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Quark-counting and hadron-size effects for total cross sections*

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We show that a natural explanation of the various hadron cross-section sizes, in particular of the small size of $\sigma_T(\psi p)$, is possible in a dynamical model of the Pomeron based on quantum chromodynamics.

The approximate energy independence of hadronic total cross sections is one of the oldest problems of elementary particle physics. Recently, two dynamical models for Pomeron behavior have been proposed within the framework of quantum chromodynamics. In the model of Low[1] and Nussinov,[2] which we pursue in this paper, the forward amplitude is dominated by the exchange of two colored gluons (Fig. 1). The amplitude for one-gluon exchange is zero for color-singlet hadrons and the two-gluon amplitude is purely imaginary in the $s \to \infty$ fixed-t limit. Exchanges of more than two gluons, which might be a source of embarrassment to the model (the exchanges being C-odd as well as C-even in the t channel) are ignored or assumed to be numerically small.

A second model[3] for the Pomeron postulates that the forward amplitude is dominated by the exchange of "wee" quarks, (Fig. 2). This approach explains naturally the universality of multiplicity between hadron reactions, deep-inelastic processes, and e^+e^- annihilation and also explains fragmentation power laws, in particular, in hadronic processes.

Both of these models are consistent with Feynman's original wee-parton-exchange approach and each has attractive features. It is clearly important to pursue both approaches further in order to define their distinguishing features and to reveal possible specifically dynamical regularities of Pomeron behavior.

To this end we explore further the two-gluon-exchange model, concentrating specifically on the effects of the size of the bound-state hadrons and of the number of quarks they contain. We demonstrate, in particular, that the observed small ψ-nucleon cross section is the natural result, in this model, of the color-singlet nature of hadrons and the relatively small size of the ψ, considered as a bound state of heavy charmed quarks. The basic picture, Fig. 1, begins with incoming bound states described, in general, as a superposition of Fock-space states (valence + valence $q\bar{q} + \cdots$) which are considered as established prior to interaction. The Low-Nussinov approach, to first approximation, neglects all but the simplest valence Fock component. (In comparison, Ref. 3 presumes that it is precisely the higher $q\bar{q}$-sea Fock components which are responsible for Pomeron behavior.) One then considers the lowest-order (second-order) nonzero gluon-interaction graphs involving the component valence quarks. Certainly there are many possible corrections to this picture, but all involve *explicitly* higher-order gluon mechanisms. For instance, there are components of the bound-state Fock space which contain gluons and, of course, there are explicitly higher-order corrections to the two-gluon interaction itself such as three-gluon exchange, "vertex" corrections to a gluon attachment, etc.

Our approach differs from that appropriate to the bag model[1] in the dynamics by which the momentum brought in by one of the gluons is transferred to the outgoing gluon. We employ a standard eikonal formalism which has built-in momentum conservation at all intermediate stages, unlike the static bag-model approximation. The important differences to which this leads will be noted later.

Let us consider meson-meson scattering in the limit $s \to \infty$, t fixed. (Later, we will state how our results generalize in the somewhat more complicated cases of meson-baryon and baryon-baryon

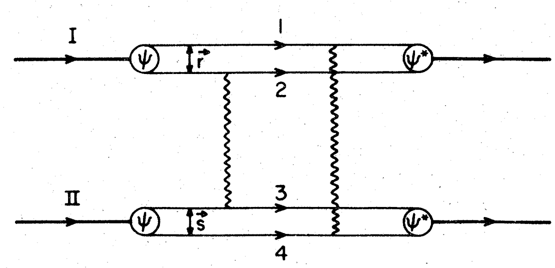

FIG. 1. The two-gluon diagram for the bare Pomeron (Refs. 1, 2).

scattering.) It is instructive to include all crossed and uncrossed ladder graphs to begin with, since these graphs sum, formally, into a simple eikonal expression. We will shortly specialize to the two-gluon-exchange term. The amplitude can be most elegantly derived by considering quark scattering from an external gluon field using null-plane field theory,[4] then tying together the scattering of the right-moving and left-moving quarks using functional differentiation. (The result for quark-quark scattering by two-gluon exchange has been obtained by Nieh and Yao[5] and by McCoy and Wu.[6])

The amplitude, normalized so that $d\sigma/dt \simeq |\mathfrak{a}|^2/16\pi s^2$, $\sigma_T = \mathfrak{a}/s$, with $\mathcal{L}_I = g\bar{\psi}\gamma_\mu\lambda^a\psi A^\mu_a$, is

$$\begin{aligned}\mathfrak{a} = -2is \int d\vec{b}\, e^{-i\vec{Q}\cdot\vec{b}} \\ \times \int_0^1 d\alpha \int d\vec{r}\, \psi_{c_1c_2}(\alpha,\vec{r})^*\psi_{a_1a_2}(\alpha,\vec{r}) \int_0^1 d\beta \int d\vec{s}\, \psi_{c_3c_4}(\beta,\vec{s})^*\psi_{a_3a_4}(\beta,\vec{s}) \\ \times \exp\left\{-ig^2\sum_{i=1}^{8}[\lambda_1^i\lambda_3^i V(\vec{x}_1-\vec{x}_3) + (\lambda_2^i)^T(\lambda_4^i)^T V(\vec{x}_2-\vec{x}_4) - \lambda_1^i(\lambda_4^i)^T V(\vec{x}_1-\vec{x}_4) - (\lambda_2^i)^T\lambda_3^i V(\vec{x}_1-\vec{x}_3)]\right\}.\end{aligned} \tag{1}$$

Here $\vec{Q}^2 = -t$ and vectors $\vec{V}$ with arrows denote transverse components (V^1, V^2). We will write $V^\pm = (V^0 \pm V^3)/\sqrt{2}$ for the other two components. The notation for a typical two-gluon-exchange graph is illustrated in Fig. 1. The wave function $\psi_{a_1a_2}(\alpha,\vec{r})$ describes the initial meson I. The indices a_1, a_2 are the color indices for quark 1 and antiquark 2; for a color-singlet meson state, $\psi = 3^{-1/2}\delta_{a_1a_2}\Psi$. The subscript on an $SU(3)_c$ λ matrix indicates the quark line on which it acts. (Note the minus signs for antiquark lines.) A circular color trace within each meson bound-state loop is implied. Spin and flavor indices are suppressed. (The exchanged gluon itself does not flip the quark spin though bound-state effects can.) Quark 1 carries a fraction α of the large-$P_{\rm I}^+$ $[=(s/\sqrt{2})^{1/2}]$ momentum component of meson I and quark 3 carries a fraction β of the total $P_{\rm II}^-$ $[=(s/\sqrt{2})^{1/2}]$ of the (left-moving) meson II. The impact parameter $\vec{b}$ is the transverse separation between the center of P^+ of meson I and the center of P^- of meson II.

The transverse positions of the four quarks are

$$\begin{aligned}\vec{x}_1 &= \tfrac{1}{2}\vec{b} + (1-\alpha)\vec{r}\,,\\ \vec{x}_2 &= \tfrac{1}{2}\vec{b} - \alpha\vec{r}\,,\\ \vec{x}_3 &= -\tfrac{1}{2}\vec{b} + (1-\beta)\vec{s}\,,\\ \vec{x}_4 &= -\tfrac{1}{2}\vec{b} - \beta\vec{s}\,.\end{aligned}$$

In the eikonal exponential factor, the one-gluon potential V is

$$V(\vec{x}) = -\int dx^+ dx^- \Delta_F(x^+, \vec{x}, x^-)\,. \tag{2}$$

The distinguishing feature of a non-Abelian gauge theory in this approximation is that the order of the λ matrices acting on a given quark line must be specified. The correct order is given by expanding the exponential, with the x^+ and x^- integrations (2) left undone, then x^+-ordering the λ matrices on the quark lines for meson I and x^--ordering the λ matrices for meson II. In the related case of quark-quark scattering the x^+ and x^- integrations would contain logarithmic divergences, which indicate the presence of logs factors in a more carefully derived result.[5,6]

In the special case of interest to us, two-gluon exchange between color-singlet mesons, the amplitude is proportional to the trace of the two λ matrices that act on meson I (and similarly meson II). Thus the order of the λ matrices is irrelevant, the x^+ and x^- integration are trivially done, and we obtain

$$\begin{aligned}\mathfrak{a}_{2\,\text{gluon}} = 2is\int d\vec{b}\, e^{-i\vec{Q}\cdot\vec{b}} \int_0^1 d\alpha \int d\vec{r}\, |\Psi(\alpha,\vec{r})|^2 \int_0^1 d\beta \int d\vec{s}\, |\Psi(\beta,\vec{s})|^2 \\ \times\left(\frac{4g^2}{3}\right)^2 [V(\vec{x}_1-\vec{x}_3) + V(\vec{x}_2-\vec{x}_4) - V(\vec{x}_1-\vec{x}_4) - V(\vec{x}_2-\vec{x}_3)]^2\,.\end{aligned} \tag{3}$$

The integral (2) for $V(\vec{x})$ gives

$$V(\vec{x}) = (2\pi)^{-2}\int d\vec{k}\, e^{i\vec{k}\cdot\vec{x}}(\vec{k}^2+\mu^2)^{-1}\,.$$

Here μ is a fictitious gluon mass, which we insert to mimic the confinement of the colored gluons to a region of size $1/\mu$. For instance, in the bag model, the infrared cutoff is provided by the com-

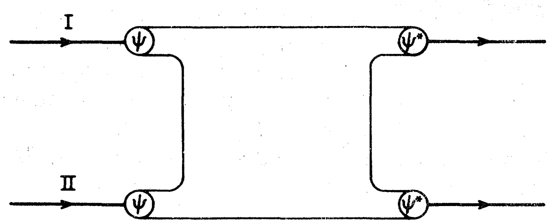

FIG. 2. The quark-exchange diagram for the bare Pomeron (Ref. 3).

bined two-bag size. (Our result for the forward amplitude is finite and independent of μ as $\mu\to 0$. In contrast the slope of $d\sigma/dt$ at $t=0$ diverges logarithmically as $\mu\to 0$.) We will take $\mu\simeq m_\pi$.

Finally the reduced wave function Ψ is normalized so that the meson's electromagnetic form factor is[7]

$$F(\vec{Q}^2)=\int_0^1 d\alpha\int d\vec{r}\,|\Psi(\alpha,\vec{r})|^2 e^{i(1-\alpha)\vec{Q}\cdot\vec{r}} \tag{4}$$

if only quark 1 is charged $[F(0)=1]$.

What are the experimental implications of Eq. (3) for forward scattering? First, we notice that $\mathcal{A}\to 0$ as the size of either of the mesons vanishes. Simply put, this is because color-singlet point particles do not radiate colored gluons, and thus cannot interact via gluon exchange. In Eq. (3) we note that if, say, $|\vec{x}_1-\vec{x}_2|=|\vec{r}|$ is small, we can write

$$\begin{aligned}[V(\vec{x}_1-\vec{x}_3)+V(\vec{x}_2-\vec{x}_4)-V(\vec{x}_1-\vec{x}_4)-V(\vec{x}_2-\vec{x}_3)]^2\\ \simeq(\vec{r}\cdot\vec{\nabla}\{V[\alpha\vec{x}_1+(1-\alpha)\vec{x}_2-\vec{x}_3]\\ -V[\alpha\vec{x}_1+(1-\alpha)\vec{x}_2-\vec{x}_4]\})^2\,.\end{aligned}$$

Thus as the size $\langle r^2\rangle_{\rm I}$ of meson I tends to zero the amplitude vanishes like[2] $\mathcal{A}\propto\langle\vec{r}^{\,2}\rangle_{\rm I}$.

In the present case of a small color-singlet meson, this cancellation in the amplitude occurs because the quark and the antiquarks couple to the color gluon with opposite signs. The same cancellation occurs for color-singlet baryons in the limit of zero size, due to the identity

$$\epsilon_{\bar a bc}\lambda_{\bar a a}+\epsilon_{a\bar b c}\lambda_{\bar b b}+\epsilon_{ab\bar c}\lambda_{\bar c c}=0\ .$$

Similarly the amplitude will vanish like r^2 for small r in higher-order gluon exchange; there is one factor of r for the first (in x^+) gluon absorbed and one factor of r for the last.

The two-gluon forward amplitude (3) can be rewritten in a useful momentum-space form:

$$\begin{aligned}\mathcal{A}(t=0)=\frac{is}{2}\left(\frac{4g^2}{3}\right)^2(2\pi)^{-2}\\ \times\int d\vec{k}[(\vec{k}/2)^2+\mu^2]^{-2}\\ \times 2[1-f_{\rm I}^{q\bar q}(\vec{k}^2)]2[1-f_{\rm II}^{q\bar q}(\vec{k}^2)]\ .\end{aligned} \tag{5}$$

Here $f_{\rm I}^{q\bar q}(\vec{k}^2)$ is a form factor for meson I: $f_{\rm I}^{q\bar q}(\vec{k}^2)=\langle e^{i\vec{k}/2\cdot(\vec{x}_1-\vec{x}_2)}\rangle$, where $\langle\ \rangle$ denotes the expectation value in the meson bound state. Thus $f^{q\bar q}(\vec{k}^2)$ differs from the meson's electromagnetic form factor, Eq. (4), only by the replacement $(1-\alpha)\to\frac{1}{2}$ in the exponential, as would be a good approximation for weakly bound quarks of equal mass. Thus for phenomenological purposes we can regard f as the true electromagnetic form factor and approximate it by

$$f(\vec{k}^2)\simeq\frac{\lambda^2}{\vec{k}^2+\lambda^2}\,, \tag{6}$$

where the important "size" is $1/\lambda$, the inverse form-factor scale.

We have employed a monopole form for $f(\vec{k}^2)$ as required by dimensional counting.[8] In that approach one envisions the momentum $k/2$ as entering on, say, the quark line and being transferred to the antiquark line by means of a basic interaction with dimensionless coupling constant (perhaps also gluon exchange). The Feynmann "Born" graph for this as compared to that for the meson electromagnetic form factor makes clear their equality for $\alpha=\frac{1}{2}$ and predicts monopole behavior for both. The structure of these "off-diagonal" form factors where the two gluons hit different quarks in the hadron is very different in the static bag model.[8] There each gluon probes the static charge distribution yielding the hadron form factor at each interaction. Thus, instead of $f^{q\bar q}(\vec{k}^2)$ one has $[f_{\rm I}^{q\bar q}(\vec{k}^2/4)]^2$ and, of course, the form factors predicted by the static charge distribution in the bag model are not necessarily power-law-behaved.

The two-gluon model can also be used to describe meson-baryon and baryon-baryon scattering. One uses a wave function $\psi(\alpha_1,\alpha_2,\alpha_3;\vec{r}_1,\vec{r}_2,\vec{r}_3)_{abc}=(1/\sqrt{6})\epsilon_{abc}\Psi$ to describe a baryon as a color-singlet bound state of three quarks. We omit the details of the intermediate steps here because the notation is somewhat cumbersome. The result for forward scattering of, say, a meson from a baryon is simple: in Eq. (8) one makes the replacement

$$2[1-f_{\rm II}^{q\bar q}(\vec{k}^2)]\to 3[1-f_{\rm II}^{qq}(\vec{k}^2)]\ . \tag{7}$$

A natural *quark-counting factor* appears in the result. In the baryon case the "form factor," $f_{\rm II}^{qq}(\vec{k}^2)$ is a bound-state expectation value of $e^{i\vec{k}/2\cdot(\vec{x}_i-\vec{x}_j)}$, where $\vec{x}_i-\vec{x}_j$ is the distance between some pair of the quarks. Because $f_{\rm II}^{qq}(\vec{k}^2)$ is a diquark form factor, its large-$\vec{k}^2$ behavior should reflect the short-distance behavior of the product of two quark fields: $f_{\rm II}^{qq}(\vec{k}^2)\sim 1/\vec{k}^2$. Indeed, an examination of the simplest possible Feynman diagrams appropriate to theories that obey dimensional counting, shows that f_{II}^{qq} and $f_{\text{II}}^{q\bar{q}}$ should have identical large-$\vec{k}$ behavior, as the third quark of the baryon is not probed. Even the mass scale is approximately the same for $f^{q\bar{q}}$ and f^{qq} in the dimensional-counting "Born" graphs. Therefore we will approximate $f_{\text{II}}^{qq}(\vec{k}^2)$ by the *same* monopole form used for mesons, Eq. (6). *This equality* $f_{\text{baryon}}^{qq}(\vec{k}^2)=f_{\text{meson}}^{q\bar{q}}(\vec{k}^2)$, *for similar quark type, leads to quark counting for total cross sections.* However, the cross section is also sensitive to the bound-state "size," $1/\lambda$, which may vary drastically in going from hadrons composed of u and d quarks to those that contain strange or, especially, charmed quarks.

In order to calculate the relative magnitude of various cross sections we must determine the mass scales λ appropriate to different quark types. The simplest approach is to presume the validity of vector dominance for the electromagnetic form factors of the different vector mesons $\rho\phi\psi$. The ρ form factor is dominated by the ρ, the ϕ by the ϕ, and the ψ by the ψ (given the orthogonal quark contents, Zweig's rule prohibits any mixing) yielding

$$\lambda_{u,d}=m_\rho \; \lambda_s=m_\phi \; \lambda_c=m_\psi \; . \tag{8}$$

Alternatively, dimensional-counting graphs suggest that λ is sensitive to quark masses and binding energies. In the simplest case λ is proportional to quark masses, which leads to somewhat larger differences between the λ's and thus the different cross sections. Other estimates of λ, though less directly connected to form-factor behavior, follow from bound-state-model calculations of $\langle r^2\rangle$. Linear-potential approaches[9] yield ratios for the "size" parameters similar to (8), while in the bag model $\langle r^2\rangle$ for the hadron states is not so sensitive to the quark mass, being largely determined by the bag parameter "B."[10]

Using (8) and $\mu\approx m_\pi\approx 0.14$ (the result is insensitive to μ if $\mu\leq m_\rho$) we obtain

$$\sigma_T(\psi p):\sigma_T(\phi p):\sigma_T(\rho p)\approx 1:3.7:5 \; , \tag{9}$$

which is in rough agreement with the experimental ratios 1 : 4–5 : 8 obtained by using the higher values of $\sigma_T(\phi p)$ (Ref. 11) and $\sigma_T(\psi p)$ (Ref. 12) extracted from nuclear shadowing assuming a moderate-to-small real part of the amplitude. (Larger theoretical ratios can be obtained if one takes λ proportional to quark masses.) In addition the absolute size of $\sigma_T(\rho p)$ determines the gluon coupling constant. Using the parameters above we obtain $g^2/4\pi\approx 0.53$ (in agreement with Ref. 1) for $\sigma_T(\rho p)\approx 27$ mb.

Finally we consider the generalization to states, say meson states, composed of quarks, q_1 and $\bar{q}_2$, of unequal mass. The $q_1\bar{q}_2$ form factor again has monopole behavior according to dimensional counting, but the mass scale is no longer directly related to electromagnetic-form-factor mass scales. The dimensional-counting Born graphs, in the simplest approximation, suggest that λ in $f^{q_1\bar{q}_2}$ is proportional to the reduced quark mass. Typical quark masses[13] are $m_u\approx 140$ MeV, $m_s\approx 5m_u$ so that $\lambda(s\bar{u})\approx\frac{5}{3}\lambda(u\bar{u})$. This would lead to a cross section for K^*p (or Kp) slightly less than 20 mb, in agreement with experiment. Note also that for D mesons this same approach predicts $\lambda(c\bar{u})\approx 2\lambda(u\bar{u})$ and hence a roughly 18-mb total cross section for Dp—much larger than that for ψp.

A discussion of $\vec{Q}^2$ (or t) dependence is obviously interesting, but is more strongly model-dependent. For example, the forward slope of $d\sigma/dt$, b, diverges logarithmically for $\mu\to 0$. For finite μ, b is less sensitive to size effects than σ_T.

Large-t dependence (at fixed angle) is discussed for meson-meson scattering by Landshoff.[14] Finally, unitarization, eikonalization, and/or s-channel iteration of the bare Pomeron amplitude that we have considered strongly affects the t dependence.[15] Thus we leave t dependence to a future work.

In conclusion we see that, at the very least, dynamical pictures of the Pomeron may yield strikingly strong dependence of the magnitudes of total cross sections on the size of the scattering hadrons. To what extent lowest-order gluon calculations are numerically reliable is open to question, but it is clear that the important cancellation mechanism is preserved in higher orders. Thus a natural dynamical explanation of the small-ψp cross section is possible in quantum-chromodynamics (QCD) approaches to Regge behavior. More conventional explanations[16] based on the "f-dominated" Pomeron picture, and the charm analogs, have also been given but do not appear related to the approach given here. The alternative (within the QCD context) Pomeron model of Brodsky and Gunion[3] will yield results similar to those presented here since the higher-quark-number Fock-space components (i.e., the $q\bar{q}$ sea of the wave function), which are responsible for Pomeron behavior in that model, arise via pair creation following colored-gluon emission. Colored-gluon emission from the valence state is clearly less probable the smaller the "chargeless" ground or valence states. A quantitative estimation of this effect is currently under way.

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