Leptonic octets and heavy-lepton decays in an SU(3) gauge theory*

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(Received 6 August 1976)

Unified weak-electromagnetic gauge theories based on the group SU(3) are discussed in some detail. In particular, the consequences of assigning the electron, the muon, and the newly discovered lepton of Perl *et al.* to an octet are explored. Almost uniquely, this leads to the prediction that the Perl particle decays preferentially into muons and neutrinos rather than into electrons and neutrinos. This means that in the e^+e^- annihilation experiments there should be more $\mu^+\mu^-$ pairs than e^+e^- pairs, after subtraction of background. Such a situation would be very difficult to achieve naturally in the framework of an SU(2)×U(1) gauge theory. The hadron sector of the theory and neutral-current reactions are also discussed as well as some general considerations on the construction of weak-interaction gauge theories with large groups.

I. SU(3) AS A WEAK-ELECTROMAGNETIC GAUGE GROUP

One motivation for considering the weak-electromagnetic group of nature to contain SU(3) is the same as that which, a long time ago, led to extending isospin to "strong" SU(3): the desire to accommodate a large variety of particles in as few multiplets as possible. Now that there is experimental evidence[1] for the lepton of Perl *et al.* as well as strong indications from neutrino production experiments that more quarks are needed,[2] the search for larger groups becomes more interesting. Another motivation for looking at SU(3) is the hope of naturally getting different kinds of predictions from the SU(2)×U(1) theory. We shall see that this hope is realized.

A certain amount of discussion of SU(3) as the unified gauge group has already been given in the literature. For example, Ueda and one of us have discussed[3] an SU(3)×U(1) scheme involving both leptons and quarks[4] which includes the following triplet fields:

$$\begin{pmatrix} q_{1L} \\ q_{2L}(\theta) \\ q_{3L}(\theta) \end{pmatrix}, \quad \begin{pmatrix} \nu_L \\ e_L \\ e'_L \end{pmatrix}, \quad \begin{pmatrix} \nu_\mu \\ \mu_L \\ \mu'_L \end{pmatrix},$$

where L stands for left-handed projection, $q_{2L}(\theta)$ and $q_{3L}(\theta)$ are Cabibbo-angle-rotated quarks, and e' and μ' are negatively charged heavy leptons. In this model suppression of undesired effects of strangeness-changing neutral currents could be gotten by taking one gauge boson to have an extremely large mass. A variant[5] of this model banished strangeness-changing neutral currents completely by taking the quark multiplet to be the same as the strong one. $|\Delta S|=1$ decays then arose by mixing of the π^+-like and K^+-like gauge bosons.

The above schemes were based on three quarks; now that the existence of $\psi(3100)$ makes at least the charmed quark likely, the most natural extension is to consider six quarks distributed in two triplets.[6] Note also that neither e' nor μ' above could be identified with the Perl particle (in the absence of gauge-boson mixing) since the decay $P\to e+2\nu$ is then not allowed in the theory. This provides us with a motivation to drastically modify the lepton assignments. A hint in this direction was given by a previous suggestion[7] that if all leptons (including both left- and right-hand projections) were assigned the quantum numbers of leptonic-quark composites, the extra Abelian gauge group in the theory would not be necessary and there would then be only a single gauge coupling constant. This feature comes about because the quarks and the presumably fictitious leptonic quarks then have the same charge structure, enabling the photon to transform as a pure member of SU(3). The natural representation-assignment possibilities for the leptons are then either the octet or the decuplet. Actually, as we shall see, the decuplet is not an acceptable possibility. Thus we are led to a vectorlike[8] gauge model with leptons transforming as a left-handed and as a right-handed[9] octet. The same idea has also been recently proposed by Fritzsch and Minkowski.[10] However, they do not discuss the assignment of the Perl particle and its neutrino to the octet and the interesting experimental consequences of such an assignment.

The details of the gauge-boson and lepton octet assignments are given in Sec. II. This section also contains some general considerations on the embedding of the weak interactions in a higher symmetry group. In Sec. III the pure leptonic decay modes of the Perl particle are discussed. The remarkable feature emerges that decays into muons and neutrinos are favored over decays into electrons and neutrinos because of the presence of additional gauge bosons in the SU(3) theory. This means that more unaccompanied $\mu^+\mu^-$ pairs than e^+e^- pairs should be seen in the e^+e^- annihilation experiments, after subtracting background. In Sec. IV, a preliminary discussion of the generation of masses by Higgs mesons in a (complicated) gauge theory is given. It is shown that we have *a priori* freedom to choose fermion masses arbitrarily but, of course, other considerations may limit this freedom. In the present theory there is no simple pattern to the lepton masses so this freedom is useful. Specific illustrations for our model are given in the Appendix. Finally, in Sec. V the hadronic sector is introduced and discussed. It is noted that hadronic decays of the Perl particle can be used to determine some basic parameters of the theory. Compatibility of this model with a recent experiment on neutrino-induced neutral-current reactions is discussed.

II. GAUGE-BOSON AND LEPTON ASSIGNMENTS

Our notation for gauge bosons is the same as that of Ref. 3. There are now eight gauge bosons described in conventional three-dimensional tensor notation as follows:

$$W^b_{a\mu} \text{ with } W^a_{a\mu}=0,$$
$$A_\mu = -(\tfrac{3}{2})^{1/2}W^1_{1\mu} = +(\tfrac{3}{2})^{1/2}(W^2_{2\mu}+W^3_{3\mu}), \tag{2.1}$$
$$Z_\mu = \frac{1}{\sqrt{2}}(W^2_{2\mu}-W^3_{3\mu}).$$

A_μ, the photon field, is a U-spin singlet while Z_μ is the $U_3=0$ member of a U-spin triplet. We will take the mass matrix of the gauge boson to be diagonal for states of definite U_3 and Y_U. This means that $W^3_{2\mu}$ and $W^2_{3\mu}$ are diagonal. Actually we have the freedom to be more general and consider the definite CP combinations $W^3_{2\mu} \pm W^2_{3\mu}$ to be states of different mass. However, this additional freedom is not necessary for our present purposes. $W^2_{1\mu}$ and $W^1_{2\mu}$ are to be identified as the mediators of the ordinary weak interactions. Their mass is the lower limit of the old SU(3) × U(1) theory[3]:

$$m(W^2_1) = 43.1 \text{ GeV}. \tag{2.2}$$

We shall consider $m(W^3_2)$ and $m(W^3_1)$ to be of the same order of magnitude as $m(W^2_1)$. This results in the possibility of a more interesting variety of reactions taking place; if these masses were extremely large, the physics of this model would be more similar to the SU(2) × U(1) models. The gauge coupling constant, g, can be conveniently specified by giving the covariant derivative D_μ acting on a fundamental triplet f_a of the theory:

$$D_\mu f_a = \partial_\mu f_a - 2igW^b_{a\mu}f_b, \tag{2.3}$$
$$g = -|e|/\sqrt{6},$$

where $|e|$ is the magnitude of the electron charge.

Now let us discuss the lepton representation assignment. It may be instructive first to show why the decuplet is not viable. This follows because the interactions for $\mu \to e + 2\nu$, $K \to \pi\mu\nu$, and $K \to \pi e\nu$ are all considered to be mediated by W^2_1, which couples to the various fields with relative strengths (after taking Cabibbo's factor into account) given by the isotopic-spin-lowering operator. Remembering from angular momentum theory the formula

$$J_-|l,m\rangle = [(l+m)(l-m+1)]^{1/2}|l,m-1\rangle,$$

and noting that the right-hand side depends on l as well as m, we see that the universality of weak interactions demands that the pairs $e_L\nu_e$, $\mu_L\nu_\mu$, and $q_{1L}q_{2L}(\theta)$ each belong to an isomultiplet of the same dimension. If we take $q_{1L}q_{2L}(\theta)$ to belong to an isodoublet then the lepton pairs must do the same. (We assume no lepton mixing angles.) Since the decuplet contains only one isodoublet, it cannot accommodate both the electron and muon pairs. Hence it is ruled out for a simple theory of the present type. It is furthermore clear that similar considerations hold for a general embedding of the weak interactions in a large gauge group.

We immediately see that the octet is a possible representation for unifying the leptons since it contains two isodoublets. Also the positions of the $\mu\nu_\mu$ and $e\nu_e$ pairs are essentially fixed. Then the Perl particle can only transform like a Σ^+ or a Σ^-. We shall choose the Σ^+ possibility and denote the Perl particle by P^+. With the simplest hadron assignment (see Sec. V for further discussion) X^- should be produced by muon neutrinos on ordinary hadron targets so one expects[11] $m(X^-) \gtrsim 7.5$ GeV. The remaining ambiguity concerns its neutrino (or associated light particle) which experiment indicates must be present. The various possibilities will be considered here. To discuss these assignments in more detail it is helpful to refer to Fig. 1, which shows the left- and right-handed octets as well as the directions in which the various intermediate bosons act. Note that as usual $\nu^c_\mu \equiv C\bar{\nu}_\mu$, where $C^{-1}\gamma_\mu C = -\gamma^T_\mu$, $C^\dagger = C^{-1}$, and $C^T = -C$. T_L and T_R together are taken to make up a new heavy neutral lepton.

We denote the lepton representations by the fields ψ^b_{La} and ψ^b_{Ra} with, for example, $e^-_L = \psi^1_{L3}$ and $\mu^+_L = \psi^3_{L1}$. It is convenient to classify the Σ^0-like and Λ-like particles according to the V-spin subgroup. We

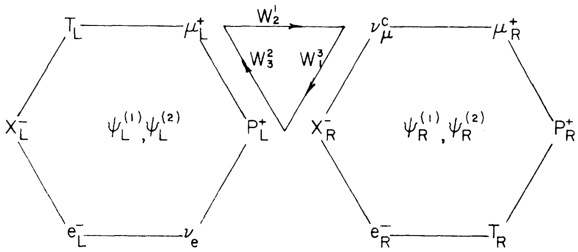

FIG. 1. Left- and right-handed lepton octets and group directions of the gauge bosons which change quantum numbers.

therefore set

$$\psi_{L,R}^{(1)} = \frac{1}{\sqrt{2}}(\psi_1^1 - \psi_3^3)_{L,R},$$
$$\psi_{L,R}^{(2)} = -(\tfrac{3}{2})^{1/2}(\psi_{L,R})_2^2 = +(\tfrac{3}{2})^{1/2}(\psi_1^1 + \psi_3^3)_{L,R}. \tag{2.4}$$

It is very important that the V-spin-triplet member $\psi^{(1)}$ be a heavy lepton. If it had zero mass, the gauge boson W_1^3 would be responsible for an additional contribution to muon decay ($\mu^+ \to e^+ + 2\psi^{(1)}$) which would destroy weak-interaction universality. On the other hand, the V-spin singlet $\psi^{(2)}$ is not connected to either μ or e by W_1^3. There are then three possibilities for the Perl neutrino, ν_P:

(i) 4-component: $\nu_P^c = \psi_L^{(2)} + \psi_R^{(2)}$,

(ii) 2-component usual: $\nu_P^c = \psi_R^{(2)}$, (2.5)

(iii) 2-component unusual: $\nu_P^c = \psi_L^{(2)}$.

In case (i) there are no additional particles left over. In cases (ii) and (iii), on the other hand, we have, counting antiparticles, four additional degrees of freedom left over. One way to acceptably incorporate these into the theory would be to add a 2-component SU(3)-singlet field, which of course would not couple to the gauge bosons, to the theory. For example, in case (ii) we might add a right-handed particle χ_R which would join together with $\psi_L^{(2)}$ to make a single massive field. Another possibility, (perhaps more elegant) would be to interpret [for case (ii)] $\psi_L^{(2)}$ and $\overline{\psi}_L^{(2)}$ as a massive Majorana particle M by setting $\psi_L^{(2)} \equiv (1+\gamma_5)/2M$, with M constrained by $M = C\overline{M}$. Note that M will not contribute to neutrinoless double-β decay via W_1^3. Also, since it is a V-spin singlet, it will turn out, for reasonable choices of hadron structure, not to contribute[12] to $K^+ \to \pi^+ e^+ e^-$ either.

Next we give the lepton interaction Lagrangian:

$$\begin{aligned}
\mathfrak{L}_{\text{leptons}} &= 2ig\overline{\psi}_{Lb}^a\gamma_\mu(W_{a\mu}^c\psi_{Lc}^b - W_{c\mu}^b\psi_{La}^c) + (L \to R) \\
&= -\sqrt{6}igA_\mu(\overline{\mu}^+\gamma_\mu\mu^+ + \overline{P}^+\gamma_\mu P^+ - \overline{e}^-\gamma_\mu e^- - \overline{X}^-\gamma_\mu X^- \\
&\quad - 2\sqrt{2}igZ_\mu(\overline{\nu}_e\gamma_\mu\nu_e - \overline{\nu}_\mu^c\gamma_\mu\nu_\mu^c + \tfrac{1}{2}\overline{e}^-\gamma_\mu e^- - \tfrac{1}{2}\overline{\mu}^+\gamma_\mu\mu^+ - \overline{T}\gamma_\mu\gamma_5 T + \tfrac{1}{2}\overline{P}^+\gamma_\mu P^+ - \tfrac{1}{2}\overline{X}^-\gamma_\mu X^-) \\
&\quad - 2ig\Big\{W_{1\mu}^2\Big[\overline{\nu}_e\gamma_\mu\left(\frac{1+\gamma_5}{2}\right)e^- - \overline{\mu}^+\gamma_\mu\left(\frac{1-\gamma_5}{2}\right)\nu_\mu^c - \overline{\mu}^+\gamma_\mu\left(\frac{1+\gamma_5}{2}\right)T + \overline{T}\gamma_\mu\left(\frac{1-\gamma_5}{2}\right)e^- + \frac{1}{\sqrt{2}}\overline{P}^+\gamma_\mu\psi^{(1)} \\
&\qquad - \frac{1}{\sqrt{2}}\overline{\psi}^{(1)}\gamma_\mu X^- + (\tfrac{3}{2})^{1/2}\overline{P}^+\gamma_\mu\psi_L^{(2)} + (\tfrac{3}{2})^{1/2}\overline{P}^+\gamma_\mu\psi_R^{(2)} - (\tfrac{3}{2})^{1/2}\overline{\psi}_L^{(2)}\gamma_\mu X^- - (\tfrac{3}{2})^{1/2}\overline{\psi}_R^{(2)}\gamma_\mu X^-\Big] \\
&\qquad + W_{1\mu}^3\Big[\overline{T}\gamma_\mu\left(\frac{1+\gamma_5}{2}\right)X^- - \overline{P}^+\gamma_\mu\left(\frac{1+\gamma_5}{2}\right)\nu_e + \overline{\nu}_\mu^c\gamma_\mu\left(\frac{1-\gamma_5}{2}\right)X^- - \overline{P}^+\gamma_\mu\left(\frac{1-\gamma_5}{2}\right)T + \sqrt{2}\,\overline{\mu}^+\gamma_\mu\psi^{(1)} - \sqrt{2}\,\overline{\psi}^{(1)}\gamma_\mu e^-\Big] \\
&\qquad + W_{2\mu}^3\Big[\overline{\mu}^+\gamma_\mu P^+ - \overline{X}^-\gamma_\mu e^- + \frac{1}{\sqrt{2}}\overline{T}\gamma_\mu\left(\frac{1+\gamma_5}{2}\right)\psi^{(1)} - (\tfrac{3}{2})^{1/2}\overline{T}\gamma_\mu\psi_L^{(2)} - \frac{1}{\sqrt{2}}\overline{\psi}^{(1)}\gamma_\mu\left(\frac{1-\gamma_5}{2}\right)T + (\tfrac{3}{2})^{1/2}\overline{\psi}_{2R}\gamma_\mu T \\
&\qquad - \frac{1}{\sqrt{2}}\overline{\psi}^{(1)}\gamma_\mu\left(\frac{1+\gamma_5}{2}\right)\nu_e + (\tfrac{3}{2})^{1/2}\overline{\psi}_L^{(2)}\gamma_\mu\nu_e + \frac{1}{\sqrt{2}}\overline{\nu}_\mu^c\gamma_\mu\left(\frac{1-\gamma_5}{2}\right)\psi^{(1)} - (\tfrac{3}{2})^{1/2}\overline{\nu}_\mu^c\gamma_\mu\psi_R^{(2)}\Big] - \text{H.c.}\Big\}
\end{aligned} \tag{2.6}$$

Note that

$$\frac{G}{\sqrt{2}} = \frac{g^2}{m^2(W_1^2)}, \tag{2.7}$$

where G is the Fermi constant.

The most straightforward application of (2.6) is to use the existing experimental information on pure lepton-scattering experiments to bound $m(Z)$. The effective Lagrangian for $\overline{\nu}_e e^- \to \overline{\nu}_e e^-$ is seen to be

$$\mathfrak{L}_{\text{eff}}(\overline{\nu}_e e^- \to \overline{\nu}_e e^-) = -\frac{G}{\sqrt{2}}\overline{\nu}_e\gamma_\mu(1+\gamma_5)\nu_e \times \overline{e}^-\gamma_\mu(C_V + C_A\gamma_5)e^-, \tag{2.8a}$$

where

$$C_V = 1 + \frac{2m^2(W_1^2)}{m^2(Z)}, \tag{2.8b}$$

$$C_A = 1. \tag{2.8c}$$

The experiments[13] have been analyzed for compatible ranges of C_V and C_A by Chen and Lee.[14] From their results we get $m(Z) \gtrsim 86$ GeV. The effective Lagrangian for $\bar{\nu}_\mu e \to \bar{\nu}_\mu e$, a process which has been observed,[15] is

$$\mathcal{L}_{\rm eff}(\bar{\nu}_\mu e \to \bar{\nu}_\mu e) = -\sqrt{2}\, G \frac{m^2(W_1^2)}{m^2(Z)} \bar{\nu}_\mu \gamma_\mu (1+\gamma_5) \nu_\mu \times \bar{e}\gamma_\mu e^- . \tag{2.9}$$

Analysis of this reaction[16] gives in our case $m(Z) \lesssim 137$ GeV, so altogether we have

$$86 \text{ GeV} \lesssim m(Z) \lesssim 137 \text{ GeV}. \tag{2.10}$$

III. LEPTONIC DECAY MODES OF THE PERL PARTICLE

One of the most fascinating features of the present theory is the purely leptonic decay modes of the Perl particle. As can be seen from (2.6), there are many decay channels open, so the pattern may be quite complicated. We therefore make a simplifying assumption, to be discussed later in this section, that the massive neutral leptons, the $T, \psi^{(1)}$, and perhaps another depending on the ν_P choice in (2.5), are sufficiently heavy that they cannot be decay products of the P^+. Even with this assumption the P^+ decays still present us with an interesting pattern which differs drastically from a straightforward extension of the SU(2) $\times$ U(1) theory. In the first place, since P^+ belongs to an isotriplet rather than an isodoublet and its associated neutrino is a linear combination of an isotriplet and an isosinglet, the strength of the interaction mediated by W_1^2 is different from a universal theory. In the second place, there are now additional decay channels mediated by W_2^3. These affect the muon decay but not the electron decay. The precise difference depends on the choice of the Perl neutrino in (2.5). In each case there exists a relation between $m(W_2^3)$ and a possible experimental enhancement of the muonic decay mode relative to the electronic one.

First we list from (2.6) the possible leptonic decays of P^+ into either an electron or muon plus neutrinos for each ν_P choice in (2.5):

(i) $P^+ \to e^+\nu_e\psi^{(2)}$ via W_1^2

$\to \mu^+\nu_\mu\psi^{(2)}$ via W_1^2 and W_2^3

$\to \mu^+\nu_e\bar{\psi}^{(2)}$ via W_2^3,

(ii) $P^+ \to e^+\nu_e\psi_R^{(2)}$ via W_1^2

$\to \mu^+\nu_\mu\psi_R^{(2)}$ via W_1^2 and W_2^3,

(iii) $P^+ \to e^+\nu_e\psi_L^{(2)}$ via W_1^2

$\to \mu^+\nu_\mu\psi_L^{(2)}$ via W_1^2

$\to \mu^+\nu_e\bar{\psi}_L^{(2)}$ via W_2^3.

The effective Lagrangian densities for these cases are

$$\mathcal{L}_{(\mathrm{i})} = \frac{\sqrt{6}g^2}{m^2(W_1^2)} \bar{\psi}^{(2)}\gamma_\alpha P^+[\bar{\mu}^+\gamma_\alpha(1-\gamma_5)\nu_\mu^c - \bar{\nu}_e\gamma_\alpha(1+\gamma_5)e^-] + \frac{\sqrt{6}g^2}{m^2(W_2^3)} \bar{\mu}^+\gamma_\alpha P^+[\bar{\psi}^{(2)}\gamma_\alpha(1-\gamma_5)\nu_\mu^c - \bar{\nu}_e\gamma_\alpha(1+\gamma_5)\psi^{(2)}], \tag{3.1i}$$

$$\mathcal{L}_{(\mathrm{ii})} = (\tfrac{3}{2})^{1/2} \frac{g^2}{m^2(W_1^2)} \bar{\psi}_R^{(2)}\gamma_\alpha(1-\gamma_5)P^+[\bar{\mu}^+\gamma_\alpha(1-\gamma_5)\nu_\mu^c - \bar{\nu}_e\gamma_\alpha(1+\gamma_5)e^-] + \sqrt{6}\frac{g^2}{m^2(W_2^3)} \bar{\mu}^+\gamma_\alpha P^+\bar{\psi}_R^{(2)}\gamma_\alpha(1-\gamma_5)\nu_\mu^c, \tag{3.1ii}$$

$$\mathcal{L}_{(\mathrm{iii})} = (\tfrac{3}{2})^{1/2} \frac{g^2}{m^2(W_1^2)} \bar{\psi}_L^{(2)}\gamma_\alpha(1+\gamma_5)P^+[\bar{\mu}^+\gamma_\alpha(1-\gamma_5)\nu_\mu^c - \bar{\nu}_e\gamma_\alpha(1+\gamma_5)e^-] - \sqrt{6}\frac{g^2}{m^2(W_2^3)} \bar{\mu}^+\gamma_\alpha P^+\bar{\nu}_e\gamma_\alpha(1+\gamma_5)\psi_L^{(2)}. \tag{3.1iii}$$

By straightforward perturbation calculation based on (3.1i)–(3.1iii) we get the appropriate widths for unpolarized P^+ decay. Setting for convenience

$$r = m^2(W_1^2)/m^2(W_2^3) \tag{3.2}$$

and neglecting the masses of the final-state leptons, the results are:

(i) 4-*component* ν_P.

$$\Gamma(P^+ \to e^+\nu_e\psi^{(2)}) = \frac{G^2m^5}{64\pi^3},$$
$$\Gamma(P^+ \to \mu^+\nu_\mu\psi^{(2)}) = \frac{G^2m^5}{64\pi^3}(1+r+r^2), \tag{3.3}$$
$$\Gamma(P^+ \to \mu^+\nu_e\bar{\psi}^{(2)}) = \frac{G^2m^5r^2}{64\pi^3};$$

(ii) 2-*component usual* ν_P.

$$\Gamma(P^+ \to e^+\nu_e\psi_R^{(2)}) = \frac{G^2m^5}{128\pi^3},$$
$$\Gamma(P^+ \to \mu^+\nu_\mu\psi_R^{(2)}) = \frac{G^2m^5}{128\pi^3}(1+2r+2r^2); \tag{3.4}$$

(iii) 2-*component unusual* ν_P.

$$\Gamma(P^+ \to e^+\nu_e\psi_L^{(2)}) = \frac{G^2m^5}{128\pi^3},$$
$$\Gamma(P^+ \to \mu^+\nu_\mu\psi_L^{(2)}) = \frac{G^2m^5}{128\pi^3}, \tag{3.5}$$
$$\Gamma(P^+ \to \mu^+\nu_e\bar{\psi}_L^{(2)}) = \frac{G^2m^5r^2}{64\pi^3}.$$

In the above m is the mass of P^+. Note that if we were to use a naive universal theory for the Perl particle and its neutrino, the numerical factor in (3.4), for example, would be $\frac{2}{3}(\frac{1}{128})=\frac{1}{192}$ rather than $\frac{1}{128}$. For any choice of ν_P we notice from above the startling result that

$$\Gamma(P^+ \to \mu^+ + \text{neutrinos}) \geq \Gamma(P^+ \to e^+ + \text{neutrinos}), \tag{3.6}$$

with the equality sign holding only for $m(W_2^3)=\infty$. This result is a consequence of our assignment of P^+ to the same octet as the electron and the muon. Furthermore, this result will continue to hold if the gauge group is increased to SU(3) × U(1) as in the old theory but the lepton representations remain octets. This follows because the additional U(1) only affects the A_μ and Z_μ couplings.

Hence (3.6) is a good test for a theory of this type. In the e^+e^- annihilation experiments[1] one extracts from the experimental data the ratio

$$R=\frac{\text{number of weak-interaction } e^+e^- \text{ pairs without hadrons}}{\text{number of weak-interaction } \mu^+\mu^- \text{ pairs without hadrons}}. \tag{3.7}$$

Assuming that the unobserved particles are neutrinos, we may predict this ratio in terms of r for each choice of ν_P from (3.3)–(3.5):

$$R_{(\text{i})}=(1+r+2r^2)^{-2}, \tag{3.8i}$$

$$R_{(\text{ii})}=(1+2r+2r^2)^{-2}, \tag{3.8ii}$$

$$R_{(\text{iii})}=(1+2r^2)^{-2}. \tag{3.8iii}$$

[Eq. (3.6) is of course the same as $R \leq 1$]. At present the data are in a very preliminary stage and the statistics are poor. If we were to take seriously the result[17]

$$R=0.57\pm 0.31$$

we would find for cases (i), (ii), and (iii), respectively, $m(W_2^3)\simeq 90$ GeV, 113 GeV, and 68 GeV. If the experimental result $R<1$ holds up it would then be interesting to compute the angular distributions[18] from Eqs. (3.1) to try to discriminate among the different ν_P choices.

In the above discussion we assumed the massive neutral leptons to play no role in Perl lepton decays. If they were sufficiently light they could affect R in (3.7) by means of decays of the form

$$P^+ \xrightarrow[\text{via } W_1^2,\, W_1^3,\, \text{or } W_2^3]{} (e^+ \text{ or } \mu^+) + (\text{massive neutral lepton}) + \nu,$$

followed by

$$\text{massive neutral lepton} \xrightarrow[\text{via } W_2^3]{} \begin{cases} 3\,\nu\text{'s} \\ (\text{another massive neutral lepton}) + 2\,\nu\text{'s} \xrightarrow[\text{via } W_2^3]{} 3\,\nu\text{'s}. \end{cases} \tag{3.9}$$

(Another similar possibility corresponds to replacing the neutrino in the first step by a massive neutral lepton.) Reference to (2.6) shows that there are many possible reactions of this type and the first step of the chain can be mediated by any of W_1^2, W_1^3, and W_2^3. Again there are more interaction terms resulting in muons than in electrons (since W_2^3-mediated amplitudes go only into muons while W_1^2 and W_1^3 treat muons and electrons symmetrically), so it is likely that we would still have $R<1$. However, the situation is complicated to analyze in detail at the present stage because there are several unknown masses and interferences between different terms in the same amplitude make simple counting unreliable.

What *can* be said about the masses of the massive neutral leptons? We must have $m(T)\gtrsim m(K)$, otherwise T should have been seen in the W_1^2-mediated reaction $K^+\to Te^+$. In the future this lower bound might be raised if D^+ or $F^+\to Te^+$ are not observed. Since T has decay modes like $e^+e^-\nu_e$ or $\mu^+\mu^-\overline{\nu}_\mu$, etc., it would, if it had low mass, take part in reactions where $e^+e^-\to P^+\overline{P}^+\to$ (6 charged muons and/or electrons + neutrinos). This distinctive signature might be sufficient to rule out the possibility of a low mass T. For $\psi^{(1)}$, $m(\psi^{(1)}) \gtrsim \frac{1}{2}m(\mu)$ is required to avoid the W_1^3-mediated reaction $\mu^+\to 2\psi^{(1)}+e^+$. The nonobservation of D^+ or $F^+\to\psi^{(1)}e^+$ could raise this lower bound if we choose a hadron model such that the virtual transitions $D^+\to W_1^3$ or $F^+\to W_1^3$ are possible. It is more difficult to give an experimental restriction on our possible massive Majorana lepton since it has no direct electron or muon couplings. If it had low mass it would show up in the decays of hadrons containing new quantum numbers.

Finally, we mention how our discussion is to be modified if the linear combinations $W_{2\mu}^3 \pm W_{3\mu}^2$ have different masses. Then, for the leptonic decays of the Perl particle we should replace $g^2/m^2(W_2^3)$ by $\frac{1}{2}g^2(1/m_A{}^2+1/m_B{}^2)$, where m_A and m_B are the two masses. In addition there will now be the decay possibilities $P^+ \to \mu^+\bar{\nu}_e\psi_L^{(2)}$ and $P^+ \to \mu^+\bar{\nu}_\mu\bar{\psi}_R^{(2)}$. These amplitudes will be suppressed by a factor $(m_A{}^2 - m_B{}^2)/(m_A{}^2 + m_B{}^2)$. Since these extra modes involve muons rather than electrons our conclusion would be unaltered.

IV. GENERATION OF LEPTON MASSES

Since the electron, muon, and Perl particle all belong to the same octet, one might at first expect to get a lepton mass formula somewhat analogous to the Gell-Mann–Okubo relation. However, the symmetry breaking is much more drastic in the present case and except for the electric charge, Q, there seems to be no remnant of any approximate symmetry. (For example, the required mass pattern looks nothing like I-, U-, or V-spin invariance.)

On a deeper level one might hope to generate all masses of the theory by dynamical symmetry breaking. This of course is extremely difficult to implement, so it seems more reasonable to adopt the usual approach of introducing auxiliary Higgs bosons. Even this approach is very complicated in the present case if one wishes to achieve an essentially arbitrarily given lepton mass matrix with "natural" restrictions[19] on the interactions of the Higgs particles. Thus we will be content just to mention and illustrate the essentially obvious point that if we are allowed complete freedom as to the choice of Higgs fields and their coupling constants and vacuum expectation values then it is always possible, for any gauge theory, to achieve at the tree level an arbitrary fermion mass matrix. Note that we are disregarding, for simplicity, any constraints which might arise from the Higgs potential function and any requirements which may arise from avoiding pseudo-Goldstone particles. To see this, note that the mass terms in the Lagrangian after spontaneous breaking are of the form (for any gauge theory)

$$-\sum_{A,B} m_{AB}\bar{R}_A L_B + \text{H.c.}, \qquad (4.1)$$

where L_B is the (in general) reducible representation of left-handed fermion fields, R_A is the right-handed representation, and the sum goes over the subspace of zero charge. We want to demonstrate that m_{AB} can be an arbitrary matrix. For each AB pair in the $Q=0$ subspace it is always possible to introduce a boson field ${}_{AB}\Phi^{CD}$ (reducible, in general) such that

$$\sum_{C,D} \bar{R}_C L_D\, {}_{AB}\Phi^{CD} \qquad (4.2)$$

is a group invariant. Note that in (4.2) the C and D summations are unrestricted. The field ${}_{AB}\Phi^{CD}$ in general contains the complex conjugate of all irreducible representations in the $\bar{R}_C L_D$ product. Now we choose the gauge-invariant Yukawa-interaction part of the Lagrangian to be

$$-\sum_{A,B} g_{AB} \sum_{C,D} \bar{R}_C L_D\, {}_{AB}\Phi^{CD} + \text{H.c.}, \qquad (4.3)$$

where the A and B summations are restricted to the $Q=0$ subspace and the g_{AB} are a set of arbitrary coupling constants. Choosing the vacuum expectation values of the bosons as

$$\langle {}_{AB}\Phi^{CD}\rangle_0 = \delta_A^C \delta_B^D V_{CD} \quad \text{(no sum)}, \qquad (4.4)$$

where the pairs AB run over the $Q=0$ subspace and the V_{AB} are a set of arbitrary constants, we see that by substituting (4.4) in (4.3) and comparing with (4.1)

$$m_{AB} = g_{AB} V_{AB} \quad \text{(no sum)}$$

and is indeed arbitrary.

Thus, at our preliminary stage of looking at the SU(3) lepton octet there is no immediate objection to assigning arbitrary lepton masses [or, for that matter, mixing angles, since m_{AB} in (4.1) is not necessarily diagonal]. It might be hoped that some particularly simple set of bosons belonging to some simple irreducible rperesentations might suffice. However, it turns out that to achieve our pattern of lepton masses at tree level all possible nontrivial irreducible representations which might contribute (i.e., 8, 10, $\overline{10}$, and 27) are required. This is illustrated explicitly in the Appendix. The vacuum expectation values of the Higgs fields given there are compatible with the gauge-boson mass pattern discussed in Sec. II.

V. HADRONS AND FURTHER DISCUSSION OF PERL-PARTICLE DECAYS

The theoretical description of weak interactions involving hadrons is more difficult than the lepton case owing to the model dependence of attempts to describe the strong interactions. Furthermore the most relevant experiments are highly fluid at present. Hence we shall be brief here, discussing only some characteristic features of the present model.

As in the leptonic sector, all charged particles must belong to multiplets of the same dimension if we wish to have invariance under the simple group SU(3). This requires a vectorlike model; the case of two triplets is almost unique if we adopt the Glashow-Iliopoulos-Maiani (GIM) mechanism[20] and has been previously given by Fritzsch and Minkowski.[10] There are altogether four quark triplets, counting both left and right projections, which we denote as follows:

$$Q_L = \begin{pmatrix} q_{1L} \\ q_{2L}\cos\theta + q_{3L}\sin\theta \\ q_{5L} \end{pmatrix}, \quad Q'_L = \begin{pmatrix} q_{4L} \\ q_{3L}\cos\theta - q_{2L}\sin\theta \\ q_{6L} \end{pmatrix},$$
$$Q_R = \begin{pmatrix} q_{1R} \\ q_{6R} \\ q_{2R}\cos\theta' + q_{3R}\sin\theta' \end{pmatrix}, \quad Q'_R = \begin{pmatrix} q_{4R} \\ q_{5R} \\ q_{3R}\cos\theta' - q_{2R}\sin\theta' \end{pmatrix}. \tag{5.1}$$

Here q_1 to q_4 are the usual quarks while q_5 and q_6 are new ones, with $Q = -\frac{1}{3}$. θ is the Cabibbo angle and θ' is another mixing angle; q_5 and q_6 mixing could also be introduced if desired. We shall regard (5.1) as a tentative identification. The results of neutral-current experiments may force us to modify this assignment—this point will be taken up later. Note that according to the result of the preceding section all quark masses and Cabibbo-like mixing angles are arbitrary and cannot be predicted at the tree level. This is also illustrated in the Appendix.

The hadron interaction-Lagrangian density is

$$\begin{aligned}\mathcal{L} = 2ig\{ & W^2_{1\alpha}[\bar{q}_{1L}\gamma_\alpha(q_{2L}\cos\theta + q_{3L}\sin\theta) + \bar{q}_{4L}\gamma_\alpha(q_{3L}\cos\theta - q_{2L}\sin\theta) + \bar{q}_{1R}\gamma_\alpha q_{6R} + \bar{q}_{4R}\gamma_\alpha q_{5R}] \\ & + W^3_{1\alpha}[\bar{q}_{1L}\gamma_\alpha q_{5L} + \bar{q}_{4L}\gamma_\alpha q_{6L} + \bar{q}_{1R}\gamma_\alpha(q_{2R}\cos\theta' + q_{3R}\sin\theta') + \bar{q}_{4R}\gamma_\alpha(q_{3R}\cos\theta' - q_{2R}\sin\theta')] \\ & + W^3_{2\alpha}[(\bar{q}_{2L}\cos\theta + \bar{q}_{3L}\sin\theta)\gamma_\alpha q_{5L} + (\bar{q}_{3L}\cos\theta - \bar{q}_{2L}\sin\theta)\gamma_\alpha q_{6L} \\ & \quad + \bar{q}_{6R}\gamma_\alpha(q_{2R}\cos\theta' + q_{3R}\sin\theta') + \bar{q}_{5R}\gamma_\alpha(q_{3R}\cos\theta' - q_{2R}\sin\theta')] - \text{H.c.}\} \\ & + \sqrt{2}\, igZ_\alpha(\bar{q}_2\gamma_\alpha\gamma_5 q_2 + \bar{q}_3\gamma_\alpha\gamma_5 q_3 - \bar{q}_5\gamma_\alpha\gamma_5 q_5 - \bar{q}_6\gamma_\alpha\gamma_5 q_6) \\ & + \text{photon terms.}\end{aligned} \tag{5.2}$$

First note that there will be an extra (right-handed current)^2 contribution to the ordinary nonleptonic decays proportional to $1/m^2(W^3_1)\cos\theta'\sin\theta'$. If we wish to retain the usual picture of these decays we should either take θ' small or $m(W^3_1)$ large. Actually these quantities can in principle be measured by decays of the P^+-like $P^+ \to \pi^+ +$ neutrino and $P^+ \to K^+ +$ neutrino which may proceed either by W^2_1 with emission of a Perl neutrino or by W^3_1 with emission of ν_e. One finds for the decay widths

$$\Gamma(P^+ \to \pi^+ + \nu) = \frac{G^2F_\pi^{\,2}}{16\pi}\,\frac{(m^2 - m_\pi^{\,2})^2}{m}\,[\tfrac{3}{2}(K)\cos^2\theta + r'^2\cos^2\theta'], \tag{5.3}$$

where the factor K is 1 for either kind of 2-component Perl neutrino in (2.5) and $K = 2$ for a 4-component Perl neutrino. Also

$$r' \equiv \frac{m^2(W^2_1)}{m^2(W^3_1)} \tag{5.4}$$

and $F_\pi \simeq m_\pi$ is the pion decay constant. For the case of K^+ decay we should make the following replacements in (5.3):

$$\begin{aligned} & m_\pi \to m_K, \\ & F_\pi \to F_K, \\ & \cos\theta \to \sin\theta, \\ & \cos\theta' \to \sin\theta'. \end{aligned} \tag{5.5}$$

In this discussion we have again for simplicity assumed that T is sufficiently massive so that reactions like

$$P^+ \xrightarrow[\text{via } W^3_1]{} \pi^+ + T, \qquad T \xrightarrow[\text{via } W^3_2]{} 3\nu\text{'s}$$

do not take place.

Note that we have now either specified (or in the case of Z, bounded) all gauge-boson masses or shown how they may be gotten from measurements of P^+ decays.

From (5.2) we notice that the Z-mediated neutral current has a pure axial-vector form. This would seem to disagree with the results of a recent experiment[21] by Benvenuti *et al.*, which indicates the presence of V-A interference. If this experiment holds up, the vector like models based on SU(2) × U(1) with only quark doublets may be ruled out. However, the present vectorlike model with only quark triplets is not necessarily ruled out since there are extra contributions to neutral-current events mediated by the boson W_2^3. These contributions have either $V+A$ or $V-A$ form as may be seen from the following effective interaction Lagrangian:

$$\mathcal{L}=\cdots-(\tfrac{3}{2})^{1/2}\frac{g^2}{m^2(W_2^3)}\overline{\psi}_R^{(2)}\gamma_\alpha(1-\gamma_5)\nu_\mu^c \times[(\overline{q}_2\cos\theta+\overline{q}_3\sin\theta)\gamma_\alpha(1+\gamma_5)q_5+(\overline{q}_3\cos\theta-\overline{q}_2\sin\theta)\gamma_\alpha(1+\gamma_5)q_6 +\overline{q}_6\gamma_\alpha(1-\gamma_5)(q_2\cos\theta'+q_3\sin\theta')+\overline{q}_5\gamma_\alpha(1-\gamma_5)(q_3\cos\theta'-q_2\sin\theta')] +\text{H.c.} \tag{5.6}$$

It should be noted that if the unusual choice [case (iii) in (2.5), wherein $\overline{\psi}_R^{(2)}$ would be a massive Majorana particle rather than a neutrino] is made for the Perl neutrino, this mechanism is not possible. An amusing point here is that $\overline{\nu}_\mu$ preferentially produces q_6 from q_2 while ν_μ preferentially produces q_5 from q_2. Thus differences in the masses of q_5 and q_6 should result in different energy behavior for ν_μ and $\overline{\nu}_\mu$ production experiments. Additional reactions similar to the above wherein the incoming neutrino produces massive heavy neutral leptons via W_2^3 may also be counted in as neutral-current events, among other things, in the experiments at high energies.

Finally, it seems to be instructive, although perhaps not necessary, to consider another way of introducing V,A interference terms in the hadron neutral current. Consider Q_L and Q_L' as given by (5.1) but introduce a new quark q_7 with the remaining particles assigned as follows:

Triplets:

$$Q_R=\begin{pmatrix} q_{1R} \\ q_{6R} \\ q_{7R}\cos\theta'+q_{3R}\sin\theta' \end{pmatrix},\quad Q_R'=\begin{pmatrix} q_{4R} \\ q_{5R} \\ q_{3R}\cos\theta'-q_{7R}\sin\theta' \end{pmatrix}; \tag{5.7}$$

Singlets:

q_{2R}, q_{7L}.

Because charged quarks are now assigned to singlets, this would correspond to a vectorlike SU(3) $\times$ U(1) gauge theory. There is now a singlet gauge boson which, however, does not couple to neutrinos in the lepton octets. The photon would now be a mixture of an SU(3)-octet and an SU(3)-singlet field so we would have a new mixing angle, similar to the Weinberg angle, in the theory. The formulas for $m(W_1^2)$ and $|e|$ are now modified to those of Ref. 3. The curcial point is that the Z couplings are now given by

$$\mathcal{L}=\cdots+2igZ_\alpha[\overline{q}_3\gamma_\alpha\gamma_5q_3-\overline{q}_5\gamma_\alpha\gamma_5q_5 -\overline{q}_6\gamma_\alpha\gamma_5q_6+\overline{q}_2\gamma_\alpha\tfrac{1}{2}(1+\gamma_5)q_2 -\overline{q}_7\gamma_\alpha\tfrac{1}{2}(1-\gamma_5)q_7]. \tag{5.8}$$

There are now also additional neutral currents coupling to another gauge boson, but these do not couple to neutrinos so we will not write them down. Note that if we had attempted to switch q_{1R} with q_{7R} the neutral current would not have been affected since the U-spin triplet member, Z, does not couple to q_1 which is a U-spin singlet.

It is very interesting to note that if the assignment (5.8) is adopted more varied possibilities open up for the Perl particle. With (5.1) one could argue that X^- could be produced by $\overline{\nu}_\mu$ on ordinary hadronic targets via W_1^3 and hence should be more massive than about 7.5 GeV by the Barish experiment.[11] This rules out the identification of X^- as P^-. Such an argument evidently does not apply with equal force to the assignment (5.7). We could then have four Perl particles, P^+, $\overline{P}^+$, P^-, and $\overline{P}^-$. As we have seen, P^+ and $\overline{P}^+$ prefer to decay into muons rather than electrons. P^- and $\overline{P}^-$, on the other hand, prefer electrons to muons. Depending on their relative masses, different patterns for the ratio R (see Sec. III) as a function of energy would be observed in e^+e^- annihilation experiments.

ACKNOWLEDGMENTS

We would like to thank Dr. Steven Borchardt and Dr. Mark Singer for helpful discussions.

APPENDIX

As a first step we give the Higgs fields needed to generate lepton masses. No constraints arising from the form of the potential will be assumed at this stage. Note that our choice is more illustrative than unique. There are, in order, two octets, a decuplet, an antidecuplet, and four different 27's:

$$\rho_b^a, \rho_b'^a, \eta_{abc}, \overline{\eta}^{abc}, \xi_{cd}^{ab}, \xi_{cd}'^{ab}, \pi_{cd}^{ab}, \chi_{cd}^{ab}. \tag{A1}$$

Under CP these go to, in order,

$$\rho_a^b, \rho_a'^b, \overline{\eta}^{abc}, \eta_{abc}, \xi_{ab}^{cd}, \xi_{ab}'^{cd}, \pi_{ab}^{cd}, -\chi_{ab}^{cd}. \tag{A2}$$

(Notice the minus sign for the last case.) The components with nonzero vacuum expectation values are given below (taking trace and symmetry conditions into account):

$$\begin{aligned}
&\langle\rho_2^2\rangle_0=\langle\rho_3^3\rangle_0=-\tfrac{1}{2}\langle\rho_1^1\rangle_0=t_1,\\
&\langle\rho_1'^1\rangle_0=\langle\rho_3'^3\rangle_0=-\tfrac{1}{2}\langle\rho_2'^2\rangle_0=t_2,\\
&\langle\eta_{123}\rangle_0=\langle\overline{\eta}^{123}\rangle_0=W,\\
&\langle\xi_{12}^{12}\rangle_0=\langle\xi_{13}^{13}\rangle_0=-\tfrac{1}{2}\langle\xi_{11}^{11}\rangle_0=-\tfrac{1}{2}u_1,\\
&\langle\xi_{22}^{22}\rangle_0=\langle\xi_{33}^{33}\rangle_0=u_2,\\
&\langle\xi_{23}^{23}\rangle_0=\tfrac{1}{2}u_1-u_2,\\
&\langle\xi_{12}'^{12}\rangle_0=\langle\xi_{23}'^{23}\rangle_0=-\langle\xi_{13}'^{13}\rangle_0=-\tfrac{1}{2}\langle\xi_{22}'^{22}\rangle_0=-\tfrac{1}{2}v,\\
&\langle\pi_{33}^{22}\rangle_0=\langle\pi_{22}^{33}\rangle_0=x.\\
&\langle\chi_{33}^{22}\rangle_0=-\langle\chi_{22}^{33}\rangle_0=ix.
\end{aligned} \tag{A3}$$

The vacuum expectation values of the π_{cd}^{ab} and χ_{cd}^{ab} fields are related in order that, as we have assumed, the gauge bosons $W_{2\alpha}^3$ and $W_{3\alpha}^2$ rather than their linear combinations be diagonal. The lepton Yukawa terms in the Lagrangian are

$$\begin{aligned}
\mathcal{L}=&-l_1\overline{\psi}_{Lb}^a\psi_{Ra}^b-l_2\overline{\psi}_{Lb}^a\psi_{Rc}^b\rho_a^c-l_3\overline{\psi}_{Lb}^a\psi_{Ra}^c\rho_c^b-l_4\overline{\psi}_{Lb}^a\psi_{Rc}^b\rho_a'^c-l_5\overline{\psi}_{Lb}^a\psi_{Ra}^c\rho_c'^b-l_6\overline{\psi}_{Lb}^a\psi_{Rd}^c\xi_{ac}^{bd}\\
&-l_7\overline{\psi}_{Lb}^a\psi_{Rd}^c\xi_{ac}'^{bd}-l_8(\overline{\psi}_{Lc}^a\psi_{Rd}^b+\overline{\psi}_{Rc}^a\psi_{Ld}^b)\epsilon^{ecd}\eta_{eab}-l_9\overline{\psi}_{Lb}^a\psi_{Rd}^c\pi_{ac}^{bd}-l_{10}\overline{\psi}_{Lb}^a\gamma_5\psi_{Rd}^c\chi_{ac}^{bd}+\text{H.c.}
\end{aligned} \tag{A4}$$

One then gets, to tree order, the following masses:

$$\begin{aligned}
&m(e)=l_1+l_2t_1-2l_3t_1+l_4t_2+l_5t_2-\tfrac{1}{2}l_6u_1+\tfrac{1}{2}l_7v-2l_8W,\\
&m(\mu)=l_1-2l_2t_1+l_3t_1+l_4t_2+l_5t_2-\tfrac{1}{2}l_6u_1+\tfrac{1}{2}l_7v+2l_8W,\\
&m(P^+)=l_1-2l_2t_1+l_3t_1+l_4t_2-2l_5t_2-\tfrac{1}{2}l_6u_1-\tfrac{1}{2}l_7v-2l_8W,\\
&m(X^-)=l_1+l_2t_1-2l_3t_2-2l_4t_2+l_5t_2-\tfrac{1}{2}l_6u_1-\tfrac{1}{2}l_7v+2l_8W,\\
&m(\psi^{(1)})=l_1-\tfrac{1}{2}(l_2+l_3)t_1+(l_4+l_5)t_2+l_6(u_1+\tfrac{1}{2}u_2)-\tfrac{1}{2}l_7v,\\
&m(\psi^{(2)})=l_1+\tfrac{1}{2}(l_2+l_3)t_1-(l_4+l_5)t_2+\tfrac{3}{2}l_6u_2+\tfrac{3}{2}l_7v,\\
&m(T)=(l_9-l_{10})x,
\end{aligned} \tag{A5}$$

together with the following constraints:

$$l_9=-l_{10},\quad (l_2+l_3)t_1=l_6(u_1-u_2),(l_4-l_5)t_2=-\tfrac{4}{3}l_8W,\quad (l_4+l_5)t_2=3l_6u_1-4l_6u_2+2l_1-l_7v. \tag{A6}$$

The constraints in (A6) eliminate cross terms. There are eleven independent parameters (each is a product of a coupling constant and a vacuum expectation value) for seven masses in (A5) and four constraints in (A6). Hence each mass can be arbitrarily chosen. Note that we choose $m(\psi^{(2)})=0$. As it stands (A5) is then suitable for describing the situation with a 4-component Perl neutrino. To generate mass for a possible Majorana lepton we would have to add additional invariant Yukawa terms like $\psi_L C^{-1}\psi_L$. As stated in Sec. IV, all irreducible representations which can contribute are present. Inspection of (A5) and (A6) shows that one could not leave out any of them and still get an arbitrary mass spectrum.

To generate quark masses we need an addition to ρ_a^b in (A1) two new octets described as follows:

$$\begin{aligned}
&\phi_a^b \underset{CP}{\rightarrow} \phi_b^a,\quad \chi_a^b \underset{CP}{\rightarrow} -\chi_b^a;\\
&\langle\phi_2^3\rangle_0=\langle\phi_3^2\rangle_0=s,\quad \langle\chi_2^3\rangle_0=-\langle\chi_3^2\rangle_0=is.
\end{aligned} \tag{A7}$$

As before, $\langle\phi_2^3\rangle_0$ and $\langle\chi_2^3\rangle_0$ are related. The quark Yukawa terms in the Lagrangian are then

$$\begin{aligned}
\mathcal{L}=&-h_1\overline{Q}_L^aQ_{Rb}\phi_a^b-h_2\overline{Q}_L'^aQ'_{Rb}\phi_a^b-h_3\overline{Q}_L^aQ'_{Rb}\phi_a^b-h_4\overline{Q}_L'^aQ_{Rb}\phi_a^b-ih_5\overline{Q}_L^a\gamma_5Q_{Rb}\chi_a^b-ih_6\overline{Q}_L'^a\gamma_5Q'_{Rb}\chi_a^b\\
&-ih_7\overline{Q}_L^a\gamma_5Q'_{Rb}\chi_a^b-ih_8\overline{Q}_L'^a\gamma_5Q_{Rb}\chi_a^b-h_9\overline{Q}_L^aQ_{Ra}-h_{10}\overline{Q}_L'^aQ'_{Ra}-h_{11}\overline{Q}_L^aQ_{Rb}\rho_a^b-h_{12}\overline{Q}_L'^aQ'_{Rb}\rho_a^b+\text{H.c.}
\end{aligned} \tag{A8}$$

There are essentially twelve parameters. We get six constraints and six arbitrary masses for any choice of mixing angles:

$$
\begin{aligned}
&m_1 = h_9 - 2t_1 h_{11},\\
&m_2 = s(h_1+h_5)\cos\theta\cos\theta' + s(h_2+h_6)\sin\theta\sin\theta' - s(h_3+h_7)\cos\theta\sin\theta' - s(h_4+h_8)\sin\theta\cos\theta',\\
&m_3 = s(h_1+h_5)\sin\theta\sin\theta' + s(h_2+h_6)\cos\theta\cos\theta' + s(h_3+h_7)\sin\theta\cos\theta' + s(h_4+h_8)\cos\theta\sin\theta',\\
&m_4 = h_{10} - 2t_1 h_{12},\\
&m_5 = s(h_3 - h_7),\\
&m_6 = s(h_4 - h_8),\\
&h_1 = h_5,\\
&h_2 = h_6,\\
&h_9 = -h_{11}t_1,\\
&h_{10} = -h_{12}t_1,\\
&(h_1 - h_2) = -\tfrac{1}{2}\cot(\theta+\theta')[h_3+h_4+h_7+h_8],\\
&(h_1 + h_2) = \tfrac{1}{2}\cot(\theta-\theta')[h_3-h_4+h_7-h_8].
\end{aligned}
\qquad (A9)
$$

*Work supported in part by the U.S.Energy Research and Development Administration (ERDA).

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