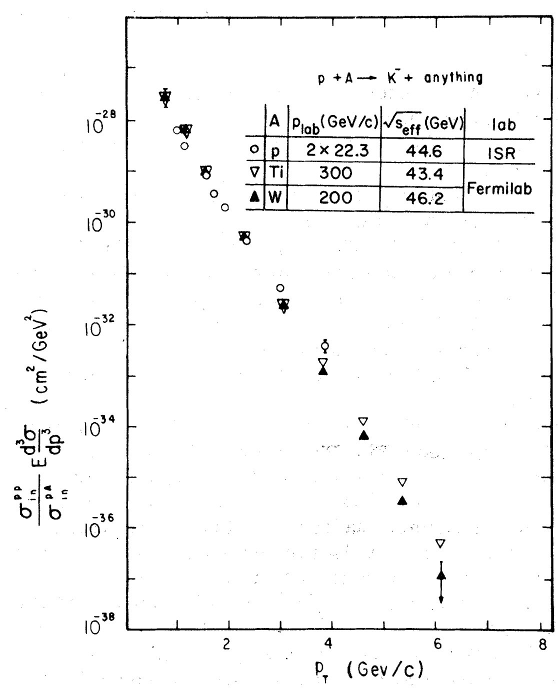

FIG. 4. The same as in Fig. 1, for K^-.

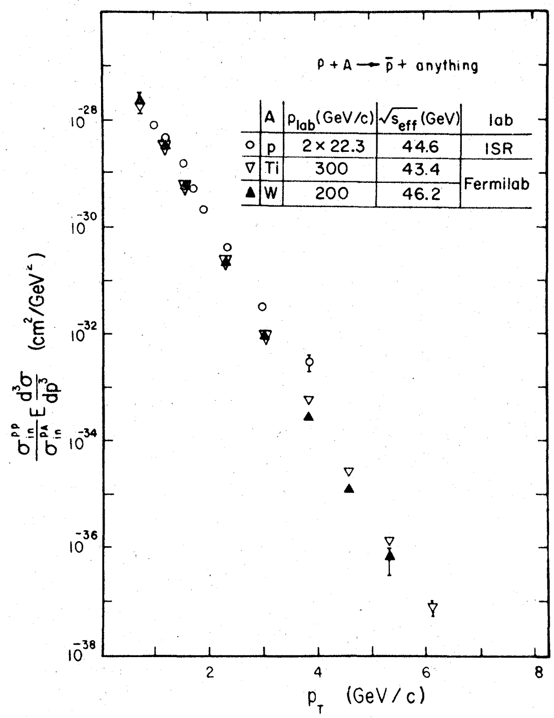

FIG. 5. The same as in Fig. 1, for $\bar{p}$.

CONCLUSIONS

The success of the model for the average multiplicity, the Koba-Nielsen-Olesen function, the dispersion, the dependence of the average multiplicity on the number of knocked-out protons,[3] and for the high-p_T inclusive cross section has the following important implication: The average effective energy squared available for particle production in particle-nucleus collisions is $A^{1/3}$ times larger than the energy available in particle-particle collisions at the same laboratory momentum. At Fermilab with 400-GeV protons incident on uranium one has an average effective energy squared of $s_{\rm eff} \simeq 2A^{1/3}mp_{\rm lab} \simeq 4600\ \mathrm{GeV}^2$. There is of course a non-negligible probability for obtaining energy-squared values beyond 10^4 GeV^2, as indicated by events with a large number of knocked-out protons.[3] Moreover, for high-energy nucleus-nucleus interactions[13] $s_{\rm eff} = A_1^{1/3}A_2^{1/3}s$.

Consequently experiments with high-energy particles incident on nuclear targets or experiments with nuclear beams with high energy per nucleon incident on nuclear targets can provide information on the energy domains of future accelerators much before the future machines will be actually constructed. In particular, possible heavy particles may be produced by the cumulative effect discussed here, using nuclear targets at present accelerators.

---

*Research supported in part by the Israel Commission for Basic Research.

†In partial fulfillment of the requirements for the D.Sc. degree.

‡On sabbatical leave from the Technion-Israel Institute of Technology, Haifa, Israel.

[1]J. W. Cronin *et al.*, Phys. Rev. D 11, 3105 (1975).

[2]It is not clear that a power of A describes the invariant inclusive cross section for all energies and p_T down to $A=1$. The form $A^{\alpha(p_T)}$ is used in Ref. 1 as an additional assumption to extrapolate to $A=1$.

[3]G. Berlad, A. Dar, and G. Eilam, Phys. Rev. D 13, 161 (1976).

[4]A similar model was suggested by A. Z. Patashinskii, Zh. Eksp. Teor. Fiz. Pis'ma Red. 19, 654 (1974) [JETP Lett. 19, 338 (1974)]. However, this author predicts $R_A = \overline{\langle i \rangle}^{1/4}$ for the ratio of the average multiplicity in particle-nucleus collisions to the average multiplicity in particle-particle collisions, where i is the number of nucleons in an array. This relation holds only approximately in our model which predicts $R_A = \langle i^{1/4} \rangle$.

Similar ideas have also been proposed earlier by F. C. Roesler and C. B. A. McCusker, Nuovo Cimento 10, 127 (1953); W. Heitler and C. H. Terreaux, Proc. Phys. Soc. London A66, 929 (1953); D. S. Narayan and K. V. L. Sarma, Prog. Theor. Phys. 31, 93 (1964); L. D. Landau, in *Collected Papers of L. D. Landau*, edited by D. Ter Harr (Pergamon, London, 1965); K. Gottfried, in *High Energy Physics and Nuclear Structure*, edited by G. Tibell (North-Holland, Amsterdam, 1974); A. Dar, in Proceedings of the ICTP Topical Meeting on High Energy Reactions Involving Nuclei, Trieste, 1974 (unpublished); F. Takagi, Lett. Nuovo Cimento 14, 559 (1975); S. Fredriksson, Nucl. Phys. B111, 167 (1976); Meng Ta-chung, Phys. Rev. D 15, 197 (1977).

[5]Z. Koba, H. B. Nielsen, and P. Olesen, Nucl. Phys. B40, 317 (1972).

[6]A. Białas and W. Czyż [Phys. Lett. 58B, 325 (1975)] consider a model of the type described in Ref. 3, to which they add an additional assumption alien to our model. This assumption results in predictions which are different from ours.

[7]If one considers a deterministic model instead, then the number of nucleons in an array located at impact parameter b is given by $i(b) = \sigma T(b)$. R_A calculated from such a model is then very similar to R_A obtained from the probabilistic model of Ref. 3. The maximal number of knocked-out protons is then smaller than

the number allowed in the probabilistic model.

[8]B. Alper *et al.*, Nucl. Phys. B87, 19 (1975); G. Jarlskog, in *Phenomenology of Hadronic Structure*, proceedings of the Xth Rencontre de Moriond, 1975, edited by J. Tran Thanh Van (CNRS, Paris, 1975), p. 203.

[9]We have used $\sigma_{\rm in}^{pA}/\sigma_{\rm in}^{pp}=1.4A^{0.69}$ ($A\gtrsim 10$) as in W. Busza, in *High Energy Physics and Nuclear Structure—1975*, proceedings of the Sixth International Conference, Santa Fe and Los Alamos, edited by D. E. Nagle *et al.* (A.I.P., New York, 1975), p. 211.

[10]The most meaningful comparison is between data for different A, all with $A\gtrsim 10$, as is clear from the comparison of $R_A=\sum_{i=1}^{A}P(i,A)i^{1/4}$ with $R_A=\text{const}\times A^{1/12}$; the const should be larger than 1, and the approximate form works for $A\gtrsim 10$.

[11]We have checked that this is the case for various parametrizations of $E\,d^3\sigma^{pp}/dp^3$.

[12]The model has been recently applied to the data of Ref. 1 by S. Fredriksson, Nucl. Phys. B111, 167 (1976). This author uses $Ed^3\sigma^{pA}/dp^3\sim A^{\overline{\alpha(p_T)}}$ as a starting point (see Ref. 2), thus obtaining $s_{\rm eff}=\text{const}\times A^{1/3}s$, where const$<1$, in disagreement with our results.

[13]The study of cumulative phenomena in nuclei as observed in the production of particles by high-energy heavy ions is reviewed in A. M. Baldin, in *High Energy Physics and Nuclear Structure—1975* (Ref. 9), p. 621.